

# TWO-WAY INTERFERENCE-LIMITED AF RELAYING WITH SELECTION-COMBINING

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## ABSTRACT

We investigate the performance of two-way interference-limited amplify-and-forward (AF) relaying systems with selection-combining (SC) over Nakagami- $m$  fading channels. In particular, a tight lower bound on the end-to-end outage probability (OP) is derived in closed-form, while a useful expression is presented for the asymptotically low outage regime. Some special cases of practical interest (e.g., no interference power and Rayleigh fading channels) are also studied. The numerical results provide important physical insights into the implications of model parameters on the system performance.

**Index Terms**— Amplify-and-forward, interference limited systems, outage probability, two-way selective relaying.

## 1. INTRODUCTION

The development of cooperative diversity schemes has been motivated by the high data rate demands of wireless applications. In this context, in amplify-and-forward (AF) relaying schemes, the relay just amplifies the received signal before sending it to the destination (without doing any demodulating and decoding of the received signal) [1]. On this basis, AF relaying systems have low implementation complexity and are easy to deploy [2]. Furthermore, AF relaying has been adopted in the LTE-A standard by the 3GPP group [3].

In several recent works, co-channel interference (CCI) has been a dominant factor in the context of wireless relaying (see e.g., [4–7] and references therein). In two-way relaying systems and during the first time slot, two nodes transmit simultaneously to the relay, and the relay will broadcast data to the designated destinations in the second time slot. In [8], the optimal outage performance of a two-way relaying network in the interference-free case was investigated. The authors in [9] obtained the outage probability (OP) and symbol error probability (SEP) of interference-limited systems over Rayleigh fading channels, where they worked with the upper bound of the harmonic mean. Furthermore, in [10], the authors examined the outage performance of dual-hop AF relaying systems with CCI over independent, non-identically distributed (i.n.i.d.) Nakagami- $m$  fading channels, while they extended

their work to two-way relaying systems in [11], where they considered the impact of interference only at the relay; in addition, they approximated the probability distribution function (PDF) of the sum of interferers' powers by a gamma random variable (RV). In [12], the authors examined the OP of two-way AF relaying systems with CCI over Nakagami- $m$  fading channels, where the relay was not subject to interference. Opportunistic relaying or selective relaying is a method to enhance the performance of cooperative system by selecting one relay to transmit. In [13–20], authors investigated different performance metrics of the two-way interference-free selective relaying systems. To the best of our knowledge, there is no work considering two-way interference-limited AF relaying systems with selection-combining. As a matter of fact, the most important differences between the work presented here and [11, 12] are: 1) In [12], only one relay exists in the network; moreover closed-form results were derived only for Rayleigh fading channels, 2) In [11], only one relay exists in the network, while the relay gain does not contain the interferers' effect.

The contributions of this paper can be summarized as follows:

- We consider a two-way multi-relay dual-hop configuration, where the source nodes are subject to noise only, while relays are affected by multiple interferers. All channels are assumed to experience Nakagami- $m$  fading [21]. For this scenario, a tight closed-form lower bound on the OP is derived.
- In order to get some additional insights into the impact of system parameters, we consider the asymptotically low outage regime, where the diversity order is obtained. Finally, we particularize our results to some cases of practical interest.

*Notation:* Throughout this paper, we use  $f_h(\cdot)$  and  $F_h(\cdot)$  to denote the PDF and cumulative distribution function (CDF) of a RV  $h$ , respectively. Also  $\Gamma(n) = \int_0^\infty e^{-t} t^{n-1} dt$  is the gamma function [22, Eq. (8.310.1)], and  $\Gamma(b, x) = \int_x^\infty e^{-t} t^{b-1} dt$  and  $\gamma(b, x) = \Gamma(n) - \Gamma(b, x)$  are the upper and lower incomplete gamma functions [22, Eq. (8.350.2, 8.350.1)], respectively. The operator  $E[\cdot]$  stands for expectation, while  $\Pr(\cdot)$  denotes probability.

## 2. SYSTEM MODEL AND FADING STATISTICS

We consider a cooperative relaying system that is composed of two single-antenna source nodes ( $S1$  and  $S2$ ), which exchange information via the  $k$ -th relay  $R_k$  where  $k = 1, \dots, K$ . Moreover,  $R_k$  experiences CCI from  $N_k$  users in the network. Additionally,  $f_k$  is the channel coefficient between  $S1$  and  $R_k$  and vice versa (i.e., the  $S1 \rightarrow R_k$  and  $R_k \rightarrow S1$  links) and  $g_k$  is the channel coefficient between  $S2$  and  $R_k$  which is reciprocal (i.e., the  $S2 \rightarrow R_k$  and  $R_k \rightarrow S2$  links). Also,  $h_{k,i}$  is the channel coefficient between  $R_k$  and the  $i$ -th ( $i = 1, \dots, N_k$ ) interferer at  $R_k$ . Additionally,  $P_k$ ,  $P_{s_1}$  and  $P_{s_2}$  are the transmitted powers of  $R_k$ ,  $S1$  and  $S2$ , respectively. Furthermore,  $P_{k,i}$  is the power of the  $i$ -th CCI signal impairing  $R_k$ , and  $\sigma^2$  denotes the noise variance at all nodes. Hence, the instantaneous SNRs for the  $S1 \rightarrow R_k$ ,  $S2 \rightarrow R_k$ ,  $R_k \rightarrow S1$ , and  $R_k \rightarrow S2$  links are given by  $\gamma_{1k} = \frac{P_{s_1}|f_k|^2}{\sigma^2}$ ,  $\gamma_{2k} = \frac{P_{s_2}|g_k|^2}{\sigma^2}$ ,  $\gamma_{3k} = \frac{P_k|f_k|^2}{\sigma^2}$  and  $\gamma_{4k} = \frac{P_k|g_k|^2}{\sigma^2}$ , respectively. Also the instantaneous interference-to-noise ratio for the  $i$ -th CCI at  $R_k$  is given by  $\gamma_{k,i} = \frac{P_{k,i}|h_{k,i}|^2}{\sigma^2}$ .

As was previously mentioned, in this paper we assume that the amplitude of all links follows the Nakagami- $m$  distribution, where  $m \geq 0.5$  represents the fading severity parameter [23]. We recall that the PDF and CDF of the Nakagami- $m$  fading channels, are given by [21, Eqs. (2.21, 9.272)], respectively. As such, the distribution of the corresponding SNRs are Gamma RVs, where the shape parameter is  $m$  and the scale parameter is  $\Omega/m$ , where  $\Omega$  is the average SNR per symbol. For the case under consideration, the shape parameters of  $f_k$ ,  $g_k$  and  $h_{k,i}$  are respectively  $m_{1k}$ ,  $m_{2k}$  and  $m_{k,i}$ , while the scale parameters are  $1/a_k$ ,  $1/b_k$  and  $1/\beta_k$ . Note that  $a_k \triangleq \frac{m_{1k}}{\Omega_{1k}}$ ,  $b_k \triangleq \frac{m_{2k}}{\Omega_{2k}}$  and  $\beta_k \triangleq \frac{m_{k,i}}{\Omega_{k,i}}$ . The signal received at the relay is as  $y_{kn} = y_k + n$  where  $y_k$  is as follows

$$y_k = \sqrt{P_{s_1}}f_k x_{s1} + \sqrt{P_{s_2}}g_k x_{s2} + \sum_{i=1}^{N_k} \sqrt{P_{k,i}}h_{k,i}x_{k,i} \quad (1)$$

where  $x_{s1}$ ,  $x_{s2}$  and  $x_{k,i}$  are the signals generated from  $S1$ ,  $S2$  and the  $i$ -th interferer affecting the relay, respectively, while  $n$  is the additive white Gaussian noise (AWGN) at the relay. The amplification factor of the  $k$ -th relay is defined as

$$G_k^{-1} \triangleq \sqrt{P_{s_1}|f_k|^2 + P_{s_2}|g_k|^2 + \sum_{i=1}^{N_k} P_{k,i}|h_{k,i}|^2 + \sigma^2}. \quad (2)$$

Since  $S1$  is aware of its transmitted signal, it can perfectly eliminate the self-interference term. The received SINR at  $S1$  from the signal transmitted by  $R_k$  can then be expressed as

$$\gamma_{S1k} = \frac{P_k P_{s_2} G_k^2 |f_k|^2 |g_k|^2}{G_k^2 P_k |f_k|^2 \left( \sum_{i=1}^{N_k} P_{k,i} |h_{k,i}|^2 + \sigma^2 \right) + \sigma^2}. \quad (3)$$

By assuming  $P_{s_1} = P_k = P_s$ , the received SINR at  $S1$  can be further simplified according to

$$\gamma_{S1k} = \frac{\gamma_{1k}\gamma_{2k}}{\gamma_{1k} + \gamma_{2k} + (\gamma_{1k} + 1) \left( \sum_{i=1}^{N_k} \gamma_{k,i} + 1 \right)}. \quad (4)$$

In a similar way the received signal at  $S2$  can be derived, though the expression is omitted due to space limitations.

## 3. PERFORMANCE ANALYSIS

By setting  $\gamma_{rk} \triangleq \sum_{i=1}^{N_k} \gamma_{k,i}$ , the received SINR at  $S1$  can be tightly upper bounded in the interference-limited regime, according to

$$\gamma_{S1k} \leq \frac{\frac{\gamma_{1k}\gamma_{2k}}{\gamma_{rk}+1}}{\frac{\gamma_{1k}+\gamma_{2k}}{\gamma_{rk}+1} + \gamma_{1k}} = \frac{\frac{\gamma_{1k}\gamma_{2k}}{\gamma_{rk}+2}}{\gamma_{1k} + \frac{\gamma_{2k}}{\gamma_{rk}+2}} = \frac{XY}{X+Y} \quad (5)$$

where  $X_k \triangleq \gamma_{1k}$ ,  $Y_k \triangleq \frac{\gamma_{2k}}{\gamma_{rk}+2}$ . Note that a similar expression can be derived for the received SINR at  $S2$  when  $P_{s_2} = P_k = P_s$ . It is well known that the  $\min(X, Y)$  is a tight upper bound of  $\frac{XY}{X+Y}$ ; in fact, as  $X$  and  $Y$  go to infinity the bound becomes exact. Hence, we use this bound for all derivations henceforth. Then, the upper bounded SINR at  $S1$  and  $S2$  can be expressed as

$$\begin{aligned} \gamma_{S1k}^{\text{up}} &= \min \left( \gamma_{1k}, \frac{\gamma_{2k}}{\gamma_{rk}+2} \right), \\ \gamma_{S2k}^{\text{up}} &= \min \left( \gamma_{2k}, \frac{\gamma_{1k}}{\gamma_{rk}+2} \right). \end{aligned} \quad (6)$$

The end-to-end SINR for the  $k$ -th relay of this system can be written as

$$\gamma_{SCk} = \min(\gamma_{S1k}, \gamma_{S2k}) \leq \min(\gamma_{S1k}^{\text{up}}, \gamma_{S2k}^{\text{up}}) \triangleq \gamma_{SCk}^{\text{up}}. \quad (7)$$

Finally, the end-to-end SINR of this system can be written as

$$\gamma_{e2e} = \max_k \gamma_{SCk} \leq \max_k \gamma_{SCk}^{\text{up}} \triangleq \gamma_{e2e}^{\text{up}}. \quad (8)$$

To compute the OP of the end-to-end SINR, we first need to derive the OP of  $X_k$  and  $Y_k$ . In general, it is known (see e.g., [24]) that the sum of  $L$  i.i.d. Gamma RVs with common shape parameter  $k$  and scale parameter  $\theta$  (i.e.  $g(k, \theta)$ ), is also a Gamma RV with parameters  $kL$  and  $\theta$ . Hence, the PDF of  $\gamma_{rk}$  can be written as  $g(m_k, 1/\beta_k)$ , where  $m_k \triangleq \sum_{i=1}^{N_k} m_{k,i}$ .

The CDFs of  $X_k$  and  $Y_k$  are given by:

**Proposition 1** *The CDFs of  $X_k$  and  $Y_k$  are respectively*

$$\begin{aligned} F_{X_k}(z) &= 1 - \frac{\Gamma(m_{1k}, a_k z)}{\Gamma(m_{1k})}, \quad F_{Y_k}(z) = 1 - \frac{\beta_k^{m_k}}{\Gamma(m_k)} e^{-2b_k z} \\ &\times \sum_{i=0}^{m_{2k}-1} \sum_{j=0}^i \binom{i}{j} \frac{(b_k z)^i 2^{i-j}}{i!} \frac{\Gamma(j+m_k)}{(b_k z + \beta_k)^{j+m_k}}. \end{aligned} \quad (9)$$

**Proof 1** See Section 8.1.

From (9), it is clear that  $m_{2k}$  should be integer. After computing the CDFs of  $X_k$  and  $Y_k$ , we now proceed to derive the CDFs of  $\gamma_{S_1k}^{\text{up}}$  and  $\gamma_{S_2k}^{\text{up}}$  via the next proposition.

**Proposition 2** The CDF of  $\gamma_{S_1k}^{\text{up}}$  is given by

$$F_{\gamma_{S_1k}^{\text{up}}}(z) = 1 - \frac{\Gamma(m_{1k}, a_k z)}{\Gamma(m_{1k})} \frac{\beta_k^{m_k}}{\Gamma(m_k)} e^{-2b_k z} \times \sum_{i=0}^{m_{2k}-1} \sum_{t=0}^i \frac{(b_k z)^i 2^{i-t}}{i!} \binom{i}{t} \frac{\Gamma(m_k + t)}{(\beta_k + b_k z)^{m_k+t}}. \quad (10)$$

**Proof 2** Due to space limitations, the proof is omitted.

Note that a similar expression can be found for the CDF of  $\gamma_{S_2k}^{\text{up}}$ . With these results in our hands, we can now evaluate the CDF of the upper bounded end-to-end SINR.

**Proposition 3** The CDF of the upper bounded end-to-end SINR of the  $k$ -th relay,  $\gamma_{S_{Ck}}^{\text{up}}$ , is given by

$$F_{\gamma_{S_{Ck}}^{\text{up}}}(z) = 1 - \frac{e^{-2(a_k+b_k)z}}{2^{t+l-i-j}} \frac{\beta_k^{2m_k}}{(\Gamma(m_k))^2} \sum_{i=0}^{m_{1k}-1} \sum_{j=0}^{m_{2k}-1} \sum_{t=0}^i \sum_{l=0}^j \binom{i}{t} \binom{j}{l} \frac{a_k^i b_k^j z^{i+j}}{i! j!} \frac{\Gamma(m_k + t) \Gamma(m_k + l)}{(a_k z + \beta_k)^{m_k+t} (b_k z + \beta_k)^{m_k+l}}. \quad (11)$$

**Proof 3** See Section 8.2.

We can infer from (11) that  $m_{1k}$  and  $m_{2k}$  should take integer values.

**Proposition 4** The CDF of the upper bounded end-to-end SINR,  $\gamma_{e2e}^{\text{up}}$ , is given by

$$F_{\gamma_{e2e}^{\text{up}}}(z) = \sum_{k=0}^K \frac{(-1)^k}{k!} \sum_{n_1, \dots, n_k} \sum_{i_k}^{m_{1k}-1} \sum_{j_k}^{m_{2k}-1} \sum_{t_k}^{i_k} \sum_{l_k}^{j_k} \prod_{p=1}^k \left[ \binom{i_p}{t_p} \binom{j_p}{l_p} \left( \frac{\beta_{n_p}^{m_{n_p}}}{\Gamma(m_{n_p})} \right)^2 \frac{\Gamma(m_{n_p} + l_p)}{(b_{n_p} z + \beta_{n_p})^{m_{n_p} + l_p}} \times \frac{(a_{n_p})^{i_p} (b_{n_p})^{j_p} (2z)^{\sum_{p=1}^k (i_p + j_p)}}{i_p! j_p! 2^{\sum_{p=1}^k (t_p + l_p)} e^{\sum_{p=1}^k 2(a_{n_p} + b_{n_p})z}} \frac{\Gamma(m_{n_p} + t_p)}{(a_{n_p} z + \beta_{n_p})^{m_{n_p} + t_p}} \right] \quad (12)$$

where  $\sum_{n_1, \dots, n_k} \triangleq \sum_{n_1=1}^K \dots \sum_{n_k=1}^K$ ,  $\sum_{i_k}^{m_{1k}} \triangleq \sum_{i_1=1}^{m_{11}} \dots \sum_{i_k=1}^{m_{1k}}$ .

**Proof 4** See Section 8.3.

Hereafter, we investigate the lower bounded OP for two-way interference-limited systems based on (12). The OP is the probability that either the  $S_1$  to relay link SINR or the  $S_2$  to relay link SINR falls below a certain threshold,  $\gamma_{\text{th}}$ . By using (12), we can now obtain the following lower bound on the exact OP of the system

$$P_{\text{out}}(\gamma_{\text{th}}) \geq P_{\text{out}}^{\text{lb}}(\gamma_{\text{th}}) = F_{\gamma_{e2e}^{\text{up}}}(\gamma_{\text{th}}). \quad (13)$$

Note that, (13) can be easily evaluated since it includes finite summations of elementary functions.

#### 4. ASYMPTOTIC PERFORMANCE ANALYSIS

Since the results of the previous section provide limited physical insights, we now focus on the asymptotically low outage regime. In this regime,  $\gamma_{\text{th}}$  tends to zero and we can approximate the PDF distribution of the end-to-end SINR around the origin via a Taylor's series. For the sake of simplicity we assume that  $m_{1k} = m_1$  and  $m_{2k} = m_2$ . More importantly, it can be shown that the diversity order for fixed interference power over Nakagami- $m$  fading channels is equal to  $\min(m_1, m_2) \times K$ . We now elaborate on the case of Rayleigh fading, by setting all  $m$  parameters to one while the interference power,  $P_{k,i}$ , is assumed constant; then, the asymptotic OP for Rayleigh fading channels is obtained as

$$P_{\text{out}}^{\infty}(\gamma_{\text{th}}) = \sum_{k=0}^K \frac{(-1)^k}{k!} \sum_{n_1, \dots, n_k} \sum_{p=0}^k \frac{1}{p!} \sum_{r_1, \dots, r_p}^k \prod_{q=1}^p \left[ (a_{n_{r_q}} + b_{n_{r_q}}) \left( \frac{N_{n_{r_q}}}{\beta_{n_{r_q}}} + 2 \right) \right] \gamma_{\text{th}}^K + o(\gamma_{\text{th}}^K). \quad (14)$$

Note that, in this case  $\gamma_{e2e}^{\text{up}} = \min(\frac{\gamma_{1k}}{2}, \frac{\gamma_{2k}}{2})$ , while the diversity order is  $K$  which is in agreement with  $\min(m_1, m_2) \times K$ .

#### 5. PRACTICAL CASES OF INTEREST

In this section, we particularize the previously reported results to some practical cases of interest. We begin with the case of no interference:

*a) Interference-free* ( $N_k = 0 \quad \forall k = 1, \dots, K$ )

When we set  $(P_{k,i} = 0 \quad \forall k = 1, \dots, K)$ , (13) simplifies to

$$P_{\text{out}}^{\text{lb}}(\gamma_{\text{th}}) = 1 + \sum_{k=1}^K \frac{(-1)^k}{k!} \sum_{n_1, \dots, n_k} \sum_{i_k}^{m_{1k}-1} \sum_{j_k}^{m_{2k}-1} \sum_{p=1}^k \prod \left[ \frac{\beta_{n_p}^{2m_{n_p}} \binom{i_p}{t_p} \binom{j_p}{l_p} (a_{n_p})^{i_p} (b_{n_p})^{j_p} (2\gamma_{\text{th}})^{\sum_{p=1}^k (i_p + j_p)}}{e^{2 \sum_{p=1}^k (a_{n_p} + b_{n_p}) \gamma_{\text{th}}} i_p! j_p! 2^{\sum_{p=1}^k (t_p + l_p)}} \right] \quad (15)$$

where, in this case,  $\gamma_{S_1k}^{\text{up}} = \min(\gamma_{1k}, \frac{\gamma_{2k}}{2})$ , which is a tight upper bound for  $\frac{\gamma_{1k} \gamma_{2k}}{2\gamma_{1k} + \gamma_{2k}}$  while  $\gamma_{S_2k}^{\text{up}} = \min(\gamma_{2k}, \frac{\gamma_{1k}}{2})$ .

### b) Rayleigh fading channels

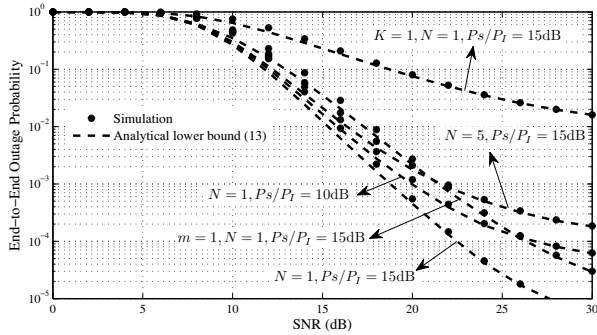
For Rayleigh fading channels, after some simple manipulations, (13) turns into

$$P_{\text{out}}^{\text{lb,ray}}(\gamma_{\text{th}}) = \sum_{k=0}^K \frac{(-1)^k}{k!} \sum_{n_1, \dots, n_k} e^{-2 \sum_{p=1}^k (a_{n_p} + b_{n_p}) \gamma_{\text{th}}} \prod_{p=1}^k \left[ \frac{\beta_{n_p}^{2N_{n_p}}}{(a_{n_p} \gamma_{\text{th}} + \beta_{n_p})^{N_{n_p}} (b_{n_p} \gamma_{\text{th}} + \beta_{n_p})^{N_{n_p}}} \right]. \quad (16)$$

Note that the diversity order in this case is equal to  $K$ .

## 6. SIMULATION RESULTS

In this section, the presented theoretical results are validated by a set of Monte-Carlo simulations, where we assume that  $N_k = N$ ,  $P_{k,i} = P_I$ ,  $m_{1k} = 2$ ,  $m_{2k} = 1$ ,  $m_{k,i} = 1.5$ ,  $\Omega_{1k} = \Omega_{2k} = 1$ ,  $\Omega_{k,i} = 0.1 \forall k = 1, 2, 3$ . Figure 1 illustrates the analytical lower bound for the OP (13), where  $P_s/P_I$  is kept constant. We observe that by increasing the number of relays and the fading parameter  $m$  or decreasing the number of interferers, the OP decreases too. Also, as the SNR increases, the OP reaches an error floor since the effect of interference becomes dominant.



OP against the average SNR.

## 7. CONCLUSIONS

In this paper, we investigated the performance of a dual-hop two-way SC AF relaying, where all relay nodes are impaired by CCI. More specifically, we have derived new tight lower bounds for the OP of the system at arbitrary SINRs. Simplified results in the low outage regime were also deduced.

## 8. APPENDIX

### 8.1. Proof of Proposition 8.1

Mathematically speaking, the CDF of  $Y_k$  can be written as

$$F_{Y_k}(z) = \Pr(Y \leq z) = F_{\gamma_2}(z(\gamma_{Rk} + 1) | \gamma_{Rk})$$

$$= \frac{\beta_k^{m_k}}{\Gamma(m_k)} \int_0^\infty \left[ \frac{\gamma(m_{2k}, b_k z(x+2))}{\Gamma(m_{2k})} \right] \frac{x^{m_k-1}}{e^{\beta_k x}} dx. \quad (17)$$

Assuming integer values for  $m_{2k}$ , (17) can be rewritten as

$$F_{Y_k}(z) = 1 - \frac{\beta_k^{m_k}}{\Gamma(m_k)} \sum_{i=0}^{m_{2k}-1} \sum_{j=0}^i \frac{(b_k z)^i 2^{i-j}}{j!(i-j)! e^{2b_k z}} \int_0^\infty \frac{x^{j+m_k-1}}{e^{(b_k z + \beta_k)x}} dx.$$

Using [22, Eq. (17.13.3)], the above integral can be evaluated to yield (9).

### 8.2. Proof of Proposition 3

Since  $X_k$  and  $Y_k$  are dependent to each other, we have

$$\begin{aligned} F_{\gamma_{SC_k}^{\text{up}}}(z) &= 1 - \Pr\left(\min(\gamma_{S1k}^{\text{up}}, \gamma_{S2k}^{\text{up}}) \geq z\right) = 1 - \\ &\Pr\left(\min\left(\gamma_{1k}, \frac{\gamma_{2k}}{\gamma_{rk} + 2}\right) \geq z, \min\left(\gamma_{2k}, \frac{\gamma_{1k}}{\gamma_{rk} + 2}\right) \geq z\right) \\ &= 1 - E_{\gamma_{rk}} \left[ \Pr\left(\gamma_{1k} \geq z(\gamma_{rk} + 2) \mid \gamma_{rk}\right) \right] \\ &E_{\gamma_{rk}} \left[ \Pr\left(\gamma_{2k} \geq z(\gamma_{rk} + 2) \mid \gamma_{rk}\right) \right]. \end{aligned} \quad (18)$$

The two expectations can be evaluated using Proposition 1.

### 8.3. Proof of Proposition 4

Since  $\gamma_{SC_k}^{\text{up}}$  in (8) are independent, we can write the CDF of the upper bounded end-to-end SINR as

$$F_{\gamma_{e2e}^{\text{up}}}(z) = \prod_{k=1}^K F_{\gamma_{SC_k}^{\text{up}}}(z). \quad (19)$$

To compute (19), we can use the following expansion

$$\prod_{k=1}^K (1 - x_k) = \sum_{k=0}^K \frac{(-1)^k}{k!} \sum_{n_1, \dots, n_k} \prod_{p=1}^k x_{n_p} \quad (20)$$

where  $x_k$  is defined as

$$\begin{aligned} x_k &\triangleq e^{-2(a_k + b_k)z} \left( \frac{\beta_k^{m_k}}{\Gamma(m_k)} \right)^2 \sum_{i=0}^{m_{1k}-1} \sum_{j=0}^{m_{2k}-1} \sum_{t=0}^i \sum_{l=0}^j \binom{i}{t} \\ &\times \binom{j}{l} \frac{(a_k z)^i (b_k z)^j}{i! j! 2^{t+l-i-j}} \frac{\Gamma(m_k + t)}{(a_k z + \beta_k)^{m_k+t}} \frac{\Gamma(m_k + l)}{(b_k z + \beta_k)^{m_k+l}}. \end{aligned} \quad (21)$$

Now using the following identity

$$\prod_{l=1}^k \left( e^{-\alpha_l \gamma} \sum_{i=0}^{m_l} \frac{\alpha_l^i \gamma^i}{i!} \right) = e^{-\sum_{l=1}^k \alpha_l \gamma} \sum_{i_k}^{m_k} \left( \prod_{l=1}^k \frac{\alpha_l^{i_l}}{i_l!} \gamma^{\sum_{l=1}^k i_l} \right) \quad (22)$$

and applying (20) and (22) on (19), we can obtain (12) after some manipulations.

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