MU-MIMO Precoding for VLC with Imperfect CSI

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Abstract—**This paper investigates the performance of different precoding schemes for a multi-user MIMO VLC system with channel estimation errors, an assumption that is commonly neglected in the literature. In particular, dirty paper coding, channel inversion, and block diagonalization, are considered for interference mitigation under imperfect channel state information. The impact of the variation of the beam angles of the transmitters and the field of view (FOV) of the receivers on the system performance is also examined. Simulation results reveal that, dirty paper coding provides the best performance under perfect channel state information (CSI). However, under imperfect CSI, suboptimal linear precoding schemes will give better performance. Furthermore, tuning the transmitting angles and the FOVs can significantly improve the system performance.**

Index Terms—**Block diagonalization, channel inversion, dirty paper, optical wireless communication, precoding, visible light communication.**

I. INTRODUCTION

Visible light communication (VLC) is a rapidly emerging technology that derives benefit from the tremendous developments in solid state lighting devices, such as light emitting diodes (LEDs), and its integration with wireless communication technology [1]. A VLC system is an optical wireless (OW) communication system that uses the visible light for transmission [2], [3]. For OW links utilizing LEDs, the transmitter (TX) modulates the intensity of the optical signal at sufficiently high frequencies, without affecting the illumination function of the LEDs, a process called intensity modulation. Direct detection of the decoded signals is applied at the photo detector (PD) of the receiver (RX), which converts the incoming light stream into an electrical signal [4].

VLC is considered a promising candidate to complement conventional indoor radio frequency (RF) communication due to its many attractive advantages, the most important being:

- The widespread deployment of LEDs for illumination purposes.
- VLC signals do not interfere with RF systems.
- The available huge unregulated BW in the visible part of the spectrum.
- Light signals do not penetrate through walls, which allows high order of spatial density to be achieved.
- It is a safe and green form of communications.

Recently, there has been a growing interest in VLC from both research and industrial communities. Multiple-input multiple-output (MIMO) configurations have been investigated for VLC in [5], [6], where multiple LED sources are used for illumination in a typical room environment. In these papers, the presence of multi-user interference (MUI) was not considered. However, the broadcasting scenario, where multiple users are active, is more realistic in VLC applications. The work in [7], [8] focused on the multi-user downlink transmission with MUI cancellation. For the special case where the TX consists of multiple LED arrays and all users employ a single PD, multiple-input single-output (MISO) precoding schemes have been investigated in [7] to eliminate interference at each user terminal. Furthermore, a MIMO scenario with block diagonalization precoding was adopted in [8] to eliminate MUI. However, all previous works on precoding in VLC systems assume perfect knowledge of the channel fading coefficients at both the TX and the RX side. However, these coefficients must be accurately estimated, which is not the case in practical scenarios, thereby leading to performance degradation. In general, the performance of MIMO precoding techniques significantly depends on the knowledge of the channel state information (CSI) at the TX side.

In this paper, we investigate the performance of precoding techniques for MU-MIMO VLC systems under imperfect CSI. Specifically, three different transmission precoding schemes, namely, dirty paper coding (DPC), channel inversion (CI), and block diagonalization (BD), are implemented and their performance is studied under imperfect CSI. To the best of the authors' knowledge, the performance of these precoding techniques considering also channel estimation errors have not been studied in the literature so far.

The contribution of this paper is summarized as follows:

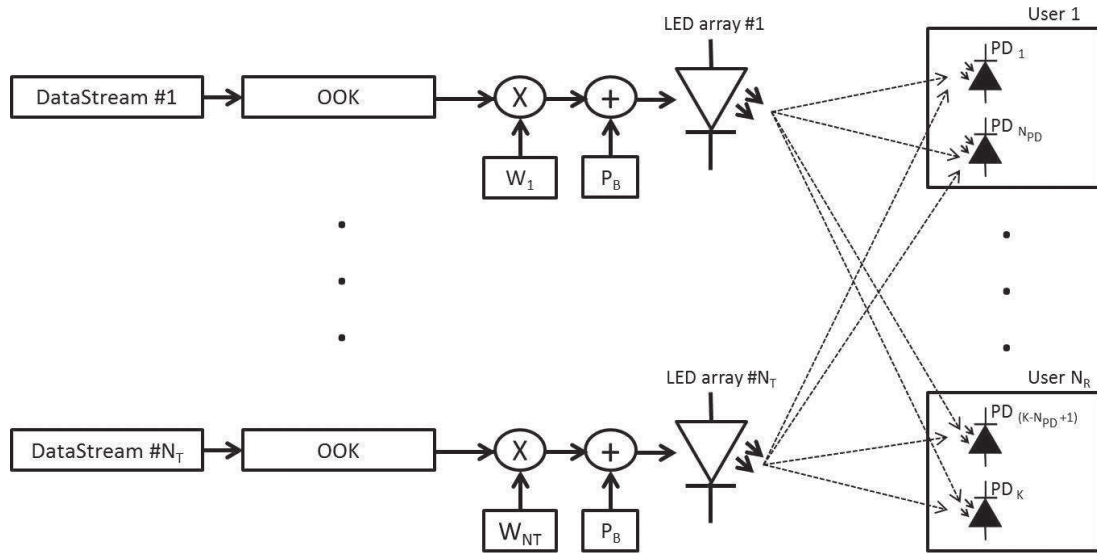- We present the design of DPC, CI, and BD precoding

Fig. 1. Precoding MU-MIMO VLC model.

schemes in MU-MIMO VLC systems.

- We evaluate the bit error rate (BER) of these schemes under different values of channel estimation errors.
- We investigate the effect of TXs and RXs correlation on system performance by changing the TX beam angles and the RX field of view (FOV).

The rest of the paper is organized as follows. In Section II, the optical MU-MIMO model is presented. Section III investigates the precoding design for MU-MIMO VLC systems, while simulation results are discussed in Section IV. Finally, Section V concludes the paper.

II. SYSTEM AND CHANNEL MODEL

We consider a precoded MU-MIMO VLC system that consists of N_T transmitting LED arrays, each array equipped with L LEDs. At the RX side, the number of user terminals is N_R, each comprised of N_{PD} receiving PDs. The total number of PDs in the system is $K = N_R \times N_{PD}$. Let the vector $\mathbf{x} \in \mathbb{R}^{N_T \times 1}$ represent the information signal, which is transmitted after being multiplied by a precoding matrix, $\mathbf{W} \in \mathbb{R}^{N_T \times N_T}$. The precoded signal $\mathbf{Wx}$ must be real, nonnegative, and lies in the linear region of the LED operation. To ensure this, a DC bias, $\mathbf{p}_B \in \mathbb{R}^{N_T \times 1}$, is added at the input of the LED prior to transmission. The precoded signal is then transmitted through the MIMO channel $\mathbf{H} \in \mathbb{R}^{K \times N_T}$.

The transmission model is shown in Fig. 1, and the overall input-output system model can be written as

$$\mathbf{y} = \gamma P_{LED} \mathbf{H}(\mathbf{Wx} + \mathbf{p}_B) + \mathbf{z}, \qquad (1)$$

where $\mathbf{y} \in \mathbb{R}^{K \times 1}$ is the received signal vector, γ is the detector responsivity, P_{LED} is the power emitted by the LED array, and $\mathbf{z} \in \mathbb{R}^{K \times 1}$ denotes the noise vector with statistically independent and identically distributed (iid) entries drawn from a circularly-symmetric complex Gaussian distribution of

zero mean and variance given in (7). Decoding of the received signals is performed using maximum likelihood decision rule.

The downlink optical propagation channel is modeled by the line of sight (LOS) path and each element $h_{i,j}$ of $\mathbf{H}$ reads

$$h_{i,j} = \sum_{l=1}^{L} \frac{A_i}{d_{j,i,l}^2} R_o(\varphi_{j,i,l}) g(\phi_{j,i,l}) \cos \phi_{j,i,l}, \, 0 \le \phi_{j,i,l} \le \phi_c, \qquad (2)$$

while $h_{i,j} = 0$ for $\phi_{j,l} > \phi_c$. In (2), $i = 1, 2, \ldots K$, $j = 1, 2, \ldots N_T$, A_i is the RX PD area, $d_{j,i,l}$ depicts the distance between the lth LED in the jth TX and the ith PD, $\varphi_{j,i,l}$ is the angle of emergence with respect to the TX axis, $\phi_{j,i,l}$ is the angle of incidence with respect to the RX axis, ϕ_c is the PD's FOV angle, and $g(\phi_c)$ is the gain of the optical concentrator

$$g(\phi_c) = \frac{n^2}{\sin^2 \phi_c} \qquad (3)$$

with n being the refractive index. Furthermore, $R_o(\varphi_{j,i,l})$ is the Lambertian radiant intensity of each LED, given by

$$R_o(\varphi_{j,i,l}) = \frac{m+1}{2\pi} \cos^m \varphi_{j,i,l}, \qquad (4)$$

where m is the order of Lambertian emission defined as

$$m = \frac{\ln(2)}{\ln(\cos(\varphi_{1/2}))} \qquad (5)$$

with $\varphi_{1/2}$ being the TX semi-angle at half power. Finally, the channel matrix gain $\mathbf{H}$ is given by

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & h_{1,2} & \ldots & h_{1,N_T} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,N_T} \\ \vdots & \vdots & \vdots & \vdots \\ h_{K,1} & h_{K,2} & \ldots & h_{KN_T} \end{bmatrix}. \qquad (6)$$

The noise generated at the PDs has statistically iid entries of zero mean and variance

$$\sigma^2 = \sigma_{sh}^2 + \sigma_{th}^2 \tag{7}$$

with σ_{sh}^2 and σ_{th}^2 being the variances of the shot noise and the thermal noise, respectively [1].

Channel Estimation Error

Precoding matrix design depends on CSI available at the TX. Thus, estimation errors are highly critical for the performance of the system. We consider imperfect CSI at the TX obtained via an RF uplink channel. It is assumed that the uplink channel is noiseless (i.e., RXs and TXs have the same estimates of the channel coefficients). Assuming least mean square estimator (LMSE), we can write

$$\mathbf{H} = \hat{\mathbf{H}} + e, \tag{8}$$

where $\hat{\mathbf{H}}$ is the channel estimate and e is the estimation error modeled as zero-mean Gaussian random variable with variance σ_e. In this case, the received signal can be expressed as

$$\mathbf{y} = \gamma P_{LED} \mathbf{H}(\mathbf{W}(\hat{\mathbf{H}})\mathbf{x} + \mathbf{p}_B) + \mathbf{z}, \tag{9}$$

where $\mathbf{W}(\hat{\mathbf{H}})$ is the precoding matrix based on the channel estimate $\hat{\mathbf{H}}$.

III. PRECODING DESIGN FOR MU-MIMO VLC

The key impediment in MU-MIMO systems is the interference due to the simultaneous transmission of parallel data streams, which significantly degrades the overall system performance. Accordingly, pre-processing algorithms are required to eliminate the interference induced by the channel. In the following, we study the performance of three precoding schemes in a VLC system under imperfect CSI.

A. Dirty Paper Coding (DPC)

The DPC technique, which was first introduced by Costa [9], is analogous to writing on a dirty paper, where the dirt is indistinguishable from ink. The idea is that if the TX knows where the dirt (interference) is, it can communicate as much information as if the channel was clean (interference-free). DPC has been applied to MIMO systems for interference suppression [10]. In [7], DPC was implemented to eliminate interference in a MU-MISO VLC system.

According to DPC, the transmitted data is efficiently pre-coded, such that the impact of interference is diminished subject to some interference known at the TX. In more detail, the interference caused by the first up to $k-1$ PDs is eliminated during the precoding of the kth PD signal. This noncausal interference cancelation requires the product $\hat{\mathbf{H}}\mathbf{W}_{DPC}(\hat{\mathbf{H}})$ to be a lower triangular matrix, with $\mathbf{W}_{DPC}(\hat{\mathbf{H}})$ being the precoding matrix. To do so, channel estimate $\hat{\mathbf{H}} \in \mathbb{R}^{K \times N_T}$ can be LQ-decomposed as

$$\hat{\mathbf{H}} = \mathbf{LQ}, \tag{10}$$

where $\mathbf{L} \in \mathbb{R}^{K \times N_T}$ is a lower triangular matrix and $\mathbf{Q} \in \mathbb{R}^{N_T \times N_T}$ is an orthogonal matrix. Then, the required precoding matrix can be obtained as

$$\mathbf{W}_{DPC}(\hat{\mathbf{H}}) = \mathbf{Q}^{-1}. \tag{11}$$

The LQ factorization process consists of two different stages. In the first stage, the channel matrix is transformed into Hessenberg form through similarity transformations. Then, QR iterations are applied.

The overall complexity of this algorithm is $O(n^3)$, therefore, it cannot be applied to large matrices due to the prohibitive memory requirements [11]. Moreover, the QR algorithm computes all eigenvalues, which increases the computational complexity. In order to avoid the excessive complexity of the nonlinear DPC scheme, many suboptimal linear precoding techniques have been developed, such as CI and BD [10].

B. Channel Inversion (CI)

Channel inversion is a well-known linear beamforming scheme that has been widely investigated for RF MIMO links due to its low computational complexity [10], [12]. In VLC systems, CI with zero forcing has been used in [7] to eliminate the interference in a multi-user MISO scenario. In the present paper, we generalize CI to support multi-PDs user equipments (MIMO scenario), where user terminals do not only suffer from multi-user interference, but also from the interference between the PDs of the same user.

Channel inversion precoder inverts the downlink channel, resulting in multiple independent subchannels for each PD. This can be realized using the concept of generalized inverses [13]. The CI precoding matrix $\mathbf{W}_{CI}(\hat{\mathbf{H}}) \in \mathbb{R}^{N_T \times N_T}$ can be obtained as

$$\mathbf{W}_{CI}(\hat{\mathbf{H}}) = \beta\hat{\mathbf{H}}^{-1}, \tag{12}$$

where β is a constant used to fit the maximum transmitted power constraint of the LEDs, given by

$$\beta = \sqrt{\frac{N_T}{\mathrm{Tr}(\hat{\mathbf{H}}^{-1}(\hat{\mathbf{H}}^{-1})^H)}} \tag{13}$$

with $\mathrm{Tr}(.)$ denoting the trace of a matrix. After removing the DC component, the received symbol at the ith PD is given by

$$y_i = \frac{1}{\beta}\left[\mathbf{h}_i \sum_{k=1}^{K} \mathbf{w}_{CI_k}(\hat{\mathbf{h}})x_k + z_i\right], \, i = 1, 2, \ldots, K$$

$$= \frac{1}{\beta}\mathbf{h}_i\mathbf{w}_{CI_i}(\hat{\mathbf{h}})x_i + \frac{1}{\beta}\sum_{\substack{k=1 \\ k \neq i}}^{K} \mathbf{h}_i\mathbf{w}_{CI_k}(\hat{\mathbf{h}})x_k + \frac{1}{\beta}z_i, \quad (14)$$

where z_i is the noise variance, evaluated from (7). The received signal is divided by β to compensate for the multiplication performed at the TX. In channel inversion precoding, the goal is to make $\mathbf{h}_i\mathbf{w}_{CI_k}(\hat{\mathbf{H}})$ equal to zero, which guarantees an interference-free transmission.

C. Block Diagonalization (BD)

Block diagonalization is a linear precoding technique that has been proposed in the literature as a generalization of CI [12]. Unlike CI, BD only deals with multi-user interference, without considering the interference between the PDs of the same user. As a result, noise enhancement in BD is significantly reduced. The interference between the PDs of the same user is eliminated by post-equalization in the RX.

By applying BD precoding at the VLC TX, the MU-MIMO VLC channel is decoupled into multiple parallel single user-MIMO channels. This is done when the channel matrix of one user (involving the multiple PDs) is block-diagonalized, that is

$$\hat{\mathbf{H}}_u \mathbf{W}_{BD_v}(\hat{\mathbf{H}}) = \mathbf{0}_{N_{PD} \times N_{PD}}, \quad (15)$$

where $\hat{\mathbf{H}}_u \in \mathbb{R}^{N_{PD} \times N_T}$ is the channel matrix estimate for the uth user, $\mathbf{W}_{BD_v}(\hat{\mathbf{H}}) \in \mathbb{R}^{N_T \times N_T}$ is the precoding matrix for user $v \neq u$, and N_{PD} is the number of the PDs of each user.

The received signal at the uth user should be MUI free, i.e.,

$$y_u = \mathbf{H}_u \mathbf{W}_{BD_v}(\hat{\mathbf{H}}) x_u + z_u, \quad u = 1, 2, \ldots N_R, \quad (16)$$

where N_R is the number of users.

The precoding matrix $\mathbf{W}_{BD_v}(\hat{\mathbf{H}})$ that satisfies (15) can be obtained as follows. First, we form the channel matrix $\widetilde{\mathbf{H}}_u$ that contains all channel estimate matrices except that of uth user,

$$\widetilde{\mathbf{H}}_u = [\hat{\mathbf{H}}_1^H \cdots \hat{\mathbf{H}}_{u-1}^H \hat{\mathbf{H}}_{u+1}^H \cdots \hat{\mathbf{H}}_{N_R}^H]^H. \quad (17)$$

Then, we design the desired precoding matrix $\mathbf{W}_{BD_v}(\hat{\mathbf{H}})$ to lie in the null space of $\widetilde{\mathbf{H}}_u$. If we assume that

$$K = N_R \times N_{PD} = N_T \quad (18)$$

that is, the total number of PDs used by all user equipments is the same as the number of transmitting LED arrays, then, the singular value decomposition (SVD) of $\widetilde{\mathbf{H}}_u$ can be written as

$$\widetilde{\mathbf{H}}_u = \widetilde{\mathbf{U}}_u \widetilde{\Lambda}_u [\widetilde{\mathbf{V}}_u^{non-zero} \widetilde{\mathbf{V}}_u^{zero}], \quad (19)$$

where the vectors of $\widetilde{\mathbf{V}}_u^{non-zero} \in \mathbb{R}^{(K-N_{PD,u}) \times N_T}$ and $\widetilde{\mathbf{V}}_u^{zero} \in \mathbb{R}^{N_{PD,u} \times N_T}$ correspond to the non-zero and zero singular values, respectively. Thus, $\widetilde{\mathbf{V}}_u^{zero}$ forms an orthogonal basis for the null space of matrix $\widetilde{\mathbf{H}}_u$. In other words, when a signal is emitted in the direction of $\widetilde{\mathbf{V}}_u^{zero}$, only the uth user receives the signal, while all other users receive nothing.

Block diagonalization has higher computational complexity, compared to channel inversion precoding. This mainly comes from the SVD operation, which depends on the number of users in the system as well as the size of each user's channel matrix. One limitation of BD is that it cannot be applied to scenarios where the total number of PDs is greater than the number of TXs.

IV. SIMULATION RESULTS AND DISCUSSION

In this section, the performance of the precoding schemes introduced in Section III is investigated via simulations in practical VLC scenarios. Four LED TX arrays are located in the center of the ceiling in a typical $4 \times 4 \times 3$ m^3 room, with 0.2 m spacing on the x- and y-axis between the TXs.

TABLE I
SIMULATION PARAMETERS

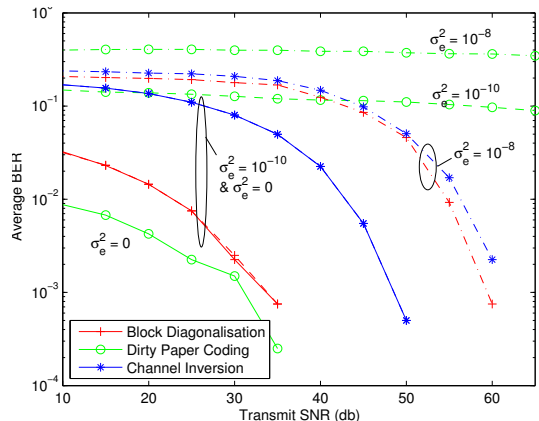| Description	Value
Number of LEDs per Array	60×60
Transmitted optical power per LED	10 mW
Transmitter Semi-angle	60 deg
Field of View	60 deg
Physical Area of PD	1.0 cm^2
PD Responsively	0.54 A/W
Refractive Index of PD Lens	1.5
Gain of Optical Filter	1.0
Data Rate	100 Mbps

Fig. 2. BER for different error variances and correlated transmitters.

Two user terminals, each with two PDs, are placed at a height of 0.75 m with 0.1 m separation on the x- and y-axis. This configuration constitutes a $4 \times [2, 2]$ MU-MIMO VLC system based on the model presented in Section II. We ensure that the LEDs will have the same illumination level in the three precoding schemes to guarantee fairness in the simulations. The parameters used for the simulations are listed in Table I.

In the following, we examine the performance of the three precoding techniques when all transmitting LEDs have the same transmitting semi-angles, $\varphi_{1/2}$. In this case, the MIMO links are highly correlated, which establish a worst-case scenario regarding interference mitigation. Fig. 2 shows the average BER with different error variances, $\sigma_e^2 = 0, 10^{-8}$ and 10^{-10}, knowing that the channel coefficients are in the order of 10^{-5}. It is evident that DPC precoding yields the best performance under perfect CSI. This is because DPC forms a lower triangular matrix $\mathbf{HW}_{DPC}$ to eliminate the noncausal interference, which makes it immune to the noise generated at the RX. However, DPC is highly sensitive to imperfect CSI and it performs worse under low and high channel estimation errors. This is due to the amplification of channel estimation error caused by the multiplication by the lower triangular matrix $\mathbf{L}$ in (10). On the other hand, BD performs better than CI under zero or low estimation errors, yet, the performance gap between the two precoding techniques is narrow when high estimation error exists.

Next, we investigate the effect of changing the beam width of the transmitting LEDs by using different transmitting semi-angles $\varphi_{1/2}$, which determines the light spread coming from
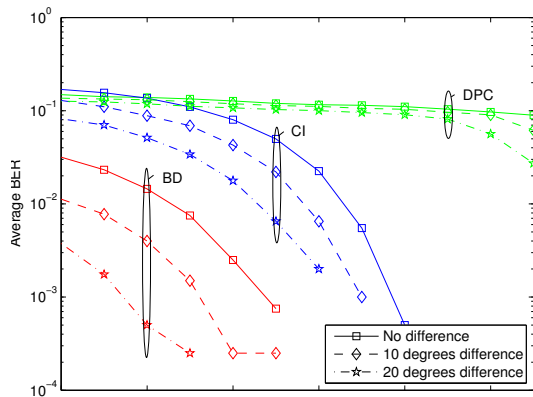
Fig. 3. BER for various transmit angles differences and $\sigma_e^2 = 10^{-10}$.
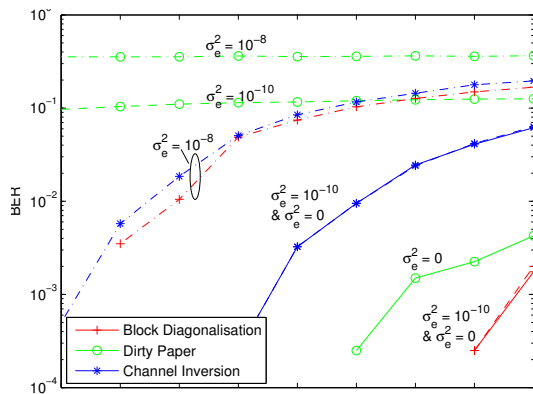

Fig. 4. BER with respect to the FOV of PDs.

the light source. A small $\varphi_{1/2}$ produces hard-edge beam of light, i.e., spot LED lights, where a clear boundary exists between light and dark. On the contrary, lights with wide $\varphi_{1/2}$ emit soft-edge beams that fade gradually. It was shown in [14] that human eye can accommodate the adjustments in light intensity caused by changing the width of the LED beams. We test the performance of the precoding schemes under imperfect CSI when the TXs' semi-angles are not equal and differ by $10°$ and $20°$, and compare it to the case of high correlated TXs induced by equal transmitting angles. Fig. 3 shows that having different transmitting angles significantly improves BER performance for BD and CI precoded MIMO systems. However, the improvement is minor in the case of DPC and it still suffers from high BER due to its sensitivity the channel estimation error.

To gain more insight, we examine changing the FOVs of the RXs' PDs. Fig. 4 shows the average BER when the FOV of the one PD for each user is fixed to $15°$ while the second FOV takes the values from $20°$ to $60°$. As the FOV of the second PD increases, more correlation is established among the two PDs. As can be seen, the less the correlation between the FOVs of the same user terminal, the better the performance is. In Fig. 4, BD outperforms DPC under perfect CSI while CI remains the worst. This can be explained as follows: reducing the FOV of the first PD of each user terminal will increase the gain of this PD, which in turn lessen the noise enhancement

existing on the first PD only in the case of CI. However, in the case of BD, noise enhancement is reduced on both PDs as the channel is block-diagonalized. Resulting in a significant performance boost that causes BD to outperform DPC.

V. Conclusion

In this paper, we have investigated the performance of three precoding schemes in a MU-MIMO-VLC system under channel estimation errors. Simulation results have shown that the dirty paper coding based precoding can achieve the optimal performance under perfect CSI when the MIMO channels are highly correlated. However, dirty paper coding scheme suffers from severe performance loss under channel estimation errors. Moreover, simulations have demonstrated that suboptimal linear solutions give good performance under imperfect CSI. Furthermore, it was shown that using different transmitting angles can reduce the correlation between the MIMO channels and significantly improve performance. Finally, the FOVs can be set to different tunings to reduce the correlation among the photo-detectors of the users, which yields expressive performance gain in the case of block diagonalization precoding.

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