Two-Way Interference-Limited AF Relaying over Nakagami-m Fading Channels

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***Abstract*—We investigate the performance of two-way interference-limited amplify-and-forward relaying systems over independent but non-identically distributed (i.n.i.d.) Nakagami-m fading channels. A tight closed-form lower bound on the outage probability of the system is derived, along with a simplified expression in the asymptotically low outage regime. Some special cases of practical interest (e.g., no interference power and interference-limited case) are also examined. The theoretical and numerical results provide important physical insights into the implications of the model parameters on the system performance.**

I. Introduction

The demand for high data rate and reliable communications in modern wireless systems have shifted research interest towards cooperative diversity schemes. These systems utilize some basic techniques, such as Amplify-and-Forward (AF) and decode-and-forward (DF). More specifically, in AF relaying schemes, the relay simply amplifies the received signal from the source and transmits it to the destination [1]. On this basis, AF relaying systems are attractive from a practical viewpoint. For this reason, we hereafter focus on the performance of AF relaying.

As wireless networks evolve towards high load deployments with tight cellular frequency reuse, inter-cell co-channel interference (CCI) emerges as a critical factor with potentially severe impact on the network performance. For this reason, CCI has recently been investigated in the context of wireless relaying. Considering CCI at the relay, the outage probability (OP) was analyzed in [2] for Nakagami-m fading, with interference affecting only the relay. The impact of beamforming and CCI at the relay was analyzed in [3]. In [4], the OP and symbol error probability (SEP), with a single interferer impairing both the relay and destination, were analyzed.

In conventional relaying systems, the spectral efficiency (SE) is low due to the fact that the entire communications occupies two time slots. Therefore, two-way relaying techniques have received significant attention in recent years, since they can double the SE compared to conventional relaying systems. In two-way relaying systems, two nodes transmit to the relay simultaneously in the first time slot, whilst in the second time slot, the relay will transmit data to a designated destination. The authors in [5] obtained the OP and SEP of interference-limited systems over Rayleigh fading channels, where they utilized the upper bound of the harmonic mean. However, they introduced a simplifying assumption of independence between two dependent random variables (RVs) to derive their closed-form results for the OP and SEP. In [6], the authors examined the outage performance of two-hop AF relaying systems with CCI over i.n.i.d. Nakagami-m fading channels. They extended their work to two-way relaying systems in [7], where they assumed interference only at the relay; they also approximated the probability distribution function (PDF) of the sum of interferers' powers by a gamma RV. In [8], the authors investigated the OP of two-way AF relaying systems with CCI over Nakagami-m fading channels, where the relay was not subject to interference.

While all previous works have improved our knowledge on the performance of two-way interference-limited AF relaying systems, the most important differences between the work presented here and [7], [8] are:

1) In [8], interference affects only the source nodes and the relay is subject to noise, while all closed-form OP results are limited to Rayleigh fading channels, and
2) In [7], interference affects only the relay, while the relay gain does not contain the interferers' effect.

Motivated by the above mentioned limitations of [7] and [8], we herein pursue a detailed performance analysis of dual-hop two-way AF relaying systems, *where CCI affects both the source nodes and the relay.* The paper contributions can be summarized as follows:

- We consider a two-way dual-hop configuration, where all nodes are affected by multiple interferers. This is a practical but complicated setup, which has been scarcely investigated in the literature. All channels are assumed to experience Nakagami-m fading [9]. In particular, a tight closed-form lower bound on the OP is derived that extends and complements several previous results in the literature.
- In order to get some additional insights into the impact of system parameters, such as the fading severity parameters and the number of interferers, we consider the asymptotically low outage regime where new tractable expressions are deduced. Finally, we particularize our own results to the cases of no interference power and interference-limited case.

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Notation: Throughout this paper, we use $f_h(.)$ and $F_h(.)$ to denote the PDF and cumulative distribution function (CDF) of a RV h, respectively. Also, $\mathfrak{g}\,(a,b)$ represents a Gamma distribution, where a and b are the shape and scale parameters, respectively. The operator $\mathbb{E}[.]$ stands for expectation, while $\Pr(.)$ denotes probability.

II. System Model and Fading Statistics

We consider a cooperative system that consists of two nodes $S1$ and S_2 that exchange information via R (see Fig. 1).

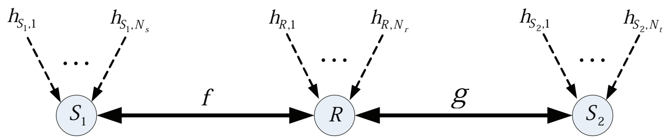

Fig. 1. Schematic illustration of the cooperative system under consideration.

Moreover, S_1, S_2 and R experience N_s, N_t and N_r CCI from other users in the network, respectively. Additionally, f is the channel coefficient between S_1 and R and vice versa (i.e., the $S_1 \to R$ and $R \to S_1$ links) and g is the channel coefficient between S_2 and R which is reciprocal (i.e., the $S_2 \to R$ and $R \to S_2$ links). Also, $h_{R,i}$, $h_{S_1,j}$ and $h_{S_2,k}$ are the channel coefficients between R, S_1 and S_2 and the i-th ($i = 1, ..., N_r$), j-th ($j = 1, ..., N_s$) and k-th ($k = 1, ..., N_t$) interferer at R, S_1 and S_2, respectively. Additionally, P_R, P_{S_1} and P_{S_2} are the transmitted power from R, S_1 and S_2, respectively. Furthermore, P_{Ri}, P_{S_1j} and P_{S_2k} is the power of the i-th, j-th and k-th CCI signal at R, S_1 and S_2, respectively, while σ^2 denotes the noise variance at all nodes. Hence, the instantaneous SNR for $S_1 \to R$, $S_2 \to R$, $R \to S_1$, and $R \to S_2$ links are given by $\gamma_1 = \frac{P_{S_1}|f|^2}{\sigma^2}$, $\gamma_2 = \frac{P_{S_2}|g|^2}{\sigma^2}$, $\gamma_3 = \frac{P_R|f|^2}{\sigma^2}$ and $\gamma_4 = \frac{P_R|g|^2}{\sigma^2}$, respectively. Also the instantaneous SNR for the i-th CCI at R, j-th CCI at S_1 and k-th CCI at S_2 are given by $\gamma_{R,i} = \frac{P_{Ri}|h_{R,i}|^2}{\sigma^2}$, $\gamma_{S_1,j} = \frac{P_{S_1j}|h_{S_1,j}|^2}{\sigma^2}$ and $\gamma_{S_2,k} = \frac{P_{S_2k}|h_{S_2,k}|^2}{\sigma^2}$, respectively.

As was previously mentioned, in this paper we assume that the amplitude of all links follows the Nakagami-m distribution, where m represents the fading severity parameter [9]. In this case, the distribution of $|f|^2$, $|g|^2$, $\gamma_{S_1,j}$, $\gamma_{S_2,k}$ and $\gamma_{R,i}$ is respectively given by $\mathfrak{g}\,(m_1, 1/a)$, $\mathfrak{g}\,(m_2, 1/b)$, $\mathfrak{g}\,(m_s, 1/\alpha)$, $\mathfrak{g}\,(m_t, 1/\eta)$ and $\mathfrak{g}\,(m_r, 1/\beta)$, where $a \triangleq \frac{m_1}{\Omega_1}$, $b \triangleq \frac{m_2}{\Omega_2}$, $\alpha \triangleq \frac{m_s\sigma^2}{\Omega_s P_{S_1j}}$, $\beta \triangleq \frac{m_r\sigma^2}{\Omega_r P_{Ri}}$ and $\eta \triangleq \frac{m_t\sigma^2}{\Omega_t P_{S_2k}}$, while Ω is the average SNR per symbol. We recall that the PDF and CDF of the Nakagami-m fading channels, are given by [9, Eqs. (2.21, 9.272)], respectively. Then, the signal received at the relay can be expressed as

$$y_R = \sqrt{P_{S_1}} f x_{S_1} + \sqrt{P_{S_2}} g x_{S_2} + \sum_{i=1}^{N_r} \sqrt{P_{Ri}} h_{R,i} x_{R,i} + n_R \tag{1}$$

where x_{S_1}, x_{S_2} and $x_{R,i}$ are the signals generated from S_1, S_2 and the i-th interferer at the relay, respectively while n_R is the additive white Gaussian noise (AWGN) at the relay. After amplification at the relay by a variable gain factor G, the signal received at S_1 can be written as

$$\widetilde{y}_{S_1} = \sqrt{P_R} G y_R + \sum_{j=1}^{N_s} \sqrt{P_{S_1j}} h_{S_1,j} x_{S_1,j} + n_{S_1} \tag{2}$$

where $x_{S_1,j}$ is the signal generated from the j-th interferer at S_1 and n_{S_1} is the AWGN at the relay, while the amplification gain is as follows

$$G \triangleq \frac{1}{\sqrt{P_{S_1}|f|^2 + P_{S_2}|g|^2 + \sum\limits_{i=1}^{N_r} P_{Ri}|h_{R,i}|^2 + \sigma^2}}. \tag{3}$$

Since S_1 can eliminate the self-interference term [10], the received SINR at S_1 can then be expressed as

$$\gamma_{S_1} = \frac{P_R P_{S_2} G^2 |f|^2 |g|^2}{\left[G^2 P_R |f|^2 \sum\limits_{i=1}^{N_r} P_{Ri}|h_{R,i}|^2 + \sum\limits_{j=1}^{N_s} P_{S_1j}|h_{S_1,j}|^2 + \sigma^2\left(G^2 P_R |f|^2 + 1\right)\right]}. \tag{4}$$

After some manipulations, γ_{S_1} can be rewritten as

$$\gamma_{S_1} = \frac{\dfrac{P_R P_{S_2}|f|^2|g|^2}{\left[\sum\limits_{j=1}^{N_s} P_{S_1j}|h_{S_1,j}|^2+\sigma^2\right]\left[\sum\limits_{i=1}^{N_r} P_{Ri}|h_{R,i}|^2+\sigma^2\right]}}{\dfrac{P_{S_1}|f|^2+P_{S_2}|g|^2}{\left[\sum\limits_{i=1}^{N_r} P_{Ri}|h_{R,i}|^2+\sigma^2\right]} + \dfrac{P_R|f|^2}{\left[\sum\limits_{j=1}^{N_s} P_{S_1j}|h_{S_1,j}|^2+\sigma^2\right]} + 1}. \tag{5}$$

In a similar way we can derive the corresponding expression for γ_{S_2}, though the expression is omitted due to space limitations.

III. Performance Analysis

By assuming $P_{S_1} = P_{S_2} = P_S$, and defining $\varrho \triangleq \frac{P_R}{P_S}$, $\gamma_S = \sum_{j=1}^{N_s} \gamma_{S_1,j} + 1$ and $\gamma_R = \sum_{i=1}^{N_r} \gamma_{R,i} + 1$, the received SINR at S_1 can be tightly upper bounded, in the interference-limited regime, according to

$$\gamma_{S_1} \leq \frac{\frac{\varrho\gamma_1\gamma_2}{\gamma_S\gamma_R}}{\frac{\gamma_1+\gamma_2}{\gamma_R} + \frac{\varrho\gamma_1}{\gamma_S}} = \frac{\frac{\varrho\gamma_1\gamma_2}{(\gamma_S+\varrho\gamma_R)\gamma_S}}{\frac{\gamma_1}{\gamma_S} + \frac{\gamma_2}{(\gamma_S+\varrho\gamma_R)}} = \varrho\frac{XY}{X+Y} \tag{6}$$

where $X \triangleq \frac{\gamma_1}{\gamma_S}$, $Y \triangleq \frac{\gamma_2}{\gamma_S+\varrho\gamma_R}$. Note that, a similar expression can be derived for the received SINR at S_2 where $\gamma_T = \sum_{k=1}^{N_t} \gamma_{S_2,k} + 1$. It is well known that the $\min(X,Y)$ is a tight upper bound of $\frac{XY}{X+Y}$; in fact, when X and Y go to infinity this bound becomes exact. Hence, we use this upper bound for all our derivations henceforth. Therefore, the upper bounded SINR at S_1 and S_2 can be expressed as

$$\gamma_{S_1} \leq \gamma_{S_1}^{\text{up}} = \varrho \min\left(\frac{\gamma_1}{\gamma_S}, \frac{\gamma_2}{\gamma_S+\varrho\gamma_R}\right) \tag{7}$$

$$\gamma_{S_2} \leq \gamma_{S_2}^{\text{up}} = \varrho \min\left(\frac{\gamma_2}{\gamma_T}, \frac{\gamma_1}{\gamma_T+\varrho\gamma_R}\right). \tag{8}$$

Finally, the end-to-end SINR of this system can be written as

$$\gamma_{e2e} = \min\left(\gamma_{S_1}, \gamma_{S_2}\right) \leq \min\left(\gamma_{S_1}^{\text{up}}, \gamma_{S_2}^{\text{up}}\right) \triangleq \gamma_{e2e}^{\text{up}}. \tag{9}$$

Note that some works, such as [5], derived analytical expressions based on γ_{S_1}, which is not the true end-to-end SINR of a two-way relaying system. To compute the OP of the end-to-end SINR, we should first derive the CDFs of the intermediate variables X and Y. In general, it is known that the sum of L i.i.d Gamma RVs, with shape parameter k and scale parameter θ, is also a Gamma RV, with parameters kL and θ. Then, by defining $\gamma_s \triangleq \sum_{j=1}^{N_s} \gamma_{S_1,j}$, $\gamma_t \triangleq \sum_{k=1}^{N_t} \gamma_{S_2,k}$ and $\gamma_r \triangleq \sum_{i=1}^{N_r} \gamma_{R,i}$, the distribution of γ_s, γ_t and γ_r can be written as $\mathfrak{g}\left(N_s m_s, 1/\alpha\right)$, $\mathfrak{g}\left(N_t m_t, 1/\eta\right)$ and $\mathfrak{g}\left(N_r m_r, 1/\beta\right)$, respectively. The CDFs of X and Y are given by the following proposition:

Proposition 1: The CDFs of X and Y are respectively

$$F_X(z) = 1 - \frac{\alpha^{N_s m_s}}{\Gamma(N_s m_s)} e^{-\frac{az}{\bar{\gamma}}} \sum_{i=0}^{m_1-1} \sum_{j=0}^{i} \binom{i}{j} \frac{(az)^i}{\bar{\gamma}^i i!} \times \frac{\Gamma(j+N_s m_s)}{\left(\frac{az}{\bar{\gamma}}+\alpha\right)^{j+N_s m_s}} \tag{10}$$

$$F_Y(z) = 1 - \frac{\alpha^{N_s m_s}}{\Gamma(N_s m_s)} \frac{\beta^{N_r m_r}}{\Gamma(N_r m_r)} \sum_{i=0}^{m_2-1} \sum_{j=0}^{i} \sum_{k=0}^{i-j} \binom{i}{j} \binom{i-j}{k} \frac{\varrho^k (az)^i}{\bar{\gamma}^i i!} \frac{(\varrho+1)^{i-j-k}}{e^{\frac{2bz}{\bar{\gamma}}}} \frac{\Gamma(j+N_s m_s)}{\left(\frac{bz}{\bar{\gamma}}+\alpha\right)^{j+N_s m_s}} \frac{\Gamma(k+N_r m_r)}{\left(\frac{\varrho bz}{\bar{\gamma}}+\beta\right)^{k+N_r m_r}}. \tag{11}$$

where $\Gamma(n)$ is the gamma function defined as $\Gamma(n) = \int_0^\infty e^{-t} t^{n-1}\, dt$ [11, Eq. (8.310.1)], while $\bar{\gamma} = \frac{P_S}{\sigma^2}$ is the average SNR per symbol.

Proof: See Appendix A. ■

After computing the CDFs of X and Y, we now proceed to derive the CDFs of $\gamma_{S_1}^{\text{up}}$ and $\gamma_{S_2}^{\text{up}}$:

Proposition 2: The CDFs of $\gamma_{S_1}^{\text{up}}$ and $\gamma_{S_2}^{\text{up}}$ are given by

$$F_{\gamma_{S_1}^{\text{up}}}(z) = 1 - \frac{\alpha^{m_s N_s}}{\Gamma(m_s N_s)} \frac{\beta^{m_r N_r}}{\Gamma(m_r N_r)} e^{-((\varrho+1)b+a)\frac{z}{\varrho\bar{\gamma}}} \sum_{i=0}^{m_1-1} \sum_{j=0}^{m_2-1} \sum_{k=0}^{j} \sum_{l=0}^{i} \sum_{t=0}^{j-k} \frac{(\varrho+1)^{j-k-t}}{i!j!} \binom{j}{k} \binom{j-k}{t} \binom{i}{l} \frac{(az)^i (bz)^j}{\varrho^{i+j-t} \bar{\gamma}^{i+j}} \frac{\Gamma(m_s N_s + l + k)}{\left(\frac{az}{\varrho\bar{\gamma}} + \frac{bz}{\varrho\bar{\gamma}} + \alpha\right)^{m_s N_s + l + k}} \frac{\Gamma(m_r N_r + t)}{\left(\beta + \frac{bz}{\bar{\gamma}}\right)^{m_r N_r + t}}. \tag{12}$$

$$F_{\gamma_{S_2}^{\text{up}}}(z) = 1 - \frac{\eta^{m_t N_t}}{\Gamma(m_t N_t)} \frac{\beta^{m_r N_r}}{\Gamma(m_r N_r)} e^{-((\varrho+1)a+b)\frac{z}{\varrho\bar{\gamma}}} \sum_{i=0}^{m_2-1} \sum_{j=0}^{m_1-1} \sum_{k=0}^{j} \sum_{l=0}^{i} \sum_{t=0}^{j-k} \frac{(\varrho+1)^{j-k-t}}{i!j!} \binom{j}{k} \binom{j-k}{t} \binom{i}{l} \frac{(az)^j (bz)^i}{\varrho^{i+j-t} \bar{\gamma}^{i+j}} \frac{\Gamma(m_t N_t + l + k)}{\left(\frac{az}{\varrho\bar{\gamma}} + \frac{bz}{\varrho\bar{\gamma}} + \eta\right)^{m_t N_t + l + k}} \frac{\Gamma(m_r N_r + t)}{\left(\beta + \frac{az}{\bar{\gamma}}\right)^{m_r N_r + t}}. \tag{13}$$

Proof: A detailed proof is relegated in [12]. ■

We can now evaluate the CDF of the end-to-end SINR.

Proposition 3: The CDF of the upper bounded end-to-end SINR of two-way interference-limited relaying over Nakagami-m fading channels is given by

$$F_{\gamma_{e2e}^{\text{up}}}(z) = 1 - \left(\mathcal{P}_{11}(z) + \mathcal{P}_{12}(z)\right)\left(\mathcal{P}_{21}(z) + \mathcal{P}_{22}(z)\right) \tag{14}$$

where $\mathcal{P}_{11}(z)$, $\mathcal{P}_{12}(z)$, $\mathcal{P}_{21}(z)$ and $\mathcal{P}_{22}(z)$ are defined in (15)-(18) at the top of next page.

Proof: See Appendix B. ■

Note that m_1, m_2, $m_s N_s$ and $m_t N_t$ should have integer values. We can now analytically evaluate the OP for two-way interference-limited systems. The OP is the probability that the SINR falls bellow a certain normalized threshold, $\gamma_{\text{th}} = \frac{\gamma_0}{\bar{\gamma}}$, where γ_0 is a finite threshold value. Since (14) yields the CDF of the upper bounded end-to-end SINR, we can now obtain the following lower bound on the exact OP of the system

$$P_{\text{out}}(\gamma_{\text{th}}) \geq P_{\text{out}}^{\text{lb}}(\gamma_{\text{th}}) = 1 - \mathcal{P}_1(\gamma_0)\mathcal{P}_2(\gamma_0) \tag{19}$$

where $\mathcal{P}_1(\gamma_0) \triangleq \mathcal{P}_{11}(\gamma_0) + \mathcal{P}_{12}(\gamma_0)$, $\mathcal{P}_2(\gamma_0) \triangleq \mathcal{P}_{21}(\gamma_0) + \mathcal{P}_{22}(\gamma_0)$. Although (19) consists of long expressions, it can be easily evaluated since it includes finite summations of elementary functions.

IV. Asymptotic Outage Analysis

Since the exact results of the previous section provide limited physical insights, we now focus on the asymptotically low outage regime. Clearly, as SNR increases, γ_{th} tends to zero and we can approximate the PDF distribution of the end-to-end SINR around the origin via a Taylor's series. Note that the following approximations are useful in our analysis [11, Eqs (1.211.1,1.110)]

$$(1+\gamma_{\text{th}})^{-n} = \sum_{i=0}^{K} \binom{-n}{i} \gamma_{\text{th}}^i + o\left(\gamma_{\text{th}}^K\right),$$
$$e^{\gamma_{\text{th}}} = \sum_{i=0}^{K} \frac{\gamma_{\text{th}}^i}{i!} + o\left(\gamma_{\text{th}}^K\right), \quad \text{as } \gamma_{\text{th}} \to 0 \tag{20}$$

where n and K are positive integers. Recall that, in the low outage regime the proposed upper bound on the end-to-end SINR becomes exact and we can precisely predict the diversity order. As a matter of fact, after some simple manipulations and by keeping only the dominant term for the terms γ_{th}^n, the sum of these terms in (19) becomes zero when $n < \min(m_1, m_2)$. Therefore, it can be shown that the diversity order for fixed interference power over Nakagami-m fading channels is equal to $\min(m_1, m_2)$. This implies that, when all interferers' powers (i.e. P_{Ri}, $P_{S_1 j}$ and $P_{S_2 k}$) are kept constant, interference does not affect the diversity order. However, when the interference power is growing large while the ratio of transmit powers of both sources versus the interferers' powers is kept constant (a scenario corresponding to the special case b in Section V), the performance of the system cannot be improved due to the interference becoming dominant; as such, the diversity order in this case is equal to 0. These results are consistent with [5] and [7].

For the sake of clarity, however, we herein elaborate on the case of Rayleigh fading, by setting all m parameters to

$$\mathcal{P}_{11}(z) \triangleq \frac{\beta^{m_r N_r}}{\Gamma(m_r N_r)} \frac{\eta^{m_t N_t}}{\Gamma(m_t N_t)} \frac{\alpha^{m_s N_s}}{\Gamma(m_s N_s)} e^{-\left(\frac{az}{\bar{\gamma}}\left(\frac{1}{\varrho}+1\right)+\varrho\alpha\right)} \sum_{l=0}^{m_1-1} \sum_{r=0}^{l} \sum_{i=0}^{m_s N_s+r-1} \sum_{j=0}^{i} \sum_{k=0}^{i-j} \binom{l}{r} \binom{i}{j} \binom{i-j}{k} \frac{a^l z^l}{\varrho^{l+k-i} \bar{\gamma}^l l! i!}$$
$$\times \frac{\Gamma(m_s N_s + r)}{\left(\alpha + \frac{az}{\varrho\bar{\gamma}}\right)^{m_s N_s + r - i}} \frac{\Gamma(m_r N_r + j)}{\left(\varrho\alpha + \frac{az}{\bar{\gamma}} + \beta\right)^{m_r N_r + j}} \frac{\Gamma(m_t N_t + k)}{\left(\alpha + \frac{az}{\varrho\bar{\gamma}} + \eta\right)^{m_t N_t + k}} \tag{15}$$

$$\mathcal{P}_{12}(z) \triangleq \frac{\beta^{m_r N_r}}{\Gamma(m_r N_r)} \frac{\eta^{m_t N_t}}{\Gamma(m_t N_t)} e^{-\frac{az}{\bar{\gamma}}\left(\frac{1}{\varrho}+1\right)} \sum_{l=0}^{m_1-1} \sum_{j=0}^{l} \sum_{k=0}^{l-j} \binom{l}{j} \binom{l-j}{k} \frac{a^l z^l (\varrho+1)^{l-j-k}}{\varrho^{l-j} \bar{\gamma}^l l!} \frac{\Gamma(m_r N_r + j)}{\left(\frac{az}{\bar{\gamma}} + \beta\right)^{m_r N_r + j}} \frac{\Gamma(m_t N_t + k)}{\left(\frac{az}{\varrho\bar{\gamma}} + \eta\right)^{m_t N_t + k}}$$
$$- \frac{\eta^{m_t N_t}}{\Gamma(m_t N_t)} \frac{\beta^{m_r N_r}}{\Gamma(m_r N_r)} e^{-\left(\frac{az}{\bar{\gamma}}\left(\frac{1}{\varrho}+1\right)+\varrho\alpha\right)} \sum_{l=0}^{m_1-1} \sum_{r=0}^{l} \sum_{i=0}^{m_s N_s-1} \sum_{j=0}^{i} \sum_{w=0}^{l-r} \sum_{k=0}^{i-j} \binom{i}{j} \binom{l}{r} \binom{l-r}{w} \binom{i-j}{k}$$
$$\times \frac{a^l z^l \alpha^i (\varrho+1)^{l-r-w}}{\varrho^{l+k-i-r} \bar{\gamma}^l l! i!} \frac{\Gamma(m_r N_r + j + r)}{\left(\frac{az}{\bar{\gamma}} + \varrho\alpha + \beta\right)^{m_r N_r + j + r}} \frac{\Gamma(m_t N_t + k + w)}{\left(\frac{az}{\varrho\bar{\gamma}} + \alpha + \eta\right)^{m_t N_t + k + w}} \tag{16}$$

$$\mathcal{P}_{21}(z) \triangleq \frac{\beta^{m_r N_r}}{\Gamma(m_r N_r)} \frac{\alpha^{m_s N_s}}{\Gamma(m_s N_s)} e^{-\frac{2bz}{\bar{\gamma}}\left(\frac{1}{\varrho}+1\right)} \sum_{l=0}^{m_2-1} \sum_{j=0}^{l} \sum_{k=0}^{l-j} \binom{l}{j} \binom{l-j}{k} \frac{b^l z^l (\varrho+1)^{l-j-k}}{\varrho^{l-j} \bar{\gamma}^l l!} \frac{\Gamma(m_r N_r + j)}{\left(\frac{bz}{\bar{\gamma}} + \beta\right)^{m_r N_r + j}} \frac{\Gamma(m_s N_s + k)}{\left(\frac{bz}{\varrho\bar{\gamma}} + \alpha\right)^{m_s N_s + k}}$$
$$- \frac{\beta^{m_r N_r}}{\Gamma(m_r N_r)} \frac{\alpha^{m_s N_s}}{\Gamma(m_s N_s)} e^{-\left(\frac{bz}{\bar{\gamma}}\left(\frac{1}{\varrho}+1\right)+\varrho\eta\right)} \sum_{l=0}^{m_2-1} \sum_{r=0}^{l} \sum_{i=0}^{m_t N_t-1} \sum_{j=0}^{i} \sum_{w=0}^{l-r} \sum_{k=0}^{i-j} \binom{i}{j} \binom{l}{r} \binom{l-r}{w} \binom{i-j}{k}$$
$$\times \frac{b^l z^l \eta^i (\varrho+1)^{l-r-w}}{\varrho^{l+k-i-r} \bar{\gamma}^l l! i!} \frac{\Gamma(m_r N_r + j + r)}{\left(\frac{bz}{\bar{\gamma}} + \varrho\eta + \beta\right)^{m_r N_r + j + r}} \frac{\Gamma(m_s N_s + k + w)}{\left(\frac{bz}{\varrho\bar{\gamma}} + \alpha + \eta\right)^{m_s N_s + k + w}} \tag{17}$$

$$\mathcal{P}_{22}(z) \triangleq \frac{\beta^{m_r N_r}}{\Gamma(m_r N_r)} \frac{\eta^{m_t N_t}}{\Gamma(m_t N_t)} \frac{\alpha^{m_s N_s}}{\Gamma(m_s N_s)} e^{-\left(\frac{bz}{\bar{\gamma}}\left(\frac{1}{\varrho}+1\right)+\varrho\eta\right)} \sum_{l=0}^{m_2-1} \sum_{r=0}^{l} \sum_{i=0}^{m_t N_t+r-1} \sum_{j=0}^{i} \sum_{k=0}^{i-j} \binom{l}{r} \binom{i}{j} \binom{i-j}{k} \frac{b^l z^l}{\varrho^{l+k-i} \bar{\gamma}^l l! i!}$$
$$\times \frac{\Gamma(m_t N_t + r)}{\left(\eta + \frac{bz}{\varrho\bar{\gamma}}\right)^{m_t N_t + r - i}} \frac{\Gamma(m_r N_r + j)}{\left(\varrho\eta + \frac{bz}{\bar{\gamma}} + \beta\right)^{m_r N_r + j}} \frac{\Gamma(m_s N_s + k)}{\left(\alpha + \frac{bz}{\varrho\bar{\gamma}} + \eta\right)^{m_s N_s + k}}. \tag{18}$$

one. Then, the simplified OP in the low outage regime can be written as

$$P_{\text{out}}^{\infty}(\gamma_{\text{th}}) = \Bigg((a+b)\left(\frac{\varrho+1}{\varrho} + \frac{N_r}{\beta}\right) + \frac{aN_t}{\varrho\eta} + \frac{bN_s}{\varrho\alpha}$$
$$+ \frac{\beta^{N_r}}{\Gamma(N_r)} \frac{\eta^{N_t}}{\Gamma(N_t)} e^{-\varrho\alpha} \sum_{i=0}^{N_s-1} \sum_{j=0}^{i} \sum_{k=0}^{i-j} \frac{a\alpha^{i-1}(N_s - i)}{\varrho^{k-i+1} i!} \binom{i}{j}$$
$$\times \binom{i-j}{k} \frac{\Gamma(N_r + j)}{(\varrho\alpha + \beta)^{N_r + j}} \frac{\Gamma(N_t + k)}{(\eta + \alpha)^{N_t + k}} + \frac{\beta^{N_r} \alpha^{N_s}}{\Gamma(N_r)\Gamma(N_s)}$$
$$\times e^{-\varrho\eta} \sum_{i=0}^{N_t-1} \sum_{j=0}^{i} \sum_{k=0}^{i-j} \frac{b\eta^{i-1}(N_t - i)}{\varrho^{k-i+1} i!} \binom{i}{j} \binom{i-j}{k}$$
$$\times \frac{\Gamma(N_r + j)}{(\varrho\eta + \beta)^{N_r + j}} \frac{\Gamma(N_s + k)}{(\alpha + \eta)^{N_s + k}} \Bigg) \gamma_{\text{th}} + o(\gamma_{\text{th}}). \tag{21}$$

As anticipated, the diversity order is equal to 1 for the particular case under consideration. Note that (21) is different from [5, Eq. (12)] because, as previously mentioned, the authors therein worked on the OP of γ_{S_1} and also assumed that X and Y are independent. In this paper, however, this assumption has been relaxed. When interference exists only at the relay (i.e. $N_s = N_t = 0$), (21) reduces to

$$P_{\text{out}}^{\infty}(\gamma_{\text{th}}) = (a+b)\left(\frac{\varrho+1}{\varrho} + \frac{N_r}{\beta}\right)\gamma_{\text{th}} + o(\gamma_{\text{th}}). \tag{22}$$

We can see that by increasing N_r and the nodes' power or by reducing the interference power, the OP will reduce. For the interference-free system (i.e. $N_s = N_t = N_r = 0$), (21) further reduces to

$$P_{\text{out}}^{\infty}(\gamma_{\text{th}}) = \frac{\varrho+1}{\varrho}(a+b)\gamma_{\text{th}} + o(\gamma_{\text{th}}). \tag{23}$$

V. Special Cases

In this section, we particularize the previously reported results to some practical cases of interest. We begin with the case of no interference

a) Interference-free ($N_s = N_r = N_t = 0$).

When $P_{S_1 i} = P_{S_2 i} = P_{Ri} = P_I = 0$, (19) simplifies to

$$P_{\text{out}}^{\text{lb}}(\gamma_{\text{th}}) = 1 - e^{-(\varrho+1)(a+b)\gamma_{\text{th}}} \sum_{i=0}^{m_1-1} \sum_{j=0}^{m_2-1} \frac{(\varrho+1)^{i+j} a^i b^j \gamma_{\text{th}}^{i+j}}{i! j!} \tag{24}$$

where, in this case, $\gamma_{S_1}^{\mathrm{up}} = \varrho \min\left(\gamma_1, \frac{\gamma_2}{\varrho+1}\right)$, which is a tight upper bound for $\frac{\varrho\gamma_1\gamma_2}{(\varrho+1)\gamma_1+\gamma_2}$ while $\gamma_{S_2}^{\mathrm{up}} = \varrho \min\left(\gamma_2, \frac{\gamma_1}{\varrho+1}\right)$. Note that the derived CDF in (12), when $N_s = N_r = 0$, is a tight lower bound for [13, Eq. (4)].

b) Interference-limited case:

For simplicity, we assume that $a = b, \alpha = \beta = \eta, m_1 = m_2, m_s N_s = m_t N_t$. By setting $P_{S_1 j} = P_{S_2 k} = P_{Ri} = P_I$, $P_S = P_R$ and $\frac{P_S}{P_I} = \rho \geq 1$ where $P_S, P_I \to \infty$, (19) after some manipulations simplifies to

$$P_{\mathrm{out}}^{\mathrm{lb}}(\gamma_{\mathrm{th}}) = 1 - \left(\mathcal{P}_3(\gamma_{\mathrm{th}}) + \mathcal{P}_4(\gamma_{\mathrm{th}})\right)^2 \tag{25}$$

where

$$\mathcal{P}_3(\gamma_{\mathrm{th}}) \triangleq \frac{1}{\Gamma(m_r N_r)} \frac{1}{[\Gamma(m_s N_s)]^2} \sum_{l=0}^{m_1-1} \sum_{i=0}^{m_s N_s+l-1} \sum_{j=0}^{i} \binom{i}{j} \frac{\left(\frac{m_1\Omega_s}{\rho m_s \Omega_f}\right)^l \gamma_{\mathrm{th}}^l}{l! i!} \frac{\Gamma(m_s N_s + l)}{\left(1 + \frac{m_1\Omega_s}{\rho m_s\Omega_f}\gamma_{\mathrm{th}}\right)^{m_s N_s + l - j - k}}$$
$$\times \frac{\Gamma(m_r N_r + j)}{\left(2 + \frac{m_1\Omega_s}{\rho m_s\Omega_f}\gamma_{\mathrm{th}}\right)^{m_r N_r + j}} \frac{\Gamma(m_s N_s + i - j)}{\left(2 + \frac{m_1\Omega_s}{\rho m_s\Omega_f}\gamma_{\mathrm{th}}\right)^{m_s N_s + i - j}}$$

$$\mathcal{P}_4(\gamma_{\mathrm{th}}) \triangleq \frac{1}{\Gamma(m_r N_r)} \frac{1}{\Gamma(m_s N_s)} \left(\sum_{l=0}^{m_1-1} \sum_{j=0}^{l} \binom{l}{j} \frac{\gamma_{\mathrm{th}}^l}{l!} \right.$$
$$\times \frac{\Gamma(m_r N_r + j)}{\left(\frac{m_1\Omega_s}{\rho m_s\Omega_f}\gamma_{\mathrm{th}} + 1\right)^{m_r N_r + j}} \frac{\Gamma(m_s N_s + l - j)}{\left(\frac{m_1\Omega_s}{\rho m_s\Omega_f}\gamma_{\mathrm{th}} + 1\right)^{m_s N_s + l - j}}$$
$$\times \left(\frac{m_1\Omega_s}{\rho m_s\Omega_f}\right)^l - \sum_{l=0}^{m_1-1} \sum_{r=0}^{l} \sum_{i=0}^{m_s N_s - 1} \sum_{j=0}^{i} \binom{i}{j} \binom{l}{r}$$
$$\times \frac{\left(\frac{m_1\Omega_s}{\rho m_s\Omega_f}\right)^l \gamma_{\mathrm{th}}^l}{l! i!} \frac{\Gamma(m_r N_r + j + r)}{\left(\frac{m_1\Omega_s}{\rho m_s\Omega_f}\gamma_{\mathrm{th}} + 2\right)^{m_r N_r + j + r}}$$
$$\left. \times \frac{\Gamma(m_s N_s + l - r + i - j)}{\left(\frac{m_1\Omega_s}{\rho m_s\Omega_f}\gamma_{\mathrm{th}} + 2\right)^{m_s N_s + l - r + i - j}} \right)$$

where $\Omega_s = \mathbb{E}[|h_{S_1,j}|^2]$ and $\Omega_1 = \mathbb{E}[|f|^2]$. The diversity order is equal to 0 which means that the OP will saturate when the ratio of signal to interference power is constant.

VI. Simulation Results

In order to verify our analytical results, we now compare them against Monte-Carlo simulations. Without loss of generality, we assume that $N_t = N_r = N_s = N$.

Figure 2 demonstrates the analytical lower bound for the OP in (19) along with the low outage approximation, where P_I is a constant (i.e. a scenario corresponding to high SNR as well). We should mention that the OP is plotted against the average SNR $\bar{\gamma}$. As expected, the diversity order for Nakagami-m and Rayleigh fading channels are respectively equal to 2 and 1 (i.e. minimum of m_1 and m_2). As can be seen, the OP increases as the power of interference, P_I, increases. More importantly, the proposed lower bound yields excellent accuracy across the entire SNR range and becomes exact at high SNRs. Likewise, the asymptotic outage approximation can very efficiently predict the exact OP.

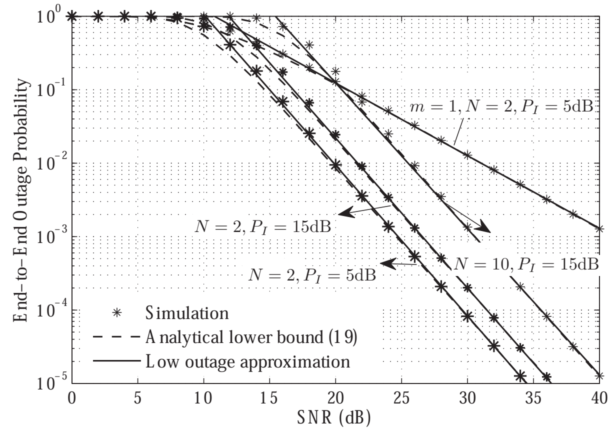

Fig. 2. OP against the average SNR (constant P_I) ($\gamma_0 = 3, m_1 = m_t = 2, m_2 = m_s = 3, m_r = 2.5, \Omega_1 = \Omega_2 = 1, \Omega_s = \Omega_r = \Omega_t = 0.01$).

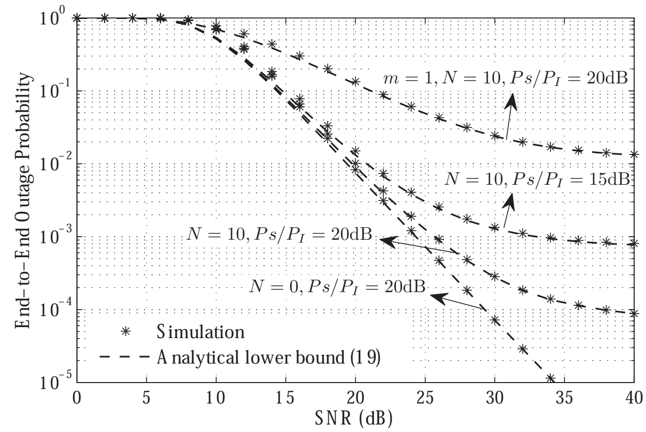

Fig. 3. OP against the average SNR (constant P_S/P_I) .

Figure 3 illustrates the analytical lower bound for the OP, where P_S/P_I is kept constant, while $\varrho = 1$. We observe that by increasing the number of interferers, the OP will be increased. Also, as the SNR increases, the OP reaches an error floor since the effect of interference becomes dominant. Note that the diversity order in this case is equal to 0. Moreover, all curves are close to each other in the low SNR regime which is anticipated since in this regime, the effect of interference is dominated by the AWGN noise.

VII. Conclusion

In this paper, we investigated the performance of a dual-hop two-way AF relay system over i.n.i.d. Nakagami-m fading channels, where all nodes are impaired by CCI. More specifically, we have derived a new tight lower bound for the OP, while asymptotic OP results were also deduced along with simplified results for some special scenarios. Note that the presented results extend several previous results reported in the literature.

APPENDIX A
PROOF OF PROPOSITION 1

Mathematically speaking, the CDFs of X and Y can be written as

$$F_X(z) = \Pr(X \le z) = \Pr(\gamma_1 \le z\gamma_S |\gamma_S) = F_{\gamma_1}(z\gamma_S |\gamma_S)$$
$$F_Y(z) = \Pr(Y \le z) = \Pr(\gamma_2 \le z(\gamma_S + \varrho\gamma_R) \mid \gamma_S, \gamma_R)$$
$$= F_{\gamma_2}(z(\gamma_S + \varrho\gamma_R) \mid \gamma_S, \gamma_R).$$

The CDFs of X and Y can be written in integral form as

$$F_X(z) = \int_0^\infty F_{\gamma_1}(z\gamma_S |\gamma_S) f_{\gamma_S}(\gamma_S) d\gamma_S \tag{26}$$

$$F_Y(z) = \int_0^\infty\int_0^\infty F_{\gamma_2}(z(\gamma_S + \varrho\gamma_R) |\gamma_S\gamma_R) \times f_{\gamma_S}(\gamma_S) f_{\gamma_R}(\gamma_R) d\gamma_S d\gamma_R. \tag{27}$$

Defining $\gamma_S = \gamma_s + 1$ and $\gamma_R = \gamma_r + 1$, substituting the PDFs of γ_s and γ_r and the CDF of γ_1 into (26), we get

$$F_X(z) = \frac{\alpha^{N_s m_s}}{\Gamma(N_s m_s)} \int_0^\infty \left[1 - \frac{\Gamma\left(m_1, \frac{az}{\bar\gamma}(x+1)\right)}{\Gamma(m_1)}\right] \frac{x^{N_s m_s - 1}}{e^{\alpha x}} dx. \tag{28}$$

Using [11, Eq. (8.352.2)] for integer m_1, (28) becomes

$$F_X(z) = 1 - \frac{\alpha^{N_s m_s}}{\Gamma(N_s m_s)} \sum_{i=0}^{m_1-1} \frac{(az)^i}{\bar\gamma^i i!} \times \int_0^\infty e^{-\frac{az}{\bar\gamma}(x+1)} (x+1)^i x^{N_s m_s - 1} e^{-\alpha x} dx. \tag{29}$$

Using the definition of binomial coefficients, (29) becomes

$$F_X(z) = 1 - \frac{\alpha^{N_s m_s}}{\Gamma(N_s m_s)} \sum_{i=0}^{m_1-1} \sum_{j=0}^{i} \frac{(az)^i}{\bar\gamma^i i!} \binom{i}{j} e^{-\frac{az}{\bar\gamma}} \times \int_0^\infty e^{-\left(\frac{az}{\bar\gamma}+\alpha\right)x} x^{j+N_s m_s - 1} dx. \tag{30}$$

Using [11, Eq. (17.13.3)], (30) can be written as in (10). Defining $\gamma_T = \gamma_t + 1$, (27) can be expanded, for integer m_2, as

$$F_Y(z) = 1 - \frac{\alpha^{N_s m_s}}{\Gamma(N_s m_s)} \frac{\beta^{N_r m_r}}{\Gamma(N_r m_r)} \sum_{i=0}^{m_2-1} \sum_{j=0}^{i} \frac{(bz)^i}{\bar\gamma^i i!} \binom{i}{j} \times \frac{\Gamma(j + N_s m_s)}{\left(\frac{bz}{\bar\gamma} + \alpha\right)^{j+N_s m_s}} \int_0^\infty \frac{(\varrho y + \varrho + 1)^{i-j}}{e^{\frac{bz}{\bar\gamma}(\varrho y + \varrho + 1)}} \frac{y^{N_r m_r - 1}}{e^{\beta y}} dy. \tag{31}$$

Once more, by using the definition of binomial coefficients and [11, Eq. (17.13.3)], (31) simplifies to (11).

APPENDIX B
PROOF OF PROPOSITION 3

Since X and Y are dependent to each other, we utilize the following methodology,

$$\begin{aligned}
F_{\gamma_{e2e}^{\mathrm{up}}}(z) &= 1 - \Pr\left(\min\left(\gamma_{S_1}^{\mathrm{up}}, \gamma_{S_2}^{\mathrm{up}}\right) \ge z\right) \\
&= 1 - \Pr\Bigg(\min\left(\frac{\varrho\gamma_1}{\gamma_s+1}, \frac{\varrho\gamma_2}{\gamma_s + \varrho\gamma_r + \varrho + 1}\right) \ge z, \\
&\qquad \min\left(\frac{\varrho\gamma_2}{\gamma_t+1}, \frac{\varrho\gamma_1}{\gamma_t + \varrho\gamma_r + \varrho + 1}\right) \ge z\Bigg) \\
&= 1 - \Pr\Bigg(\frac{\varrho\gamma_1}{\gamma_s+1} \ge z, \frac{\varrho\gamma_2}{\gamma_s + \varrho\gamma_r + \varrho + 1} \ge z, \\
&\qquad \frac{\varrho\gamma_2}{\gamma_t+1} \ge z, \frac{\varrho\gamma_1}{\gamma_t + \varrho\gamma_r + \varrho + 1} \ge z\Bigg) \\
&= 1 - \mathbb{E}_{\gamma_s,\gamma_t,\gamma_r}\Big[\Pr\Big(\varrho\gamma_1 \ge z(\gamma_s+1), \\
&\qquad \varrho\gamma_1 \ge z(\gamma_t + \varrho\gamma_r + \varrho + 1) \Big| \gamma_s,\gamma_t,\gamma_r\Big)\Big] \\
&\quad \times \mathbb{E}_{\gamma_s,\gamma_t,\gamma_r}\Big[\Pr\Big(\varrho\gamma_2 \ge z(\gamma_s + \varrho\gamma_r + \varrho + 1), \\
&\qquad \varrho\gamma_2 \ge z(\gamma_t+1) \Big| \gamma_s,\gamma_t,\gamma_r\Big)\Big] \\
&= 1 - \mathcal{P}_1(z)\mathcal{P}_2(z)
\end{aligned} \tag{32}$$

where $\mathcal{P}_1(z)$ and $\mathcal{P}_2(z)$ can be expressed as

$$\begin{aligned}
\mathcal{P}_1(z) &= \mathbb{E}_{\gamma_s,\gamma_t,\gamma_r}\Big[\Pr\Big(\varrho\gamma_1 \ge z(\gamma_s+1), \\
&\qquad \gamma_s + 1 \ge \gamma_t + \varrho\gamma_r + \varrho + 1 \Big| \gamma_s,\gamma_t,\gamma_r\Big)\Big] \\
&\quad + \mathbb{E}_{\gamma_s,\gamma_t,\gamma_r}\Big[\Pr\Big(\varrho\gamma_1 \ge z(\gamma_t + \varrho\gamma_r + \varrho + 1), \\
&\qquad \gamma_t + \gamma_r + \varrho + 1 \ge \gamma_s + 1 \Big| \gamma_s,\gamma_t,\gamma_r\Big)\Big] \\
\mathcal{P}_2(z) &= \mathbb{E}_{\gamma_s,\gamma_t,\gamma_r}\Big[\Pr\Big(\varrho\gamma_2 \ge z(\gamma_s + \varrho\gamma_r + \varrho + 1), \\
&\qquad \gamma_s + \varrho\gamma_r + \varrho + 1 \ge \gamma_t + 1 \Big| \gamma_s,\gamma_t,\gamma_r\Big)\Big] \\
&\quad + \mathbb{E}_{\gamma_s,\gamma_t,\gamma_r}\Big[\Pr\Big(\varrho\gamma_2 \ge z(\gamma_t+1), \\
&\qquad \gamma_t + 1 \ge \gamma_s + \varrho\gamma_r + \varrho + 1 \Big| \gamma_s,\gamma_t,\gamma_r\Big)\Big].
\end{aligned}$$

Now, $\mathcal{P}_1(z)$ can be written in integral form according to

$$\mathcal{P}_1(z) = \mathbb{E}_{\gamma_t,\gamma_r}\Big[\int_{\gamma_t+\varrho\gamma_r+\varrho}^{\infty} \int_{\varrho^{-1}z(y+1)}^{\infty} f_{\gamma_1}(x) f_{\gamma_s}(y)\, dx dy\Big] + \mathbb{E}_{\gamma_t,\gamma_r}\Big[\int_0^{\gamma_t+\varrho\gamma_r+\varrho} \int_{\varrho^{-1}z(\gamma_t+\varrho\gamma_r+\varrho+1)}^{\infty} f_{\gamma_1}(x) f_{\gamma_s}(y)\, dx dy\Big]. \tag{33}$$

Utilizing [11, Eq. (2.33.10)] and [11, Eq. (8.350.4)], the first term in (33) can be obtained as

$$\mathcal{P}_{11}(z) = \mathbb{E}_{\gamma_t,\gamma_r}\Big[\frac{\alpha^{m_s N_s}}{\Gamma(m_s N_s)} \frac{a^{m_1}}{\bar\gamma^{m_1}\Gamma(m_1)} \int_{\gamma_t+\varrho\gamma_r+\varrho}^{\infty} \int_{\varrho^{-1}z(y+1)}^{\infty} x^{m_1-1} \times e^{-\frac{ax}{\bar\gamma}} dx y^{m_s N_s - 1} e^{-\alpha y} dy\Big]$$

The above expression can be alternatively written as

$$\mathcal{P}_{11}(z) = \mathbb{E}_{\gamma_t,\gamma_r}\Big[\frac{\alpha^{m_sN_s}}{\Gamma(m_sN_s)} \times \int_{\gamma_t+\varrho\gamma_r+\varrho}^{\infty} \frac{\Gamma\left(m_1, \frac{az}{\varrho\bar{\gamma}}(y+1)\right)}{\Gamma(m_1)} y^{m_sN_s-1}e^{-\alpha y}dy\Big].$$

For integer values of m_1, the above expression becomes

$$\mathcal{P}_{11}(z) = \mathbb{E}_{\gamma_t,\gamma_r}\Big[\frac{\alpha^{m_sN_s}}{\Gamma(m_sN_s)}\sum_{l=0}^{m_1-1}\frac{a^lz^l}{\varrho^l\bar{\gamma}^l l!}\int_{\gamma_t+\varrho\gamma_r+\varrho}^{\infty}(y+1)^l \times e^{-\frac{az}{\varrho\bar{\gamma}}(y+1)}y^{m_sN_s-1}e^{-\alpha y}dy\Big] = \mathbb{E}_{\gamma_t,\gamma_r}\Big[\frac{\alpha^{m_sN_s}e^{-\frac{az}{\varrho\bar{\gamma}}}}{\Gamma(m_sN_s)} \sum_{l=0}^{m_1-1}\sum_{r=0}^{l}\binom{l}{r}\frac{a^lz^l}{\varrho^l\bar{\gamma}^l l!}\int_{\gamma_t+\gamma_r+1}^{\infty} e^{-\left(\frac{az}{\varrho\bar{\gamma}}+\alpha\right)y}y^{m_sN_s+r-1}dy\Big] \tag{34}$$

where the second equality is obtained by using the definition of binomial coefficients. By solving the integral in (34) utilizing [11, Eq. (2.33.10)], and assuming integer values for m_sN_s and applying [11, Eq. (8.352.2)]), we have

$$\mathcal{P}_{11}(z) = \mathbb{E}_{\gamma_t,\gamma_r}\Big[\frac{\alpha^{m_sN_s}e^{-\frac{az}{\varrho\bar{\gamma}}}}{\Gamma(m_sN_s)}\sum_{l=0}^{m_1-1}\sum_{r=0}^{l}\binom{l}{r}\frac{a^lz^l}{\varrho^l\bar{\gamma}^l l!} \frac{\Gamma\left(m_sN_s+r, \left(\alpha+\frac{az}{\varrho\bar{\gamma}}\right)(\gamma_t+\varrho\gamma_r+\varrho)\right)}{\left(\alpha+\frac{az}{\varrho\bar{\gamma}}\right)^{m_sN_s+r}}\Big]$$
$$= \mathbb{E}_{\gamma_t,\gamma_r}\Big[\sum_{l=0}^{m_1-1}\sum_{r=0}^{l}\sum_{i=0}^{m_sN_s+r-1}\frac{1}{i!}\binom{l}{r}\frac{a^lz^l}{\varrho^l\bar{\gamma}^l l!}\frac{\alpha^{m_sN_s}e^{-\frac{az}{\varrho\bar{\gamma}}}}{\Gamma(m_sN_s)} \frac{(\gamma_t+\varrho\gamma_r+\varrho)^i\Gamma(m_sN_s+r)}{\left(\alpha+\frac{az}{\varrho\bar{\gamma}}\right)^{m_sN_s+r-i}}e^{-\left(\alpha+\frac{az}{\varrho\bar{\gamma}}\right)(\gamma_t+\varrho\gamma_r+\varrho)}\Big]. \tag{35}$$

By using [14, Eq. (2.1.3.2)], (35) can be written as

$$\mathcal{P}_{11}(z) = \mathbb{E}_{\gamma_t}\Big[\frac{\beta^{m_rN_r}}{\Gamma(m_rN_r)}\sum_{l=0}^{m_1-1}\sum_{r=0}^{l}\sum_{i=0}^{m_sN_s+r-1}\sum_{j=0}^{i}\binom{i}{j} \binom{l}{r}\frac{a^lz^l}{\varrho^{l-j}\bar{\gamma}^l l!i!}\frac{\alpha^{m_sN_s}e^{-\frac{az}{\varrho\bar{\gamma}}-\left(\alpha+\frac{az}{\varrho\bar{\gamma}}\right)(\gamma_t+\varrho)}}{\Gamma(m_sN_s)}\Gamma(m_sN_s+r) \times \frac{(\gamma_t+\varrho)^{i-j}}{\left(\alpha+\frac{az}{\varrho\bar{\gamma}}\right)^{m_sN_s+r-i}}\int_0^{\infty}e^{-\left(\alpha\varrho+\frac{az}{\bar{\gamma}}+\beta\right)x}x^{m_rN_r+j-1}dx\Big].$$

The above expression admits the following manipulations

$$\mathcal{P}_{11}(z) = \mathbb{E}_{\gamma_t}\Big[\frac{\beta^{m_rN_r}}{\Gamma(m_rN_r)}\sum_{l=0}^{m_1-1}\sum_{r=0}^{l}\sum_{i=0}^{m_sN_s+r-1}\sum_{j=0}^{i}\binom{i}{j}\binom{l}{r} \times \frac{a^lz^l}{\varrho^{l-j}\bar{\gamma}^l l!i!}\frac{\alpha^{m_sN_s}e^{-\frac{az}{\varrho\bar{\gamma}}-\left(\alpha+\frac{az}{\varrho\bar{\gamma}}\right)(\gamma_t+\varrho)}}{\Gamma(m_sN_s)}\frac{(\gamma_t+\varrho)^{i-j}}{\left(\alpha+\frac{az}{\varrho\bar{\gamma}}\right)^{m_sN_s+r-i}} \times \frac{\Gamma(m_rN_r+j)}{\left(\varrho\alpha+\frac{az}{\bar{\gamma}}+\beta\right)^{m_rN_r+j}}\Big] = \frac{\beta^{m_rN_r}}{\Gamma(m_rN_r)}\frac{\eta^{m_tN_t}}{\Gamma(m_tN_t)}\frac{\alpha^{m_sN_s}}{\Gamma(m_sN_s)}$$
$$\times e^{-\frac{az}{\varrho\bar{\gamma}}-\left(\varrho\alpha+\frac{az}{\bar{\gamma}}\right)}\sum_{l=0}^{m_1-1}\sum_{r=0}^{l}\sum_{i=0}^{m_sN_s+r-1}\sum_{j=0}^{i}\sum_{k=0}^{i-j}\binom{l}{r}\binom{i}{j} \times \binom{i-j}{k}\frac{\Gamma(m_sN_s+r)}{\left(\alpha+\frac{az}{\varrho\bar{\gamma}}\right)^{m_sN_s+r-i}}\frac{\Gamma(m_rN_r+j)}{\left(\varrho\alpha+\frac{az}{\bar{\gamma}}+\beta\right)^{m_rN_r+j}} \times \frac{a^lz^l}{\varrho^{l+k-i}\bar{\gamma}^l l!i!}\int_0^{\infty}e^{-\left(\alpha+\frac{az}{\varrho\bar{\gamma}}+\eta\right)y}y^{m_tN_t+k-1}dy. \tag{36}$$

By integrating over γ_t in (36) using [11, Eq. (3.351.3)], we arrive to $\mathcal{P}_{11}(z)$ in (14). Likewise, $\mathcal{P}_{12}(z)$, $\mathcal{P}_{21}(z)$ and $\mathcal{P}_{22}(z)$ can be derived for integer m_2 and m_tN_t.

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