Performance Analysis of Single Relay Selection in Rayleigh Fading

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***Abstract*—We provide closed-form expressions for the outage and bit error probability (BEP) of uncoded, threshold-based opportunistic relaying (OR) and selection cooperation (SC), at arbitrary signal to noise ratios (SNRs) and number of available relays, assuming decode-and-forward relays and Rayleigh fading channels. Numerical results demonstrate that SC performs slightly better in terms of outage probability; in terms of BEP, both systems may outperform one another, depending on the SNR threshold that determines the set of relays that participate in the forwarding process.**

***Index Terms*—Cooperative diversity, fading channels.**

I. Introduction

Opportunistic Relaying (OR) and Selection cooperation (SC) are two similar yet different relay selection methods proposed for cooperative diversity systems [1]- [2]. Their operation is based upon the selection of only a single relay out of the set of the available ones, achieving full-order spatial diversity while avoiding the reduction in spectral efficiency that the orthogonal transmissions of the "all-participate" systems entail (see e.g., [3]- [5]).

In [1], the authors proposed the OR method, showing that the diversity-multiplexing tradeoff of OR is identical with that of the distributed space-time coding (DSTC) systems [6]. In [2], a high signal to noise ratio (SNR) approximation for the outage probability of SC was derived, demonstrating that SC outperforms DSTC in terms of outage probability. An approximate outage analysis of SC in the low to medium SNR region can be found in [7]; other works on relay selection algorithms include [8]- [10].

An outage-based comparison of OR and SC was conducted in [11], where it was proven that, given the fact that the source-destination channel is not taken into account, the outage probabilities of SC and OR are identical. This analysis, however, limits the comparison of these two systems only to scenarios where the source-destination signal is negligible. This scenario may not be the case in practical applications where relaying transmissions are used to improve the quality of service over an existing link[1] (see, e.g., [12], [7]).

In this letter, we provide a) closed form expressions for the outage probability of uncoded, threshold-based[2] OR and SC at arbitrary SNR, number of available relays and source-destination channel conditions, and b) approximate closed form bit-error-probability (BEP) expressions for both OR and SC, assuming BPSK modulation. Based upon these expressions, we perform a numerical performance comparison of these two systems, showing that SC performs slightly better in terms of outage probability; in terms of BEP, both systems may outperform one another, depending on the SNR threshold that determines the set of relays participating in the forwarding process.

II. System Model

We consider the OR and SC models proposed respectively in [1] and [2], where a source node S communicates with a destination one, D, with the help of L independent decode-and-forward (DF) relaying terminals, denoted by R_i, $i \in \{1, ..., L\}$. The instantaneous SNRs of the S-D, S-R_i and R_i-D channels are denoted by γ_{SD}, γ_{SR_i} and γ_{R_iD}, respectively. All relays are assumed to operate in the half-duplex relaying mode, hence each transmission slot is divided in two subslots, corresponding to the S-R_i and R_i-D communication intervals, respectively. Moreover, it is assumed that *uncoded modulation is used*, so that the relays cannot detect any erroneous detection. However, they employ the so-called *threshold-based DF relaying*, i.e., if the received SNR is lower than a given threshold, denoted here by T, they remain idle in the second subslot.

In both SC and OR, only a single relay out of the L available ones is selected to forward the decoded information. However, their modes of operation are different:

- in SC, all relays listen to S, and only those with $\gamma_{SR_i} > T$ demodulate the received signal, forming the decoding set $\mathcal{C}$. In the second subslot, only the relay with the highest γ_{R_iD} (provided that $R_i \in \mathcal{C}$) transmits to the destination.
- in OR, the participating (best) relay R_b is selected according to $b = \arg\max_{i\in\{1,...,L\}} \min(\gamma_{SR_i}, \gamma_{R_iD})$, i.e., the selected relay is that with the highest $\min(\gamma_{SR_i}, \gamma_{R_iD})$. If $\gamma_{SR_b} < T$, however, none of the relays transmit in the second subslot.

In both systems the selection may be implemented either in a distributed fashion [1], provided that relays can communicate with each other, or by appropriate feedback broadcasted by the destination. The destination is assumed to combine the signals incident from the source and the selected relay (during the

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[1]We note that the reader should not be confused from the fact that in [11], the authors used the terms reactive and proactive opportunistic relaying to refer to what it is termed here as SC and OR, respectively.

[2]The term threshold-based relaying refers to the case where the relays forward only if the received SNR is greater than a specified threshold.

first and second subslot, respectively), into a time-diversity maximal ratio combiner (MRC). The fading in each channel is assumed to be independent, slow and Rayleigh distributed, hence γ_{SD}, γ_{SR_i} and γ_{R_iD} are exponential random variables with parameter $1/\overline{\gamma}_{SD}$, $1/\overline{\gamma}_{SR_i}$ and $1/\overline{\gamma}_{R_iD}$, respectively, where the overbar $\overline{(\cdot)}$ denotes expectation.

III. Performance Analysis of Selection Cooperation

A. Outage Probability

Let $\mathcal{O}$ denote the outage event. The outage probability of selection cooperation is given by [2]

$$\Pr\{\mathcal{O}\} = \sum_{\mathcal{C}\in\mathcal{P}(\mathcal{R})} \Pr\{\mathcal{C}\}\Pr\{\mathcal{O}\,|\mathcal{C}\}, \tag{1}$$

where $\mathcal{R}$ denotes the set of the available relays, i.e., $\mathcal{R} = \{R_1, R_2, ..., R_L\}$ and $\mathcal{P}(\cdot)$ stands for the power set of its argument, i.e., the set of all its subsets. The decoding set $\mathcal{C}$ is defined as $\mathcal{C} = \{R_i : \gamma_{SR_i} \geq T,\ i \in \{1, ..., L\}\}$, where $T = 2^{2r} - 1$ represents the outage threshold SNR for a given target rate r. The probability of $\mathcal{C}$ is expressed as

$$\Pr\{\mathcal{C}\} = \prod_{i:R_i\in\mathcal{C}} e^{-\frac{T}{\overline{\gamma}_{SR_i}}} \prod_{i:R_i\notin\mathcal{C}} \left(1 - e^{-\frac{T}{\overline{\gamma}_{SR_i}}}\right). \tag{2}$$

Given a decoding set $\mathcal{C}$, an outage occurs if $\gamma_{SD} + \max_{i:R_i\in\mathcal{C}}(\gamma_{R_iD}) < T$, hence

$$\Pr\{\mathcal{O}\,|\mathcal{C}\} = \int_0^T \frac{e^{-\frac{x}{\overline{\gamma}_{SD}}}}{\overline{\gamma}_{SD}} \prod_{i:R_i\in\mathcal{C}} \left(1 - e^{-\frac{T-x}{\overline{\gamma}_{R_iD}}}\right) dx. \tag{3}$$

Product Expansion: Consider the set

$$\mathcal{A} = \left\{\overline{\gamma}_{R_iD} : R_i \in \mathcal{C},\ i \in \{1, ..., L\}\right\}, \tag{4}$$

with cardinality equal to that of $\mathcal{C}$, i.e., $|\mathcal{A}| = |\mathcal{C}|$. Furthermore, let $\mathcal{A}_{n,k}$ denote the nth k-subset of $\mathcal{A}$ (excluding the empty set $\varnothing$), i.e., the nth subset of $\mathcal{A}$ that contains exactly k elements ($k = 1, ..., |\mathcal{C}|,\ n = 1, ..., \binom{|\mathcal{C}|}{k}$). The elements of $\mathcal{A}_{n,k}$ are denoted by $\phi_{k,n,j},\ j = 1, ..., k$. For the reader's convenience, more details on the relation between $\phi_{j,n,k}$ and $\overline{\gamma}_{R_iD}$ (assuming that the numbering of the relays is modified so that $i : R_i \in \mathcal{C}$) can be found in Table I.

Considering the above, the product of exponentials in (3) can be expanded for $\mathcal{C} \neq \varnothing$ as

$$\prod_{i:R_i\in\mathcal{C}} \left(1 - e^{-\frac{T-x}{\overline{\gamma}_{R_iD}}}\right) = 1 + \sum_{k=1}^{|\mathcal{C}|} (-1)^k \sum_{n=1}^{\binom{|\mathcal{C}|}{k}} \prod_{j=1}^{k} e^{-\frac{T-x}{\phi_{k,n,j}}}$$
$$= 1 + \sum_{k=1}^{|\mathcal{C}|} (-1)^k \sum_{n=1}^{\binom{|\mathcal{C}|}{k}} e^{-(T-x)\sum_{j=1}^{k}\frac{1}{\phi_{k,n,j}}}. \tag{5}$$

Therefore, substituting (5) in (3) yields

$$\Pr\{\mathcal{O}\,|\mathcal{C}\} = 1 - e^{-\frac{T}{\overline{\gamma}_{SD}}} + \sum_{k=1}^{|\mathcal{C}|} (-1)^k \sum_{n=1}^{\binom{|\mathcal{C}|}{k}} \frac{e^{-\frac{T}{\overline{\gamma}_{SD}}} - e^{-T\sum_{j=1}^{k}\frac{1}{\phi_{k,n,j}}}}{\overline{\gamma}_{SD}\sum_{j=1}^{k}\left(\frac{1}{\phi_{k,n,j}}\right) - 1}, \tag{6}$$

and thus the outage probability is derived by substituting (2) and (6) in (1). Note that for the special case of $\mathcal{C} = \varnothing$, it holds $\Pr\{\mathcal{O}\,|\mathcal{C} = \varnothing\} = 1 - e^{-\frac{T}{\overline{\gamma}_{SD}}}$.

B. Bit Error Probability

Throughout this letter, BPSK modulation is assumed; however, the presented analysis can be easily extended to yield the BEP of other modulation schemes. Let $\mathcal{E}^{S\text{-}D}$, $\mathcal{E}^{S\text{-}R_i}$ and $\mathcal{E}^{R_i\text{-}D}$ denote the event that the S-D, S-R_i and R_i-D links lead to an error on symbol (bit) detection, respectively. Let $\mathcal{E}^{S\text{-}D,R_i\text{-}D}$ denote the probability that the MRC output with inputs the S-D and the R_i-D channels leads to an error, provided that the same symbol is transmitted from S and R_i; let $\mathcal{S}^i$ stand for the event of selecting R_i. The BEP of SC can be expressed as

$$\Pr\{\mathcal{E}\} = \sum_{\mathcal{C}\in\mathcal{P}(\mathcal{R})} \Pr\{\mathcal{C}\}\Pr\{\mathcal{E}\,|\mathcal{C}\}. \tag{7}$$

Denoting with $R_b \in \mathcal{C}$ the selected "best" relay, the conditional BEP (conditioned on the set $\mathcal{C} \neq \varnothing$) in eq. (7) can be approximated by

$$\Pr\{\mathcal{E}\,|\mathcal{C}\} \approx \left(1 - \Pr\left\{\mathcal{E}^{S\text{-}R_b}\right\}\right)\Pr\left\{\mathcal{E}^{S\text{-}D,\,R_b\text{-}D}\right\} + \Pr\left\{\mathcal{E}^{S\text{-}R_b}\right\}\left(1 - \Pr\left\{\mathcal{E}^{R_b\text{-}D}\right\}\right), \tag{8}$$

where we have used the fact that the MRC output is dominated by the R_b-D link, hence the conditional error proability at the MRC ouput at the destination given the event of $\mathcal{E}^{S\text{-}R_b}$ is approximated by $1 - \Pr\left\{\mathcal{E}^{R_b\text{-}D}\right\}$. Consequently, (8) can be re-written as

$$\Pr\{\mathcal{E}\,|\mathcal{C}\} \approx \sum_{i:R_i\in\mathcal{C}} \left[\Pr\left\{\mathcal{E}^{S\text{-}R_i} \cap \mathcal{S}^i\,|\mathcal{C}\right\} + \Pr\left\{\mathcal{E}^{S\text{-}D,\,R_i\text{-}D} \cap \mathcal{S}^i\,|\mathcal{C}\right\} - \Pr\left\{\mathcal{E}^{S\text{-}R_i} \cap \mathcal{E}^{S\text{-}D,\,R_i\text{-}D} \cap \mathcal{S}^i\,|\mathcal{C}\right\} - \Pr\left\{\mathcal{E}^{S\text{-}R_i} \cap \mathcal{E}^{R_i\text{-}D} \cap \mathcal{S}^i\,|\mathcal{C}\right\}\right]. \tag{9}$$

Considering that the summation in (9) concerns all $R_i \in \mathcal{C}$ (i.e., $R_i : \gamma_{SR_i} \geq T$), it holds

$$\Pr\left\{\mathcal{E}^{S\text{-}R_i} \cap \mathcal{S}^i\,|\mathcal{C}\right\} = \int_T^\infty \frac{\mathrm{erfc}\left(\sqrt{x}\right)}{2\overline{\gamma}_{SR_i} e^{\frac{x}{\overline{\gamma}_{SR_i}}}} \Pr\left\{\mathcal{S}^i\,|\mathcal{C}\right\} dx = \frac{\Pr\left\{\mathcal{S}^i\,|\mathcal{C}\right\} I_1\left(\frac{1}{\overline{\gamma}_{SR_i}}, 1, T\right)}{2\overline{\gamma}_{SR_i}} \tag{10}$$

where the auxiliary function $I_1(\cdot,\cdot,\cdot)$ is defined as (see [13, eq. (8)])

$$I_1(\alpha, \beta, \omega) = \int_\omega^\infty \exp(-\alpha x)\,\mathrm{erfc}\left(\sqrt{\beta x}\right) dx = \frac{e^{-\alpha\omega}\mathrm{erfc}\left(\sqrt{\beta\omega}\right)}{a} - \frac{\sqrt{\beta}\mathrm{erfc}\left(\sqrt{(\alpha+\beta)\,\omega}\right)}{\alpha\sqrt{\alpha+\beta}}, \tag{11}$$

and the conditional probability of selecting the R_i relay as

$$\Pr\left\{\mathcal{S}^i\,|\mathcal{C}\right\} = \int_0^\infty \frac{e^{-\frac{x}{\overline{\gamma}_{R_iD}}}}{\overline{\gamma}_{R_iD}} \prod_{j:R_j\in\mathcal{C};\ j\neq i} \left(1 - e^{-\frac{x}{\overline{\gamma}_{R_jD}}}\right) dx. \tag{12}$$

In order to transform the product in (12) into a summation of exponential terms, for each $R_i \in \mathcal{C}$ we consider the set

$$\mathcal{A}_i = \left\{\overline{\gamma}_{R_jD} : R_j \in (\mathcal{C} \setminus \{R_i\})\right\}, \tag{13}$$

where $A \backslash B$ denotes the relative complement of the event B in the event A. Further, we denote with $\psi^i_{k,n,m}$ the mth element

TABLE I
THE RELATION BETWEEN $\phi_{j,n,k}$ AND $\overline{\gamma}_{R_iD}$, FOR $k \in \{1, ..., |\mathcal{C}|\}$, $n \in \left\{1, ..., \binom{|\mathcal{C}|}{k}\right\}$ AND $j \in \{1, ..., k\}$. NOTE THAT THE SUBSCRIPT i REFERS TO ALL $R_i \in \mathcal{C}$.

k/n	$n=1$	$n=2$	...	$n=\binom{\lvert\mathcal{C}\rvert}{k}$
$k=1$	$\phi_{1,1,1}=\overline{\gamma}_{R_1D}$	$\phi_{1,2,1}=\overline{\gamma}_{R_2D}$	...	$\phi_{1,\lvert\mathcal{C}\rvert,1}=\overline{\gamma}_{R_{\lvert\mathcal{C}\rvert}D}$
$k=2$	$\phi_{2,1,1}=\overline{\gamma}_{R_1D}$ $\phi_{2,1,2}=\overline{\gamma}_{R_2D}$	$\phi_{2,2,1}=\overline{\gamma}_{R_2D}$ $\phi_{2,2,2}=\overline{\gamma}_{R_3D}$	...	$\phi_{2,\binom{\lvert\mathcal{C}\rvert}{2},1}=\overline{\gamma}_{R_{\lvert\mathcal{C}\rvert}D}$ $\phi_{2,\binom{\lvert\mathcal{C}\rvert}{2},2}=\overline{\gamma}_{R_1D}$
...	...	...	...	...
$k=\lvert\mathcal{C}\rvert-1$	$\phi_{\lvert\mathcal{C}\rvert-1,1,1}=\overline{\gamma}_{R_1D}$ $\phi_{\lvert\mathcal{C}\rvert-1,1,2}=\overline{\gamma}_{R_2D}$ ... $\phi_{\lvert\mathcal{C}\rvert-1,1,\lvert\mathcal{C}\rvert-1}=\overline{\gamma}_{R_{\lvert\mathcal{C}\rvert-1}D}$	$\phi_{\lvert\mathcal{C}\rvert-1,2,1}=\overline{\gamma}_{R_2D}$ $\phi_{\lvert\mathcal{C}\rvert-1,2,2}=\overline{\gamma}_{R_3D}$ ... $\phi_{\lvert\mathcal{C}\rvert-1,2,\lvert\mathcal{C}\rvert-1}=\overline{\gamma}_{R_{\lvert\mathcal{C}\rvert}D}$	...	$\phi_{\lvert\mathcal{C}\rvert-1,\lvert\mathcal{C}\rvert,1}=\overline{\gamma}_{R_{\lvert\mathcal{C}\rvert}D}$ $\phi_{\lvert\mathcal{C}\rvert-1,\lvert\mathcal{C}\rvert,2}=\overline{\gamma}_{R_1D}$ ... $\phi_{\lvert\mathcal{C}\rvert-1,\lvert\mathcal{C}\rvert,\lvert\mathcal{C}\rvert-1}=\overline{\gamma}_{R_{\lvert\mathcal{C}\rvert-2}D}$
$k=\lvert\mathcal{C}\rvert$	–	–	–	$\phi_{\lvert\mathcal{C}\rvert,1,1}=\overline{\gamma}_{R_1D}$ $\phi_{\lvert\mathcal{C}\rvert,1,2}=\overline{\gamma}_{R_2D}$ ... $\phi_{\lvert\mathcal{C}\rvert,1,\lvert\mathcal{C}\rvert}=\overline{\gamma}_{R_{\lvert\mathcal{C}\rvert}D}$

of the nth k-subset of $\mathcal{A}_i$; we note that the the relation between $\psi^i_{k,n,m}$ and $\overline{\gamma}_{R_jD}$ is identical with that of $\phi_{k,n,j}$ and $\overline{\gamma}_{R_iD}$ (where $i : R_i \in \mathcal{C}$, $j : R_j \in \mathcal{C} \setminus \{R_i\}$) shown in Table I. As a result, using (5) and trivial integrations, (12) yields

$$\Pr\left\{\mathcal{S}^i \left|\mathcal{C}\right.\right\} = 1+\sum_{k=1}^{|\mathcal{C}|-1}\sum_{n=1}^{\binom{|\mathcal{C}|-1}{k}} \frac{(-1)^k}{1+\overline{\gamma}_{R_iD}\sum_{m=1}^{k}\frac{1}{\psi^i_{k,n,m}}}. \quad (14)$$

The second term in (9) can be expressed as

$$\begin{aligned}&\Pr\left\{\mathcal{E}^{S\text{-}D,\ R_i\text{-}D} \cap \mathcal{S}^i \left|\mathcal{C}\right.\right\}\\ &= \int_0^\infty\int_0^\infty \frac{\operatorname{erfc}\left(\sqrt{y+z}\right) e^{-\frac{y}{\overline{\gamma}_{R_iD}}} e^{-\frac{z}{\overline{\gamma}_{SD}}}}{2\overline{\gamma}_{R_iD}\overline{\gamma}_{SD}}\\ &\quad\times \prod_{j:R_j\in\mathcal{C};\ j\neq i}\left(1-e^{-\frac{y}{\overline{\gamma}_{R_jD}}}\right)dydz. \end{aligned} \quad (15)$$

Using the product expansion as before, and the auxiliary function $I_2(\cdot,\cdot,\cdot,\cdot)$ defined as (please refer to Appendix)

$$I_2(\alpha,b,\beta,\omega) = \int_0^\infty\int_\omega^\infty \frac{\operatorname{erfc}\left(\sqrt{\beta(x+y)}\right)}{e^{\alpha x}e^{by}}dxdy$$
$$= \begin{cases} \frac{e^{-\alpha\omega}\operatorname{erfc}\left(\sqrt{\beta\omega}\right)}{\alpha b} - \frac{\sqrt{\beta}\operatorname{erfc}\left(\sqrt{\omega(\alpha+\beta)}\right)}{\alpha(b-a)\sqrt{\alpha+\beta}} \\ \quad + \frac{\sqrt{\beta}e^{(b-\alpha)\omega}\operatorname{erfc}\left(\sqrt{\omega(b+\beta)}\right)}{b(b-\alpha)\sqrt{b+\beta}}, \quad a\neq b \\ \frac{e^{-\alpha\omega}\operatorname{erfc}\left(\sqrt{\beta\omega}\right) - \frac{\sqrt{\beta}\operatorname{erfc}\left(\sqrt{(a+\beta)\omega}\right)}{\sqrt{a+\beta}}}{\alpha^2} \\ \quad - \frac{\frac{2e^{-(\alpha+\beta)\omega}\sqrt{(\alpha+\beta)\omega}}{\sqrt{\pi}} + (1-2(\alpha+\beta)\omega)\operatorname{erfc}\left(\sqrt{(\alpha+\beta)\omega}\right)}{2\alpha\beta^{-1/2}(\alpha+\beta)^{3/2}}, \quad a=b \end{cases} \quad (16)$$

eq. (15) yields

$$\begin{aligned}\Pr\left\{\mathcal{E}^{S\text{-}D,\ R_i\text{-}D} \cap \mathcal{S}^i \left|\mathcal{C}\right.\right\} &= \frac{I_2\left(\frac{1}{\overline{\gamma}_{R_iD}}, \frac{1}{\overline{\gamma}_{SD}}, 1, 0\right)}{2\overline{\gamma}_{R_iD}\overline{\gamma}_{SD}} \quad (17)\\ &+ \sum_{k=1}^{|\mathcal{C}|-1}\sum_{n=1}^{\binom{|\mathcal{C}|-1}{k}} \frac{I_2\left(\frac{1}{\overline{\gamma}_{R_iD}}+\sum_{m=1}^{k}\frac{1}{\psi^i_{k,n,m}}, \frac{1}{\overline{\gamma}_{SD}}, 1, 0\right)}{2\overline{\gamma}_{R_iD}\overline{\gamma}_{SD}(-1)^k}.\end{aligned}$$

Working similarly, the rest of the terms in (9) are derived as follows

$$\begin{aligned}&\Pr\left\{\mathcal{E}^{S\text{-}R_i} \cap \mathcal{E}^{S\text{-}D,\ R_i\text{-}D} \cap \mathcal{S}^i \left|\mathcal{C}\right.\right\}\\ &= \frac{I_1\left(\frac{1}{\overline{\gamma}_{SR_i}}, 1, T\right)}{2\overline{\gamma}_{SR_i}} \Pr\left\{\mathcal{E}^{S\text{-}D,\ R_i\text{-}D} \cap \mathcal{S}^i \left|\mathcal{C}\right.\right\}, \end{aligned} \quad (18)$$

$$\begin{aligned}&\Pr\left\{\mathcal{E}^{S\text{-}R_i} \cap \mathcal{E}^{R_i\text{-}D} \cap \mathcal{S}^i \left|\mathcal{C}\right.\right\} = \frac{I_1\left(\frac{1}{\overline{\gamma}_{SR_i}}, 1, T\right)}{2\overline{\gamma}_{SR_i}}\\ &\times\left(\frac{I_1\left(\frac{1}{\overline{\gamma}_{R_iD}}, 1, 0\right)}{2\overline{\gamma}_{R_iD}}\right.\\ &\left.+\sum_{k=1}^{|\mathcal{C}|-1}\sum_{n=1}^{\binom{|\mathcal{C}|-1}{k}} \frac{I_1\left(\frac{1}{\overline{\gamma}_{R_iD}}+\sum_{m=1}^{k}\frac{1}{\psi^i_{k,n,m}}, 1, 0\right)}{2\overline{\gamma}_{R_iD}(-1)^k}\right), \end{aligned} \quad (19)$$

where $\Pr\left\{\mathcal{E}^{S\text{-}D,\ R_i\text{-}D} \cap \mathcal{S}^i \left|\mathcal{C}\right.\right\}$ is given in (17). Consequently, a closed-form expression for the BEP of SC is derived by inserting (10), (17) (18) and (19) in (9), in conjunction with (7). Note that for the special case of $\mathcal{C} = \varnothing$, it holds $\Pr\left\{\mathcal{E}\left|\mathcal{C}=\varnothing\right.\right\} = I_1(1/\overline{\gamma}_{SD}, 1, 0)/2\overline{\gamma}_{SD}$.

IV. PERFORMANCE ANALYSIS OF OPPORTUNISTIC RELAYING

Let us define the random variable γ_i as

$$\gamma_i := \min\left(\gamma_{SR_i}, \gamma_{R_iD}\right),\ i = \{1, ..., L\}, \quad (20)$$

which is also exponentially distributed with parameter equal to the sum of the parameters of γ_{SR_i} and γ_{R_iD} i.e., $1/\overline{\gamma}_i = 1/\overline{\gamma}_{SR_i} + 1/\overline{\gamma}_{R_iD}$. Moreover, for each $R_i \in \mathcal{R}$, we consider the set

$$\mathcal{B}_i = \left\{\overline{\gamma}_j : R_j \in (\mathcal{R} \setminus \{R_i\})\right\}, \quad (21)$$

and we denote with $\eta^i_{k,n,m}$ the mth element of the nth k-subset of $\mathcal{B}_i$; we note that the relation between $\eta^i_{k,n,m}$ and $\overline{\gamma}_j$ is identical with that of $\phi_{k,n,j}$ and $\overline{\gamma}_{R_iD}$ ($i : R_i \in \mathcal{C}$, $j : R_j \in \mathcal{R} \setminus \{R_i\}$) shown in Table I.

A. Outage Probability

The outage probability of OR can be expressed as

$$\begin{aligned}
&\Pr\{\mathcal{O}\}\\
&=\sum_{i:R_i\in\mathcal{R}}\left[\Pr\left\{\gamma_{SR_i}<T\cap\gamma_{SD}<T\cap\mathcal{S}^i\right\}\right.\\
&\left.+\Pr\left\{\gamma_{SR_i}>T\cap\gamma_{SD}+\gamma_{R_iD}<T\cap\mathcal{S}^i\right\}\right]\\
&=\sum_{i:R_i\in\mathcal{R}}\left[\left(\left(\int_0^T\frac{\Pr\left\{\mathcal{S}^i\left|\gamma_{SR_i}=x\right.\right\}}{\overline{\gamma}_{SR_i}e^{\frac{x}{\overline{\gamma}_{SR_i}}}}dx\right)\left(1-e^{-\frac{T}{\overline{\gamma}_{SD}}}\right)\right.\right.\\
&\left.+e^{-\frac{T}{\overline{\gamma}_{SR_i}}}\int_0^T\frac{1-e^{-\frac{T-y}{\overline{\gamma}_{SD}}}}{\overline{\gamma}_{R_iD}e^{\frac{y}{\overline{\gamma}_{R_iD}}}}\prod_{j:\overline{\gamma}_j\in\mathcal{B}_i}\left(1-e^{-\frac{y}{\overline{\gamma}_j}}\right)dy\right],
\end{aligned}\quad (22)$$

where the conditional probability of selecting R_i, conditioned on γ_{SR_i}, is given by

$$\begin{aligned}
&\Pr\left\{\mathcal{S}^i\left|\gamma_{SR_i}=x\right.\right\}\\
&=\Pr\left\{\min\left(\gamma_{SR_i},\gamma_{R_iD}\right)\geq\max_{j:\overline{\gamma}_j\in\mathcal{B}_i}\gamma_j\left|\gamma_{SR_i}=x\right.\right\}\\
&=\int_0^x\frac{e^{-\frac{y}{\overline{\gamma}_{R_iD}}}}{\overline{\gamma}_{R_iD}}\prod_{j:\overline{\gamma}_j\in\mathcal{B}_i}\left[1-\exp\left(-\frac{y}{\overline{\gamma}_j}\right)\right]dy\\
&+\int_x^\infty\frac{e^{-\frac{y}{\overline{\gamma}_{R_iD}}}}{\overline{\gamma}_{R_iD}}\prod_{j:\overline{\gamma}_j\in\mathcal{B}_i}\left[1-\exp\left(-\frac{x}{\overline{\gamma}_j}\right)\right]dy\\
&=\sum_{k=1}^{L-1}\sum_{n=1}^{\binom{L-1}{k}}\frac{1-\exp\left(-x\left[\frac{1}{\overline{\gamma}_{R_iD}}+\sum_{m=1}^k\left(\frac{1}{\eta^i_{k,n,m}}\right)\right]\right)}{(-1)^k\left(1+\overline{\gamma}_{R_iD}\sum_{m=1}^k\left(\frac{1}{\eta^i_{k,n,m}}\right)\right)}\\
&+1+\sum_{k=1}^{L-1}\sum_{n=1}^{\binom{L-1}{k}}\frac{\exp\left(-x\left[\frac{1}{\overline{\gamma}_{R_iD}}+\sum_{m=1}^k\left(\frac{1}{\eta^i_{k,n,m}}\right)\right]\right)}{(-1)^k}.
\end{aligned}\quad (23)$$

Therefore, using (23), (5) and trivial integrations, (22) yields (24) shown at the top of next page.

B. Bit Error Probability

Considering that the selected relay forwards the demodulated signal only if the received SNR is greater than T, and using the same approximation as that used in (8), the approximate BEP of OR is derived as it is shown in (25) at the top of next page.

In (25), the probability of the intersection of the events $\mathcal{E}^{S\text{-}R_i}$, $\mathcal{S}^i$ and $\gamma_{SR_i}\geq T$ is given by

$$\begin{aligned}
&\Pr\left\{\mathcal{E}^{S\text{-}R_i}\cap\mathcal{S}^i\cap\gamma_{SR_i}\geq T\right\}\\
&=\int_T^\infty\frac{\operatorname{erfc}\left(\sqrt{x}\right)e^{-\frac{x}{\overline{\gamma}_{SR_i}}}}{2\overline{\gamma}_{SR_i}}\Pr\left\{\mathcal{S}^i\left|\gamma_{SR_i}=x\right.\right\}dx.
\end{aligned}\quad (26)$$

Hence, combining (23) and (26) we obtain

$$\begin{aligned}
&\Pr\left\{\mathcal{E}^{S\text{-}R_i}\cap\mathcal{S}^i\cap\gamma_{SR_i}\geq T\right\}=\frac{I_1\left(\frac{1}{\overline{\gamma}_{SR_i}},1,T\right)}{2\overline{\gamma}_{SR_i}}\\
&+\sum_{k=1}^{L-1}(-1)^k\sum_{n=1}^{\binom{L-1}{k}}\left[\frac{I_1\left(\frac{1}{\overline{\gamma}_i}+\sum_{m=1}^k\frac{1}{\eta^i_{k,n,m}},1,T\right)}{2\overline{\gamma}_{SR_i}}\right.\\
&\left.+\frac{I_1\left(\frac{1}{\overline{\gamma}_{SR_i}},1,T\right)-I_1\left(\frac{1}{\overline{\gamma}_i}+\sum_{m=1}^k\frac{1}{\eta^i_{k,n,m}},1,T\right)}{2\overline{\gamma}_{SR_i}\left(1+\overline{\gamma}_{R_iD}\sum_{m=1}^k\frac{1}{\eta^i_{k,n,m}}\right)}\right].
\end{aligned}\quad (27)$$

Likewise, we may derive the probability of the intersection of the events $\mathcal{E}^{S\text{-}D,\ R_i\text{-}D}$, $\mathcal{S}^i$ and $\gamma_{SR_i}\geq T$ as

$$\begin{aligned}
&\Pr\left\{\mathcal{E}^{S\text{-}D,\ R_i\text{-}D}\cap\mathcal{S}^i\cap\gamma_{SR_i}\geq T\right\}\\
&=\int_0^\infty\int_0^T\frac{\operatorname{erfc}\left(\sqrt{y+z}\right)\prod_{j:\overline{\gamma}_j\in\mathcal{B}_i}\left(1-e^{-\frac{y}{\overline{\gamma}_j}}\right)}{2\overline{\gamma}_{R_iD}\overline{\gamma}_{SD}e^{\frac{y}{\overline{\gamma}_{R_iD}}}e^{\frac{z}{\overline{\gamma}_{SD}}}}dydz\\
&+\int_0^\infty\int_T^\infty\frac{\Pr\left\{\mathcal{S}^i\cap\gamma_{SR_i},y\geq T\left|\gamma_{R_iD}=y\right.\right\}e^{-\frac{y}{\overline{\gamma}_{R_iD}}}}{2\left[\operatorname{erfc}\left(\sqrt{y+z}\right)\right]^{-1}\overline{\gamma}_{R_iD}\overline{\gamma}_{SD}e^{\frac{z}{\overline{\gamma}_{SD}}}}dydz,
\end{aligned}\quad (28)$$

where we have used the fact that $\Pr\left\{\mathcal{S}^i\cap\gamma_{SR_i}\geq T\cap y<T\left|\gamma_{R_iD}=y\right.\right\}=\prod_{j:\overline{\gamma}_j\in\mathcal{B}_i}\left(1-e^{-\frac{y}{\overline{\gamma}_j}}\right)$.

Working similarly as in (23), we obtain

$$\begin{aligned}
&\Pr\left\{\mathcal{S}^i\cap\gamma_{SR_i},y\geq T\left|\gamma_{R_iD}=y\right.\right\}=e^{-\frac{T}{\overline{\gamma}_{SR_i}}}+\sum_{k=1}^{L-1}\sum_{n=1}^{\binom{L-1}{k}}\\
&\frac{e^{-T\left(\frac{1}{\overline{\gamma}_{SR_i}}+\sum_{m=1}^k\frac{1}{\eta^i_{k,n,m}}\right)}-e^{-y\left(\frac{1}{\overline{\gamma}_{SR_i}}+\sum_{m=1}^k\frac{1}{\eta^i_{k,n,m}}\right)}}{(-1)^k\left(1+\overline{\gamma}_{SR_i}\sum_{m=1}^k\frac{1}{\eta^i_{k,n,m}}\right)}\\
&+\sum_{k=1}^{L-1}(-1)^k\sum_{n=1}^{\binom{L-1}{k}}\exp\left(-y\left[\frac{1}{\overline{\gamma}_{SR_i}}+\sum_{m=1}^k\frac{1}{\eta^i_{k,n,m}}\right]\right).
\end{aligned}\quad (29)$$

Using the auxiliary function $I_3(\cdot,\cdot,\cdot,\cdot)$ defined as (please refer to Appendix)

$$\begin{aligned}
&I_3(\alpha,b,\beta,\omega)=\int_0^\infty\int_0^\omega\frac{\operatorname{erfc}\left(\sqrt{\beta(x+y)}\right)}{e^{\alpha x}e^{by}}dxdy\\
&=\begin{cases}
\frac{b\sqrt{\frac{\beta}{\alpha+\beta}}\operatorname{erf}\left(\sqrt{(\alpha+\beta)\omega}\right)-\alpha\sqrt{\frac{\beta}{b+\beta}}+(b-\alpha)e^{-\alpha\omega}\operatorname{erfc}\left(\sqrt{\beta\omega}\right)}{\alpha b(\alpha-b)}\\
+\frac{\alpha-b+\alpha e^{(b-\alpha)\omega}\sqrt{\frac{\beta}{b+\beta}}\operatorname{erfc}\left(\sqrt{(b+\beta)\omega}\right)}{\alpha b(\alpha-b)}, & a\neq b\\
\frac{+\alpha\sqrt{(\alpha+\beta)\beta\omega/\pi}-\left(\alpha^2\omega-\beta+\alpha(\beta\omega-3/2)\right)\operatorname{erfc}\left(\sqrt{(\alpha+\beta)\omega}\right)}{\alpha^2(\alpha+\beta)^{3/2}}\\
\frac{(\alpha+\beta)^{3/2}-\sqrt{\beta}(3\alpha/2+\beta)}{\alpha^2(\alpha+\beta)^{3/2}}-\frac{e^{-\alpha\omega}\operatorname{erfc}\left(\sqrt{\beta\omega}\right)}{\alpha^2}, & a=b
\end{cases}
\end{aligned}\quad (31)$$

from (28) and (29) we infer eq. (30) shown at the top of next page.

The probability of the intersection of the events $\mathcal{E}^{S\text{-}R_i}$,

$$\begin{aligned}
\Pr\{\mathcal{O}\} = \sum_{i:R_i\in\mathcal{R}} \Bigg[& \left(1-e^{-\frac{T}{\overline{\gamma}_{SD}}}\right)\Bigg(1-e^{-\frac{T}{\overline{\gamma}_{SR_i}}} + \sum_{k=1}^{L-1}\sum_{n=1}^{\binom{L-1}{k}} \frac{1-e^{-\frac{T}{\overline{\gamma}_{SR_i}}} - \frac{1-\exp\left(-T\left(1/\overline{\gamma}_i+\sum_{m=1}^{k}\frac{1}{\eta^i_{k,n,m}}\right)\right)}{\overline{\gamma}_{SR_i}\left(1/\overline{\gamma}_i+\sum_{m=1}^{k}\frac{1}{\eta^i_{k,n,m}}\right)}}{(-1)^k\left(1+\overline{\gamma}_{R_iD}\sum_{m=1}^{k}\left(\frac{1}{\eta^i_{k,n,m}}\right)\right)} \\
& + \sum_{k=1}^{L-1}\sum_{n=1}^{\binom{L-1}{k}} \frac{1-e^{-T\left(1/\overline{\gamma}_i+\sum_{m=1}^{k}\frac{1}{\eta^i_{k,n,m}}\right)}}{(-1)^k\,\overline{\gamma}_{SR_i}\left(1/\overline{\gamma}_i+\sum_{m=1}^{k}\frac{1}{\eta^i_{k,n,m}}\right)}\Bigg) + e^{-\frac{T}{\overline{\gamma}_{SR_i}}}\Bigg(1-e^{-\frac{T}{\overline{\gamma}_{R_iD}}} + \sum_{k=1}^{L-1}\sum_{n=1}^{\binom{L-1}{k}} \frac{1-e^{-T\left(1/\overline{\gamma}_{R_iD}+\sum_{m=1}^{k}\frac{1}{\eta^i_{k,n,m}}\right)}}{(-1)^k\left(1+\overline{\gamma}_{R_iD}\sum_{m=1}^{k}\frac{1}{\eta^i_{k,n,m}}\right)} \\
& - \frac{e^{-\frac{T}{\overline{\gamma}_{SD}}}}{\overline{\gamma}_{R_iD}}\left(\frac{1-e^{-T\left(\frac{1}{\overline{\gamma}_{R_iD}}-\frac{1}{\overline{\gamma}_{SD}}\right)}}{1/\overline{\gamma}_{R_iD}-1/\overline{\gamma}_{SD}} + \sum_{k=1}^{L-1}\sum_{n=1}^{\binom{L-1}{k}} \frac{1-e^{-T\left(\frac{1}{\overline{\gamma}_{R_iD}}-\frac{1}{\overline{\gamma}_{SD}}+\sum_{m=1}^{k}\frac{1}{\eta^i_{k,n,m}}\right)}}{(-1)^k\left(1/\overline{\gamma}_{R_iD}-1/\overline{\gamma}_{SD}+\sum_{m=1}^{k}\frac{1}{\eta^i_{k,n,m}}\right)}\right)\Bigg)\Bigg] \qquad (24)
\end{aligned}$$

$$\begin{aligned}
\Pr\{\mathcal{E}\} \approx & \sum_{i:R_i\in\mathcal{R}} \left[\Pr\left\{\left[\left(\mathcal{E}^{S\text{-}R_i}\backslash\mathcal{E}^{R_i\text{-}D}\right)\cup\left(\mathcal{E}^{S\text{-}D,\,R_i\text{-}D}\backslash\mathcal{E}^{S\text{-}R_i}\right)\right]\cap\mathcal{S}^i\cap\gamma_{SR_i}\geq T\right\} + \Pr\left\{\mathcal{E}^{S\text{-}D}\cap\mathcal{S}^i\cap\gamma_{SR_i}<T\right\}\right] \\
= & \sum_{i:R_i\in\mathcal{R}} \left[\Pr\left\{\mathcal{E}^{S\text{-}R_i}\cap\mathcal{S}^i\cap\gamma_{SR_i}\geq T\right\} + \Pr\left\{\mathcal{E}^{R_i\text{-}D}\cap\mathcal{S}^i\cap\gamma_{SR_i}\geq T\right\} - \Pr\left\{\mathcal{E}^{S\text{-}R_i}\cap\mathcal{E}^{R_i\text{-}D}\cap\gamma_{SR_i}\geq T\cap\mathcal{S}^i\right\}\right. \\
& \left.- \Pr\left\{\mathcal{E}^{S\text{-}R_i}\cap\mathcal{E}^{S\text{-}D,\,R_i\text{-}D}\cap\gamma_{SR_i}\geq T\cap\mathcal{S}^i\right\} + \Pr\left\{\mathcal{E}^{S\text{-}D}\cap\gamma_{SR_i}<T\cap\mathcal{S}^i\right\}\right]. \qquad (25)
\end{aligned}$$

$$\begin{aligned}
\Pr\left\{\mathcal{E}^{S\text{-}D,\,R_i\text{-}D}\cap\gamma_{SR_i}\geq T\cap\mathcal{S}^i\right\} = & \frac{1}{2\overline{\gamma}_{R_iD}\overline{\gamma}_{SD}}\Bigg\{I_3\left(\frac{1}{\overline{\gamma}_{R_iD}},\frac{1}{\overline{\gamma}_{SD}},1,T\right) + e^{-\frac{T}{\overline{\gamma}_{SR_i}}}I_2\left(\frac{1}{\overline{\gamma}_{R_iD}},\frac{1}{\overline{\gamma}_{SD}},1,T\right) \\
& + \sum_{k=1}^{L-1}(-1)^k\sum_{n=1}^{\binom{L-1}{k}}\Bigg[I_3\left(\frac{1}{\overline{\gamma}_{R_iD}}+\sum_{m=1}^{k}1/\eta^i_{k,n,m},\frac{1}{\overline{\gamma}_{SD}},1,T\right) + I_2\left(\frac{1}{\overline{\gamma}_i}+\sum_{m=1}^{k}1/\eta^i_{k,n,m},\frac{1}{\overline{\gamma}_{SD}},1,T\right) \\
& + \frac{I_2\left(\frac{1}{\overline{\gamma}_{R_iD}},\frac{1}{\overline{\gamma}_{SD}},1,T\right)e^{-T\left(\frac{1}{\overline{\gamma}_{SR_i}}+\sum_{m=1}^{k}\frac{1}{\eta^i_{k,n,m}}\right)} - I_2\left(\frac{1}{\overline{\gamma}_i}+\sum_{m=1}^{k}1/\eta^i_{k,n,m},\frac{1}{\overline{\gamma}_{SD}},1,T\right)}{1+\overline{\gamma}_{SR_i}\sum_{m=1}^{k}1/\eta^i_{k,n,m}}\Bigg]\Bigg\} \qquad (30)
\end{aligned}$$

$\mathcal{E}^{S\text{-}D,\,R_i\text{-}D}$, $\gamma_{SR_i}\geq T$ and $\mathcal{S}^i$ is given by

$$\begin{aligned}
&\Pr\left\{\mathcal{E}^{S\text{-}R_i}\cap\mathcal{E}^{S\text{-}D,\,R_i\text{-}D}\cap\gamma_{SR_i}\geq T\cap\mathcal{S}^i\right\} \\
&= \Pr\left\{\mathcal{E}^{S\text{-}R_i}\cap\gamma_{SR_i}\geq \max_{j:\overline{\gamma}_j\in\mathcal{B}_i}\gamma_j\cap\gamma_{SR_i}\geq T\right\} \\
&\times \Pr\left\{\mathcal{E}^{S\text{-}D,\,R_i\text{-}D}\cap\gamma_{R_iD}\geq \max_{j:\overline{\gamma}_j\in\mathcal{B}_i}\gamma_j\right\}, \qquad (32)
\end{aligned}$$

where it holds

$$\begin{aligned}
&\Pr\left\{\mathcal{E}^{S\text{-}R_i}\cap\gamma_{SR_i}\geq \max_{j:\overline{\gamma}_j\in\mathcal{B}_i}\gamma_j\cap\gamma_{SR_i}\geq T\right\} \\
&= \int_T^\infty \frac{\mathrm{erfc}\left(\sqrt{x}\right)e^{-\frac{x}{\overline{\gamma}_{SR_i}}}}{2\overline{\gamma}_{SR_i}}\prod_{j:\overline{\gamma}_j\in\mathcal{B}_i}\left(1-e^{-\frac{x}{\overline{\gamma}_j}}\right)dx \\
&= \frac{I_1\left(\frac{1}{\overline{\gamma}_{SR_i}},1,T\right)}{2\overline{\gamma}_{SR_i}} \\
&+ \sum_{k=1}^{L-1}\sum_{n=1}^{\binom{L-1}{k}}\frac{I_1\left(\frac{1}{\overline{\gamma}_{SR_i}}+\sum_{m=1}^{k}\frac{1}{\eta^i_{k,n,m}},1,T\right)}{2\overline{\gamma}_{SR_i}\left(-1\right)^k} \qquad (33)
\end{aligned}$$

and

$$\begin{aligned}
&\Pr\left\{\mathcal{E}^{S\text{-}D,\,R_i\text{-}D}\cap\gamma_{R_iD}\geq \max_{j:\overline{\gamma}_j\in\mathcal{B}_i}\gamma_j\right\} \\
&= \int_0^\infty\int_0^\infty \frac{1/2\mathrm{erfc}\left(\sqrt{y+z}\right)}{\overline{\gamma}_{R_iD}\overline{\gamma}_{SD}e^{\frac{y}{\overline{\gamma}_{R_iD}}}e^{\frac{z}{\overline{\gamma}_{SD}}}}\prod_{j:\overline{\gamma}_j\in\mathcal{B}_i}\left(1-e^{-\frac{y}{\overline{\gamma}_j}}\right)dydz \\
&= \frac{I_2\left(\frac{1}{\overline{\gamma}_{R_iD}},\frac{1}{\overline{\gamma}_{SD}},1,0\right)}{2\overline{\gamma}_{R_iD}\overline{\gamma}_{SD}} \\
&+ \sum_{k=1}^{L-1}\sum_{n=1}^{\binom{L-1}{k}}\frac{I_2\left(\frac{1}{\overline{\gamma}_{R_iD}}+\sum_{m=1}^{k}\frac{1}{\eta^i_{k,n,m}},\frac{1}{\overline{\gamma}_{SD}},1,0\right)}{2\overline{\gamma}_{R_iD}\overline{\gamma}_{SD}\left(-1\right)^k}. \qquad (34)
\end{aligned}$$

Working similarly, it is easy to derive the probability $\Pr\left\{\mathcal{E}^{S\text{-}R_i}\cap\mathcal{E}^{R_i\text{-}D}\cap\gamma_{SR_i}\geq T\cap\mathcal{S}^i\right\}$ directly from (32), by substituting the second term in its right-hand side with

$$
\begin{aligned}
&\Pr\left\{\mathcal{E}^{R_i\text{-}D} \cap \gamma_{R_iD} \geq \max_{j:\overline{\gamma}_j \in \mathcal{B}_i} \gamma_j\right\} \\
&= \int_0^{\infty} \frac{\operatorname{erfc}\left(\sqrt{y}\right)}{2\overline{\gamma}_{R_iD} e^{\frac{y}{\overline{\gamma}_{R_iD}}}} \prod_{j:\overline{\gamma}_j \in \mathcal{B}_i} \left(1 - e^{-\frac{y}{\overline{\gamma}_j}}\right) dy \\
&= \frac{I_1\left(\frac{1}{\overline{\gamma}_{R_iD}}, 1, 0\right)}{2\overline{\gamma}_{R_iD}} \\
&+ \sum_{k=1}^{L-1} \sum_{n=1}^{\binom{L-1}{k}} \frac{I_1\left(\frac{1}{\overline{\gamma}_{R_iD}} + \sum_{m=1}^{k} \frac{1}{\eta^i_{k,n,m}}, 1, 0\right)}{2\overline{\gamma}_{R_iD}\left(-1\right)^k}. \qquad (35)
\end{aligned}
$$

Finally, the last term in (25) can be evaluated using (23) as

$$
\begin{aligned}
&\Pr\left\{\mathcal{E}^{S\text{-}D} \cap \gamma_{SR_i} < T \cap \mathcal{S}^i\right\} \\
&= \int_0^{T} \frac{\Pr\left\{\mathcal{S}^i \left|\gamma_{SR_i} = x\right.\right\} e^{-\frac{x}{\overline{\gamma}_{SR_i}}}}{\overline{\gamma}_{SR_i}} dx \\
&\times \int_0^{\infty} \frac{\operatorname{erfc}\left(\sqrt{z}\right) e^{-\frac{z}{\overline{\gamma}_{SD}}}}{2\overline{\gamma}_{SD}} dz \\
&= \frac{I_1\left(\frac{1}{\overline{\gamma}_{SD}}, 1, 0\right)}{2\overline{\gamma}_{SD}} \left[1 - e^{-\frac{T}{\overline{\gamma}_{SR_i}}}\right. \qquad (36) \\
&+ \sum_{k=1}^{L-1} \sum_{n=1}^{\binom{L-1}{k}} \frac{1 - e^{-\frac{T}{\overline{\gamma}_{SR_i}}} - \frac{1-\exp\left(-T\left(\frac{1}{\overline{\gamma}_i} + \sum_{m=1}^{k} \frac{1}{\eta^i_{k,n,m}}\right)\right)}{\overline{\gamma}_{SR_i}\left(\frac{1}{\overline{\gamma}_i} + \sum_{m=1}^{k} \frac{1}{\eta^i_{k,n,m}}\right)}}{\left(1 + \overline{\gamma}_{R_iD} \sum_{m=1}^{k} \frac{1}{\eta^i_{k,n,m}}\right)\left(-1\right)^k} \\
&\left. + \sum_{k=1}^{L-1} \sum_{n=1}^{\binom{L-1}{k}} \frac{1 - \exp\left(-T\left(\frac{1}{\overline{\gamma}_i} + \sum_{m=1}^{k} \frac{1}{\eta^i_{k,n,m}}\right)\right)}{\overline{\gamma}_{SR_i}\left(\frac{1}{\overline{\gamma}_i} + \sum_{m=1}^{k} \frac{1}{\eta^i_{k,n,m}}\right)\left(-1\right)^k}\right].
\end{aligned}
$$

Therefore, a closed-form expression for the BEP of OR is derived by combining (27), (30), (32) - (36) and (25). We should note that this BEP expression can be simplified by setting $\Pr\left\{\mathcal{E}^{S\text{-}R_i} \cap \mathcal{E}^{S\text{-}D,\, R_i\text{-}D} \cap \gamma_{SR_i} \geq T \cap \mathcal{S}^i\right\} = \Pr\left\{\mathcal{E}^{S\text{-}R_i} \cap \mathcal{E}^{R_i\text{-}D} \cap \gamma_{SR_i} \geq T \cap \mathcal{S}^i\right\} \approx 0$, since, intuitively, it is very unlikely that an error on both the S-R_i and R_i-D links occurs at a transmission slot, say t, given that R_i is selected for t.

V. Numerical Performance Comparison

In this Section, a performance comparison between SC and OR is presented, in terms of outage probability and BEP, based on the closed-form expressions derived above. Also, in order to verify the validity of the aforementioned expressions, an extensive set of simulations was performed, the results of which match closely the theoretical ones, as it is shown in Figs. 1-3. The average SNRs of the S-R_i and R_i-D links are considered to follow an exponential profile with identical mean values, denoted by ε (i.e., $E\left[\overline{\gamma}_{SR_i}\right] = E\left[\overline{\gamma}_{R_iD}\right] = \varepsilon$, where $E\left[\cdot\right]$ denotes here expectation over $i \in \{1, ..., L\}$), and decay factor equal to 0.5. The average SNR $\overline{\gamma}_{SD}$ of the direct S-D channel is set equal to ε/λ; the parameter λ thus reflects the relative S-D channel quality, with respect to that of the S-R_i and R_i-D ones. The fading in all links is assumed to be independent, Rayleigh distributed.

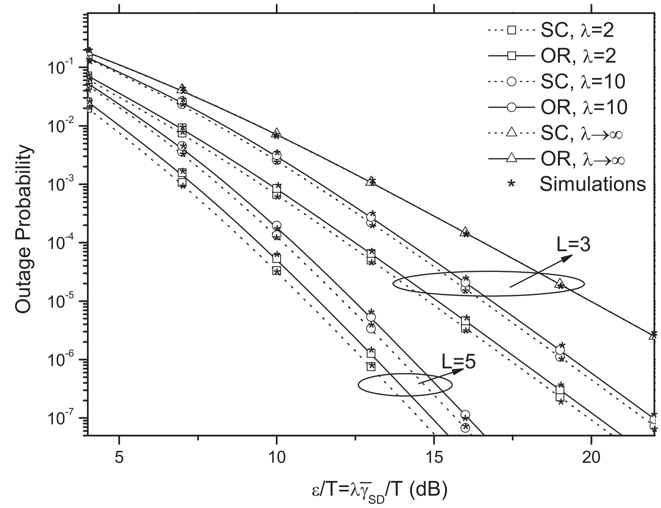

Fig. 1. Outage performance of SC and OR for some λ and L assumptions

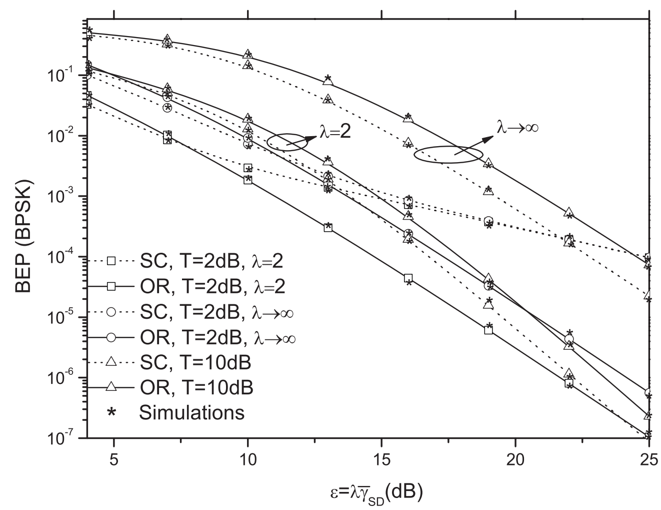

Fig. 2. BEP performance of SC and OR for $L = 3$

Fig. 1, depicts the outage performance of SC and OR for some λ and L assumptions. All curves were plotted versus the normalized value of ε with respect to the threshold SNR (which is identical here with T), so that the information they convey is directed towards the outage probability. The dotted lines correspond to SC schemes, whereas the solid ones to OR. In general, we may notice that SC slightly outperforms OR, in terms of outage probability. In fact, SC seems to take better advantage of the direct S-D channel, since the outage performance of the two schemes is identical when the S-D channel is not taken into account (or equivalently, when $\lambda \to \infty$). Recall that the latter result was derived theoretically in [11]. We also notice that the SC outage curves are very close to the approximate ones given in [2] and [7], in the high and low-to-medium SNR regimes, respectively.

In Figs. 2 and 3, we present a BEP comparison between SC and OR for BPSK modulation, assuming $L = 3$ and $L = 5$, respectively. The main result extracted from these two figures is that *the relative performance of the two studied schemes is highly affected by the threshold* T. Recall that T corresponds to a fixed value associated with the target rate r, and may

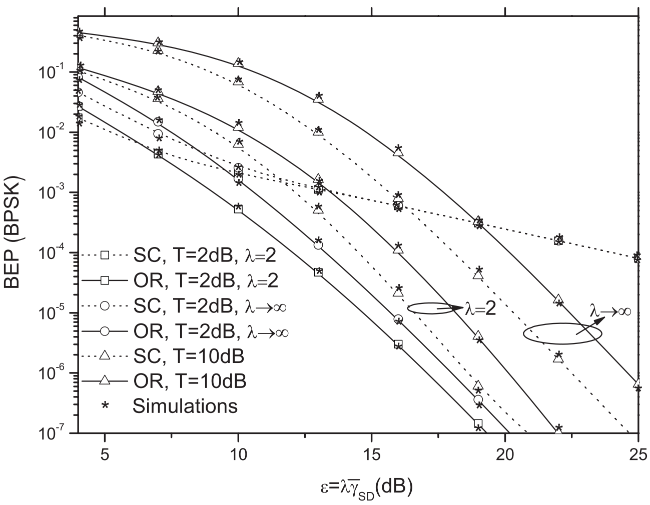

Fig. 3. BEP performance of SC and OR for $L = 5$

vary from application to application; in fact, it determines the number of relays that belong to $\mathcal{C}$ in the SC scenario, and whether the destination receives from both the source and the selected relay or only from the source, in the OR scenario. Hence, we notice the interesting result that T seems to shift the OR BEP curves, though without affecting their slopes, whereas it does so in the SC curves. This result can be explained by considering the fact that the relay selection in SC is done according to the R_i-D links, regardless of the S-R_i ones; thus, when T is low, it is likely that the S-R_i link of the selected relay leads to an error, hence strongly degrading the error performance. Contrarily, in OR both the S-R_i and R_i-D links participate in the selection process, thus the BEP performance is less affected by T. Generally speaking, we may conclude that both schemes may outperform one another in terms of BEP, depending on the SNR threshold T: Lower T values result in better OR performance; higher T values in better SC performance.

Appendix

Assuming $\alpha \neq 0$ and using integration by parts, we obtain

$$\int e^{-\alpha x} \operatorname{erfc}\left(\sqrt{\beta (x+y)}\right) dx = -\frac{e^{-\alpha x}}{\alpha} \operatorname{erfc}\left(\sqrt{\beta x}\right) + \frac{1}{\alpha} \int e^{-\alpha x} \left(-\frac{\beta e^{-\beta (x+y)}}{\sqrt{\pi}\sqrt{\beta (x+y)}} \right) dx, \tag{37}$$

where we have used the integral representation of erf$(\cdot)$ given in [14, eq. 8.251.1] to obtain the derivative of erfc$(\cdot)$. Using [14, eq. 8.251.1] again, (37) yields

$$\int e^{-\alpha x} \operatorname{erfc}\left(\sqrt{\beta (x+y)}\right) dx = -\frac{\operatorname{erfc}\left(\sqrt{\beta (x+y)}\right)}{\alpha e^{\alpha x}} - \frac{\sqrt{\beta} e^{\alpha y}}{\alpha \sqrt{\alpha+\beta}} \operatorname{erf}\left(\sqrt{\alpha+\beta}\sqrt{x+y}\right). \tag{38}$$

Using (38), $I_2(\alpha, b, \beta, \omega)$ can be written as

$$I_2(\alpha, b, \beta, \omega) = \int_0^\infty e^{-by} \left[\frac{e^{-\alpha\omega} \operatorname{erfc}\left(\sqrt{\beta (y+\omega)}\right)}{\alpha} - \frac{\sqrt{\beta} e^{\alpha y}}{\alpha\sqrt{\alpha+\beta}} \operatorname{erfc}\left(\sqrt{(\alpha+\beta)(y+\omega)}\right) \right] dy. \tag{39}$$

Hence, we may apply (38) in (39) to obtain the first part of (16), after some manipulations. For the special case of $\alpha = b$, we may use integration by parts to yield

$$\int_0^\infty \operatorname{erfc}\left(\sqrt{(\alpha+\beta)(y+\omega)}\right) dy = \frac{2\pi^{-1/2}\sqrt{(\alpha+\beta)\,\omega}}{2(\alpha+\beta)\, e^{(\alpha+\beta)\omega}} + \frac{(1 - 2(\alpha+\beta)\,\omega) \operatorname{erfc}\left(\sqrt{(\alpha+\beta)\,\omega}\right)}{2(\alpha+\beta)}, \tag{40}$$

and thus to derive the second part of (16).

The auxiliary function $I_3(\cdot,\cdot,\cdot,\cdot)$ is derived as $I_3(\alpha, b, \beta, \omega) = I_2(\alpha, b, \beta, 0) - I_2(\alpha, b, \beta, \omega)$.

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