# A Generalized Mixture of Gaussians Model for Fading Channels

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***Abstract*—The analysis of composite fading channels, which are typically encountered in wireless channels due to multipath and shadowing is quite involved, as the underlying fading distributions do not lend themselves to analysis. An example of such channels are the Nakagami/Rayleigh-Lognormal fading channels. Several simplified expressions have been proposed in the literature. In this paper, a generalized fading model for composite and non-composite fading models, based on the so-called Mixture of Gaussians (MoG) distribution, is proposed. The well-known expectation-maximization algorithm is utilized to estimate the parameters of the MoG model. Furthermore, relying on the proposed MoG model, we derive closed form expressions for several performance metrics used in wireless communication systems, including the raw moments, the amount of fading, the outage probability, the average channel capacity, and the moment generating function. In addition, the symbol error rate of *L*-branch maximum ratio combining diversity receiver is studied for linear coherent signaling schemes. Monte Carlo simulations are presented to corroborate the analytical results and to assess the accuracy of the MoG model.**

***Index Terms*—Fading channels, composite distributions, expectation-maximization algorithms, Gaussian mixture models, maximum ratio combining, performance analysis.**

## I. Introduction

Modeling most of terrestrial wireless propagation channels requires not only to account for the small scale fading (multipath), but also to account for the large scale fading (shadowing). Several statistical models, such as Rayleigh, Nakagami-*m*, Weibull and Rice distributions have been widely used to characterize the small scale fading phenomena. Large scale power variations caused by the shadowing of obstacles have been proven to follow the Lognormal distribution [1], [2]. Due to its generality, one of the most commonly used composite fading models is the Nakagami-lognormal (NL) distribution, which includes the Rayleigh-lognormal (RL) distribution as a special case. Unfortunately, the NL distribution can only be expressed as an infinite integral, due to the special form of the Lognormal distribution, making the analysis of the symbol error rate (SER), the moment generating function (MGF) and the amount of fading (AF) analytically intractable [3].

Several models have been proposed in the open literature to characterize composite fading channels. The first widely adopted model is the $K$ distribution [4], in which the Lognormal distribution is replaced by the Gamma distribution in the RL distribution. In [5], the authors propose a more generalized model known as the Generalized-$K$ ($K_G$) distribution. Although these models are in closed forms, they contain the modified Bessel function of the second kind, which complicates further analytical performance measures.

In [6], the Lognormal distribution was replaced by the Inverse-Gaussian distribution, resulting in the Rayleigh/Inverse-Gaussian (RIGD) distribution, followed by its generalized version, the $\mathcal{G}$-distribution [3]. The drawback of these distributions is the increased complexity of the mathematical form, due to a combination of a complicated algebraic form and modified Bessel function of the second kind. Another interesting work has been proposed by Atapattu *et al.* [7], where several composite models are represented as a Mixture Gamma ($M\mathcal{G}$) distribution via Gauss-Quadrature approximation. The $M\mathcal{G}$ model is more accurate than the aforementioned alternatives, and it has the advantage of simplicity. Yet, the $M\mathcal{G}$ model is not general for all wireless fading models.

In this paper, an alternative model, that represents both composite and non-composite fading channels by the Mixture of Gaussian (MoG) distribution, is proposed [8]. The approximation method is based on the well-known expectation-maximization (EM) algorithm, which was coined by Dempster *et al.* in their seminal paper [9]. Note that, due to space constraints, elaboration on the EM algorithm along with further statistical analysis on the approach adopted can be found in an extended version of this paper [8]. Here, the importance of the MoG model is demonstrated by deriving several tools for the performance analysis of single-user communications in the MoG Channel. We summarize our main contributions as follows:

- A closed form expression of the raw moments of the MoG model is derived, from which the AF becomes readily available.
- We derive the average channel capacity, which serves as

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a theoretical upper bound for the maximum rate of data transmission at an arbitrarily small SER.

- We derive an MGF expression for the MoG channel, which is then used to derive a closed form expression for the symbol error rate (SER) of *L*-branch maximum ratio combining (MRC) diversity system for various coherent signaling schemes.
- Monte Carlo simulation results are presented to corroborate the derived analytical results.

## II. Fading Channels

The NL fading model is a mixture of Nakagami-*m* distribution and Lognormal distribution, expressed as

$$f_\alpha(\alpha) = \int_0^\infty f_\alpha(\alpha|\sigma) f_\sigma(\sigma)\, d\sigma, \tag{1}$$

where $f_\alpha(\alpha|\sigma)$ is the Nakagami-*m* distribution written as

$$f_\alpha(\alpha|\sigma) = \frac{2m^m}{\sigma^m \Gamma(m)} \alpha^{2m-1} e^{-m\frac{\alpha^2}{\sigma}}. \tag{2}$$

Above, $m$ is the fading parameter, which is inversely proportional to multipath fading severity i.e., as $(m \to \infty)$ multipath severity diminishes. The parameter $\sigma$ follows a Lognormal distribution, contributing to shadowing at longer routes, modeled as

$$f_\sigma(\sigma) = \frac{\lambda}{\sqrt{2\pi}\sigma\zeta} e^{\frac{-(10\log(\sigma)-M)^2}{2\zeta^2}}, \tag{3}$$

where $\lambda = \frac{\ln 10}{10}$, $M$ and $\zeta^2$ are the mean and variance of the associated gaussian random variable $V = 10\,\log_{10}(\sigma)$. An important remark regarding the Lognormal distribution is that, while $\zeta$ essentially defines different Lognormal distributions, $M$ is effectively a scaling factor [10]. Denote $M_n = 10^{M/10}$, then it is straightforward to show that

$$f_\alpha(\alpha M_n) = \frac{1}{M_n} f_\alpha(\alpha|M=0).$$

Therefore, it is only sufficient to perform an approximation for $M = 0$ dB, and generalize the results for other scaling factors. Let $E_s$ denote the energy per symbol, and $N_0$ be the single sided power spectral density of the complex additive white Gaussian noise (AWGN). Assuming $\mathbb{E}[|\alpha^2|] = 1$, where $\mathbb{E}[.]$ denotes the expectation operator. By applying the following transformation to (1)

$$\gamma = \alpha^2 \overline{\gamma}, \tag{4}$$

where $\overline{\gamma} = \mathbb{E}[\gamma] = \frac{E_s}{N_0}$ is the average signal-to-noise ratio (SNR), we obtain the Gamma-lognormal (GL) distribution expressed as

$$f_\gamma(x) = \frac{2\lambda m^m}{\Gamma(m)\sqrt{2\pi}\zeta} \int_0^\infty \frac{x^{m-1}}{\overline{\gamma}^m \sigma^{m+1}} e^{-\frac{mx}{\overline{\gamma}\sigma}} e^{\frac{-(10\log\sigma)^2}{2\zeta^2}} \mathrm{d}\sigma. \tag{5}$$

Note that the RL distribution is a special case of NL distribution, that is when $m = 1$.

In this paper, we utilize the MoG fading model to study the performance of Nakagami-*m*, Lognormal, and NL distributions. In addition, we study the performance of the Weibull fading channel, which characterizes a multipath environment associated with mobile radio systems operating in the 800/900 MHz, where it's instantaneous SNR pdf is expressed as

$$f_\gamma(x) = \frac{\beta}{2} \left(\frac{\Gamma(1+\frac{2}{\beta})}{\overline{\gamma}}\right)^{\frac{\beta}{2}} x^{\beta/2-1} e^{-[\frac{x}{\overline{\gamma}}\Gamma(1+\frac{2}{\beta})]^{\beta/2}}, \tag{6}$$

where $\beta$ is the Weibull fading parameter and $\Gamma(.,.)$ is the incomplete upper Gamma function [11, eq. 8.350.2]. In the subsequent section, the MoG Model is introduced.

## III. The MoG Model

In [8], the envelopes of various fading models is shown to follow a finite mixture of Gaussian densities as follows

$$f_\alpha(x) = \sum_{j=1}^{C} \frac{\omega_j}{\sqrt{2\pi}\eta_j} \exp\left(-\frac{(x-\mu_j)^2}{2\eta_j^2}\right), \tag{7}$$

where $C$ corresponds to the number of components. The weight of the $j^{th}$ component is $\omega_j > 0$, with $\sum_j^C \omega_j = 1$. Parameters $\mu_j$ and $\eta_j^2$ represent the mean and variance of the $j^{th}$ component, respectively.

It is remarkable that the MoG distribution is attributed to have the "Universal-approximation" property, where it has been proven by Weiner's approximation theorem [12] that the MoG distribution can approximate any arbitrarily shaped non-Gaussian density.

### *A. The pdf of the Instantaneous SNR of the MoG Model*

By the change of variables $\gamma = \overline{\gamma}\, x^2$, the pdf of the instantaneous SNR of the MoG model can be written as

$$f_\gamma(\gamma) = \sum_{j=1}^{C} \frac{\omega_j}{\sqrt{8\pi\overline{\gamma}}\eta_j} \frac{1}{\sqrt{\gamma}} \exp\left(\frac{-(\sqrt{\gamma/\overline{\gamma}} - \mu_j)^2}{2\eta_j^2}\right). \tag{8}$$

In [8], this model is shown to approximate the following channels: The NL, RL, Nakagami-*m*, and Weibull distributions.

## IV. Performance Analysis using the Proposed MoG Model

The MoG model provides a simplifying and unifying analysis for wireless communication systems over generalized fading models. In this section, we first derive several performance metrics that can be used for the evaluation of wireless communication systems in a generalized manner. In particular, we derive expressions for the outage probability, the MGF, the raw moments of the MoG model, the AF, and the capacity. We further derive a closed form expression for the SER performance of *L*-branch MRC diversity system.

### A. Outage Probability

The outage probability is defined as $F(\gamma_{th}) = \int_0^{\gamma_{th}} f_\gamma(x)\,dx$, and it is a standard performance criterion used over fading channels. By performing the following change of variables in (8)

$$y = \frac{x-\mu_i}{\sqrt{2}\eta_i}, \tag{9}$$

and after some mathematical manipulations, the CDF of (8) can be written as

$$F(\gamma_{th}) = \sum_{i=1}^{C} \frac{\omega_i}{\sqrt{\pi}} \int_{\frac{-\mu_i}{\sqrt{2}\eta_i}}^{\frac{\sqrt{\frac{\gamma_{th}}{\overline{\gamma}}}-\mu_i}{\sqrt{2}\eta_i}} \exp(-\mathrm{y}^2)\,\mathrm{d}y. \tag{10}$$

Further simplifications yield

$$F(\gamma_{th}) = \sum_{i=1}^{C} \omega_i [Q(\frac{-\mu_i}{\eta_i}) - Q(\frac{\sqrt{\frac{\gamma_{th}}{\overline{\gamma}}}-\mu_i}{\eta_i})], \tag{11}$$

$$F(\gamma_{th}) = \sum_{i=1}^{C} \omega_i\, Q(\frac{\sqrt{\frac{\gamma_{th}}{\overline{\gamma}}}-\mu_i}{-\eta_i})], \tag{12}$$

where $Q(.)$ is the standard $Q$-function, defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(\frac{-u^2}{2})\,\mathrm{d}u$.

### B. Moment Generating Function

By definition, the MGF $M_\gamma(s) = \mathbb{E}[e^{-s\gamma}]$ is given by

$$M_\gamma(s) = \sum_{i=1}^{C} \frac{\omega_i}{\sqrt{8\overline{\gamma}\pi}\eta_i} \int_0^\infty \frac{1}{\sqrt{\gamma}} \exp(-\frac{(\sqrt{\frac{\gamma}{\overline{\gamma}}}-\mu_i)^2}{2\eta_i^2})\, e^{-\gamma s} d\gamma. \tag{13}$$

Applying the change of variables $x = \sqrt{\frac{\gamma}{\overline{\gamma}}}$, and after expanding the exponentials and considerable mathematical simplifications, we arrive at

$$M_\gamma(s) = \sum_{i=1}^{C} \frac{\omega_i}{\sqrt{2\pi}\eta_i} \int_0^\infty \exp(\frac{-(2-\beta_i)(x^2 - \frac{2\mu_i}{\beta}x + \frac{\mu_i^2}{\beta})}{2\eta_i^2}) dx, \tag{14}$$

where $\beta_i = 1 + 2\eta_i^2\overline{\gamma}s$. Then, after some mathematical manipulations, we obtain

$$M_\gamma(s) = \sum_{i=1}^{C} \frac{\omega_i \exp(\frac{\mu_i^2(1/\beta - 1)}{2\eta_i^2})}{\sqrt{\beta_i \pi}} \int_{\frac{-\mu_i}{\eta_i\sqrt{2\beta}}}^\infty \exp(-z^2) dz, \tag{15}$$

which leads to the following simple closed form expression

$$M_\gamma(s) = \sum_{i=1}^{C} \frac{\omega_i}{\sqrt{\beta_i}} \exp(\frac{\mu_i^2 s}{\beta_i}) Q(\frac{-\mu_i}{\eta_i\sqrt{\beta_i}}). \tag{16}$$

### C. Raw Moments

The $n^{th}$ raw moment of the MoG distribution by definition, is

$$\mathbb{E}[\gamma^n] = \sum_{i=1}^{C} \frac{\omega_i}{\sqrt{8\overline{\gamma}\pi}\eta_i} \int_0^\infty \frac{\gamma^n}{\sqrt{\gamma}} \exp(\frac{-(\sqrt{\frac{\gamma}{\overline{\gamma}}}-\mu_i)^2}{2\eta_i^2})\, d\gamma. \tag{17}$$

By the change of variables $x = \sqrt{\frac{\gamma}{\overline{\gamma}}}$, and after some mathematical simplifications, we can write (17) as

$$\mathbb{E}[\gamma^n] = \sum_{i=1}^{C} \omega_i \overline{\gamma}^n \int_0^\infty \frac{x^{2n}}{\sqrt{2\pi}\eta_i} \exp(\frac{-(x-\mu_i)^2}{2\eta_i^2})\, dx, \tag{18}$$

Alternatively, we can express (18) as

$$\mathbb{E}[\gamma^n] = \sum_{i=1}^{C} \omega_i \overline{\gamma}^n\, \mathbb{E}[X_i^{2n}], \tag{19}$$

where $X_i \sim \mathcal{N}(\mu_i, \zeta_i)$ is the $i^{th}$ Gaussian random variable. Using the MGF approach, (19) can be expressed as

$$\mathbb{E}[\gamma^n] = \sum_{i=1}^{C} \omega_i \overline{\gamma}^n \frac{d^{(2n)} M_X(s)}{dx^{(2n)}} \mid_{s=0}, \tag{20}$$

where $M_X(s) = \mathbb{E}\{e^{-sx}\}$ is the MGF of $X_i$ given by $\exp(\mu_i s + \frac{\eta_i^2 s^2}{2})$. Equation (20) is mathematically convenient for solving for the first few moments, where it will be utilized to calculate the AF in the preceding section.

An alternative approach that yields a closed form expression can be attained by following the same method in [13], where the $v^{th}$ raw moment of $X_i$ is derived as

$$\mathbb{E}[x_i^v] = \eta_i^v 2^{\frac{v}{2}} \frac{\Gamma(\frac{v}{2}+\frac{1}{2})}{\sqrt{\pi}}\, {}_1F_1[-\frac{v}{2}, \frac{1}{2}, -\frac{-\mu_i^2}{2\eta_i^2}], \tag{21}$$

where $v$ is an even integer (note that there is no loss in generality), and the function ${}_1F_1$ is the confluent hypergeometric function [11, eq. 9.210.1]. By substituting (21) into (19), with $v = 2n$, the $n^{th}$ raw moment of the MoG model is derived as

$$\mathbb{E}[\gamma^n] = \sum_{i=1}^{C} \omega_i \overline{\gamma}^n \eta_i^{2n} 2^n \frac{\Gamma(n+\frac{1}{2})}{\sqrt{\pi}}\, {}_1F_1[-n, \frac{1}{2}, -\frac{-\mu_i^2}{2\eta_i^2}], \tag{22}$$

### D. Amount of Fading

The AF measure was firstly introduced by Charash [14], as a measure of the severity of the fading channel. The AF requires the knowledge of only the first two moments in the corresponding fading channel, where it is defined by

$$\mathrm{AF} = \frac{\mathbb{E}[\gamma^2] - (\mathbb{E}[\gamma])^2}{(\mathbb{E}[\gamma])^2}. \tag{23}$$

By solving (20) for the first two moments, we obtain

$$\mathrm{AF}=\frac{\sum_{\mathrm{i}=1}^{\mathrm{C}}\omega_{\mathrm{i}}(\mu_{\mathrm{i}}^4+6\mu_{\mathrm{i}}^2\eta_{\mathrm{i}}^2+3\eta_{\mathrm{i}}^4)}{[\sum_{\mathrm{i}=1}^{\mathrm{C}}\omega_{\mathrm{i}}(\mu_{\mathrm{i}}^2+\eta_{\mathrm{i}}^2)]^2}-1. \quad (24)$$

### *E. Average Ergodic Channel Capacity*

When only the receiver has knowledge about the channel state information (CSI), the ergodic capacity $C_{erg}$ is expressed as

$$C_{erg}=\frac{B}{\ln 2}\int_0^\infty \ln(1+\gamma)\, f_\gamma(\gamma)\, \mathrm{d}\gamma, \quad (25)$$

where $B$ is the channel bandwidth measured in Hertz. The exact solution of (25) is intractable. Instead, a computationally simple and very accurate form can be obtained by following [15], where $\ln(1+\gamma)$ is expanded about the mean value of the instantaneous SNR $\mathbb{E}[\gamma]$, using Taylor's series, yielding

$$\ln(1+\gamma)=\ln(1+\mathbb{E}[\gamma])+\sum_{w=1}^{\infty}\frac{(-1)^{w-1}}{w}\frac{(x-\mathbb{E}[\gamma])^w}{(1+\mathbb{E}[\gamma])^w}. \quad (26)$$

Substituting (26) into (25), i.e. taking the expectation of $\ln(1+\gamma)$, the ergodic capacity can be re-written as

$$C_{erg}\approx\frac{B}{\ln 2}[\ln(1+\mathbb{E}[\gamma])-\frac{\mathbb{E}[\gamma^2]-\mathbb{E}^2[\gamma]}{2(1+\mathbb{E}[\gamma])^2}], \quad (27)$$

where

$$\mathbb{E}[\gamma]=\sum_{i=1}^{C}\omega_i(\mu_i^2+\eta_i^2), \quad (28)$$

$$\mathbb{E}[\gamma^2]=\sum_{i=1}^{C}\omega_i(\mu_i^4+6\mu_i^2\eta_i^2+3\eta_i^4). \quad (29)$$

### *F. Symbol Error Analysis*

In order to further demonstrate the significance of the MoG model, we study the performance of independent but not identically distributed (*i.n.i.d.*) *L*-branch MRC diversity receiver over various composite and non-composite fading scenarios. The MRC scheme is the optimal combining scheme at the expense of increased complexity, where the receiver requires knowledge of all channel fading parameters [1]. Here, the receiver sums up all received instantaneous SNR replicas $\gamma_k$ as follows

$$\gamma_{MRC}=\sum_{k=1}^{L}\gamma_k. \quad (30)$$

The corresponding MGF is thus

$$M_{\gamma_{MRC}}(s)=\mathbb{E}\{e^{-s\sum_{k=1}^{L}\gamma_k}\}=\prod_{k=1}^{L}M_{\gamma_k}(s), \quad (31)$$

where $M_{\gamma_k}(s)$ is derived in (16). The SER $P_s(E)$, for coherent binary signals, can be computed as follows [1]

$$P_s(E)=\mathbb{E}_{\gamma_{MRC}}[Q(\sqrt{2g\gamma_{MRC}})], \quad (32)$$

where $g$ is some constant resembling several coherent binary signals, such as the coherent binary phase shift keying (BPSK) and coherent orthogonal binary frequency shift keying (BFSK) corresponding to $g=1$ and $g=\frac{1}{2}$, respectively. By utilizing the closed form representation of the $Q$-function in [1, eq. 4.2], the SER is written as

$$P_s(E)=\int_0^{\frac{\pi}{2}}\int_0^\infty \exp(-\frac{g\gamma_{MRC}}{\sin^2(\theta)})\, f_{\gamma_{MRC}}(\gamma_{MRC})d\gamma_{MRC}\, d\theta, \quad (33)$$

The inner infinite integral in (33) is the equivalent MGF derived in (30). Hence, the SER is expressed as

$$P_s(E)=\int_0^{\frac{\pi}{2}}\prod_{k=1}^{L}\sum_{i=1}^{C}\frac{\omega_i}{\sqrt{1+\frac{2\eta_i^2\overline{\gamma}g}{\sin^2(\theta)}}}\exp(\frac{\mu_i^2\frac{g}{\sin^2(\theta)}}{1+\frac{2\eta_i^2\overline{\gamma}g}{\sin^2(\theta)}})\, d\theta. \quad (34)$$

Following a similar approach, and by utilizing [1, eq. 8.22] and [1, eq. 8.10], the SER expressions for *M*-PSK and square *M*-QAM signaling schemes are given by

$$p_s(E)=\frac{1}{\pi}\int_0^{(M-1)\pi/M}\prod_{k=1}^{L}M_{\gamma_k}(\frac{-\sin^2(\frac{\pi}{M})}{\sin^2(\theta)})d\theta, \quad (35)$$

$$P_s(E) = \frac{4}{\pi}(1-\frac{1}{\sqrt{M}})[\int_0^{\frac{\pi}{2}}\prod_{k=1}^{L}M_{\gamma_k}(\frac{g_{QAM}}{\sin^2(\theta)})\, d\theta - \int_0^{\frac{\pi}{4}}\prod_{k=1}^{L}M_{\gamma_k}(\frac{g_{QAM}}{\sin^2(\theta)})\, d\theta], \quad (36)$$

where $g_{QAM}={}^{3}/_{2(M-1)}$.

## V. Simulation Results

In this section, we address some analytical and simulation results for the outage probability, the average ergodic capacity and the SER of MRC scheme.

Fig. 1 depicts the outage probability, as in (12), versus the threshold SNR $\gamma_{th}$ for two NL scenarios, where the multipath severity is reduced from $m=1$ to $m=3$. Here, one can notice how accurate the approximation is with average SNR, i.e. $\overline{\gamma}=10$ dB. Also, it is very noticeable how the multipath fading severity affects the outage probability performance.

Fig. 2 depicts the capacity, derived in (27), for the two aforementioned scenarios. The term 'simulation' refers to cross-validating the results via the recursive adaptive simpson quadrature method performed by the aid of a mathematical package. Here, we notice that the composite channel in the second scenario exhibits a constant improvement of the capacity of about 0.2 (bits/s/Hz) throughout the whole operating average SNR.

Fig. 3 illustrates the analytical SER of an independent and indentically distributed (*i.i.d.*) 2-branch MRC diversity receiver for the coherent BPSK signaling scheme. We assume various NL scenarios, including mild and severe fading cases in both shadowing and multipath. The solid squared line represents the corresponding Monte Carlo simulation. It is quite noticeable how the multipath severity plays a greater role in determining the SER where, as observed, at $\overline{\gamma}=10$, incrementing $m$ from 1 to 2 yields an SER performance

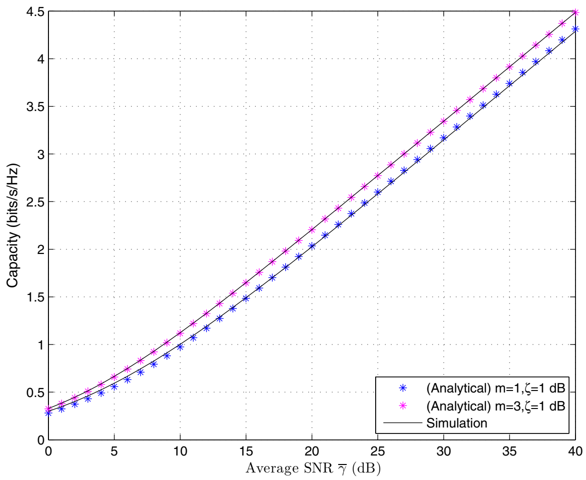


Figure 1. Analytical and simulated ergodic capacity, $C_{erg}$, for Two NL fading scenarios when B=$\frac{1}{2}$.

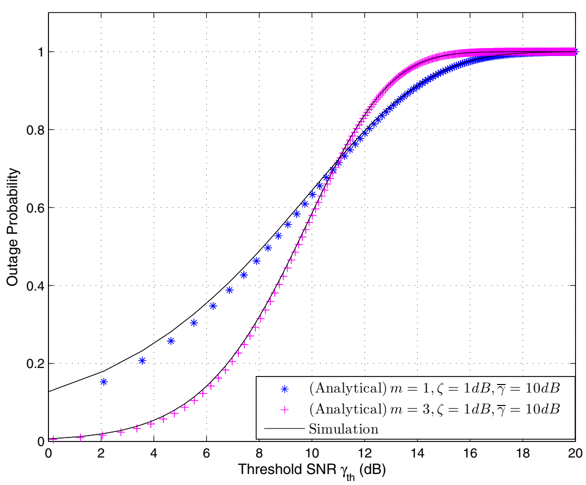


Figure 2. Analytical and simulated outage probability for two NL fading scenarios.

improvement of about an order of magnitude from $10^{-3}$ to $10^{-4}$. On the other hand, increasing $\zeta$ from 2 to 6 dB, while fixing $m$, yields a very similar SER performance.

Fig. 4 features the analytical SER of a *i.i.d.* $L$-branch MRC diversity receiver for the 16-QAM signaling scheme for the Nakagami-*m* and Weibull-*m* fading models, with $m = 3$. Here, $L$ corresponds to the number of antennas, and it is noticeable that the MoG model is still very accurate for large antenna diversity orders.

## VI. Conclusions

Inspired by the universal-approximation property held by the MoG distribution, a new simple and generalized model has been presented to approximate composite and non-composite fading models via the EM algorithm. Several analytical tools

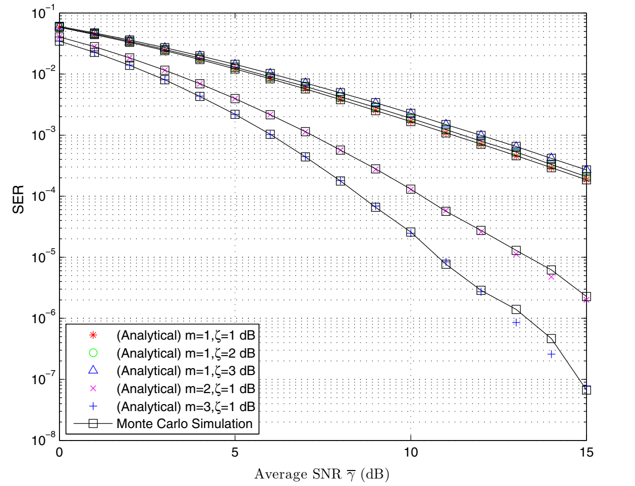


Figure 3. Analytical SER of 2-branch MRC diversity receiver for BPSK signaling scheme for RL and NL composite fading channels, with Monte Carlo simulation.

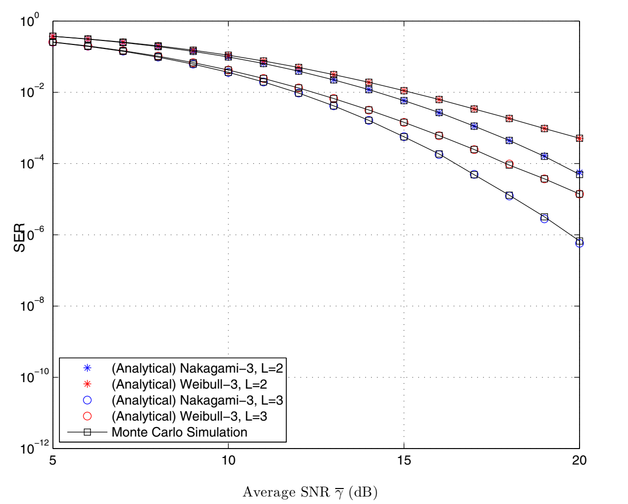


Figure 4. Analytical SER of $L$-branch MRC diversity receiver for 16-QAM signaling scheme for Nakagami-*m* and Weibull-*m* multipath fading channels, with Monte Carlo simulation.

essential for the evaluation of performance analysis of digital communications over generalized fading models were presented. We believe that this new model can be applied to different scenarios including, diversity systems, cooperative communications, and cognitive radio networks.

## Appendix A
## MoG Parameters for Selected Scenarios

The following tables provide the approximation parameters for all scenarios presented in the paper.

Table I
MoG Parameters for RL Fading Channel with $\zeta = 1\,\mathrm{dB}$

| $i$ | $w_i$ | $[\mu_i, \eta_i]$ | $w_i$ | $[\mu_i, \eta_i]$ |
|---|---|---|---|---|
| 1/6 | 0.0051164 | 0.060411,0.025507 | 0.16838 | 0.96294,0.15747 |
| 2/7 | 0.021518 | 0.14045,0.046689 | 0.14568 | 1.2183,0.20628 |
| 3/8 | 0.062099 | 0.26306,0.076786 | 0.13552 | 1.4884,0.31467 |
| 4/9 | 0.14837 | 0.43713,0.11547 | 0.040294 | 1.8528,0.43084 |
| 5/10 | 0.26982 | 0.68518,0.15515 | 0.0031988 | 2.274,0.56255 |

Table II
MoG Parameters for RL Fading Channel with $\zeta = 2\,\mathrm{dB}$

| $i$ | $w_i$ | $[\mu_i, \eta_i]$ | $w_i$ | $[\mu_i, \eta_i]$ |
|---|---|---|---|---|
| 1/6 | 0.0045723 | 0.054824,0.023214 | 0.20138 | 0.8864,0.18134 |
| 2/7 | 0.018487 | 0.12622,0.041603 | 0.20286 | 1.1829,0.25922 |
| 3/8 | 0.053141 | 0.23603,0.068681 | 0.13306 | 1.5772,0.37511 |
| 4/9 | 0.11657 | 0.39073,0.10468 | 0.034063 | 2.1289,0.53072 |
| 5/10 | 0.23304 | 0.60606,0.15207 | 0.0028262 | 2.8129,0.81739 |

Table III
MoG Parameters for RL Fading Channel with $\zeta = 3\,\mathrm{dB}$

| $i$ | $w_i$ | $[\mu_i, \eta_i]$ | $w_i$ | $[\mu_i, \eta_i]$ |
|---|---|---|---|---|
| 1/6 | 0.013628 | 0.087034,0.037077 | 0.1804 | 1.1221,0.24894 |
| 2/7 | 0.050433 | 0.19744,0.065056 | 0.1419 | 1.4935,0.35483 |
| 3/8 | 0.11614 | 0.35142,0.10021 | 0.067797 | 2.0375,0.49984 |
| 4/9 | 0.20896 | 0.55843,0.14439 | 0.017268 | 2.8308,0.75869 |
| 5/10 | 0.20232 | 0.82448,0.183 | 0.0011544 | 4.0701,1.3529 |

Table IV
MoG Parameters for NL Fading Channel with $m = 2, \zeta = 1\,\mathrm{dB}$

| $i$ | $w_i$ | $[\mu_i, \eta_i]$ | $w_i$ | $[\mu_i, \eta_i]$ |
|---|---|---|---|---|
| 1/6 | 0.018334 | 0.30455,0.08798 | 0.14674 | 1.2847,0.24108 |
| 2/7 | 0.11932 | 0.50407,0.12491 | 0.0439 | 1.4702,0.28394 |
| 3/8 | 0.31224 | 0.73853,0.15534 | 0.0062887 | 1.9316,0.36121 |
| 4/9 | 0.166 | 0.97151,0.15317 | 0.010059 | 1.6849,0.26647 |
| 5/10 | 0.1669 | 1.1294,0.20025 | 0.010214 | 1.7405,0.2531 |

Table V
MoG Parameters for NL Fading Channel with $m = 3, \zeta = 1\,\mathrm{dB}$

| $i$ | $w_i$ | $[\mu_i, \eta_i]$ | $w_i$ | $[\mu_i, \eta_i]$ |
|---|---|---|---|---|
| 1/6 | 0.028243 | 0.45395,0.1075 | 0.060344 | 1.3236,0.21195 |
| 2/7 | 0.27669 | 0.68858,0.1457 | 0.027664 | 1.4728,0.26251 |
| 3/8 | 0.26612 | 0.91492,0.14596 | 0.020771 | 1.5543,0.25319 |
| 4/9 | 0.22083 | 1.1092,0.19932 | 0.004416 | 1.7682,0.33435 |
| 5/10 | 0.080896 | 1.1454,0.16278 | 0.014031 | 1.4453,0.15585 |

Table VI
MoG Parameters for Nakagami-3 Fading Channel

| $i$ | $w_i$ | $[\mu_i, \eta_i]$ | $w_i$ | $[\mu_i, \eta_i]$ |
|---|---|---|---|---|
| 1/6 | 0.012256 | 0.40355,0.092473 | 0.10878 | 1.037,0.18181 |
| 2/7 | 0.1741 | 0.62361,0.12718 | 0.096799 | 1.1327,0.18953 |
| 3/8 | 0.12848 | 0.78859,0.11074 | 0.095857 | 1.1476,0.1952 |
| 4/9 | 0.1319 | 0.92625,0.13577 | 0.10312 | 1.3028,0.22945 |
| 5/10 | 0.11989 | 0.96646,0.15685 | 0.028822 | 1.3492,0.29308 |

Table VII
MoG Parameters for Weibull-3 Fading Channel

| $i$ | $w_i$ | $[\mu_i, \eta_i]$ | $w_i$ | $[\mu_i, \eta_i]$ |
|---|---|---|---|---|
| 1/6 | 0.0052097 | 0.17592 0.058515 | 0.11426 | 1.0215,0.19211 |
| 2/7 | 0.029342 | 0.32431 0.089484 | 0.11301 | 1.0945,0.1984 |
| 3/8 | 0.17097 | 0.54879 0.1332 | 0.097193 | 1.2318,0.24213 |
| 4/9 | 0.1445 | 0.76212 0.12345 | 0.096228 | 1.2365,0.23923 |
| 5/10 | 0.13386 | 0.9294 0.15251 | 0.095419 | 1.3711,0.27779 |

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