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User Grouping for Hybrid VLC/RF Networks with NOMA: A Coalitional Game Approach

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ABSTRACT Recently, visible light communication (VLC) networks have emerged as a promising alternative for indoor data access, due to high data rate, low implementation cost and free from radio frequency (RF) interference. However, the co-existence of VLC with the RF access points as well as the dependence of VLC to room illumination compel both technologies to work in parallel and thus, to form a hybrid heterogeneous VLC/RF network. This network offers the advantages of both technologies, namely increased capacity and ubiquitous coverage. Furthermore, non-orthogonal multiple access (NOMA) is a very promising candidate technique for the next generation of wireless networks, mainly due to its increased spectrum efficiency compared to orthogonal access schemes. However, the optimal user grouping in NOMA is a combinatorial NP-complete problem, which calls for low complexity techniques. To this end, in this paper, we propose the use of coalitional game theory, where the users served by the same access point (VLC or RF) form a single coalition, while the users can switch through coalitions based on their payoff. A novel utility function is proposed that takes into account the peculiarities of the NOMA hybrid VLC/RF network. Finally, a coalition formation algorithm is presented as well as an efficient power allocation policy. Computer simulations validate the presented analysis and reveal the effectiveness of the proposed user grouping scheme compared to an opportunistic approach.

INDEX TERMS coalitional game theory, heterogeneous network, non-orthogonal multiple access (NOMA), user grouping, visible light communications (VLC).

I. INTRODUCTION

THE need to improve wireless networking in order to accommodate the demands of the next generation of wireless networks (5G and beyond) has lead academia and industry to pursue creative solutions. The commercial use of different regions of the electromagnetic spectrum, e.g., mmWave and optical, signals a possible solution the spectrum scarcity problem [?]. More specifically, visible light communications (VLC) take advantage of the already existing infrastructure for illumination to offer ultra high data rate to indoor users users [?], [?], [?]. This type of networks has been primarily investigated as an indoor solution due to physical limitations, due to light's propagation and background solar radiation. It is important to state here that around 80% of the data traffic originates from indoor

activities [?]. As such, VLC has become a prime candidate for indoor networking, due to its vast unregulated available spectrum, low implementation cost and immunity against interference compared to conventional radio frequency (RF) systems. However, indoor VLC has to be combined with an RF network in order to support the functional limitations in the uplink scenario as well as to provide ubiquitous coverage, thus, forming a hybrid VLC/RF heterogeneous network (HetNet) [?], [?]. This kind of HetNet offers numerous research challenges in user selection, resource allocation and handover schemes.

Besides the capitalization of the available bandwidth at a different region of the electromagnetic spectrum, techniques that make a more efficient utilization of the spectrum are needed. Spectral efficiency is an important performance

metric in wireless networks, related to the quality-of-service (QoS), and it has attracted significant attention from the research community. Non-orthogonal multiple access (NOMA) has been proposed as an efficient access technique to improve spectral efficiency. The main principle of NOMA is fundamentally different from conventional orthogonal multiple access schemes, e.g., time-division multiple access (TDMA), since NOMA places together more than one user into a single orthogonal resource block. In order to accommodate the users in this manner, the transceiver implements advanced signal processing techniques, as superposition coding (SC) and successive interference cancellation (SIC). NOMA's superiority in spectral efficiency has been proven in literature, as it can achieve the capacity region of the broadcast channel. This advantage has elevated NOMA to a prime solution for the massive connectivity requirements of the next generation of wireless networks [?].

A. RELATED LITERATURE

User association and user grouping in NOMA has been established as a prominent problem for research, since in [?], [?] it was proved that pairing plays an important role in the system performance. User scheduling and grouping has been investigated in various NOMA scenarios [?], [?], [?], [?], [?], [?], [?], [?], [?]. Also, NOMA has been studied for VLC systems in [?], [?], [?], [?], [?], [?], [?], [?], [?], [?], [?], [?], focusing on optimal and suboptimal power allocation schemes. Finally, NOMA has been studied in some works with HetNets as well [?], [?], [?], [?].

In more detail, the gain of NOMA over orthogonal schemes is higher when the channel conditions of the paired users are more distinctive. Moreover, depending on metric of interest, different user grouping can be employed to reach the optimal solution [?]. To avoid decoding and resource allocation complexity, as well as interference, most works in the literature assume that users are grouped in pairs. Specifically, in [?], a matching algorithm was proposed for a NOMA system with an amplify-and-forward relay, in order to allocate users to certain subchannels through orthogonal frequency division multiple access (OFDMA). Each subchannel can be used by a maximum number of source-destination pairs at once, which saves complexity compared to an exhaustive search of possible source-destination pairs. In [?], the authors maximize the weighted sum rate through the use of a *many-to-many* matching algorithm. A user can form pairs and swap those pairs with another or choose to block a swap. In [?], conventional matching games are disregarded in favor of a matching with peer effect, since the authors investigate a device-to-device (D2D) system, while a pair of users in NOMA affects the rest of the set, and thus, the grouping. User grouping in NOMA can also be used to optimize beamforming in a mmWave network. Specifically, in [?], [?], optimization techniques and a clustering approach based on machine learning were proposed, in order to deal with this problem.

In [?], user grouping is investigated in a heterogeneous ultra dense network, where conventional grouping methods cannot be applied, due to the aggregated interference. In the same work, users choose to associate with the base station (BS) that offers them higher average power instead of the closer BS. This happens because by choosing the closest BS would lead users to crowd the lower tier BSs, since they are more densely deployed and closer to the end users.

For the first time, NOMA has been proposed for VLC networks in [?] and a comprehensive review on the subject was presented in [?]. After that, in [?], NOMA was experimentally used in VLC, while in [?], an optimal power allocation scheme was proposed to maximize the proportional fairness. In [?], the error performance of an uplink VLC network was studied by using phase pre-distortion. In [?], NOMA was studied in a VLC network with DC Offset-OFDM, while the non-linear effects of the LEDs were studied in [?]. Furthermore, the ergodic sum rate of NOMA in VLC and the effect of different type of LEDs was studied in [?]. As mentioned above, the performance of NOMA increases when the users' channel conditions differ most. In [?] NOMA was discussed as a promising multiple access scheme for VLC, while in [?] an empirical power allocation policy was proposed. The authors in [?] studied the error performance of NOMA in VLC networks, assuming imperfect channel state information. User grouping has also attracted attention in VLC networks with NOMA. Specifically, in [?] user grouping was optimized to reduce the interference in a multi-cell network. Moreover, in [?], a simple user grouping was proposed, which splits the users in two groups, according to the channel conditions and pair each strong with the corresponding weak user.

Despite the rich research of VLC with NOMA, to the best of the authors' knowledge hybrid VLC/RF networks with NOMA have not been studied yet in open literature. These networks present various peculiarities, since the two subsystems operate at an entire different region of the EM spectrum. However, this raises two major issues, namely the disparity in the capacities of the two sub-networks as well as their respective coverage, since VLC offers better capacity but limited coverage. As such, the trade-off between achievable rate and fairness becomes more prominent due to this rate asymmetry. Second, the inclusion of multiple VLC cells in the system, hence more combinations of possible groupings, makes this problem even more complex. Moreover, in this system, users are described by a vector containing their channel conditions at each access point and not a scalar. So conventional or trivial grouping schemes cannot be utilized.

B. CONTRIBUTION

In this paper, we investigate, for the first time in the literature, the practical indoor scenario of a hybrid VLC/RF network, where both VLC and RF subsystems perform NOMA. Note that due to NOMA's particularities, optimal user grouping is still an open problem of research. A hybrid VLC/RF network

creates several challenges, due to the different nature of the two subsystems, and the asymmetry in the users' achievable rate. Users that can be served by the VLC network can increase their capacity far beyond the respective users that are served by the RF, and thus, fairness is a problem. User selection/grouping in such a network plays an important role in order to avoid congestion and maximize the benefits of the hybrid system. Its significance is even greater when the access points utilize NOMA. Conventional empirical methods do not work in such a network; it is impossible to pair a strong user with a weak one, since multiple access points are at play, so users experience different channel conditions at each AP, therefore they cannot be classified as strong or weak in the system. User grouping, hence, is particularly challenging. However, in such a network with asymmetric rates, users tend to maximize their own payoff in a non-cooperative manner. In order to balance the individual rates maximization and fairness, we propose a novel utility that also takes into account the additional complexity of the NOMA scheme. In order to model all these interactions between users and the respective access points of the hybrid network we utilize the *coalitional game theory*. Each coalition is assigned to a specific access point, VLC or RF, and users can join a coalition that best suits them.

The contributions of this paper can be summarized as follows:

- Modeling the interactions of users in the hybrid VLC/RF network through the application of coalitional game theory. This leads to a coalition formation algorithm that solves the user grouping problem. The algorithm is based on the *merge-and-split* that is used to reserve complexity in combinatorial optimization problems.
- Proposing a novel utility function to be used from the users in the game, taking into account the particularities of the NOMA HetNet. This utility function assumes that there is a cost to join a coalition and so users have to team up and divide the cost among themselves to increase their payoff.
- Through the coalition formation phase, a power allocation policy is obtained, based on the cognitive radio inspired NOMA [?]. This is also based on the concept of *consent*, according to which adding a new user in the coalition does not decrease the payoff of the users who are already part of this coalition.
- Finally, computer simulations validate the presented analysis and reveal the effectiveness of the proposed user grouping scheme compared to an opportunistic approach.

C. STRUCTURE

The rest of the paper is organized as follows. Section II describes a comprehensible system model and channel model of the heterogeneous network. In Section III, the problem of user grouping is illustrated in NOMA networks and it is formulated via a game theoretic approach. In

Section IV a coalition formation algorithm is proposed to solve the problem of user grouping. Finally, in Section V, simulation results validate the proposed analysis in a plethora of scenarios and in section VI some brief conclusions are drawn.

II. SYSTEM AND CHANNEL MODEL

We consider the downlink transmission of a hybrid VLC/RF network with multiple users, consisting of a total $|\mathcal{M}| = M + 1$ access points (APs) and $N = |\mathcal{N}|$ users, where $\mathcal{M} = \{0, 1, \dots, m, \dots, M\}$, $\mathcal{N} = \{1, \dots, i, \dots, N\}$, and the operator $|\mathcal{A}|$ denotes the cardinality of set \mathcal{A} . Among $|\mathcal{M}|$ APs, M are non-interfering VLC APs, and one is an RF AP, which will be denoted as $m = 0$. We further assume that each user is served by either the RF AP or the VLC APs. Also, it is assumed that all mobile nodes are equipped with single antennas/optical receivers and each AP performs power domain NOMA, with B_m , being its bandwidth.

During the transmission phase, a total of N_m signals are transmitted to each user assigned to the m -th AP, where N_m denotes the number of users assigned to the m -th AP with $\sum_{m=0}^M N_m = N$. Then, the baseband equivalent of the received signal of a user n_m that is assigned to AP m is given by

$$y_{n_m,m} = h_{n_m,m} \sum_{i=1}^{N_m} P_{i,m} s_{i,m} + n_{n_m}, \quad (1)$$

where $h_{n_m,m}$ denotes the Rayleigh fading channel coefficient between the m -th AP and the n_m -th user, $P_{i,m}$ represents the power of the i -th user that is also assigned to the m -th AP, or square root of power for the RF case, $s_{i,m}$ denotes the message sent from the m -th AP to the i -th user, and n_{n_m} is the additive Gaussian noise at the n_m -th receiver.

A. THE VLC SUBSYSTEM

The channel power gain for the n_m -th user from the m -th VLC AP is given by [?], [?]

$$h_{n_m,m} = \frac{L_r}{d_{n_m,m}^2} r_0(\varphi_{n_m,m}) T_s(\psi_{n_m,m}) \times g(\psi_{n_m,m}) \cos(\psi_{n_m,m}), \quad (2)$$

where L_r is the area of the photo-detector and $d_{n_m,m}$ is the transmission distance from the m -th AP to the n_m -th user. Furthermore, $T_s(\psi_{n_m,m})$ is the gain of the optical filter and $g(\psi_{n_m,m})$ represents the gain of the optical concentrator, given by [?], [?]

$$g(\psi_{n_m,m}) = \begin{cases} \frac{\rho^2}{\sin^2(\Psi_{\text{fov}})}, & 0 \leq \psi_{n_m,m} \leq \Psi_{\text{fov}}, \\ 0, & \psi_{n_m,m} > \Psi_{\text{fov}}. \end{cases} \quad (3)$$

with ρ and Ψ_{fov} being the refractive index and FOV, respectively. Also in (2), $r_0(\varphi_{n_m,m})$ is the Lambertian radiant intensity of the LED, written as

$$r_0(\varphi_{n_m,m}) = \frac{\xi + 1}{2\pi} \cos^\xi \varphi_{n_m,m}, \quad (4)$$

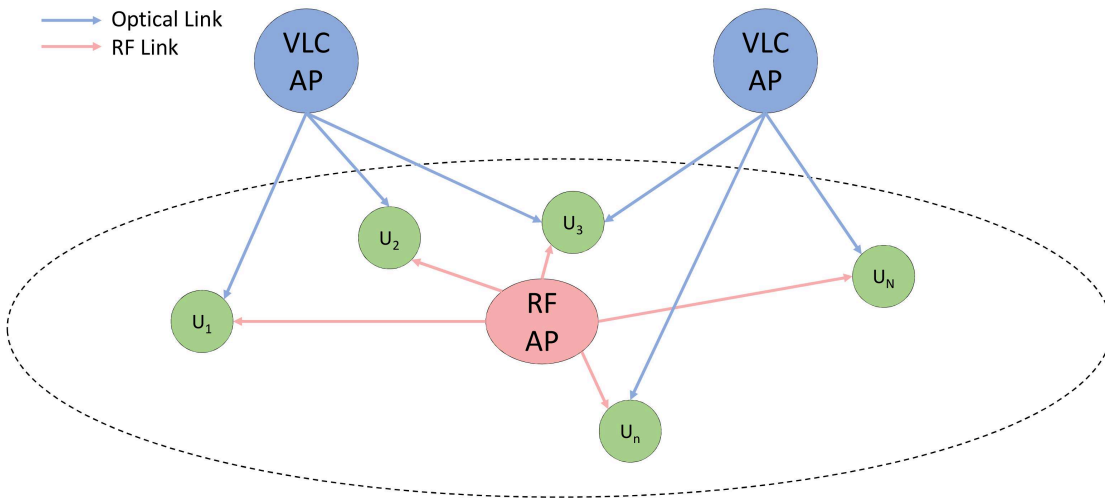


FIGURE 1. System Model.

where $\varphi_{n_m,m}$ is the irradiance angle, $\psi_{n_m,m}$ is the incidence angle, and $\xi = -\frac{1}{\log_2 \cos(\Phi_{1/2})}$, with $\Phi_{1/2}$ being the semi-angle at half luminance.

Note that the achievable rate of the VLC system is also limited by the average optical power (lighting constraint), i.e.,

$$\sum_{n_m=1}^{N_m} P_{n_m,m} \leq P_{\max}, \quad (5)$$

where P_{\max} denotes the maximum available power of each VLC AP. Thus, by applying NOMA with SIC for any user n_m , $1 \leq n_m \leq N_m$ the received signal is detected and the information for other users with better channel conditions is considered as interference. Therefore, the receiving signal-to-interference plus noise ratio (SINR) for the n_m user is given by

$$\gamma_{n_m,m} = \frac{(|h_{n_m,m}| \eta P_{n_m,m})^2}{|h_{n_m,m}|^2 \eta^2 \sum_{i=n_m+1}^{N_m} P_{i,m}^2 + \frac{1}{\rho}}, \quad (6)$$

where ρ is the transmit SNR, and η denotes the photodetector's responsivity. The N_m -th user, i.e., the user with the best channel quality decodes its own message with the following SINR

$$\gamma_{N_m,m} = \rho |h_{N_m,m}|^2 P_{N_m,m}, \quad (7)$$

if it can decode the rest of the users' messages successfully. Finally, we express the achievable rate by the n -th user through a well-known lower bound for the capacity, given in [?] as

$$R_{n_m,m} = B_m \log_2 \left(1 + \frac{e}{2\pi} \gamma_{n_m,m} \right), \quad (8)$$

where B_m is the bandwidth of the VLC system.

B. THE RF SUBSYSTEM

The path loss factor of the link between the RF AP to user n_0 is denoted by $L_{n_0,0}$, while the channel coefficient is given by the complex random variable $h_{n_0,0} \sim \mathcal{CN}(0, 1)$ with zero mean and unitary variance. As such, for the RF system, the SINR of user n_0 can be expressed as

$$\gamma_{n_0,0} = \frac{(L_{n_0,0} |h_{n_0,0}|^2 P_{n_0,0})}{L_{n_0,0} |h_{n_0,0}|^2 \sum_{i=n_0+1}^{N_0} P_{i,0} + N_0 B_0}, \quad (9)$$

where B_0 is the bandwidth of the RF system and N_0 is the power spectral density of the white noise for the RF system. Thus, the achievable rate is

$$R_{n_0,0} = B_0 \log_2 (1 + \gamma_{n_0,0}). \quad (10)$$

III. USER GROUPING IN NOMA HYBRID VLC/RF NETWORKS

In this section, we discuss the proposed cooperative protocol among the users and formulate it as a hedonic coalition formation game.

A. THE USER ASSIGNMENT AS A COALITION FORMATION GAME

NOMA excels in terms of spectral efficiency by fitting a group of users together into a single orthogonal resource block. The users in that block take advantage of the power domain and SIC. However, the weakest users have to put up with interference from the strongest ones, so a fair algorithm is needed to allocate more power to the weaker users [?]. Moreover, as more users join the same resource block, the weaker users are hindered by increased interference from the stronger users. Thus, there exists a trade-off between the benefits gained from increased spectral efficiency in the network and fairness, due to the lower data rates for the weakest users, who are susceptible to the aggregated

interference. On the other hand, in order to deal with their respective interference, stronger users utilize SIC, which increases their receivers' complexity with additional users. Following the above, there is also a trade-off between spectral efficiency of the system and complexity. The problem of users grouping in NOMA networks can be described as ways of partitioning a set of users. The subsets in this case need to have no common user, i.e., the intersection between the resulting subsets is the empty set. Moreover, the union of all the resulting subsets needs to be the set of all users, forming a complete set partition. Looking for all possible combinations of possible resulting subsets, partitions, in order to find the optimal partition is a very complex task, even for only one access point. Evidently, user grouping is a problem of paramount importance with coalitional game theory being the appropriate tool [?]. More specifically, a coalition $S \subseteq \mathcal{N}$ is a group of users connected to a specific AP, and consequently belonging to a specific NOMA group. Hence, the total number of coalition is the total number of the APs.

Definition 1: A coalitional game with non-transferable utility is defined by a pair (\mathcal{N}, V) where \mathcal{N} is the set of players and V is a mapping such that for every coalition $S \subseteq \mathcal{N}$, and $V(S)$ is a closed convex subset of \mathbb{R}^S , which contains the payoff vectors that players in S can achieve.

For the proposed game, the mapping V is defined as

$$V(S) = \{\mathbf{x}(S) \in \mathbb{R}^S | x_i(S) = u_i(S), \forall i \in S\}, \quad (11)$$

where $u_i(S)$ is the utility function of user i in coalition S .

For the formulated coalitional game, we notice that the grand coalition is seldom formed due to the following two reasons:

- Only a part of the users belong in the same coverage area of the same AP.
- As the number of users in a coalition increases, the achievable rate dramatically decreases.

Therefore, a grand coalition can only be formed in highly favorable conditions and for small networks, e.g., when no user belongs in the coverage of the VLC APs. Thus, the proposed game can be classified as a *coalition formation game*. Such a game is classified as hedonic if and only if: a) the payoff of any player depends solely on the members of the coalition to which the player belongs, and b) the coalitions form as a result of the preferences of the players over their possible coalitions' set.

Definition 2: A coalitional structure or a coalition partition is defined as the set $\mathcal{S} = \{S_1, \dots, S_l\}$, which partitions the players set \mathcal{N} , i.e., $\forall k \in \{1, \dots, l\}, S_k \subseteq \mathcal{N}$ are disjoint coalitions such that $\cup_{k=1}^l S_k = \mathcal{N}$.

Definition 3: For any player $i \in \mathcal{N}$, a preference relation or order \succeq_i is defined as a complete, reflexive and transitive binary relation over the set of all coalitions that player i can possibly form, i.e., the set $\{S_k \subseteq \mathcal{N} : i \in S_k\}$.

Consequently, given two coalitions $S_1 \subseteq \mathcal{N}$ and $S_2 \subseteq \mathcal{N}$, such that a user $i \in \mathcal{N}$ can belong to either of them, i.e.,

$i \in S_1$ and $i \in S_2$, the relation $S_1 \succeq_i S_2$ implies that user i prefers coalition S_1 over S_2 based on player i 's payoff function. Furthermore, using the asymmetric counterpart of \succeq_i , denoted by \succ_i , then $S_1 \succ_i S_2$, indicates that user i strictly prefers coalition S_1 over S_2 , then also the payoff function of user i in S_1 is strictly larger than the payoff function of user i in S_2 . Given the set of players \mathcal{N} and the preference relation \succeq_i for every player $i \in \mathcal{N}$, a hedonic coalition formation game is defined by the pair (\mathcal{N}, \succ) .

To join a coalition S , a user i requires the consent of the users that are already in coalition S . The concept of consent is that, if the new coalition with user i is formed, i.e., $S \cup \{i\}$, the payoff of the rest of the users that were originally part of coalition S will not decrease. Thus, when we use the preference operator, we will imply that user i is allowed to move to a new coalition. So the preference relation can be written as

$$S_1 \succeq_i S_2 \Leftrightarrow w_i(S_1) \geq w_i(S_2), \quad (12)$$

where $S_1, S_2 \subseteq \mathcal{N}$ are any two coalitions that contain user i , i.e., $i \in S_1$ and $i \in S_2$. The payoff function w_i is defined for any $i \in \mathcal{N}$ and any coalition S such that $i \in S$ follows

$$w_i(S) = \begin{cases} u_i(S) & \text{if } (w_j(S) \geq w_j(S \setminus \{i\}), \forall j \in S \setminus \{i\}), \\ -\infty, & \text{otherwise.} \end{cases} \quad (13)$$

As such, the proposed game is modeled as a (\mathcal{N}, \succ) hedonic coalition formation game, with the preference relation \succeq_i given by (12) for any user $i \in \mathcal{N}$.

B. THE UTILITY FUNCTION

The utility function of user n that belongs to a coalition S_m is given by

$$u_n(S_m) = R_{n,m} - \kappa_{S_m}(n_m), \quad (14)$$

where $R_{n,m}$ is the achievable rate of the n -th user in coalition S_m , normalized to the bandwidth of its system, and $\kappa_{n,m}$ is the cost that user n_m pays to join said coalition. This cost is fixed for every coalition, i.e., every access point has its own cost that needs to be paid by the users connected to it so they can be served. Following that, stronger users have the incentive to let weaker users join the coalition, despite the apparent loss in power from which it would have to suffer, since, then, they can share the cost with another user.

A weaker user has less incentive to move to a coalition, purely based on rate. That happens because, the weaker users has to put up with the aggregated interference from the stronger users and also pay an extra cost. So, in order to achieve a better payoff, it would need more power from the access point. The stronger users then would have to give up that power, so the weaker user can join them. There is not interference cost the stronger users because of the use of SIC. In order to balance this, a cost/reward function should exist in the payoff function, so that it rewards the stronger users for accepting the weaker ones. However, that would lead to accumulating all users in the same coalition,

forming always the grand coalition, which is impractical for a NOMA scenario. So, weaker users need to pay a cost for the hindrance they cause to the stronger ones. As such we have a special case of the utility function:

C. A SPECIAL CASE FOR THE COST FUNCTION

Next, we examine the special case of a specific distribution of the cost to the users of a coalition. The cost can be expressed as

$$\Xi(S_m) = \sum_{n_m=1}^{N_m} \kappa(n_m, m), \quad (15)$$

and the cost that each users pays can be given in a recurring way by the following expressions:

$$\kappa(n_m, m) = \lambda^{i-1} \kappa_0, \quad (16)$$

where i is the order of user n_m in the group and κ_0 is a standard cost that can be given by (15). Finally, λ is a parameter that decides the distribution of the cost to users based on their ordering. As such, we can see the following cases:

- $0 < \lambda < 1$ Weaker users are paying more to join the coalition. This makes it easier for a strong user to accept them, since they end up taking more of the cost, and add no interference, due to SIC, to the strong users. Weaker users are more picky about the coalition they want to join, since they would need a lot of power to overcome the cost and the interference.
- $\lambda = 1$ The cost is distributed equally to all users in a coalition.
- $\lambda > 1$ Stronger users pay more in the coalition to cover the total cost. As such, weaker users join more easily. This could have a negative impact on total sum rate?

D. POWER ALLOCATION

Power allocation is critical in NOMA systems, due to the trade-off between high throughput and fairness among users. In most optimization problems with power allocation in NOMA, the authors tend to maximize a given utility function based on the whole group of users, as the sum rate or the minimum rate. In the present paper, we consider the case that each user cares to maximize their power, thus, maximizing their payoff. Following that, the need of a power allocation policy is needed, to prevent a user from accumulating all the available power. In this setting, power is allocated within the game. Each user, at their turn, gets to move through the other coalitions and examine which is the best coalition they can join, based on their payoff. To do so, they need to get consent from the other users, i.e., the users that already belong to a certain coalition check how much power they would need to keep their payoff constant, and the remaining power goes to the user whose turn it is. As such, every user tends to maximize their own benefit at each turn, while making sure not to decrease the payoff of other users, which can be guaranteed via the utility function explained in the section above. A new user joining the coalition means that

the others users would be alleviated of some portion of the cost they need to pay to stay in that coalition. This leads them to need less power to keep their payoff constant. The power that is accumulated can be offered to the new user. Of course, there are situations where that power would not be enough to get the user to join the coalition, given that an extra user would put extra strain on the weaker users who struggle with the aggregated interference. Assuming users are sorted in a descending order, the interference can be calculated for each user as:

$$I_i^{\text{RF}} = \sum_{n=1}^{i-1} |h_i|^2 p_n, \quad (17)$$

for the RF users, and

$$I_i^{\text{VLC}} = \sum_{n=1}^{i-1} h_i^2 \eta^2 p_n^2, \quad (18)$$

for the VLC users. As such, power coefficients are given by:

$$p_i^{\text{RF}} = \frac{2^{\theta_i/B_0} - 1}{|h_i|^2} \left(\frac{N_0 B_0}{P_{\max}^2} + I_i^{\text{RF}} \right), \quad (19)$$

$$p_i^{\text{VLC}} = \frac{2\pi}{e} \frac{2^{\theta_i/B_m} - 1}{\eta^2 h_i^2} \left(\frac{\sigma^2}{P_{\max}^2} + I_i^{\text{VLC}} \right), \quad (20)$$

for the RF and VLC users, respectively. Also, θ is given by $\theta_i = u_i^{\text{old}} + \kappa_i$, where u_i is the utility of user i .

Algorithm 1 Power Allocation Policy

- 1: Calculate the power coefficients \mathbf{p}
 - 2: **Init.** Sort Channel Vector h in *descending* order.
 - 3: **Step 1:** Calculate the new power coefficients of users already in the coalition
 - 4: **for each user i in h**
 - 5: Calculate new cost κ_i
 - 6: Set $\theta = \text{payoff}_i^{\text{old}} + \kappa_i$
 - 7: Calculate Interference I according to (17) or (18) respectively.
 - 8: Calculate power coefficient p_i according to (19) or (20) respectively.
 - 9: **Step 2:** Calculate the power coefficient of new user
 - 10: $p_{\text{new}} = 1 - \sum_i p_i$
 - 11: Return the power coefficient vector \mathbf{p}
-

E. A COALITION FORMATION ALGORITHM

In the aforementioned scenario, it is clear that the maximum number of coalitions is the same with the number of existing APs. So, instead of the generic merge-and-split rule, that is usually applied in this kind of games [?], we opted for the following rule that is also used in [?].

Definition 4: Switch Rule: Given a partition $\mathcal{S} = \{S_1, \dots, S_l\}$ of the user's set \mathcal{N} , user i decides to leave its current coalition S_m , for some $m \in \mathcal{M}$ and join another coalition $S_k \in \mathcal{S}$, $S_k \neq S_m$, hence forming $\mathcal{S}' = \{\mathcal{S} \setminus \{S_m, S_k\}\} \cup \{S_m \setminus \{i\}, S_k \cup \{i\}\}$, if and only if

$S_k \cup \{i\} \succ_i S_m$. Hence, $\{S_m, S_k\} \rightarrow \{S_m \setminus \{i\}, S_k \cup \{i\}\}$ and $\mathcal{S} \rightarrow \mathcal{S}'$.

The switch rule provides a mechanism for users to change coalitions, in order to find a more favorable group with which to perform NOMA. However, it is necessary for the rest of the users to give their consent to the user to join their coalition, meaning that the switch can happen only if the utility of the rest of the users does not decrease after the switch.

As such, in this algorithm we have three stages. The first part is the initialization process. Given its ubiquitous presence, the RF AP serves as the first coalition that is formed. Every user is assigned to the RF AP at first. This stage usually yields low payoff for most users, so we proceed next to the learning stage. In this stage, the coalition formation game is played between the users, in order to find the better partition of the set. By applying the switch rule, each user can change the AP to which they are assigned.

The convergence of the coalition formation algorithm is guaranteed as follows:

Theorem 1: Beginning with any initial network partition $\mathcal{S}_{\text{init}}$, the hedonic coalition formation stage of the proposed algorithm always converges to a final network partition \mathcal{S}_f composed of a number of disjoint coalitions of users assigned to specific APs.

Proof: The proof is similar to the proof of [?, Theorem 1]. ■

Definition 5: A partition \mathcal{S} is Nash-stable if $\forall i \in \mathcal{N}$ s.t. $i \in S_m, S_m \in \mathcal{S}, S_m \succeq_i S_k \cup \{i\}$ for all $S_k \in \mathcal{S}$.

Hence, partition \mathcal{S} is Nash-stable if no user has an incentive to move from its current coalition to another coalition in \mathcal{S} , or it does not have the consent of the users of the other coalition to move there.

Proposition 1: Any partition \mathcal{S}_f resulting from the coalition formation phase of the proposed algorithm is Nash-stable.

Proof: If the partition \mathcal{S}_f resulting from the proposed algorithm is not Nash-stable, then $\exists i \in \mathcal{N}$ with $i \in S_m, S_m \in \mathcal{S}_f$ and a coalition $S_k \in \mathcal{S}_f$, such that $S_k \cup \{i\} \succ_i S_m$. Hence, user i can perform the switch operation which contradicts with the fact that \mathcal{S}_f is the result of the convergence of the proposed algorithm. Thus, any partition \mathcal{S}_f resulting from the hedonic coalition formation stage is Nash-stable and the proposition is proved. ■

Following the convergence of the hedonic coalition formation stage to a Nash-stable partition, the third and last stage of the algorithm entails that the users in each group are assigned to an AP, performing NOMA. In this stage, we assume that the users of a specific coalition share all orthogonal resources, e.g., spectrum or time window, to calculate their achievable data rate.

The proposed algorithm can be implemented in a distributed manner, since as already explained, the switch operation can be performed by each user independently of any centralized entity. To perform a switch, the user needs to calculate its payoff, given the possible data rate that it can achieve in a coalition, and also obtain the rest of the

information needed, such as whether it has the consent of the users in the new coalition, and the size of said coalition through the backbone. Once the switch is identified, the user can leave its current coalition and join the new one.

Algorithm 2 Coalition Formation Algorithm

```

1: Init. Connect all users to RF AP.
2: while  $\|\mathbf{u}_{\text{old}} - \mathbf{u}_{\text{new}}\| < \epsilon$ 
3:   for each user  $i \in \mathcal{N}$ 
4:     if  $i$  is connected to RF
5:        $\text{payoff}_i^{\text{RF}} = u_i$ 
6:        $\text{payoff}_i^{\text{VLC}} = \text{checkVLC}$ 
7:       Choose the VLC AP  $m$  that gives the best payoff.
8:     else
9:       Find the VLC AP  $m$  to which user  $i$  is connected.
10:       $\text{payoff}_i^{\text{RF}} = \text{checkRF}$ 
11:       $\text{payoff}_i^{\text{VLC}_m} = u_i$ 
12:       $\text{payoff}_i^{\text{VLC}_{m'}} = \text{checkVLC}$  for  $m' \neq m$ .
13:    end if
14:    Move user  $i$  to the AP that provides the maximum
      payoff.
15:     $\mathbf{u}_{\text{old}} = \mathbf{u}$ 
16:    Update all users' utility.
17:     $\mathbf{u}_{\text{new}} = \mathbf{u}$ 
18:  end for
19: end while

```

IV. SIMULATIONS AND DISCUSSION

In this section, we present the simulation results of the proposed algorithm in the previous section. We set up a network of M VLC APs, and 1 RF AP in the center of the room. The room dimensions are $10 \times 10 \times 4$ m³ and the users have random locations, according to a uniform distribution. For the sake of simplicity, each optical receiver is considered to be facing towards the ceiling of the room. Simulations were performed for $M = 2$. The position of VLC APs in each scenario needs to reflect the practical position of lamps in a room, although the RF AP is positioned at the center of the room. As such, the VLC APs' position can be described by $(\pm x_0, 0)$. In each case, $x_0 = y_0 = 2.5$ m.

The parameters used in the simulations are given in the following table. Also, the following path loss model is used [?].

$$L_{n_0,0}(d_{n_0,0}) = L(d_0) + 10\kappa \log_{10}(d_{n_0,0}/d_0), \quad (21)$$

where $L(d_0) = 68$ dB is the reference path loss at a reference distance, $d_0 = 1$ m, and $\kappa = 1.6$ is the path loss exponent.

TABLE 1. Parameters in simulations.

P_{max}	9 W	ρ	1.5
η	0.53 A/W	Ψ_{FOV}	$\pi/2$
σ^2	5×10^{-22} A ²	$\Phi_{1/2}$	$\pi/3$
B_m	40 MHz	p_{max}	1 W
L_r	1 cm ²	B_0	20 MHz
$T_s(\psi)$	1	N_0	4.002×10^{-21} A ² /W

A. AN ILLUSTRATIVE EXAMPLE

In order to present clearly the proposed algorithm, next, we describe an illustrative example. In this case, we have a total of five users in the hybrid VLC/RF network, which consists of two VLC APs and one RF AP. The channels of the users are given in Table 2.

TABLE 2. Channel conditions in the example

Users	$L_0 h_0 ^2 (\times 10^{-6})$	$h_1 (\times 10^{-5})$	$h_2 (\times 10^{-5})$
1	0.1086	0.0176	0.3314
2	0.0393	0	0.2437
3	0.0149	0	0.1152
4	0.0166	0.0046	0
5	0.0152	0	0

The proposed algorithm converges to the following grouping:

- Users 1 and 3 are served by the VLC AP 2. As we can observe, User 1 is the strongest user of that AP and it has been paired with the weakest (non-zero), in terms of channel gain user of that AP.
- User 2, despite experiencing good channel conditions at the VLC AP 2, chooses to remain connected to the RF AP. That is understandable given that User 3 experiences the best conditions at the RF AP.
- User 4 is served by the VLC AP 1. It is the only user that can be served by this AP since, the rest of the users' field-of-view (FoV) is not wide enough to be able to connect to that AP.
- Finally, Users 2 and 5 form a pair and are served by the RF AP as a strong user and a weak user. As it has been stated in [?], the gain of NOMA is greater when the users' channels differ.

The final grouping offers increased data rate for users connected to the VLC, while it salvages the rate of the weaker users which would congest due to increased interference.

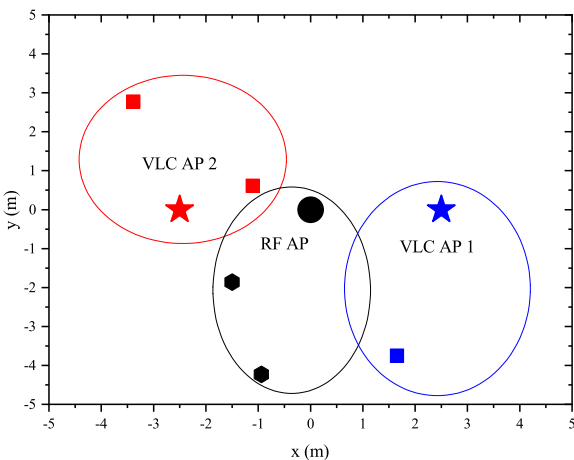


FIGURE 2. Formed Coalitions in an illustrative example

B. MONTE CARLO SIMULATIONS

In order to evaluate the system's performance, we validate the proposed method through Monte Carlo simulations for

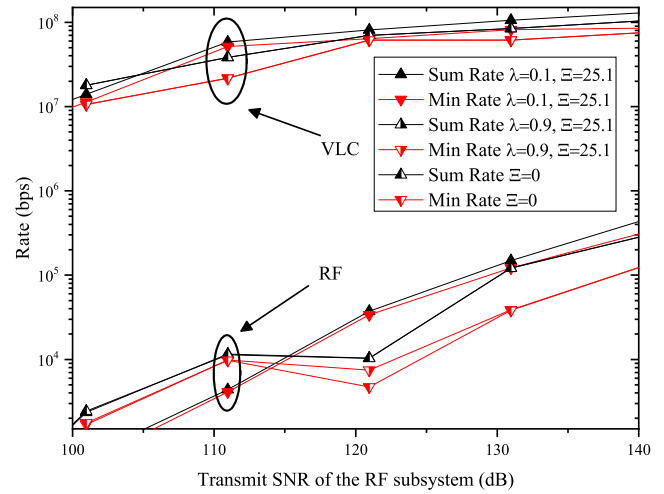


FIGURE 3. Rate vs Transmit SNR RF ($P_{max}^{VLC} = 9P_{max}^{RF}$)

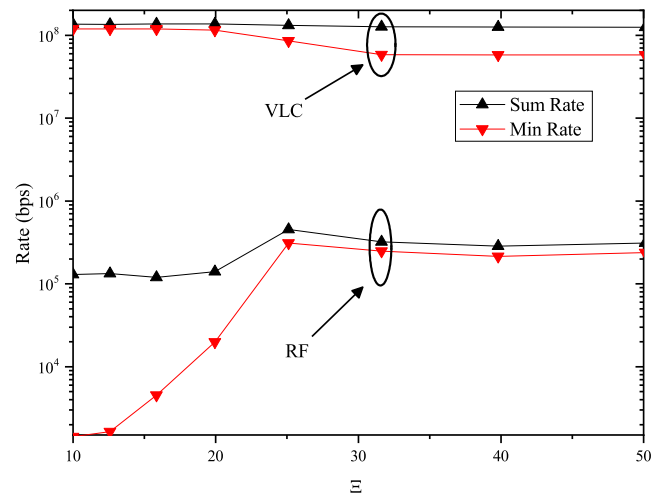


FIGURE 4. Data Rate vs Ξ for $\lambda = 0.1$

various channel realizations and users' positions. Some conclusions that can be drawn are:

- Some users that are served by the RF AP cannot connect to VLC, since, due to their FoV, they get very bad channel conditions.
- Some users prefer to stay connected to the RF AP, because they experience great channel conditions.

The selection of the value of cost Ξ is significant as it quantifies the gain and loss of cooperation between the players. The total cost that needs to be shared between the players in a coalition needs to be adjusted for their expected data rate in order to have an impact on the game. The value of Ξ needs to be in the same order of magnitude as the rate in the utility function. Otherwise, it won't play a significant role in the game. For example, a user that is served by a VLC AP needs a motive to let another user to join the coalition. Otherwise, the first user will not give its *consent* to another user to join. So, as Ξ gets similar to the achieved data rate

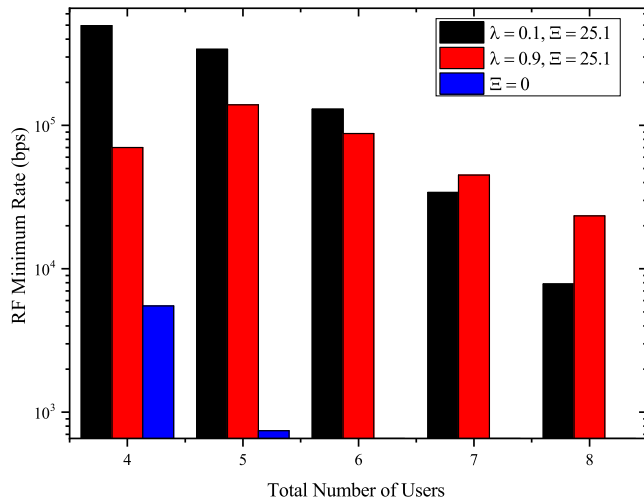


FIGURE 5. Minimum Rate of RF vs Total Number of Users

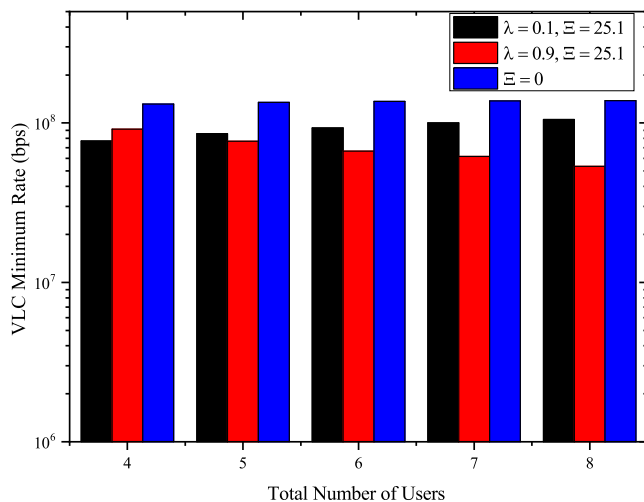


FIGURE 6. Minimum Rate of VLC vs Total Number of Users

of users, coalitions of more than one player are formed in the VLC APs, decongesting the RF AP and increasing the system performance. On the other hand, if Ξ is greater than the data rates, the data rate of the users are disregarded in the coalition forming process and the system performance drops. Finally, choosing $\Xi = 0$ leads to an opportunistic scheme where each user ignores the social welfare and only searches to maximize their own rate. This opportunistic scheme is used as a benchmark.

In Fig. 3, the sum and minimum rates are plotted for different values of the RF subsystem transmit SNR. The maximum power of VLC is given by $P_{\max}^{\text{VLC}} = 9p_{\max}^{\text{RF}}$ and the VLC bandwidth is given by $B_m = 2B_0$, $m \neq 0$. Moreover, we compare the results of different values of the parameters that appear in the game, Ξ and λ . The value of $\lambda = 0.1$ signifies that the cost of a coalition falls mainly on the weaker users. As such, it is easier for stronger users to

accept them. It can be observed that there is an increasing trend with the SNR for both the sum rate and minimum rate of both systems in the hybrid network. Furthermore, the same rates are presented for $\lambda = 0.9$, which means that the cost may fall more on the weaker users, but the price is generally similar. In this scenario, weaker users have more incentive to move to a coalition, since they would not pay as much to join, but the stronger users have less incentive to let them. It can be observed that for lower values of transmit SNR this method offers better rates for the RF users. This is not the case for the VLC rate, however. For higher values of transmit SNR this method also falls behind with the RF users as well. For both of those schemes, we assumed a cost of $\Xi = 25.1$, which is shown to be a good choice for user fairness according to Fig. 4. Finally, in Fig. 3, we present an opportunistic scheme, where $\Xi = 0$. This scheme generally performs worse in any case than our proposed analysis. While, the RF achievable rates are similar, the respective VLC ones are lower than the rest.

In Fig. 4, the effect of the value of Ξ can be observed in the rate of the system. For low values of Ξ , the minimum rate of RF is very low, while the sum rate and minimum rate of VLC are similar. This happens because there no groups forming the VLC APs; each VLC AP served one user. The value of Ξ is not high enough so the strong user lets another user in its coalition. However, when Ξ gets higher, the minimum rate of RF increases significantly, while the VLC rate drops a little, suggesting that coalitions of more than one user are forming in the VLC APs, decongesting the RF system. Finally, as Ξ gets even higher, the coalition forming driving factor is mainly Ξ , since it is greater than the spectrum efficiency of each user, thus playing the bigger role in choosing coalitions. So, very high values of Ξ effectively remove the influence of rate on the users' utility function and the formed coalitions end up with less achievable system throughput.

Moreover, in Fig. 5 and Fig. 6, we examine the minimum rate of the RF system and the VLC system, respectively, for different number of users in the system. The simulation parameters are given by Table 1. For low number of users, the superiority of the method with $\lambda = 0.1$ is obvious in terms of RF minimum rate. As new users are added in the system, though, the method with $\lambda = 0.9$ gets an advantage over the other. Finally, it can be seen that the opportunistic scheme has a detrimental effect on minimum rate, especially as new users are added in the system. In the case of VLC, for the most part, method with $\lambda = 0.1$ outperforms $\lambda = 0.9$, but the achievable rates do not diverge a lot. However, in the case of no cost, it can be seen that the minimum rate is higher. This happens because, there are usually no more than one user in each formed VLC coalition. So this value, while technically is the minimum rate of the VLC system, does not offer any information about the fairness in the VLC system, rather it can be considered a benchmark. Note that the minimum rate of the proposed method is not much lower than the essentially maximum rate of the system, so user fairness is guaranteed in the proposed schemes.

V. CONCLUSION

In this paper, we have investigated a hybrid VLC/RF network with multiple VLC APs and one RF AP, with the assumption that each AP performs NOMA. We have studied the optimal user grouping problem through coalitional game theory. A novel utility function was proposed, which takes into account the peculiarities of the NOMA system and the non-cooperative nature of most users. A special case of the cost function has been investigated, where the cost for each user is calculated based on the ordering of the NOMA group. Simulations are provided for various values of the parameters encountered in the proposed approach to show the versatility to a number of different scenarios. The proposed algorithm clearly outperforms the standard opportunistic (non-cooperative) scheme, while the simulations have also illustrated the effectiveness and robustness of the proposed method with respect to the number of users in the network.



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