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On the Uplink Sum Rate of SCMA System with Randomly Deployed Users

Zheng Yang, Xianfu Lei, *Member, IEEE,* Zhiguo Ding, *Senior Member, IEEE,* Pingzhi Fan, *Fellow, IEEE,* and George K. Karagiannidis, *Fellow, IEEE*

***Abstract*—We study the average sum rate of uplink sparse code multiple access (SCMA) systems, in a regular hexagon cell with uniformly distributed users. The theoretical results demonstrate that SCMA can achieve higher average sum rate than orthogonal frequency division multiple access (OFDMA) and so, it can be efficiently used in fifth generation (5G) wireless networks. Finally, Monte Carlo simulations are applied to validate the accuracy of the theoretical analysis.**

***Index Terms*—Sparse code multiple access, stochastic geometry, average sum rate.**

I. Introduction

RECENTLY, sparse code multiple access (SCMA) has received special attention, as a potential air interface technology that can satisfy the requirements of fifth generation (5G) networks, such as massive connectivity, ultra-low latency, and high throughput [1]. Specifically, SCMA is a multi-dimensional modulation scheme, where coded bits are directly mapped to complex sparse codewords [2]. Particularly, thanks to the sparsity of SCMA codewords, the receiver can utilize message passing algorithm (MPA) to detect users' signals with a near-optimal quality of detection, even if the number of the users is larger than the spreading factor [2].

There are a few works in the open literature on different design aspects of a SCMA system. More specifically, based on lattice constellations, a sub-optimal SCMA codebook design was proposed in [3]. In order to further improve the system throughput, power sharing and user pairing have been investigated in multi-user downlink SCMA systems [4]. In [5], the average sum rate of downlink single cell non-orthogonal multiple access system is investigated, where users randomly deployed in a disk. Recently, the authors of the present paper have studied the impact of factor graph on the performance of SCMA systems, assuming that distances between users and base station (BS) are fixed [6].

Z. Yang was with the Institute of Mobile Communications, Southwest Jiaotong University, Chengdu 610031, China, and is now with the College of Photonic and Electronic Engineering, Fujian Normal University, Fuzhou 350007, China. (email: zyfjnu@163.com).

X. Lei and P. Fan are with the Institute of Mobile Communications, Southwest Jiaotong University, Chengdu 610031, China. (emails: xflei81@gmail.com, p.fan@ieee.org)(Corresponding author: Xianfu Lei).

Z. Ding is with the School of Computing and Communications, Lancaster University, LA1 4YW, UK. (e-mail: z.ding@lancaster.ac.uk).

G. K. Karagiannidis is with the Provincial Key Laboratory of Information Coding and Transmission, Southwest Jiaotong University, Chengdu 610031, China, and also with the Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, Thessaloniki GR-54124, Greece (e-mail: geokarag@auth.gr).

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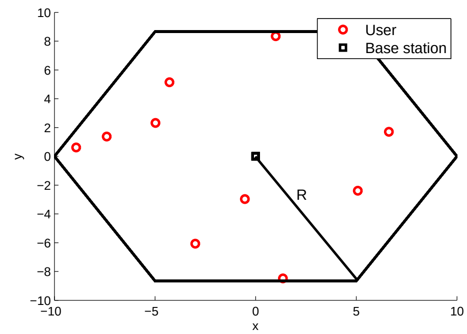

Fig. 1. The network model: users are random deployed in a hexagon cell and base station is located at the origin.

In this letter, we assume that the mobile users are randomly deployed in a realistic regular hexagon cell, with the BS located at the origin. Its contribution can be summarized as follows:

- We study the average sum rate of uplink SCMA systems, and the challenge of this work is to characterize the probability density function (PDF) of the distance between the users and the BS, when a bounded path loss model without singularity is assumed. Thanks to stochastic geometry, taking into account both small-scale fading and large-scale path loss, exact analytical expressions for the average sum rate are derived.
- For comparison purposes, an analytical expression for the average sum rate of orthogonal frequency division multiple access (OFDMA) systems is also presented. The analytical results demonstrate that SCMA can achieve better performance than OFDMA and so it can be efficiently used in fifth generation (5G) wireless networks. Finally, Monte Carlo simulations are also presented to verify the accuracy of the analytical results under different system settings.

II. System Model

Consider a single-cell uplink SCMA system, where BS is located at the center of a regular hexagon cell $\mathcal{R}$ with a hexagon side length of R, and M users are uniformly distributed inside $\mathcal{R}$, as shown in Fig. 1. All users and the BS are assumed to be equipped with a single antenna. Furthermore, we assume that the M users are multiplexed over K subcarriers. According

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to the SCMA, the system should be overloaded, i.e., $M \geq K$. The transmitted symbols of the m-th user is denoted by $\mathbf{x}_m = [x_{m1}\ x_{m2} \cdots x_{mK}]^T$, which is a sparse vector with N non-zero symbols. Let $\mathbf{y} = [y_1\ y_2 \cdots y_K]^T$ be the received signal at the BS. Then,

$$\mathbf{y} = \sum_{m=1}^{M} \sqrt{P/N}\mathrm{diag}(\mathbf{f}_m)\mathrm{diag}(\mathbf{h}_m)\mathbf{x}_m + \mathbf{w}, \tag{1}$$

where P denotes user's transmit power, which is considered to be equally allocated to N subcarriers. The channel coefficient vector associated with the m-th user is $\mathbf{h}_m = [h_{m1}\ h_{m2} \cdots h_{mK}]^T$, where $h_{mk} = g_{mk}/d_m^{\frac{\alpha}{2}}$, and g_{mk} is the Rayleigh fading channel gain from the m-th user to the k-th subcarrier with $g_{mk} \sim \mathcal{CN}(0,1)$, d_m is the distance from the m-th user to the BS, and α denotes the path loss factor. Furthermore, $\mathbf{f}_m = [f_{m1}\ f_{m2} \cdots f_{mK}]^T$, is the subcarrier allocation index associated with the m-th user, with $f_{mk} = 1$, if $x_{mk} > 0$, and $f_{mk} = 0$, if $x_{mk} = 0$. Finally, $\mathbf{w} \sim \mathcal{CN}(0, \mathbf{I})$ is the additive white Gaussian noise (AWGN).

It is well known that multiple-access channel can achieve the Shannon capacity, by using successive interference cancellation (SIC) with the ideal users' codes [7]. In this work, we would like to investigate the performance limit of the SCMA systems so that we pay attention to the information theoretical sum rate, while the optimal channel codes and optimal SCMA codebooks are assumed. The analysis can serve as a baseline for the performance with practical codes. Therefore, the sum rate of each subcarrier is equivalent to that of the multiple access channel (MAC), and can be written as [7], [8]

$$R_k = \log_2\Big(1 + \frac{P}{N}\sum_{m\in\tau_k}|h_{mk}|^2\Big), \tag{2}$$

where τ_k is the set of the users that are active in k-th subcarrier.

For the K subcarriers, the average sum rate of SCMA is then given by

$$\overline{R} = \sum_{k=1}^{K}\mathbf{E}\left(\log_2\Big(1 + \frac{P}{N}\sum_{m\in\tau_k}|h_{mk}|^2\Big)\right). \tag{3}$$

In this work, we consider both small-scale fading and large-scale path loss. Particularly, the following realistic path loss model is considered.

Definition 1: The path loss function of the Euclidean distance $d = \parallel x - y \parallel$, between x and y is [9]

$$l(d) = \begin{cases} d^{-\alpha}, & \text{if } d > R_0; \\ R_0^{-\alpha}, & \text{otherwise}, \end{cases} \tag{4}$$

where $R_0 > 0$ is the reference distance, i.e., a guard zone around the BS.

It can be observed from (4) that when the Euclidean distance, d, between the user and the BS is large, e.g., $d > R_0$, the path loss function, $l(d) = d^{-\alpha}$, is a function of d. For this case, we derive the PDF of d in the following Lemma.

Lemma *1:* When the distance, d, from the center to uniformly distributed node is constrained as, $R_0 < d \leq R$, then the PDF of d is given by

$$f(d) = \begin{cases} \frac{2\pi d}{S}, & R_0 < d \leq \frac{\sqrt{3}R}{2}; \\ \frac{6d}{S}\left(\frac{\pi}{3} - 2\arccos\left(\frac{\sqrt{3}R}{2d}\right)\right), & \frac{\sqrt{3}R}{2} < d \leq R, \end{cases} \tag{5}$$

where $S = 3\sqrt{3}R^2/2 - \pi R_0^2$.

Proof: Following similar steps as in [10], (5) can be easily derived and the proof of Lemma 1 is completed. ■

Note that the difference between (5) and [10, eq. (5)] is that we remove the area of the disk with radius R_0 in the hexagonal.

III. Average sum rate of uplink SCMA systems

Next, based on the path loss model in (4) and the PDF of d in (5), exact analytical expressions are presented for the average sum rates of SCMA and OFDMA.

Theorem *1:* The average sum rate of the uplink SCMA system is given by

$$\begin{aligned} \overline{R_S} &= \frac{1}{\ln 2}\sum_{k=1}^{K}\int_0^\infty \frac{e^{-s}}{s}\Bigg[1 - \Bigg(\frac{a_1}{sP/N + R_0^\alpha} \\ &+ \frac{a_2 N}{sP}\Big(R^{\alpha+2}H_\alpha\Big(\frac{NR^\alpha}{sP}\Big) - R_0^{\alpha+2}H_\alpha\Big(\frac{NR_0^\alpha}{sP}\Big)\Big) \\ &- \frac{8}{\sqrt{3}R^2}\int_{\frac{\sqrt{3}R}{2}}^{R}\frac{x^{\alpha+1}\arccos\left(\frac{\sqrt{3}R}{2x}\right)}{sP/N + x^\alpha}dx\Bigg)^{|\tau_k|}\Bigg]ds, \end{aligned} \tag{6}$$

where $H_\alpha(x) = {}_2F_1\big(1, 2/\alpha+1; 2/\alpha+2; -x\big)$ is a specific form of the Gauss hypergeometric function [11, eq. (9.111)], $|\tau_k|$ is the number of users that are active on the k-th subcarrier, $a_1 = 2\pi R_0^{2+\alpha}/(3\sqrt{3}R^2)$, and $a_2 = 4\pi/(3\sqrt{3}R^2(\alpha+2))$.

Proof: See Appendix A. ■

Although (6) is an exact analytical expression, it does not provide a useful insight for the sum rate. However, when the composite channel model is degraded to Rayleigh fading, the result in (6) reduces to a closed-form expression as follows.

Corollary 1: When $R = R_0$, i.e., the channels from users to the BS are Rayleigh fading, and the average sum rate $\overline{R_S^0}$ of uplink SCMA is given by

$$\begin{aligned} \overline{R_S^0} = &\frac{e^{\frac{R^\alpha N}{P}}}{\ln 2}\sum_{k=1}^{K}\sum_{i=1}^{|\tau_k|}\sum_{j=0}^{i-1}\binom{|\tau_k|}{i}\binom{i-1}{j}(-1)^{i-1-j} \\ &\times \mathrm{E}_{|\tau_k|-j}\Big(\frac{R^\alpha N}{P}\Big), \end{aligned} \tag{7}$$

where $\mathrm{E}_n(x)$ is the exponential integral [11].

Proof: See Appendix B. ■

Next, for comparison purposes, we also derive the average sum rate for OFDMA systems, where K subcarriers can support K users at most, due to its orthogonality nature. To ensure the fairness of the comparison, the transmit power of each user in OFDMA systems is, $P_o = PM/K$, so that the total transmit power for SCMA and OFDMA systems to be the same.

Theorem *2:* The average sum rate for OFDMA systems is given by

$$\begin{aligned} \overline{R_O} = &\frac{1}{\ln 2}\sum_{k=1}^{K}\int_0^\infty \frac{e^{-s}}{s}\Bigg[1 - \Bigg(\frac{a_1}{sPM/K + R_0^\alpha} \\ &+ \frac{a_2 K}{sPM}\Big(R^{\alpha+2}H_\alpha\Big(\frac{KR^\alpha}{sPM}\Big) - R_0^{\alpha+2}H_\alpha\Big(\frac{KR_0^\alpha}{sPM}\Big)\Big) \\ &- \frac{8}{\sqrt{3}R^2}\int_{\frac{\sqrt{3}R}{2}}^{R}\frac{x^{\alpha+1}\arccos\left(\frac{\sqrt{3}R}{2x}\right)}{sPM/K + x^\alpha}dx\Bigg)\Bigg]ds. \end{aligned} \tag{8}$$

Proof: Following the same procedure as in Theorem 1, (8) can be easily derived and the proof is completed. ■

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Note that (8) has the same form with (6). The reason is that OFDMA can be regarded as a special case of SCMA, when there is only one user for each subcarrier.

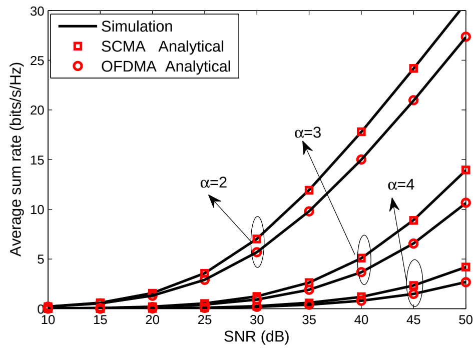

Fig. 2. Average sum rates of different multiple access schemes.

IV. Numerical Results and Discussion

In this section, we present some computer simulation results to evaluate the theoretical analysis. The parameters used in the simulations are set as follows. The hexagon side length is, $R = 50\ m$, the reference distance, $R_0 = 5\ m$, and the numbers of users and subcarriers are $M = 6$ and $K = 4$, respectively. In Fig. 2, we assume the dimension of the codewords to be, $N = 2$, and the factor graph is given by [2]

$$\mathbf{F} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}. \tag{9}$$

The average sum rates of the multiple access schemes as a function of the SNR are depicted in Fig. 2. As it is evident from this figure, the average sum rate of SCMA is superior to that of OFDMA, especially in the high SNR regime. This is because there are more than one user overloaded on each subcarrier in SCMA, which improves the sum rate, compared to OFDMA systems, where each subcarrier is utilised by a single user. Furthermore, as expected, the larger the path loss factor α is, the worse of the average sum rate of the multiple access schemes becomes. In addition, it can be observed that the analytical results given by (6) and (8) match quite well with the Monte Carlo simulations.

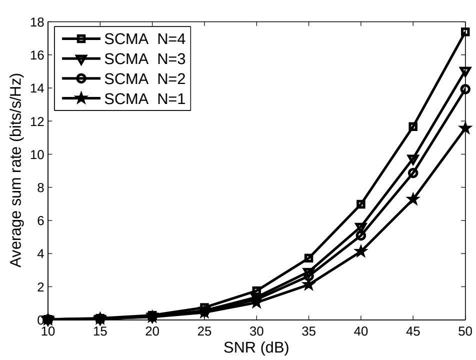

Fig. 3. Effect of N on the average sum rate of uplink SCMA systems with $\alpha = 3$.

The effect of the number of the non-zero symbols, N, on the average sum rate of uplink SCMA is demonstrated in Fig. 3. It can be observed that the average sum rate is a monotonically increasing function with respect to N. The reason is that the number of overloaded users on each subcarrier increases, as N gets larger, and thus τ_k also increase in (3). On the other hand, the implementation complexity of SCMA is higher than that of OFDMA, due to its complex detection process. In SCMA systems, the multi-user interference on each subcarrier needs to be eliminated, by applying the advanced receiver [2]. Therefore, as the number of subcarrier N increases, the complexity of SCMA becomes higher as well. However, this complexity is moderate and can be manageable at the receiver [2].

V. Conclusions

In this letter, we have derived an analytical expression for the average sum rate of SCMA systems, and theoretically have shown that the performance of SCMA is superior to OFDMA. The PDF of the random distances between the users and the BS derived in this letter could be applied into the multiple input multiple output (MIMO) systems. However, due to the precoding schemes used for MIMO, the PDF of the combined channel gain will be more complex and makes the analysis more challenging. We will consider this in our future work.

Appendix A

Proof of Theorem 1

The average rate of R_k in (2) is given by [12]

$$\begin{aligned} \overline{R_k} &= \mathbf{E}\Big(\log_2\left(1 + PI_k/N\right)\Big) \\ &= \frac{1}{\ln 2}\int_0^{\infty} \frac{e^{-s}}{s}\Big(1 - \mathcal{L}_{I_k}(sP/N)\Big)ds, \end{aligned} \tag{10}$$

where $\mathcal{L}_{I_k}(\cdot)$ denotes the Laplace transform, and $I_k = \sum_{m\in\tau_k} |h_{mk}|^2$.

The term, $\mathcal{L}_{I_k}(sP/N)$, in (10) can be evaluated as

$$\begin{aligned} \mathcal{L}_{I_k}(sP/N) &= \mathbf{E}\Big(\exp\left(-sPI_k/N\right)\Big) \\ &= \mathbf{E}\Big(\exp\Big(-s\frac{P}{N}\sum_{m\in\tau_k}|h_{mk}|^2\Big)\Big) \\ &= \mathbf{E}\Big(\prod_{m\in\tau_k}\exp\Big(-s\frac{P}{N}|h_{mk}|^2\Big)\Big) \\ &\overset{(a)}{=} \Big(\mathbf{E}\Big(\exp\left(-sP|h_{mk}|^2/N\right)\Big)\Big)^{|\tau_k|} \\ &\triangleq \Big(\mathcal{L}_{|h_{mk}|^2}(sP/N)\Big)^{|\tau_k|}, \end{aligned} \tag{11}$$

where (a) follows from the fact that all users with access to subcarrier k are independent and identically distributed (i.i.d), and $|\tau_k|$ is the number of elements in τ_k.

In order to evaluate $\mathcal{L}_{|h_{mk}|^2}(sP/N)$ in (11), the PDF of $|h_{mk}|^2$ is needed. Based on the path loss function in (4) we mentioned above, the complementary cumulative distribution function (CCDF) of $|h_{mk}|^2 = |g_{mk}|^2 d_m^{-\alpha}$ can be expressed as

$$\begin{aligned} \overline{F}_Z(z) &= \Pr\{|g_{mk}|^2 d_m^{-\alpha} > z\} \\ &= \begin{cases} e^{-zR_0^{\alpha}}, & 0 < d \le R_0; \\ \int_{R_0}^{R} e^{-zx^{\alpha}} f_d(x)dx, & R_0 < d \le R. \end{cases} \end{aligned} \tag{12}$$

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By using (5), $\overline{F}_Z(z)$ in (12) can be further evaluated as

$$\begin{aligned}\overline{F}_Z(z) &= \Pr\{0 < d \le R_0\} e^{-zR_0^\alpha} \\ &\quad + \Pr\{R_0 < d \le R\} \int_{R_0}^{R} e^{-zx^\alpha} f_d(x) dx \\ &= p_1 e^{-zR_0^\alpha} + (1-p_1) \int_{R_0}^{R} e^{-zx^\alpha} f_d(x) dx, \quad (13)\end{aligned}$$

where $\Pr\{0 < d \le R_0\} \triangleq p_1 = 2\pi R_0^2/(3\sqrt{3}R^2)$ is the probability that a user is located in the disk of radius R_0, and $\Pr\{R_0 < d \le R\} \triangleq 1 - p_1$.

Calculating the derivative of the $\overline{F}_Z(z)$ in (13), the PDF of $|h_{mk}|^2$ is given by

$$\begin{aligned}f_Z(z) &= p_1 R_0^\alpha e^{-zR_0^\alpha} + (1-p_1) \int_{R_0}^{R} x^\alpha e^{-zx^\alpha} f_d(x) dx \\ &= \frac{2\pi R_0^{2+\alpha}}{3\sqrt{3}R^2} e^{-zR_0^\alpha} + \frac{4\pi}{3\sqrt{3}R^2} \int_{R_0}^{R} x^{\alpha+1} e^{-zx^\alpha} dx \\ &- \frac{8}{\sqrt{3}R^2} \int_{\frac{\sqrt{3}R}{2}}^{R} x^{\alpha+1} e^{-zx^\alpha} \arccos\Big(\frac{\sqrt{3}R}{2x}\Big) dx. \quad (14)\end{aligned}$$

Now, by using (14),

$$\begin{aligned}&\mathcal{L}_{|h_{mk}|^2}(sP/N) \\ &= \int_0^\infty \exp\big(-sPz/N\big) f_Z(z) dz \\ &= \frac{2\pi R_0^{2+\alpha}}{3\sqrt{3}R^2} \int_0^\infty e^{-\frac{Psz}{N}} e^{-zR_0^\alpha} dz \\ &+ \frac{4\pi}{3\sqrt{3}R^2} \int_0^\infty e^{-\frac{Psz}{N}} \int_{R_0}^{R} x^{\alpha+1} e^{-zx^\alpha} dx dz \\ &- \frac{8}{\sqrt{3}R^2} \int_0^\infty e^{-\frac{Psz}{N}} \int_{\frac{\sqrt{3}R}{2}}^{R} x^{\alpha+1} e^{-zx^\alpha} \arccos\Big(\frac{\sqrt{3}R}{2x}\Big) dx dz \\ &= \frac{2\pi R_0^{2+\alpha}}{3\sqrt{3}R^2} \frac{1}{sP/N + R_0^\alpha} + \frac{4\pi}{3\sqrt{3}R^2} \underbrace{\int_{R_0}^{R} \frac{x^{\alpha+1}}{sP/N + x^\alpha} dx}_{Q_1} \\ &- \frac{8}{\sqrt{3}R^2} \underbrace{\int_{\frac{\sqrt{3}R}{2}}^{R} \frac{x^{\alpha+1}}{sP/N + x^\alpha} \arccos\Big(\frac{\sqrt{3}R}{2x}\Big) dx}_{Q_2}. \quad (15)\end{aligned}$$

The integral Q_1 in (15) can be evaluated as

$$\begin{aligned}Q_1 &= \int_{R_0}^{R} \frac{x^{\alpha+1}}{sP/N + x^\alpha} dx \\ &= \int_0^{R} \frac{x^{\alpha+1}}{sP/N + x^\alpha} dx - \int_0^{R_0} \frac{x^{\alpha+1}}{sP/N + x^\alpha} dx. \quad (16)\end{aligned}$$

By using [11, eq. (9.111)], the integrals, Q_{1i} $(i = 1, 2)$ in (16) can be evaluated as

$$\begin{aligned}Q_{1i} &= \frac{Nr^{\alpha+2}}{\alpha sP} \int_0^1 \frac{t^{\frac{2}{\alpha}}}{1 + \frac{Nr^\alpha}{sP} t} dt \\ &= \frac{Nr^{\alpha+2}}{(\alpha+2)sP} {}_2F_1\Big(1, 1+\frac{2}{\alpha}; 2+\frac{2}{\alpha}; \frac{-Nr^\alpha}{sP}\Big). \quad (17)\end{aligned}$$

It seems that it is difficult to evaluate the integral Q_2 in closed-form in (15), since it contains the arc-cosine function. However, it can be numerically evaluated through well-known mathematical packages, as Matlab or Mathematica. Substituting (16) and (17) into (10), the proof is completed.

APPENDIX B
PROOF OF COROLLARY 1

When $R = R_0$, the PDF of $|h_{mk}|^2$ in (14) is $f_Z(z) = R^\alpha e^{-zR^\alpha}$. The term, $\mathcal{L}_{|h_{mk}|^2}(sP/N)$, in (11), can be then written as

$$\mathcal{L}_{|h_{mk}|^2}(sP/N) = \frac{1}{sP/(R^\alpha N) + 1}. \quad (18)$$

By using (10) and (18),

$$\begin{aligned}\overline{R_k} &= \frac{1}{\ln 2} \int_0^\infty \frac{e^{-s}}{s} \Big(1 - \Big(\frac{1}{sP/(R^\alpha N) + 1}\Big)^{|\tau_k|}\Big) ds \quad (19) \\ &= \frac{1}{\ln 2} \sum_{i=1}^{|\tau_k|} \binom{|\tau_k|}{i} \Big(\frac{P}{R^\alpha N}\Big)^i \underbrace{\int_0^\infty \frac{e^{-s} s^{i-1} ds}{\big(sP/(R^\alpha N) + 1\big)^{|\tau_k|}}}_{Q_3}.\end{aligned}$$

Let, $x = sP/(R^\alpha N) + 1$, then the integral Q_3 in (19) can be further processed as

$$\begin{aligned}Q_3 &= a_3^i e^{a_3} \int_1^\infty \frac{e^{-a_3 x}(x-1)^{i-1} ds}{x^{|\tau_k|}} \\ &= a_3^i e^{a_3} \sum_{j=0}^{i-1} \binom{i-1}{j} (-1)^{i-1-j} \int_1^\infty \frac{e^{-a_3 x} ds}{x^{|\tau_k| - j}} \\ &= a_3^i e^{a_3} \sum_{j=0}^{i-1} \binom{i-1}{j} (-1)^{i-1-j} \mathrm{E}_{|\tau_k| - j}(a_3), \quad (20)\end{aligned}$$

where $a_3 = R^\alpha N/P$. Substituting (20) into (19), the proof is completed.

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