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Spectrum Allocation and Power Control in Full-Duplex Ultra-Dense Heterogeneous Networks

Guobin Zhang, Haijun Zhang, *Senior Member, IEEE*, Zhu Han, *Fellow, IEEE*, and George K. Karagiannidis, *Fellow, IEEE*

***Abstract*—Full duplex ultra-dense network (FDUDN) has been envisioned as a promising network paradigm for spectrum efficiency enhancement. This paper presents a novel joint spectrum and power management scheme, which maximizes the total capacity of FDUDN, under given Quality-of-Service (QoS) and cross-tier interference constraints. The proposed scheme is decomposed into inter-cell and intra-cell allocation. A novel approach, which performs initial capacity-maximization allocation and successive adjustment based on the constraints, is proposed for the allocation of the inter-cell subchannels. The inter-cell power control is formulated as a non-convex optimization problem and variable substitution is used to transform it into a convex one. Furthermore, we solve this problem through a low-complexity heuristic scheme, which utilizes the water-filling theorem in inter-cell power allocation. Finally, a time-sharing relaxation method is adopted to solve the intra-cell subchannel allocation problem and reduce the computation complexity. Computer simulation demonstrates the enhancement effect of the proposed scheme in terms of the capacity, spectrum efficiency and power efficiency.**

***Index Terms*—Ultra dense networks, full duplex, spectrum allocation, power control, interference mitigation.**

I. Introduction

THE fifth generation (5G) mobile communication network is supposed to be a heterogeneous network with various access technologies [1]. 5G network topology includes macro base stations (BSs), which are responsible for basic coverage, and low-power BSs, which undertake hot spot or indoor coverage. As expected, in the 5G systems the density of various kind of low-power nodes will be 10 times higher than those of 4G system, which makes ultra-dense networks (UDNs) to be as the main paradigm [2], [3]. Due to the advantage of wireless coverage and spectrum efficiency enhancement, UDNs are considered as a key technology of the 5G system to cope with the rapidly increasing demand of data transmissions [4], [5].

Meantime, the full duplex (FD) technology, which allows a device to simultaneously transmit and receive in the same spectrum, attracts wide attentions of both academia and industry in the 5G area research [6], [7]. Due to the high potential of theoretically doubling the spectrum efficiency and realizing flexible spectrum usage, the FD technology has been recognized as a promising solution to tap wireless spectrum resources adequately [8], [9].

Recently, researchers proposed to construct UDN with FD BSs and showed that they can achieve significant increases in user rate and network throughput [10]–[12]. The new paradigm has several main advantages: *i*) UDNs typically provide low transmit power, hence the self-interference in FD nodes can be easily mitigated into a sufficiently low level; *ii*) As FD reuses the same spectrum in uplink and downlink, the complexity of the spectrum management in UDN are efficiently reduced; *iii*) FD-UDN system can obtain the dual performance gains of FD and UDN technologies.

In spite of the potential enhancement in system performance, FD-UDN poses a great challenge in dealing with the severe and complex interference. It is well known that in heterogeneous UDNs, there exists severe interference between networks, due to the irregular construction of a large number of small cells. Additionally, when the FD technology is adopted in UDNs, there will exist residual self-interference, even if the most is eliminated by advanced cancellation techniques [13]. Therefore, in FD-UDN, the interference environment will be complicated, and wireless resource management becomes very important in order to reduce the data loss and guarantee the service requirements.

A. Literature

Resource management of heterogeneous networks has been a hot spot of research in recent years. In [14], a two-tier network with macrocell and femtocell was constructed and an optimal decentralized spectrum allocation policy was proposed. The authors in [15] formulated a flow-based framework for joint optimization of resource allocation in a heterogeneous network, which is consisted of a macro cell and several pico cells. In [16], the authors studied the energy-efficient resource allocation problems in heterogeneous cognitive radio networks with femtocells. Furthermore, in [17] and [18], resource

Guobin Zhang is with School of Electrical Engineering and Intelligentization, Dongguan University of Technology, 523808, China (e-mail: guobinzh@163.com).

Haijun Zhang is with Beijing Advanced Innovation Center for Materials Genome Engineering, Beijing Engineering and Technology Research Center for Convergence Networks and Ubiquitous Services, Institute of Artificial Intelligence, University of Science and Technology Beijing, Beijing, 100083, China (e-mail: haijunzhang@ieee.org).

Zhu Han is with the University of Houston, Houston, TX 77004 USA (e-mail: zhan2@uh.edu), and also with the Department of Computer Science and Engineering, Kyung Hee University, Seoul, South Korea, 446-701.

George K. Karagiannidis is with Aristotle University of Thessaloniki, Thessaloniki , Greece (e-mail: geokarag@auth.gr).

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allocation schemes were proposed for orthogonal frequency-division multiple access (OFDMA) based cognitive femtocells, in order to maximize the total capacity of all femtocell users under a given Quality-of-Service (QoS) and co-tier/cross-tier interference constraints. In [19], the joint transmission time and power allocation problems were studied in a multiple input multiple output (MIMO) cognitive network, in order to maximize the total capacity of the secondary users under a fairness constraint. Finally, Stackelberg game and cooperative bargaining game are used in [20] and [21], respectively, to deal with the resource allocation in small cell networks.

In the aforementioned works, there exist two main problems: *i*) The co-tier interference between small cells is not fully considered in the interference model or not treated well in the process of resource management [13]-[18]. Specifically, when the co-tier interference is introduced in the interference model, the optimization problem becomes non-convex and needs to be solved by unusual approach. *ii*) The spectrum and power allocation were directly performed between all the links of the small cells, without layering [14]-[18]. This scheme is efficient for the networks with small number of cells. However, when the number of cells increases, the scheme results in extremely large computation complexity, which consequently makes the allocation schemes hard to be realized in practice.

Recently, significant efforts were devoted to UDN resource management. The authors in [22] proposed a distributed scheduling with interference-aware power control for uplink of UDN operating with time-division duplex. In [23], a joint clustering and inter-cell resource allocation was proposed, based on game theory and graph-coloring algorithms, respectively. In [24], the authors studied the power control problem in UDNs by proposing a novel game with dynamic pricing in order to obtain the maximal sum-rate of all small cells. The authors in [25] proposed an approach for joint power control and user scheduling to optimize energy efficiency, by formulating the problem as a dynamic stochastic game. A Nash cooperative game framework is built in [26] to maximize the energy efficiency with the sub-optimal spectrum efficiency equilibria.

Meanwhile, with the deployment of the FD technology including the self-interference cancellation mechanism [27], resource management of a FD network achieves particular attention, in the context of time scheduling, channel allocation and power control. The authors in [28] proposed a resource allocation scheme to adjust the transmit power, which maximizes the downlink channel capacity, while simultaneously guaranteeing the uplink channel QoS. In [29], time slot scheduling and power allocation scheme of single FD cell were proposed, while in [30], the authors studied the pairing choice and channel allocation of uplink and downlink users in a FD system with multiple micro cells. In [31], subchannel and power allocation in FD relay cellular networks was proposed, to maximize energy efficiency. Moreover, the tradeoff between energy efficiency and spectral efficiency for FD cellular networks is addressed in [32]. In most of the above works, the research on resource management was mainly focused on a single cell or the network with a small number of users [28], [29], and the solutions for complex FD networks have not been fully investigated. Especially the interference model for a complex FD network has not been constructed well, without the fully consideration of the inter-cell, intra-cell, cross-tier interference, etc [31]. Furthermore, as FD-UDN is a newly proposed paradigm [11], resource management for FD-UDN is not fully investigated and there has no efficient scheme of spectrum and power allocation proposed yet in the open literature.

B. Contributions

As UDN and FD are promising technologies for the 5G networks, the joint development of them can bring further enhancement of system performance [11], [12]. In this paper, we investigate the resource management problem of this new type of network, and explore efficient schemes of subchannel and power allocation in order to obtain capacity maximization and spectrum efficiency enhancement.

The main contributions of this paper can be summarized as follow:

- We present a new interference model for FD-UDN, taking into consideration the inter-cell interference, self-interference, and cross-tier interference. Then, the optimization problem of subchannel allocation and power control is formulated and solved in order to maximize the sum-rate of the small cells.
- As the direct and unified allocation for the links of small cells will result in large complexity in FD-UDN, we decompose the resource allocation into inter-cell and intra-cell, while the inter-cell allocation is further decomposed into subchannel allocation and power control, and the subsequent intra-cell allocation performs the subchannel allocation between links. Additionally, the average channel gains are used in the inter-cell interference calculation to obtain an approximate result and reduce the complexity.
- In order to solve the non-convex problem of inter-cell power allocation, we transform it into a convex one and solve it through the Lagrangian dual decomposition. Moreover, we apply the water-filling theorem in the inter-cell power allocation and propose a heuristic scheme to reduce computation complexity. Additionally, a time-sharing relaxation based algorithm is proposed to solve the intra-cell subchannel allocation problem with much lower complexity.

C. Structure

The rest of the paper is organized as follows. In Section II, we construct the network model, provide the capacity computation and formulate the optimization problem to maximize the sum rates of uplink and downlink of all the small cells. In Sections III and IV, the inter-cell sub-channel and power allocation and the intra-cell sub-channel allocation schemes are given, respectively, which include the mathematical solution of the optimization problem and the detailed algorithm. The algorithm complexity is analyzed in Section V. The simulations for analysis and validation are given in Section VI, followed by the conclusion in Section VII.

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II. System Model And Problem Formulation

A. System Model

We consider a two-tier heterogeneous OFDMA network [24], [25], as shown in Fig. 1. The macro cell is the first tier network, and multiple small cells (i.e. pico-, micro-, femtocells) constitute the second tier network. The small cells are densely and uniformly distributed in the macro cell, and use licensed spectrum under the restraint of the interference to macro cell.

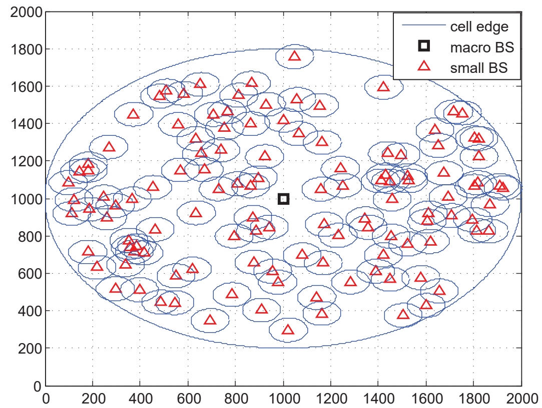

Fig. 1. System model of an UDN consisting of one macro cell and multiple small cells.

A block fading channel is considered and the channel fading is supposed to be composed of path loss, shadowing fading, and frequency selective Rayleigh fading. The path loss h_p is constructed as $h_p \propto \left(\frac{d}{d_0}\right)^n$, where d is the transmission distance, d_0 is the referred distance and n is the pass loss exponent. The shadowing fading h_s obeys lognormal distribution with the probability function of $P(h_s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(h_s-a)^2}{2\sigma^2}\right)$, where a is logarithm mean and σ^2 is the standard deviation. The probability distribution of the Rayleigh fading h_f is expressed as $P(h_f) = \frac{h_f}{\sigma^2} \exp\left(-\frac{h_f^2}{2\sigma^2}\right)$ with uniform variance σ^2. Assume that the UDN system has a bandwidth of B, which is divided into N subchannels. The channel fading remains constant within a subchannel, but varies cross different subchannels. The number of small cells is M. The small cells share all of the subchannels and each subchannel can be reused by multiple small cells. Each small cell has multiple communication links. One user is associated to one BS and composes a link, including both uplink and downlink, with the BS [10], [33]. One link can use multiple subchannels simultaneously. Both the BSs and users are configured to communicate by FD mode.

The signal transmission of the FD-UDN is subjected to inter-cell interference, self-interference introduced by the FD mode, interference from macro-cell due to the subchannel sharing and the additive white Gaussian noise (AWGN). Here we consider inter-cell interference between all small cells which use the same subchannel, which is in better accordance with the actual scenario than only part of inter-cell interference are considered. Also with such interference calculation, the effectiveness of the subchannel allocation, which means if the subchannels are reused in the cells with large mutual interference or if the reuse is excessive to introduce severe inter-cell interference, can be verified. The inter-cell interference includes two parts, the first originates from the BSs and the other from the users.

We use link l to indicate the link of a user with BS, which includes both uplink and downlink. Then the uplink and downlink signal to interference plus noise ratios (SINRs) of link l in cell m in subchannel n can be formulated, respectively, as (1) and (2), where $p_{m,n,l}^{\mathrm{up}}$ and $p_{m,n,l}^{\mathrm{dw}}$ denote the uplink and downlink power of link l in a small cell m on subchannel n, respectively. $h_{m,n,l}$ indicates the channel gain of link l in small cell m on subchannel n. Considering the same inter-cell channel gain in uplink and downlink, we use $h_{m',m,n}$ to denote the channel gain between cell m' and m in subchannel n. The $h_{m',m,n}$ includes not only short term fading but shadowing fading. The self-interference gain of link l of cell m in subchannel n is denoted by $g_{m,n,l}$ [10]. σ^2 denotes the AWGN power. L_m is used to denote the number of links of cell m. $I_{m,n}$ indicates the interference of the macro cell to cell m on subchannel n.

The uplink and downlink rate of link l in cell m in subchannel n can be written, respectively, as

$$R_{m,n,l}^{\mathrm{up}} = \log_2(1 + SINR_{m,n,l}^{\mathrm{up}}), \tag{3}$$

$$R_{m,n,l}^{\mathrm{dw}} = \log_2(1 + SINR_{m,n,l}^{\mathrm{dw}}). \tag{4}$$

Therefore, the total network capacity is

$$C = \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{l=1}^{L_m} \left(R_{m,n,l}^{\mathrm{up}} + R_{m,n,l}^{\mathrm{dw}}\right). \tag{5}$$

$$SINR_{m,n,l}^{\mathrm{up}} = \frac{p_{m,n,l}^{\mathrm{up}} h_{m,n,l}}{\underbrace{\sum_{m' \neq m} \sum_{l=1}^{L_{m'}} \left(p_{m',n,l'}^{\mathrm{up}} + p_{m',n,l'}^{\mathrm{dw}}\right) h_{m',m,n}}_{Inter-cell \quad Interference} + \underbrace{p_{m,n,l}^{\mathrm{dw}} g_{m,n,l}}_{Self-Interference} + \sigma^2 + I_{m,n}}, \tag{1}$$

$$SINR_{m,n,l}^{\mathrm{dw}} = \frac{p_{m,n,l}^{\mathrm{dw}} h_{m,n,l}}{\underbrace{\sum_{m' \neq m} \sum_{l=1}^{L_{m'}} \left(p_{m',n,l'}^{\mathrm{up}} + p_{m',n,l'}^{\mathrm{dw}}\right) h_{m',m,n}}_{Inter-cell \quad Interference} + \underbrace{p_{m,n,l}^{\mathrm{up}} g_{m,n,l}}_{Self-Interference} + \sigma^2 + I_{m,n}}, \tag{2}$$

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B. Problem Formulation

In this paper, we aim to maximize the network capacity under the constraints of the uplink and downlink transmit power, the interference limit to macro cell and capacity requirement of each small cell. To determine the subchannel allocation between the cells and links, we introduce $a_{m,n,l}$ as the subchannel allocation indicator, $a_{m,n,l} = 1$ denotes the subchannel n is allocated to link l of cell m, and $a_{m,n,l} = 0$ otherwise. Let $\mathcal{M} = \{1, 2, ..., M\}$, $\mathcal{N} = \{1, 2, ..., N\}$, $\mathcal{L}_m = \{1, 2, ..., L_m\}$. Define $\mathbf{A} = [a_{m,n,l}]_{m\in\mathcal{M},n\in\mathcal{N},l\in\mathcal{L}_m}$, $\mathbf{P} = \left[p^{\mathrm{up}}_{m,n,l}, p^{\mathrm{dw}}_{m,n,l}\right]_{m\in\mathcal{M},n\in\mathcal{N},l\in\mathcal{L}_m}$. The optimization problem can be formulated as

$$\max_{\mathbf{A},\mathbf{P}} \quad C = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{l=1}^{L_m} a_{m,n,l}\left(R^{\mathrm{up}}_{m,n,l} + R^{\mathrm{dw}}_{m,n,l}\right) \tag{6}$$

$$\begin{aligned}
\text{s.t.} \quad & C1 : 0 < \sum_{n=1}^{N} p^{\mathrm{up}}_{m,n,l} \le P^{\mathrm{up}}_{m,l}, \forall m, l,\\
& C2 : 0 < \sum_{n=1}^{N}\sum_{l=1}^{L_m} p^{\mathrm{dw}}_{m,n,l} \le P^{\mathrm{dw}}_{m}, \forall m,\\
& C3 : \sum_{m=1}^{M}\sum_{l=1}^{L_m} p_{m,n,l} h^{\mathrm{MB}}_{m,n} \le I^{\mathrm{th}}_{n}, \forall n,\\
& C4 : \sum_{n=1}^{N}\sum_{l=1}^{L_m} a_{m,n,l} R^{\mathrm{up}}_{m,n,l} \ge \varepsilon^{\mathrm{up}}_{m}, \forall m,\\
& C5 : \sum_{n=1}^{N}\sum_{l=1}^{L_m} a_{m,n,l} R^{\mathrm{dw}}_{m,n,l} \ge \varepsilon^{\mathrm{dw}}_{m}, \forall m,\\
& C6 : \sum_{l=1}^{L_m} a_{m,n,l} \le 1, \forall m, n; C7 : \sum_{n=1}^{N} a_{m,n,l} \ge 1, \forall m, l,\\
& C8 : a_{m,n,l} \in \{0, 1\}, \forall m, n, l,
\end{aligned}$$

where $h^{\mathrm{MB}}_{m,n}$ indicates the channel gain from cell m to the macro BS on subchannel n. $\varepsilon^{\mathrm{up}}_{m}$ and $\varepsilon^{\mathrm{dw}}_{m}$ denote the uplink and downlink rate requirements of cell m, respectively. In (6), $C1$ and $C2$ denote that the uplink and downlink powers must be smaller than the maximal transmit power, $C3$ means that the total interference to macro cell must be smaller than maximal tolerance limitation, $C4$ and $C5$ denote that the uplink and downlink capacities of each cell must be larger than the minimal capacity requirements, respectively, $C6$ means that each subchannel can be used by one link at most in the same cell, $C7$ means that at least one subchannel is allocated to one link, and $C8$ indicates that the value of $a_{m,n,l}$ is 0 or 1. For the optimization function of (6), there are variables to be optimized in the denominator, therefore, (6) is a non-convex problem with the NP-hard nature. Moreover, due to the existence of the integer variables, (6) is a mixed-integer problem, and it is difficult to find the global optimal solution. Hence we aim to find the near optimal solution to obtain the performance enhancement over the existing algorithm. We adopt a tier-separate and variable-separate based approach to solve (6). The architecture of the decomposition of problem (6) is shown in Fig. 2. The inter-cell allocation is performed by the central management in which the macro BS roles as the central node. The intra-cell allocation is carried out by each small BS. The solution procedures are described in Sections III and IV in detail.

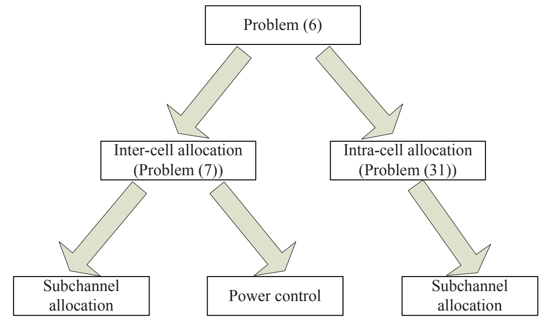

Fig. 2. Architecture of the decomposition of problem (6).

III. Inter-Cell Allocation

In the inter-cell allocation, we perform the subchannel allocation and power control between all the small cells. First, we propose a heuristic subchannel allocation algorithm to achieve near optimal solution. Then we solve the non-convex power control problem by using variable substitution and Lagrange dual method, and furthermore, we propose a heuristic algorithm based on waterfilling allocation and direct adjustment to reduce the computation complexity.

In this stage, we discard the exact channel gain of each link and use the statistical channel gains in each small cell for allocation. The statistical channel gain indicates the mean of the channel gains of all links in each subchannel and in each small cell. We use $R^{\mathrm{up}}_{m,n}$ and $R^{\mathrm{dw}}_{m,n}$ to indicate the uplink and downlink rate of subchannel n in cell m, which can be written, respectively, as

$$R^{\mathrm{up}}_{m,n} = \log_2\left(1 + \frac{p^{\mathrm{up}}_{m,n} h_{m,n}}{\sum\limits_{m'\neq m}(p^{\mathrm{up}}_{m',n} + p^{\mathrm{dw}}_{m',n}) h_{m',m,n} + p^{\mathrm{dw}}_{m,n} g_{m,n} + \sigma^2 + I_{m,n}}\right), \tag{7}$$

$$R^{\mathrm{dw}}_{m,n} = \log_2\left(1 + \frac{p^{\mathrm{dw}}_{m,n} h_{m,n}}{\sum\limits_{m'\neq m}(p^{\mathrm{up}}_{m',n} + p^{\mathrm{dw}}_{m',n}) h_{m',m,n} + p^{\mathrm{up}}_{m,n} g_{m,n} + \sigma^2 + I_{m,n}}\right). \tag{8}$$

In (7) and (8), we use $p^{\mathrm{up}}_{m,n}$ and $p^{\mathrm{dw}}_{m,n}$ to substitute $p^{\mathrm{up}}_{m,n,l}$ and $p^{\mathrm{dw}}_{m,n,l}$, respectively, as we do not take into account the allocation between links in this stage. We use $h_{m,n}$, which indicates the statistical channel information of channel n in cell m, to substitute $h_{m,n,l}$. Additionally, we use the statistical self-interference gain $g_{m,n}$ to substitute $g_{m,n,l}$. To reduce the data overhead, we only transmit the statistical channel gains from small BSs to macro BS. Moreover, in order to reduce the complexity, we only choose part links to compute the statistical inter-cell interference due to the short coverage of the small BS.

Then, we transform (6) into (9) to obtain the optimization solution for inter-cell allocation.

$$\max_{a_{m,n}, p^{\mathrm{up}}_{m,n}, p^{\mathrm{dw}}_{m,n}} \quad C = \sum_{m=1}^{M}\sum_{n=1}^{N} a_{m,n}\left(R^{\mathrm{up}}_{m,n} + R^{\mathrm{dw}}_{m,n}\right) \tag{9}$$

$$\text{s.t.} \quad C1: 0 \le p_{m,n}^{\mathrm{up}} \le \frac{\sum_{l=1}^{L_m} P_{m,l}^{\mathrm{up}}}{\sum_{n=1}^{N} a_{m,n}}, \forall m, n,$$

$$C2: 0 < \sum_{n=1}^{N} p_{m,n}^{\mathrm{dw}} \le P_m^{\mathrm{dw}}, \forall m,$$

$$C3: \sum_{m=1}^{M} \left(p_{m,n}^{\mathrm{up}} + p_{m,n}^{\mathrm{dw}}\right) h_{m,n}^{\mathrm{MB}} \le I_n^{th}, \forall n,$$

$$C4: \sum_{n=1}^{N} a_{m,n} R_{m,n}^{\mathrm{up}} \ge \varepsilon_m^{\mathrm{up}}, \forall m,$$

$$C5: \sum_{n=1}^{N} a_{m,n} R_{m,n}^{\mathrm{dw}} \ge \varepsilon_m^{\mathrm{dw}}, \forall m,$$

$$C6: a_{m,n} \in \{0, 1\}, \forall m, n.$$

As each subchannel can only be used by one link in each small cell as mentioned in Section II, we compute the total capacity of each cell by the sum of each subchannel rate of the cell, as shown in (9). We use $a_{m,n}$ to substitute $a_{m,n,l}$ to denote if subchannel n is allocated to cell m. Moreover, we set the maximal limitation of $p_{m,n}^{\mathrm{up}}$ as the average value of the total uplink power of cell m in each subchannel, as indicated in constraint $C1$.

The constrained optimization problem in (9) is still a non-convex problem. To obtain the feasible solution and reduce the complexity, we decompose the problem into two sub-problems, one of which is subchannel allocation and the other is power control.

A. Subchannel Allocation

In this stage, we allocate the subchannels between small cells based on the channel gains in each cell and interference gains between different cells. The uplink power and downlink power are determined by equal division of the maximal total power in all the subchannels of the cell.

As each power value is assumed to be a non-zero value before subchannel allocation, in order to compute the inter-cell interference correctly, we add the subchannel allocation variable in the expression of inter-cell interference, which indicates that the interference between m and $m^{'}$ in subchannel n exists only when the subchannel n is used by cells m and $m^{'}$ simultaneously. Thus the uplink and downlink rate of cell m on subchannel n is calculated, respectively, as

$$R_{m,n}^{\tilde{\mathrm{up}}} = \log_2\left(1 + \frac{p_{m,n}^{\tilde{\mathrm{up}}} h_{m,n}}{\sum\limits_{m' \ne m} a_{m',n}\left(p_{m',n}^{\tilde{\mathrm{up}}} + p_{m',n}^{\tilde{\mathrm{dw}}}\right) h_{m',m,n} + p_{m,n}^{\tilde{\mathrm{dw}}} g_{m,n} + \sigma^2 + I_{m,n}}\right), \tag{10}$$

$$R_{m,n}^{\tilde{\mathrm{dw}}} = \log_2\left(1 + \frac{p_{m,n}^{\tilde{\mathrm{dw}}} h_{m,n}}{\sum\limits_{m' \ne m} a_{m',n}\left(p_{m',n}^{\tilde{\mathrm{up}}} + p_{m',n}^{\tilde{\mathrm{dw}}}\right) h_{m',m,n} + p_{m,n}^{\tilde{\mathrm{up}}} g_{m,n} + \sigma^2 + I_{m,n}}\right), \tag{11}$$

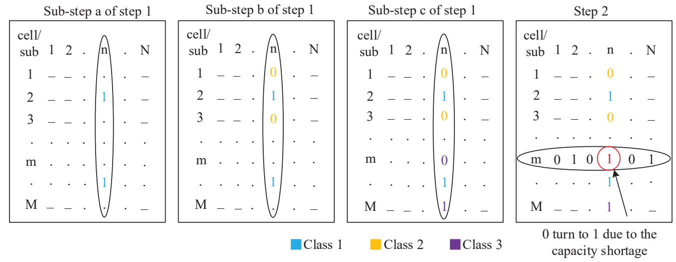

Fig. 3. Inter-cell subchannel allocation, divided into the allocation of each subchannel one by one and the adjustment of subchannel between cells.

where $p_{m,n}^{\tilde{\mathrm{up}}}$ and $p_{m,n}^{\tilde{\mathrm{dw}}}$ satisfy $p_{m,n}^{\tilde{\mathrm{up}}} = \frac{\sum_{l=1}^{L_m} P_{m,l}^{\mathrm{up}}}{\sum_{n=1}^{N} a_{m,n}}$ and $p_{m,n}^{\tilde{\mathrm{dw}}} = \frac{\sum_{l=1}^{L_m} P_{m,l}^{\mathrm{dw}}}{\sum_{n=1}^{N} a_{m,n}}$, respectively.

Then the subchannel allocation problem decomposed from problem (9) can be formulated as (12),

$$\max_{a_{m,n}} \quad C = \sum_{m=1}^{M} \sum_{n=1}^{N} a_{m,n} \left(R_{m,n}^{\tilde{\mathrm{up}}} + R_{m,n}^{\tilde{\mathrm{dw}}}\right) \tag{12}$$

$$\text{s.t.} \quad C1: \sum_{n=1}^{N} a_{m,n} R_{m,n}^{\tilde{\mathrm{up}}} \ge \varepsilon_m^{\mathrm{up}}, \forall m,$$

$$C2: \sum_{n=1}^{N} a_{m,n} R_{m,n}^{\tilde{\mathrm{dw}}} \ge \varepsilon_m^{\mathrm{dw}}, \forall m,$$

$$C3: a_{m,n} \in \{0, 1\}, \forall m, n,$$

where the constraints $C1$, $C2$ and $C3$ in (9) are removed as we don't consider power control in this stage.

It is obvious that problem (12) is a non-convex problem due to the appearance of $a_{m,n}$ in the calculation of $p_{m,n}^{\tilde{\mathrm{up}}}$ and $p_{m,n}^{\tilde{\mathrm{dw}}}$. Hence, we propose a heuristic algorithm to solve it. We decide if a subchannel is allocated to a cell or not by judging if the usage of it in the cell can enhance the total network capacity. In other words, if the added rate of the cell due to the subchannel usage in the cell is larger than the rate reduction of the whole network due to its introduced interference to other cells, it will be used. Otherwise, it will not be used.

We define $C_{m,n}^{\mathrm{Ad}}$ and $C_{m,n}^{\mathrm{Re}}$ to indicate the rate of cell m on subchannel n and the total rate reduction of other cells due to cell m's interference on subchannel n, respectively, as (13) and (14) at the top of the next page.

The algorithm includes two steps. The first is to decide the reusing cells for each subchannel in turn without the consideration of the cell capacity requirement, and the second is to adjust the results between cells according to each cell's capacity requirement. The steps are interpreted in Fig. 3 and the detailed procedure is described as follows.

- **Step 1:** For each subchannel, we decide which cells can use it. For subchannel $n (n \in \{1, 2, ..., N\})$, the procedure is described as follows.
  Sub-step a: Assuming that subchannel n is reused by all cells, for each cell compute $C_{m,n}^{\mathrm{Ad}}$ and $C_{m,n}^{\mathrm{Re}}$. If $C_{m,n}^{\mathrm{Ad}} > C_{m,n}^{\mathrm{Re}}$ is satisfied, cell m is allocated to subchannel n.

$$
\begin{aligned}
C_{m,n}^{\mathrm{Ad}} = {} & \log_2\left(1+\frac{p_{m,n}^{\mathrm{u\tilde{p}}}h_{m,n}}{\sum\limits_{m'\neq m} a_{m',n}\left(p_{m',n}^{\mathrm{u\tilde{p}}}+p_{m',n}^{\mathrm{d\tilde{w}}}\right)h_{m',m,n}+p_{m,n}^{\mathrm{d\tilde{w}}}g_{m,n}+\sigma^2+I_{m,n}}\right) \\
& +\log_2\left(1+\frac{p_{m,n}^{\mathrm{d\tilde{w}}}h_{m,n}}{\sum\limits_{m'\neq m} a_{m',n}\left(p_{m',n}^{\mathrm{u\tilde{p}}}+p_{m',n}^{\mathrm{d\tilde{w}}}\right)h_{m',m,n}+p_{m,n}^{\mathrm{u\tilde{p}}}g_{m,n}+\sigma^2+I_{m,n}}\right),
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
C_{m,n}^{\mathrm{Re}} = {} & \sum_{m'\neq m}\left(\begin{array}{l}\log_2\left(1+\frac{p_{m',n}^{\mathrm{u\tilde{p}}}h_{m',n}}{\sum\limits_{m''\neq m',m''\neq m} a_{m'',n}\left(p_{m'',n}^{\mathrm{u\tilde{p}}}+p_{m'',n}^{\mathrm{d\tilde{w}}}\right)h_{m'',m',n}+p_{m',n}^{\mathrm{d\tilde{w}}}g_{m',n}+\sigma^2+I_{m',n}}\right) \\ +\log_2\left(1+\frac{p_{m',n}^{\mathrm{d\tilde{w}}}h_{m',n}}{\sum\limits_{m''\neq m',m''\neq m} a_{m'',n}\left(p_{m'',n}^{\mathrm{u\tilde{p}}}+p_{m'',n}^{\mathrm{d\tilde{w}}}\right)h_{m'',m',n}+p_{m',n}^{\mathrm{u\tilde{p}}}g_{m',n}+\sigma^2+I_{m',n}}\right)\end{array}\right) \\
& -\sum_{m'\neq m}\left(\begin{array}{l}\log_2\left(1+\frac{p_{m',n}^{\mathrm{u\tilde{p}}}h_{m',n}}{\sum\limits_{m''\neq m'} a_{m'',n}\left(p_{m'',n}^{\mathrm{u\tilde{p}}}+p_{m'',n}^{\mathrm{d\tilde{w}}}\right)h_{m'',m',n}+p_{m',n}^{\mathrm{d\tilde{w}}}g_{m',n}+\sigma^2+I_{m',n}}\right) \\ +\log_2\left(1+\frac{p_{m',n}^{\mathrm{d\tilde{w}}}h_{m',n}}{\sum\limits_{m''\neq m'} a_{m'',n}\left(p_{m'',n}^{\mathrm{u\tilde{p}}}+p_{m'',n}^{\mathrm{d\tilde{w}}}\right)h_{m'',m',n}+p_{m',n}^{\mathrm{u\tilde{p}}}g_{m',n}+\sigma^2+I_{m',n}}\right)\end{array}\right).
\end{aligned}
\tag{14}
$$

Such part of cells are classified as class 1, whose usage of subchannel n can always enhance the network capacity.

Sub-step b: For each remaining cell except cells of class 1, we judge if its usage of subchannel n reduces the network capacity based on the usage of the cells of class 1. In each judgement we assume that the cell uses subchannel n with the cells of class 1 and compute $C_{m,n}^{\mathrm{Ad}}$ and $C_{m,n}^{\mathrm{Re}}$. If $C_{m,n}^{\mathrm{Ad}} < C_{m,n}^{\mathrm{Re}}$ exists, it is not allocated to subchannel n. Such part of cells are classified as class 2, whose usage of subchannel n always reduce the network capacity.

Sub-step c: For each remaining cell except class 1 and 2, which is classified in class 3, we determine if it uses subchannel n in turn. The cell of class 3 is assumed to use subchannel n based on the usage of class 1, the excluded usage of class 2 and the usage of the previously judged cells in class 3 with the determination of the subchannel usage. The values of $C_{m,n}^{\mathrm{Ad}}$ and $C_{m,n}^{\mathrm{Re}}$ are computed, and if $C_{m,n}^{\mathrm{Ad}} > C_{m,n}^{\mathrm{Re}}$ is met subchannel n is allocated to the cell, otherwise not.

- **Step 2:** Adjust the allocation results of step 1.
  For each cell, check if the capacity satisfies the minimal requirement. If not, assign more subchannels for it. Assume that the capacity of cell m is lower than the minimal requirement. Then, firstly, find the subchannels not allocated to it, for each subchannel assume that it is allocated to cell m and compute $C_{m,n}^{\mathrm{Ad}}$ and $C_{m,n}^{\mathrm{Re}}$; Secondly, arrange the subchannels in the order that the value of $C_{m,n}^{\mathrm{Re}}$ minus $C_{m,n}^{\mathrm{Ad}}$ turns larger; Thirdly, add subchannels according to the increasing order until the minimal requirement is satisfied.

We analyze the optimization performance of the algorithm. Sub-step a of step 1 guarantees that the usage of subchannel n by the chosen cells can definitely enhance the total capacity. Sub-step b of step 1 removes the cells whose usage of subchannel n will reduce the total capacity. Sub-step c of step 1 decides the subchannel allocation for the left cells one by one. The decision may make the result not optimal, but it can be solved by iterating sub-step c. Hence the allocation of step 1 can be optimal to maximize the total network capacity. Step 2 chooses the subchannels with the least reduction of the total capacity in the allocation for each cell. As it performs allocation for each cell one by one, the allocation for the latter cell is possible to make the inter-cell interference computation of the former cells not exactly accurate, which may cause the final result not globally optimal.

Remark 1: In the condition of enough subchannel and power resource, the results of step 1 can satisfy the rate requirements of all the users and step 2 will not make adjustments on the results of step 1, then the algorithm can obtain the optimal solution for problem (12).

Proof:We define the variable pro_m to denote the probability that the capacity of cell m meets the minimal requirement after the allocation of step 1,

$$
pro_m = \mathbb{P}\left(\sum_{n\in\mathcal{N}_m} R_{m,n}^{\tilde{up}} > \varepsilon_m^{up} \; and \sum_{n\in\mathcal{N}_m} R_{m,n}^{\tilde{dw}} > \varepsilon_m^{dw}\right). \tag{15}
$$

The lower bound $pro_m^{'}$ exists for pro_m when the rate of cell m on each subchannel is larger than the average required rate on all the channels. The calculation of $pro_m^{'}$ satisfies

$$
pro_m^{'} = \mathbb{P}\left(R_{m,n}^{\tilde{up}} > \frac{\varepsilon_m^{up}}{N_m} \; and \; R_{m,n}^{\tilde{dw}} > \frac{\varepsilon_m^{dw}}{N_m}, \forall n \in \mathcal{N}_m\right). \tag{16}
$$

From (16) $pro_m^{'}$ increases with N_m. As the allocation of each subchannel is performed separately, N_m increases with the total subchannel number. Then in the condition of adequate available subchannels N_m is big enough so as to make $pro_m^{'}$

close adequately to 1, and consequently pro_m is equal to 1. Then step 1 can satisfy the requirements of all the cells and step 2 will not make adjustments on the results of step 1, hence the global optimality can be obtained.

On the other hand, from (10) and (11), $R_{m,n}^{\tilde{\text{up}}}$ and $R_{m,n}^{\tilde{\text{dw}}}$ increase with $p_{m,n}^{\text{up}}$ and $p_{m,n}^{\text{dw}}$, and pro_m increases with the power value. Therefore, when the total power resource is large enough, pro_m is equal to 1. Thus the same conclusion can be achieved. ■

The detailed procedure of the steps in the algorithm is described in Algorithm 1.

B. Power Control

After subchannel allocation, we consider power control for each cell on each subchannel. We need to solve the optimization problem in (9) in the condition that $a_{m,n}$ has been ascertained. As there are variables of $p_{m,n}^{\text{up}}$ and $p_{m,n}^{\text{dw}}$ in the denominator of the objective function, (9) is a non-convex problem. Referring to [34], we substitute some parts of the objective function by new variables and transform the non-convex problem to a convex problem. The detailed procedure is described in Lemma 1.

Lemma 1: The objective function in (9) can be transformed into

$$C=\sum_{m=1}^{M}\sum_{n\in\mathcal{N}_m}\left(\log_2\left(1+\frac{h_{m,n}}{k_{m,n}}x_{m,n}^{\text{up}}\right)+\log_2\left(1+\frac{h_{m,n}}{k_{m,n}}x_{m,n}^{\text{dw}}\right)\right), \tag{17}$$

where $\mathcal{N}_m=\left\{n_1, n_2, ..n_i., n_{|\mathcal{N}_m|}\right\}$ indicates the subchannels allocated to cell m, $k_{m,n}$ denotes the average gain of cell m's interference to other cells in subchannel n, and $x_{m,n}^{\text{up}}$ and $x_{m,n}^{\text{dw}}$ are expressed, respectively, as

$$x_{m,n}^{\text{up}} = \frac{p_{m,n}^{\text{up}}k_{m,n}}{\sum\limits_{m'=1}^{M} p_{m',n}^{\text{up}}k_{m',n} + \delta_{m,n}^{\text{dw}}}, \tag{18}$$

$$x_{m,n}^{\text{dw}} = \frac{p_{m,n}^{\text{dw}}k_{m,n}}{\sum\limits_{m'=1}^{M} p_{m',n}^{\text{dw}}k_{m',n} + \delta_{m,n}^{\text{up}}}, \tag{19}$$

by $\delta_{m,n}^{\text{dw}}$ and $\delta_{m,n}^{\text{up}}$ satisfying

$$\delta_{m,n}^{\text{dw}} = \sum_{m'\neq m} p_{m',n}^{\text{dw}}k_{m',n}+p_{m,n}^{\text{dw}}g_{m,n} + \sigma^2 + I_{m,n}, \tag{20}$$

and

$$\delta_{m,n}^{\text{up}} = \sum_{m'\neq m} p_{m',n}^{\text{up}}k_{m',n}+p_{m,n}^{\text{up}}g_{m,n} + \sigma^2 + I_{m,n}, \tag{21}$$

respectively.

Proof: The proof is provided in Appendix A. ■

Based on (18) and (19), we obtain the expression of $p_{m,n}^{\text{up}}$ and $p_{m,n}^{\text{dw}}$ dependent with $x_{m,n}^{\text{up}}$ and $x_{m,n}^{\text{dw}}$, which is given in Lemma 2.

Lemma 2: The transmit power of cell m in subchannel n satisfies the following equations:

$$p_{m,n}^{\text{up}} = \frac{\delta_{m,n}^{\text{dw}}x_{m,n}^{\text{up}}}{\left(1-\sum\limits_{m=1}^{M}x_{m,n}^{\text{up}}\right)k_{m,n}}, \forall m, n\in\mathcal{N}_m, \tag{22}$$

Algorithm 1 Detailed procedure of inter-cell subchannel allocation

Require:
1. Allocate each subchannel between cells.
Initialize $\mathcal{M}_1$, $\mathcal{M}_2$ and $\mathcal{M}_3$ as ϕ.
For each subchannel perform (1), (2) and (3) :
(1) Find cells of class 1, which is indicated by $\mathcal{M}_1$.
Assume subchannel n is used by each cell.
for m = 1 to M **do**
  Compute $C_{m,n}^{\text{Ad}}$ and $C_{m,n}^{\text{Re}}$;
  if $C_{m,n}^{\text{Ad}} > C_{m,n}^{\text{Re}}$ **then**
    $a_{m,n}=1$;
    $\mathcal{M}_1=\mathcal{M}_1\cup\{m\}$;
  end if
end for
(2) Find cells of class 2, which is indicated by $\mathcal{M}_2$.
Initialize $\mathcal{M}^{'} = \{1,2,...,M\}\backslash\mathcal{M}_1$. Define $m^{'} \in \mathcal{M}^{'}$.
Initialize $i=0$.
for $i=1$ to $\left|\mathcal{M}^{'}\right|$ **do**
  $m^{'}=\mathcal{M}^{'}(i)$;
  Assume subchannel n is used by cells $\{m^{'}\}\cup\mathcal{M}_1$;
  Compute $C_{m^{'},n}^{\text{Ad}}$ and $C_{m^{'},n}^{\text{Re}}$;
  if $C_{m^{'},n}^{\text{Ad}} < C_{m^{'},n}^{\text{Re}}$ **then**
    $a_{m^{'},n}=0$;
    $\mathcal{M}_2=\mathcal{M}_2\cup\{m^{'}\}$;
  end if
end for
(3) Determine the $a_{m,n}$ values of the cells of class 3, which is indicated by $\mathcal{M}_3=\{1,2,...,M\}\backslash(\mathcal{M}_1\cup\mathcal{M}_2)$.
Define $\mathcal{M}_3^{'}\subseteq\mathcal{M}_3$, the set of the cells which are determined to use subchannel n. Initialize $\mathcal{M}_3^{'}$ as ϕ.
Define $m^{''}\in\mathcal{M}_3$. Initialize $i=0$.
for $i=1$ to $|\mathcal{M}_3|$ **do**
  $m^{''}=\mathcal{M}_3(i)$;
  Assume subchannel n is used by cells $\{m^{''}\}\cup\mathcal{M}_1\cup\mathcal{M}_3^{'}$;
  Compute $C_{m^{''},n}^{\text{Ad}}$ and $C_{m^{''},n}^{\text{Re}}$;
  if $C_{m^{''},n}^{\text{Ad}} > C_{m^{''},n}^{\text{Re}}$ **then**
    $a_{m^{''},n}=1$;
    $\mathcal{M}_3^{'}=\mathcal{M}_3^{'}\cup\{m^{''}\}$;
  else
    $a_{m^{''},n}=0$;
  end if
end for
2. Adjust the allocation.
for m = 1 to M **do**
  $\mathcal{N}_m^{\text{id}}=\{n, n\in\{1,2,...,N\}, a_{m,n}=0\}$;
  Initialize $i=0$.
  while $R_m^{\text{up}}<\epsilon_m^{\text{up}}$ or $R_m^{\text{dw}}<\epsilon_m^{\text{dw}}$ **do**
    for $i=1$ to $\left|\mathcal{N}_m^{id}\right|$ **do**
      $n=\mathcal{N}_m^{\text{id}}(i)$;
      Compute $C_{m,n}^{\text{Ad}}$ and $C_{m,n}^{\text{Re}}$;
      $D(i)=C_{m,n}^{\text{Ad}}-C_{m,n}^{\text{Re}}$;
    end for
    Find $i^{*}=arg\min\limits_{i\in\{1,2,...,|\mathcal{N}_m^{\text{id}}|\}}\{D(i)\}$;
    $a_{m,\mathcal{N}_m^{\text{id}}(i^{*})}=1$;

  end while
end for

$$p_{m,n}^{\mathrm{dw}} = \frac{\delta_{m,n}^{\mathrm{up}} x_{m,n}^{\mathrm{dw}}}{\left(1 - \sum_{m=1}^{M} x_{m,n}^{\mathrm{dw}}\right) k_{m,n}}, \forall m, n \in \mathcal{N}_m. \tag{23}$$

Proof: We introduce (18) to substitute $x_{m,n}^{\mathrm{up}}$ in the expression on the right side of (22) and with further deduction (22) is proved. By the same argument, (23) can be derived. ■

Based on Lemma 1, we transform optimization problem (9) into (24)

$$\max_{x_{m,n}^{\mathrm{up}}, x_{m,n}^{\mathrm{dw}}} \quad C = \sum_{m=1}^{M} \sum_{n \in \mathcal{N}_m} \begin{pmatrix} \log_2\left(1 + \frac{h_{m,n}}{k_{m,n}} x_{m,n}^{\mathrm{up}}\right) \\ + \log_2\left(1 + \frac{h_{m,n}}{k_{m,n}} x_{m,n}^{\mathrm{dw}}\right) \end{pmatrix} \tag{24}$$
$$\text{s.t.} \quad C1, C2, C3, C4, C5, C6, C7, C8, C9.$$

The constraints $C1$ to $C5$ in (24) are obtained from the transform of the constraints $C1$ to $C5$ in (9) by the introduction of (22) and (23) to substitute $p_{m,n}^{\mathrm{up}}$ and $p_{m,n}^{\mathrm{dw}}$. The constraints $C6$ and $C7$ denote the ranges of $x_{m,n}^{up}$ and $x_{m,n}^{dw}$, respectively. The constraints $C8$ and $C9$ denote the limitations of $\sum_{m=1}^{M} x_{m,n}^{up}$ and $\sum_{m=1}^{M} x_{m,n}^{dw}$ to guarantee the positive value of $p_{m,n}^{up}$ and $p_{m,n}^{dw}$, respectively. For brevity they are not listed in detail here.

In [35], the power allocation of FD network are divided into uplink and downlink allocation separately. First, the downlink power allocation is solved with the uplink power fixed, and second, the uplink power is calculated under the obtained downlink power. In this paper, we adopt the approach, which separates problem (24) into two sub-problems accordingly. The first is to obtain the value of $x_{m,n}^{\mathrm{dw}}$ under the equal uplink power allocation to maximize the downlink capacity, and the second is to solve the uplink power allocation under the obtained values of $x_{m,n}^{\mathrm{dw}}$ and $p_{m,n}^{\mathrm{dw}}$ to maximize the uplink capacity.

The first sub-problem of problem (24) is formulated as

$$\max_{\left\{x_{m,n}^{\mathrm{dw}}\right\}} \quad C = \sum_{m=1}^{M} \sum_{n \in \mathcal{N}_m} \left(\log_2\left(1 + \frac{h_{m,n}}{k_{m,n}} x_{m,n}^{\mathrm{dw}}\right)\right) \tag{25}$$

$$\begin{aligned}
\text{s.t.} \quad & C1: 0 < x_{m,n}^{\mathrm{dw}} < 1, \forall m, n \in \mathcal{N}_m, \\
& C2: \sum_{m=1}^{M} x_{m,n}^{\mathrm{dw}} < 1, \forall n \in \mathcal{N}_m, \\
& C3: \sum_{n \in \mathcal{N}_m} \frac{\delta_{m,n}^{\tilde{\mathrm{up}}} x_{m,n}^{\mathrm{dw}}}{\left(1 - \sum_{m=1}^{M} x_{m,n}^{\mathrm{dw}}\right) k_{m,n}} \le P_m^{\mathrm{dw}}, \forall m, \\
& C4: \sum_{m=1}^{M} \left(p_{m,n}^{\tilde{\mathrm{up}}} + \frac{\delta_{m,n}^{\tilde{\mathrm{up}}} x_{m,n}^{\mathrm{dw}}}{\left(1 - \sum_{m=1}^{M} x_{m,n}^{\mathrm{dw}}\right) k_{m,n}}\right) h_{m,n}^{\mathrm{MB}} \le I_n^{th}, \\
& \qquad \forall n \in \mathcal{N}_m, \\
& C5: \sum_{n \in \mathcal{N}_m} \left(\log_2\left(1 + \frac{h_{m,n}}{k_{m,n}} x_{m,n}^{\mathrm{dw}}\right)\right) \ge \varepsilon_m^{\mathrm{dw}}, \forall m,
\end{aligned}$$

where $\delta_{m,n}^{\tilde{\mathrm{up}}} = \sum_{m' \neq m} p_{m',n}^{\tilde{\mathrm{up}}} h_{m',m,n} + p_{m,n}^{\tilde{\mathrm{up}}} g_{m,n} + \sigma^2 + I_{m,n}$ are satisfied. Problem (25) is a convex problem. Therefore we use the Lagrange dual decomposition method to solve it. The detailed procedure is given below. The Lagrangian of (25) can be expressed as

$$\begin{aligned}
& L\left(\left\{x_{m,n}^{\mathrm{dw}}\right\}, \lambda, \eta, \mu, \kappa\right) \\
& = \sum_{m=1}^{M} \sum_{n \in \mathcal{N}_m} \left(\log_2\left(1 + \frac{h_{m,n}}{k_{m,n}} x_{m,n}^{\mathrm{up}}\right)\right) \\
& + \sum_{n \in \mathcal{N}_m} \left(\lambda_n \left(1 - \sum_{m=1}^{M} x_{m,n}^{\mathrm{dw}}\right)\right) \\
& + \sum_{m=1}^{M} \left(\eta_m \left(\sum_{n \in \mathcal{N}_m} \left(P_m^{\mathrm{dw}} - \frac{\delta_{m,n}^{\tilde{\mathrm{up}}} x_{m,n}^{\mathrm{dw}}}{\left(1 - \sum_{m=1}^{M} x_{m,n}^{\mathrm{dw}}\right) k_{m,n}}\right)\right)\right) \\
& + \sum_{n \in \mathcal{N}_m} \mu_n^{\mathrm{up}} \left(I_n^{\mathrm{th}} - \sum_{m=1}^{M} \left(p_{m,n}^{\tilde{\mathrm{up}}} + \frac{\delta_{m,n}^{\tilde{\mathrm{up}}} x_{m,n}^{\mathrm{dw}}}{\left(1 - \sum_{m=1}^{M} x_{m,n}^{\mathrm{dw}}\right) k_{m,n}}\right) h_{m,n}^{\mathrm{MB}}\right) \\
& + \sum_{m=1}^{M} \kappa_m \left(\sum_{n \in \mathcal{N}_m} \left(\log_2\left(1 + \frac{h_{m,n}}{k_{m,n}} x_{m,n}^{\mathrm{dw}}\right)\right) - \varepsilon_m^{\mathrm{dw}}\right)
\end{aligned} \tag{26}$$

where λ, η, μ, and κ are the Lagrange multipliers (also called dual variables) vectors for the constraints $C2$ to $C5$ in (25) respectively. The boundary constraints $C1$ in (25) are absorbed in the Karush-Kuhn-Tucker (KKT) conditions [36]. Thus, the Lagrange dual function is defined as

$$g(\lambda, \eta, \mu, \kappa) = \max_{\left\{x_{m,n}^{\mathrm{dw}}\right\}} L\left(\left\{x_{m,n}^{\mathrm{dw}}\right\}, \lambda, \eta, \mu, \kappa\right). \tag{27}$$

The dual problem can be expressed as

$$\min_{\lambda, \eta, \mu, \kappa} \quad g(\lambda, \eta, \mu, \kappa) \tag{28}$$
$$\text{s.t.} \quad C1: \lambda, \eta, \mu, \kappa \ge 0.$$

The dual variables are optimized by subgradient method based on KKT conditions [17], [18] , which are expressed according to [36]. Based on the dual variables the value of $x_{m,n}^{\mathrm{dw}}$ are obtained and accordingly the value of $p_{m,n}^{\mathrm{dw}}$ are obtained by (23).

In order to reduce the computation complexity, based on the direct-power truncation (DPT) dual scheme in [37], we propose the waterfilling allocation and direct-power adjust (WADPA) scheme to solve problem (25), which is described in Proposition 1.

Proposition 1: The WADPA scheme can be used to solve problem (25), in which the traditional water-filling theorem is applied to maximize the capacity under the total sum constraint $C2$ and the direct-power adjust is adopted to fit the constraints of $C3$, $C4$ and $C5$.

Proof: We find that the objective function in (25) has the same formality as the rate expression by the Shannon theory, in which $x_{m,n}^{\mathrm{dw}}$ are corresponding to power variable and $\frac{h_{m,n}}{k_{m,n}}$ are corresponding to the SNR coefficient. Therefore, we consider $x_{m,n}^{\mathrm{dw}}$ as power variables, and use the water-filling theorem to perform the allocation of $x_{m,n}^{\mathrm{dw}}$ under the limitation of the sum of $x_{m,n}^{\mathrm{dw}}$. As $C2$ indicates that the sum of $x_{m,n}^{\mathrm{dw}}$ need to be smaller than 1, we define a variable infinitely close to 1, which is denoted by ς_n. Also we define $\mathcal{M}_n$ to denote the cells with the allocation of subchannel n and the number of the cells in $\mathcal{M}_n$ is indicated by M_n. Then the water-filling allocation can be obtained as

$$x_{m,n}^{\mathrm{dw}\ *} = \left[\frac{1}{\lambda_n} - \frac{k_{m,n}}{h_{m,n}}\right]^+, \tag{29}$$

where

$$\lambda_n = \frac{M_n}{\varsigma_n + \sum\limits_{m\in\mathcal{M}_n} \frac{k_{m,n}}{h_{m,n}}}, \tag{30}$$

is the water-filling lever for subchannnel n, and $[Z]^+ = \max\{Z, 0\}$.

Then the direct-power adjust method is utilized to fit the constraints $C3$, $C4$ and $C5$. Referring to [37], we equally split the quota P_m^{dw}, I_n^{th} and $\varepsilon_m^{\mathrm{dw}}$ in N_m subchannels or M small cells, and bound the limit of $x_{m,n}^{\mathrm{dw}}$ by the strictest constraint. Thus the substitute limitation of $x_{m,n}^{\mathrm{dw}}$ is obtained as

$$X_{m,n}^{\mathrm{d}} \le x_{m,n}^{\mathrm{dw}} \le X_{m,n}^{\mathrm{u}}, \tag{31}$$

where

$$X_{m,n}^{\mathrm{u}} = \min\left\{ \begin{array}{l} P_m^{\mathrm{dw}}/N_m/\frac{\delta_{m,n}^{\mathrm{u\tilde{p}}}}{(1-\varsigma_n)k_{m,n}}, \\ \left(I_n^{\mathrm{th}}/M/h_{m,n}^{\mathrm{MB}} - p_{m,n}^{\mathrm{u\tilde{p}}}\right)/\frac{\delta_{m,n}^{\mathrm{u\tilde{p}}}}{(1-\varsigma_n)k_{m,n}} \end{array} \right\}, \tag{32}$$

and

$$X_{m,n}^{\mathrm{d}} = \left(2^{\varepsilon_m^{\mathrm{dw}}/N_m} - 1\right)/\frac{h_{m,n}}{k_{m,n}}. \tag{33}$$

Combining with (29), we can obtain the adjust results as

$$x_{m,n}^{\mathrm{dw}\ *} = \begin{cases} \left(\frac{1}{\lambda_n} - \frac{k_{m,n}}{h_{m,n}}\right)^+, X_{m,n}^{\mathrm{d}} \le \left(\frac{1}{\lambda_n} - \frac{k_{m,n}}{h_{m,n}}\right)^+ \le X_{m,n}^{\mathrm{u}} \\ X_{m,n}^{\mathrm{d}}, \left(\frac{1}{\lambda_n} - \frac{k_{m,n}}{h_{m,n}}\right)^+ < X_{m,n}^{\mathrm{d}} \\ X_{m,n}^{\mathrm{u}}, \left(\frac{1}{\lambda_n} - \frac{k_{m,n}}{h_{m,n}}\right)^+ > X_{m,n}^{\mathrm{u}}. \end{cases} \tag{34}$$

As the adjustment of changing $\left(\frac{1}{\lambda_n} - \frac{k_{m,n}}{h_{m,n}}\right)^+$ to $X_{m,n}^{\mathrm{d}}$ is possible to make $C2$ dissatisfied, we iterate the adjustment based on the update of the water-filling results until $C2$ is satisfied. The detailed procedure is summarized in Algorithm 2. ■

After $x_{m,n}^{\mathrm{dw}}$ and $p_{m,n}^{\mathrm{dw}}$ are obtained, the same methods are adopted to solve the second sub-problem of problem (24), which are not described in detail here. Moreover, as the approach in [35] is reversible, the procedure of solving the uplink variables first and then downlink ones can also be adopted.

Algorithm 2 Detailed procedure of inter-cell downlink power allocation

Initialize ς_n as a value infinitely close to 1 and Lagrangian variables vectors λ_n;
Initialize $\mathcal{M}_n$ as the set of the cells with the allocation of subchannel n;
for n = 1 to N **do**
  repeat
    for m = 1 to M **do**
      if $m \in \mathcal{M}_n$ **then**
        calculate λ_n according to (30);
        obtain $x_{m,n}^{\mathrm{dw}\ *}$ according to (29);
        adjust $x_{m,n}^{\mathrm{dw}\ *}$ according to (34);
        if $x_{m,n}^{\mathrm{dw}\ *} = X_{m,n}^{\mathrm{d}}$ **then**
          $\mathcal{M}_n = \mathcal{M}_n \backslash \{m\}$;
        end if
        $\varsigma_n = \sum\limits_{m\in\mathcal{M}_n} x_{m,n}^{\mathrm{dw}\ *}$.
      end if
    end for
  until $\sum\limits_{m=1}^{M} x_{m,n}^{\mathrm{dw}} < 1$.
end for

IV. Intra-Cell Allocation

After the inter-cell subchannel allocation and power control, we obtain the available subchannel set for each cell and the transmit power on each subchannel. Then we consider the allocation of subchannels between different links in each cell. The rate of link l in subchannel n for cell m can be computed as

$$\begin{aligned} &R_{m,n,l} \\ &= \log_2\left(1 + \frac{p_{m,n,l}^{\mathrm{up}} h_{m,n,l}}{\sum\limits_{m'\neq m} (p_{m',n}^{\mathrm{up}} + p_{m',n}^{\mathrm{dw}}) h_{m',m,n} + p_{m,n,l}^{\mathrm{dw}} g_{m,n,l} + \sigma^2 + I_{m,n}}\right) \\ &+ \log_2\left(1 + \frac{p_{m,n,l}^{\mathrm{dw}} h_{m,n,l}}{\sum\limits_{m'\neq m} (p_{m',n}^{\mathrm{up}} + p_{m',n}^{\mathrm{dw}}) h_{m',m,n} + p_{m,n,l}^{\mathrm{up}} g_{m,n,l} + \sigma^2 + I_{m,n}}\right). \end{aligned} \tag{35}$$

Accordingly, the subchannel allocation in cell m can be formulated as

$$\max_{a_{m,n,l}} \quad C_m = \sum_{n=1}^{N} \sum_{l=1}^{L_m} (a_{m,n,l} R_{m,n,l}) \tag{36}$$

$$\begin{aligned} \text{s.t.} \quad & C1: a_{m,n,l} \in \{0,1\}, \forall n \in \mathcal{N}_m, l \in \{1,2,...,L_m\}, \\ & C2: \sum_{l=1}^{L_m} a_{m,n,l} = 1, \forall n \in \mathcal{N}_m, \\ & C3: \sum_{n\in\mathcal{N}_m} a_{m,n,l} \ge 1, \forall l \in \{1,2,...,L_m\}, \\ & C4: \sum_{n\in\mathcal{N}_m} a_{m,n,l} p_{m,n}^{\mathrm{up}} < \overline{p_{m,l}^{\mathrm{up}}}, \forall l, \end{aligned}$$

where $C2$ means that one subchannel can be used only by one link, so as to avoid the interference between links, and $C3$ means each link needs to be allocated with one subchannel at least.

Obviously, problem (36) is a linear programming problem with an integral restraint, which can be solved by simplex method. In order to reduce the complexity further, we adopt a time-sharing relaxation method to solve it [17], [38]. We relax $a_{m,n,l}$ to be a continuous real variable in the range

[0,1], and solve problem (36) by using the Lagrangian dual decomposition method. The Lagrangian function is given by

$$\begin{gathered} L\left(\left\{a_{m,n,l}\right\},\nu,\mu,\gamma\right)= \\ \sum_{n=1}^{N}\sum_{l=1}^{L_m}\left(a_{m,n,l}R_{m,n,l}\right)+\sum_{n=1}^{N}\nu_n\left(1-\sum_{l=1}^{L_m}a_{m,n,l}\right) \\ +\sum_{l=1}^{L_m}\tau_l\left(\sum_{n\in\mathcal{N}_m}a_{m,n,l}-1\right)+\sum_{l=1}^{L_m}\gamma_l\left(\overline{p_{m,l}^{\mathrm{up}}}-\sum_{n\in\mathcal{N}_m}a_{m,n,l}p_{m,n}^{\mathrm{up}}\right), \end{gathered} \tag{37}$$

where ν, τ and γ are the Lagrange multipliers vectors for the constraints C2, C3 and C4 in (36), respectively. Thus, the Lagrangian dual function is defined as

$$g\left(\nu,\tau,\gamma\right)=\max_{\left\{a_{m,n,l}\right\}}L\left(\left\{a_{m,n,l}\right\},\nu,\tau,\gamma\right). \tag{38}$$

The dual problem can be expressed as:

$$\min_{\nu,\tau,\gamma}\quad g\left(\nu,\tau,\gamma\right) \tag{39}$$

$$\text{s.t.}\quad C1:\nu,\tau,\gamma\geq 0.$$

According to the KKT conditions, the optimal solutions of problem (36), can be obtained as

$$\frac{\partial L\left(\cdots\right)}{\partial a_{m,n,l}}\begin{cases} <0, & a_{m,n,l}^{*}=0\\ =0, & 0<a_{m,n,l}^{*}<1\\ >0, & a_{m,n,l}^{*}=1. \end{cases} \tag{40}$$

In (40), the partial derivative of the Lagrangian expressed as

$$\frac{\partial L\left(\cdots\right)}{\partial a_{m,n,l}}=H_{m,n,l}-\nu_n, \tag{41}$$

where

$$H_{m,n,l}=R_{m,n,l}+\tau_l-\gamma_l p_{m,n}^{\mathrm{up}}. \tag{42}$$

Subchannel n is assigned to link l with the largest $H_{m,n,l}$ in cell m, that is

$$a_{m,n,l^*}=1|_{l^*=\max\limits_{l}H_{m,n,l}},\forall m,n. \tag{43}$$

We use the subgradient method, and update the dual variables according to

$$\tau_l^{(i+1)}=\tau_l^{(i)}-\alpha^{(i)}\left(\sum_{n\in\mathcal{N}_m}a_{m,n,l}-1\right), \tag{44}$$

$$\gamma_l^{(i+1)}=\gamma_l^{(i)}-\beta^{(i)}\left(\overline{p_{m,l}^{\mathrm{up}}}-\sum_{n\in\mathcal{N}_m}a_{m,n,l}p_{m,n}^{\mathrm{up}}\right), \tag{45}$$

where $\alpha^{(i)}$ and $\beta^{(i)}$ are the step sizes of iteration $i\,(i\in\{1,2,...,I_{\max}\})$ of τ and γ, I_{max} is the maximum number of iterations.

The above-described solution has much lower complexity, which is analyzed in Section V. We summarize the process of the solution with pseudo code in Algorithm 3.

Algorithm 3 Detailed Procedure of intra-cell subchannel allocation

```
Initialize γ_l (l ∈ {1, 2, ..., L_m}) and I_max, set i = 0;
repeat
    for l = 1 to L_m do
        update τ_l and γ_l according to (44) and (45);
        calculate H_{m,n,l} according to (42);
    end for
    for n = 1 to N_m do
        allocate subchannel n to l* according to (43);
    end for
until i = I_max.
```

V. Complexity Analysis

The asymptotic complexity of the proposed algorithms is analyzed in this section. In Algorithm 1, the computation of $C_{m,n}^{\mathrm{Ad}}$ and $C_{m,n}^{\mathrm{Re}}$ needs $O\left(MN\right)$ operations and the searches of i^* calls MN operations in the worst case. Thus, the total computation of inter-cell subchannel allocation amounts to $O\left(MN\right)$. In Algorithm 2, for each subchannel the computation of λ_n and $x_{m,n}^{\mathrm{dw}}$ entails $O\left(M\right)$ operations at most. Therefore, the total calculation of inter-cell power allocation entails $O\left(MN\right)$ operations, which include the downlink and uplink allocation. In Algorithm 3, the computations of γ_l and $R_{m,n,l}$ calls $O\left(\sum\limits_{m=1}^{M}L_m\right)$ operations, and $O\left(\sum\limits_{m=1}^{M}N_mL_m\right)$ operations are needed for the searches of l^*. Thus, the complexity of Algorithm 3 amounts to $O\left(\sum\limits_{m=1}^{M}N_mL_m\right)$, which is equal to $O\left(N\sum\limits_{m=1}^{M}L_m\right)$ in the worst case. Moreover, $O\left(M^2N\right)$ operations are entailed in gathering and computing all inter-cell channel gains in all subchannels. As a result, the total complexity of the proposed scheme is calculated as

$$\begin{aligned} &O\left(MN\right)+O\left(MN\right)+O\left(M^2N\right)+O\left(N\sum_{m=1}^{M}L_m\right)\\ &=O\left(MN\right)+O\left(M^2N\right)+O\left(N\sum_{m=1}^{M}L_m\right) \end{aligned}. \tag{46}$$

Compared with Algorithm 1 in [17], which needs $O\left(M^2N^2\left(\sum\limits_{m=1}^{M}L_m\right)^2\Delta\right)$ operations if Δ iterations is needed to converge, the proposed algorithm has much lower complexity. Algorithm 2 in [17] has lower complexity amounting to $O\left(N\left(\sum\limits_{m=1}^{M}L_m\right)\log_2\left(\sum\limits_{m=1}^{M}L_m\right)\right)+O\left(M\left(\sum\limits_{m=1}^{M}L_m\right)N\log_2N\right)+O\left(\Delta\right)$ operations with Δ iterations. The complexity is similar with the proposed algorithm but at a large sacrifice of the capacity performance compared with Algorithm 1.

VI. Simulation and Discussion

In this section, we validate the performance of the proposed resource management for FD UDN by Monte Carlo simula-

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tions. We build a two-layer FD UDN model with one macro cell and multiple small cells as Fig. 1. The macro cell's radius is set to 500 m and the radius of each small cell is set to 10 m. Small cells are distributed uniformly in the macro cell. The channel fading model is composed of path loss, shadowing fading, and frequency selective Rayleigh fading. The pathloss model is constructed as $140.7 + 36.7\log_{10}(d/1000)$ [39], where d is the transmission distance. The intra-cell and inter-cell lognormal shadowing are modeled as random variable with the standard deviation of 10 dB and 5 dB, respectively. The Rayleigh fading channel gains are modeled as i.i.d. unit-mean exponentially distributed random variables. The self-interference gain $g_{m,n,l}$ combines the self-interference cancellation coefficient and the propagation loss from transmit antenna to receive antenna. As a whole it is set to be exponential random variable with unit-mean. The carrier frequency is 2GHz and the total bandwidth is 100 MHz. The total subchannel number is 48. Moreover, the AWGN power density σ^2 is -120 dBm/Hz, the macro BS's transmit power is 20W, and the macro cell's interference limitation I_n^{th} is -100 dBm. The user number of each small cell is 30. Referring to the user experienced data rate of 50Mbps and 100Mbps in uplink and downlink in 5G system [40], the capacity requirement of each small cell is set to 1Gbit/s and 2Gbit/s for uplink and downlink, respectively.

A. Optimization Performance Evaluation of Algorithm 1

We analyze the optimization performance of step 1 in Algorithm 1 by comparing it with exhaustive search. The networks with a small number of small cells and subchannels are constructed. The downlink and uplink power of each cell is set to be 25 dBm and 20 dBm, respectively. The network capacities with the subchannel allocation by step 1 and the optimal values by exhaustive search are listed in Tab. I. The results show that step 1 achieves the optimal subchannel allocation. In step 1, sub-step a finds the cells whose usage of the subchannel always enhance the network capacity and sub-step b removes the cells whose usage of the subchannel always decrease the network capacity. The results of the two substeps are optimal. In sub-step c, the decision may not be optimal due to the influence between the previously and posteriorly processed cells. Through iterative perform of sub-step c, the non-optimality can be avoided especially in the network with a few small cells and subchannels. Moreover, we validate the performance of the traditional greedy algorithm in the scenario configured in Table I. We compute $C_{m,n}^{\mathrm{Ad}}$ and $C_{m,n}^{\mathrm{Re}}$ for each cell on each subchannel and select the link with the highest value of $C_{m,n}^{\mathrm{Ad}} - C_{m,n}^{\mathrm{Re}}$ for allocation. Based on the allocation the remaining $C_{m,n}^{\mathrm{Ad}}$ and $C_{m,n}^{\mathrm{Re}}$ are re-computed and the same allocation is performed. After multiple performances the allocation is ended when the value of $C_{m,n}^{\mathrm{Ad}} - C_{m,n}^{\mathrm{Re}}$ is negative. The results of the greedy algorithm are also shown in Table I. The approximately 15 percent reduction can be seen by the comparison with the optimal results. As the greedy algorithm compares the value of $C_{m,n}^{\mathrm{Ad}} - C_{m,n}^{\mathrm{Re}}$ and performs allocation in all the subchannels and cells from beginning to end, the latter allocation makes the former inter-cell interference computation not accurate, thus the optimality is affected largely.

We verify the performance of Algorithm 1 in Fig. 4. The cell number is set to be 45 and the subchannel number is 4 totally. Fig. 4 shows the cell distribution and the usage of each subchannel in all the cells. We take a cell randomly, which is denoted as cell 1. Fig. 4 shows that cell 1 uses subchannel 1 and 3. We analyze the subchannel allocation of the closest cells of cell 1, which are denoted as cell 2, 3 and 4. It can be found that cell 2, 3 and 4 use subchannel 4, 2 and $\{2, 4\}$, respectively. The results show that the subchannel reuse in close neighbouring cells is avoided. Moreover, it can be found that each subchannel is used by a number of scattered cells with large distance from each other, which gives a reasonable allocation to enhance the subchannel usage and avoid the severe inter-cell interference.

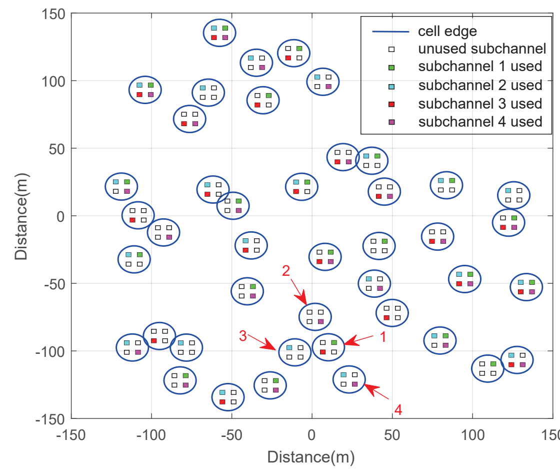

Fig. 4. The distribution of the small cells and the usage of each subchannel in all the cells.

B. Optimization Performance Evaluation of the Proposed Algorithm

The proposed algorithm, which is composed of Algorithm 1, Algorithm 2 and Algorithm 3, has suboptimal performance in maximizing the throughput. The reasons are given in two aspects. Firstly, in inter-cell subchannel allocation, Algorithm 1 obtains near optimal solution, which has been described in Section III-A. Secondly, in the solution of the non-convex problem in inter-cell power control, we use average channel gains and further adopt variable substitution, which brings optimization gap in the final answer. For analyzing the sub-optimal performance of the proposed algorithm, we compare it with Algorithm 1 in [17] and analyze the performance enhancement. [17] allocates subchannel and power between the links of multiple small cells under the assumption of the inter-cell interference as a part of AWGN. Fig. 5 compares the proposed algorithm with Algorithm 1 in [17] in term of the sum rate of the small cells. The small cell number is set to 200 and 400, respectively. The average self-interference gain of each small cell varies from -120 dB to -70 dB. It is shown that the sum rate of the small cells degrade versus the average self-interference gain. Moreover, the proposed algorithm provides

TABLE I
NETWORK CAPACITY PERFORMANCE OF STEP 1 IN ALGORITHM 1.

(small cell number, subchannel number)	$(4,3)$	$(4,4)$	$(4,5)$	$(5,3)$	$(5,4)$
network capacity by Step 1 (Mbps)	79.2	101.5	123.1	91.3	115.2
optimal network capacity by exhaust search (Mbps)	79.2	101.5	123.1	91.3	115.2
network capacity by greedy algorithm (Mbps)	71.3	89.4	106.8	78.6	98.1

superior capacity performance to Algorithm 1 in [17] by over 15 percent enhancement, which origins from the full consideration of inter-cell interference. Additionally, the capacity enhancement of the proposed algorithm degrades versus the self-interference gain. This is because we use direct truncation in the inter-cell power allocation when satisfying $C3$ in (23), and with self-interference gain increasing the coefficient of $x_{m,n}^{\mathrm{dw}}$ in $C3$ increases, which causes the decline of $x_{m,n}^{\mathrm{dw}}$ and consequently decrease the capacity enhancement.

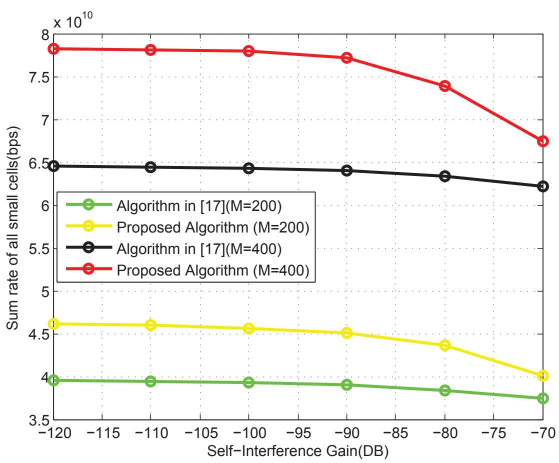

Fig. 5. Sum-rate of all the small cells of the algorithm in [17] and the proposed algorithm.

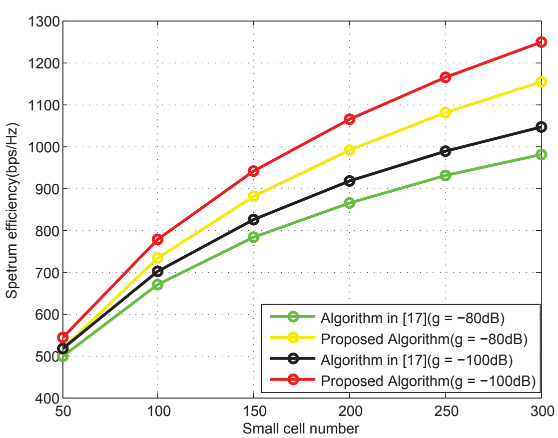

Fig. 6. Spectrum efficiency of all small cells versus small cell number.

In Fig. 6, the spectrum efficiency of Algorithm 1 in [17] and the proposed scheme are compared for different self-interference gains. The spectrum efficiency is computed by the sum rate of the small cells divided by the total bandwidth. The small cell number varies from 50 to 300. It verifies that the spectrum efficiency increases versus small cell number, and degrades versus the self-interference gain. Moreover, the increasing rate of the total capacity slows down with the cell number increasing due to the more severe inter-cell interference.

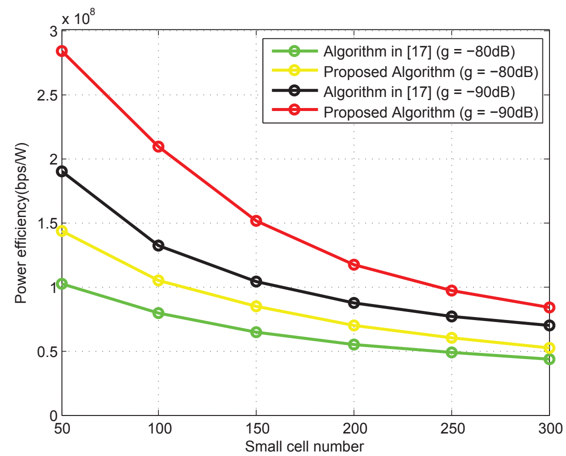

Fig. 7. Power efficiency of all the small cells versus small cell number.

In Fig. 7, the power efficiency of Algorithm 1 in [17] and the proposed algorithm with different self-interference gains are compared. The power efficiency is computed by the total capacity divided by the power sum of all the small cells. The downlink and uplink power limitation is set to be 30 dBm and 25 dBm, respectively. The cell number varies from 50 to 300. It is shown that the power efficiency degrades versus the small cell number due to the increasing inter-cell interference. The outperformance of the proposed algorithm over Algorithm 1 in [17] is shown. Approximately 20 percent enhancement in the power efficiency can be obtained. Moreover, the influence of the self-interference to the power efficiency is verified, and it is shown that the power efficiency degrades obviously when the self-interference gets larger.

C. FD Performance Evaluation

We analyze the performance gains of FD mode over HD mode with the power variation. In Fig. 8, the total capacity are compared for FD mode with different self-interference gains and HD mode as well. The maximal limitation of downlink transmission power in each small cell varies from 25dBm to 43dBm, and the uplink power limitation varies from 20dBm to 38dBm proportionally. We use g indicating the average self-interference gain of each small cell. The proposed algorithm is used for FD mode and HD mode as well. It can be seen that with the power limitation increasing the sum rate of all small cells increases. Nevertheless, the increasing rate of the sum rate degrades versus the power limitation. This can be explained as the inter-cell interference increases proportionally with the transmit power, and the power of AWGN and the interference from macro base station is unchanged, thus the ratio between the transmit power and the interference changes weaker with the power limitation increasing. The validations are in accordance with our previous research in [41], [42], which obtains the optimal power in the condition of only two neighbored femtocells existing. Additionally, for the compare of FD and HD mode, in the simulation scenarios of this paper, when the self-interference gain is lowered to -70 dBm and the downlink power limitation is larger than 40 dBm, HD mode achieves superior performance to FD mode. It verifies that when the self-interference is extremely severe, FD mode can not obtain the capacity enhancement over HD mode even though it can realize the simultaneous transmission and receipt in the same subchannel.

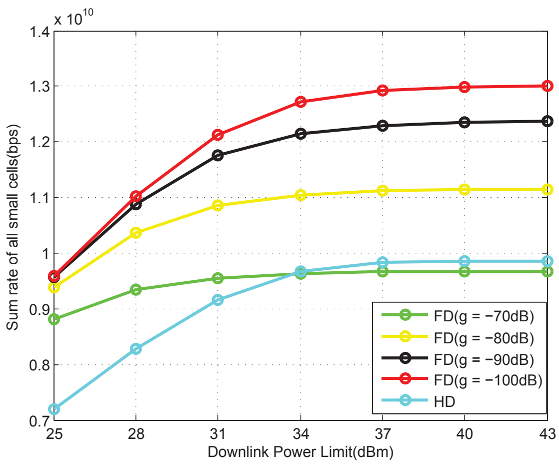

Fig. 8. Sum-rate of all small cells of FD and HD mode.

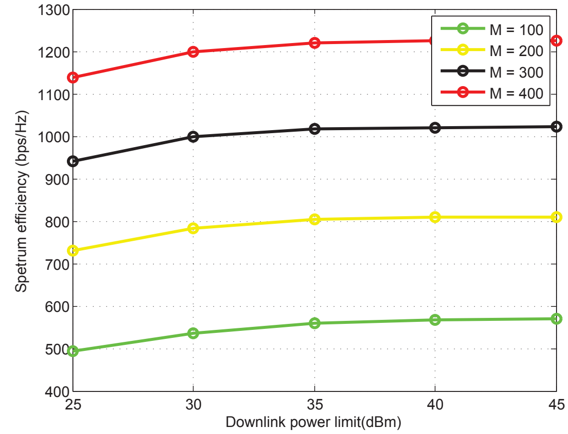

Fig. 9. Spectrum efficiency of all small cells versus downlink power limit.

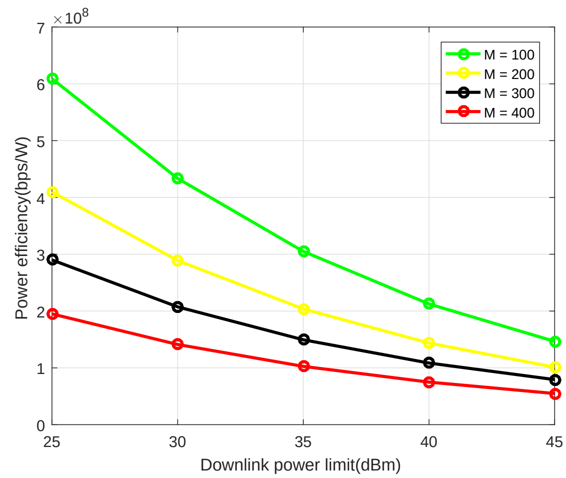

Fig. 10. Power efficiency of all small cells versus downlink power limit.

D. Performance Evaluation of Network Density Variation

We analyze the spectrum and power efficiency of the networks with different small cell number. In Fig. 9, the spectrum efficiency of all the small cells are compared for the small cell numbers from 100 to 400. The downlink transmit power limitation varies from 25 dBm to 45 dBm. The average self-interference gain of each small cell is set to -100 dB. It can be seen that the spectrum efficiency increases versus the transmit power limitation, and the increasing rate gets down when the small cell number increases.

The power efficiency performance of the proposed algorithm is analyzed in Fig. 10. To achieve the results with diversified configuration, the mean of the self-interference gain of each small cell is changed to -110 dB. The cell number is chosen as 100, 200, 300 and 400, and the downlink power limitation of each cell varies from 25 dBm to 45 dBm. From Fig. 10, it can be seen that the power efficiency degrades with the power limitation increasing due to the more severe inter-cell interference. Moreover under each power limitation the power efficiency degrades versus the cell number. The reason is that with the cell number increasing the inter-cell interference introduced to each cell is larger, and consequently the cell capacity is reduced under the same power transmission.

E. Convergency Performance Evaluation of Inter-cell Power Control

In this subsection, we analyze the convergency performance of Algorithm 2 in UDN. The downlink and uplink power of each cell is set to be 40 dBm and 35 dBm, respectively. The average self-interference gain is -100 dB. The cell number is chosen as 300 and 500. Fig. 11 shows the iteration procedure of $x_{m,n}^{\mathrm{dw}}$. It can be seen that when the cell number is not larger than 500 the total number of iteration times is smaller than 10. The required iteration times degrade with the cell number due to the reduction of the amount of the optimization variables. The results verify the fast convergency performance of Algorithm 2 in UDN.

VII. Conclusion

The paper proposed an efficient scheme of resource allocation in FDUDN, which was divided into inter-cell and intra-cell allocation. The inter-cell allocation was composed of subchannel allocation and power control. The subchannel allocation was performed by a novel approach to reduce the inter-cell interference and maximize the total network capacity. The power control was formulated as a non-convex problem

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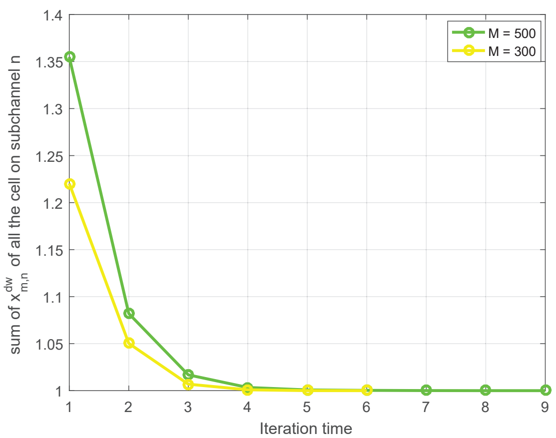

Fig. 11. Convergency of inter-cell power control.

and a novel approach based on variable substitution and water-filling allocation was proposed. The intra-cell allocation was solved by a time-sharing relaxation based algorithm with much lower complexity. The scheme aimed to maximize the total capacity of all the small cells, and the simulation results demonstrated the enhancement effect of the proposed scheme in terms of the capacity, spectrum efficiency and power efficiency.

APPENDIX A
PROOF OF LEMMA 1

As the number of small cells in UDN are large, the received total inter-cell interference in each small cell computed by the statistical inter-cell gain is close to the actual value. Hence we use the statistical inter-cell gain to compute the inter-cell interference sum for each cell. We calculate the average gain to obtain the statistical gain. The average gain of cell m's interference to other cells is denoted by $k_{m,n}$ and the expression of $k_{m,n}$ is achieved as

$$k_{m,n} = \frac{\sum\limits_{m' \neq m, m' \in \mathcal{M}_n} h_{m,m',n}}{M}. \tag{47}$$

Also $k_{m',n}$, which means the average gain of cell m' to other cells, is calculated by

$$k_{m',n} = \frac{\sum\limits_{m \neq m', m \in \mathcal{M}_n} h_{m',m,n}}{M}. \tag{48}$$

Accordingly, the objective function in (9) can be transformed into

$$C = \sum_{m=1}^{M} \sum_{n \in \mathcal{N}_m} \left(\begin{array}{c} \log_2 \left(1 + \frac{h_{m,n}}{k_{m,n}} \cdot \frac{p_{m,n}^{\mathrm{up}} k_{m,n}}{\sum\limits_{m' \neq m} p_{m',n}^{\mathrm{up}} k_{m',n} + \delta_{m,n}^{\mathrm{dw}}} \right) \\ + \log_2 \left(1 + \frac{h_{m,n}}{k_{m,n}} \cdot \frac{p_{m,n}^{\mathrm{dw}} k_{m,n}}{\sum\limits_{m' \neq m} p_{m',n}^{\mathrm{dw}} k_{m',n} + \delta_{m,n}^{\mathrm{up}}} \right) \end{array} \right). \tag{49}$$

As M is large enough, we adopt an approximate substitution which uses $\sum\limits_{m'=1}^{M} p_{m',n}^{\mathrm{up}} k_{m',n}$ to substitute $\sum\limits_{m' \neq m} p_{m',n}^{\mathrm{up}} k_{m',n}$. Then (49) is transformed into

$$C = \sum_{m=1}^{M} \sum_{n \in \mathcal{N}_m} \left(\begin{array}{c} \log_2 \left(1 + \frac{h_{m,n}}{k_{m,n}} \cdot \frac{p_{m,n}^{\mathrm{up}} k_{m,n}}{\sum\limits_{m'=1}^{M} p_{m',n}^{\mathrm{up}} k_{m',n} + \delta_{m,n}^{\mathrm{dw}}} \right) \\ + \log_2 \left(1 + \frac{h_{m,n}}{k_{m,n}} \cdot \frac{p_{m,n}^{\mathrm{dw}} k_{m,n}}{\sum\limits_{m'=1}^{M} p_{m',n}^{\mathrm{dw}} k_{m',n} + \delta_{m,n}^{\mathrm{up}}} \right) \end{array} \right). \tag{50}$$

We use new variables $x_{m,n}^{\mathrm{up}}$ and $x_{m,n}^{\mathrm{dw}}$, which are indicated in (18) and (19), to substitute the corresponding parts of expression (50), and (17) can be obtained.

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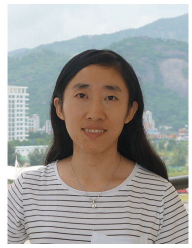
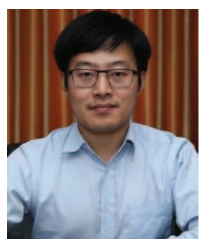


Guobin Zhang received the B.S. degree from Shandong University, in 2001, and the M.S. and Ph.D. degrees from Nanjing University of Posts and Telecommunications, in 2004, and South China University of Technology, in 2011, respectively. She is currently an assistant professor in Dongguan University of Technology, China. From 2018 to now she held the visiting scholar position with the Department of Electrical and Computer Engineering, University of Houston. Her research interests include wireless resource allocation, radio access management, optimization theory and mechine learning.



Haijun Zhang (M'13, SM'17) is currently a Full Professor in University of Science and Technology Beijing, China. He was a Postdoctoral Research Fellow in Department of Electrical and Computer Engineering, the University of British Columbia (UBC), Canada. He serves as Editor of IEEE Transactions on Communications, IEEE Transactions on Green Communications Networking, IEEE Communications Letters, and IEEE Future Networks Tech Focus. He received the IEEE CSIM Technical Committee Best Journal Paper Award in 2018 and IEEE ComSoc Young Author Best Paper Award in 2017.

Zhu Han (S'01, M'04, SM'09, F'14) received the B.S. degree in electronic engineering from Tsinghua University, in 1997, and the M.S. and Ph.D. degrees in electrical and computer engineering from the University of Maryland, College Park, in 1999 and 2003, respectively.

From 2000 to 2002, he was an R&D Engineer of JDSU, Germantown, Maryland. From 2003 to 2006, he was a Research Associate at the University of Maryland. From 2006 to 2008, he was an assistant professor at Boise State University, Idaho. Currently, he is a Professor in the Electrical and Computer Engineering Department as well as in the Computer Science Department at the University of Houston, Texas. His research interests include wireless resource allocation and management, wireless communications and networking, game theory, big data analysis, security, and smart grid. Dr. Han received an NSF Career Award in 2010, the Fred W. Ellersick Prize of the IEEE Communication Society in 2011, the EURASIP Best Paper Award for the Journal on Advances in Signal Processing in 2015, IEEE Leonard G. Abraham Prize in the field of Communications Systems (best paper award in IEEE JSAC) in 2016, and several best paper awards in IEEE conferences. Currently, Dr. Han is an IEEE Communications Society Distinguished Lecturer.

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George K. Karagiannidis (M'96, SM'03, F'14) was born in Pithagorion, Samos Island, Greece. He received the University Diploma (5 years) and Ph.D. degree, both in electrical and computer engineering from the University of Patras, in 1987 and 1999, respectively. From 2000 to 2004, he was a Senior Researcher at the Institute for Space Applications and Remote Sensing, National Observatory of Athens, Greece. In June 2004, he joined the faculty of Aristotle University of Thessaloniki, Greece, where he is currently Professor in the Electrical and Computer Engineering Department and Director of Digital Telecommunications Systems and Networks Laboratory. He is also Honorary Professor at South West Jiaotong University, Chengdu, China.

His research interests are in the broad area of Digital Communications Systems and Signal processing, with emphasis on Wireless Communications, Optical Wireless Communications, Wireless Power Transfer and Applications, Communications for Biomedical Engineering, Stochastic Processes in Biology and Wireless Security.

Dr. Karagiannidis has been involved as General Chair, Technical Program Chair and member of Technical Program Committees in several IEEE and non-IEEE conferences. In the past, he was Editor in IEEE Transactions on Communications, Senior Editor of IEEE Communications Letters, Editor of the EURASIP Journal of Wireless Communications and Networks and several times Guest Editor in IEEE Selected Areas in Communications. From 2012 to 2015 he was the Editor-in Chief of IEEE Communications Letters.

Dr. Karagiannidis is one of the highly-cited authors across all areas of Electrical Engineering, recognized from Clarivate Analytics as Web-of-Science Highly-Cited Researcher in the four consecutive years 2015-2018.