The $\kappa - \mu$ / Inverse Gamma Fading Model

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***Abstract*—Statistical distributions have been extensively used in modeling fading effects in conventional and modern wireless communications. In the present work, we propose a novel $\kappa - \mu$ composite shadowed fading model, which is based on the valid assumption that the mean signal power follows the inverse gamma distribution instead of the lognormal or commonly used gamma distributions. This distribution has a simple relationship with the gamma distribution, but most importantly, its semi heavy-tailed characteristics constitute it suitable for applications relating to modeling of shadowed fading. Furthermore, the derived probability density function of the $\kappa - \mu$ / inverse gamma composite distribution admits a rather simple algebraic representation that renders it convenient to handle both analytically and numerically. The validity and utility of this fading model are demonstrated by means of modeling the fading effects encountered in body centric communications channels, which have been known to be susceptible to the shadowing effect. To this end, extensive comparisons are provided between theoretical and respective real-time measurement results. It is shown that these comparisons exhibit accurate fitting of the new model for various measurement set ups that correspond to realistic communication scenarios.**

I. Introduction

It is widely known that fading phenomena can significantly impact signal reception, resulting in detrimental effects on the corresponding performance of wireless communication systems. This is the case for both conventional and emerging communication systems and motivated by this, the last few decades have witnessed significant contributions in the characterization and modeling of fading channels [1]–[4]. It is well known that fading is generally attributed to multipath and shadowing effects [4]–[6]. In this context, shadowing of the received signal in wireless communications systems is typically caused by two different phenomena. The first of these is *line-of-sight* (LOS) shadowing which relates to the random variation of the power of the LOS component caused by complete or partial obscuration of the direct signal path [2], [7]. The second one occurs due to *multiplicative* or *composite* shadowing which causes random fluctuations of the total power of the multipath components, i.e. both the LOS and scattered components simultaneously [8]–[19] and the references therein.

It is also widely known that multipath and shadowing effects occur simultaneously and thus, they should not be taken into account separately. Based upon this, composite fading distributions, which are composed of a combination of one multipath and one shadowing distribution, were proposed as an effective modeling approach [1], [2]. Traditionally, composite fading scenarios in wireless communications have been modeled under the assumption of Rayleigh [8] or Nakagami-m distributed [9] signal envelopes with the mean signal power assumed to follow either the lognormal or the gamma distribution. More recently, the $\kappa - \mu$ signal envelope has also been studied as it constitutes a remarkably versatile fading model which also includes as special cases other important distributions, such as the one-sided Gaussian, Rice (Nakagami-n), Nakagami-m and Rayleigh distributions [3].

To this end, the authors of [11] assumed that the mean signal power of a $\kappa-\mu$ signal envelope follows a gamma probability density function (PDF). Yet, due to the inherent mathematical complexity of the resulting integral, the derivation of a simple closed-form for the PDF of the $\kappa - \mu$ / gamma composite model was infeasible and an exact infinite series representation was derived instead. In the same context, elegant closed-form expressions for different statistical measures of the $\kappa - \mu$ shadowed distribution were derived in [2]. Nevertheless, the algebraic representation of these expressions is not particularly convenient for the derivation of tractable closed-form expressions for various performance measures of interest. Motivated by this, the present contribution explores the use of the closely related inverse gamma distribution to represent the shadowing of the mean signal power. Based on this, we subsequently utilize it in the context of composite fading conditions when multipath fading is characterized by the $\kappa-\mu$ distribution. It is noted that similar to the gamma PDF, the inverse gamma PDF can also exhibit an explicit semi heavy-tailed behavior, which as is subsequently shown, renders it

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a rather accurate fading model for accounting for shadowing effects. In addition, a simple analytic expression is derived for the PDF of $\kappa-\mu$ / inverse gamma fading model, which is expressed in a straightforward closed-form representation that is convenient to handle both analytically and numerically. As already mentioned, the proposed model will be useful in numerous applications in wireless communications. To this end, its validity and usefulness is indicatively justified through comparisons with respective real-time measurement results in the context of body centric communications.

II. The $\kappa-\mu$ / Inverse Gamma Multipath/ Shadowing Fading Distribution

A. The $\kappa-\mu$ Distribution

It is recalled that the $\kappa-\mu$ distribution constitutes a generic fading model that is capable of accounting for multipath fading in *line-of-sight* (LOS)scenarios [3]. Its flexibility is based on its two distinct fading parameters κ and μ while for a signal with envelope R, the corresponding PDF is defined as [3]

$$\hat{r}f_R(r)=\frac{2\mu(\kappa+1)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}}e^{\mu\kappa}e^{\mu(1+\kappa)\left(\frac{r}{\hat{r}}\right)^2}}\left(\frac{r}{\hat{r}}\right)^{\mu}I_{\mu-1}\left(2\mu\sqrt{\kappa(\kappa+1)}\frac{r}{\hat{r}}\right) \tag{1}$$

where $\hat{r}$ is the root-mean-square (rms) value of R and $I_v(x)$ denotes the modified Bessel function of the first kind and order v. Based on this, the corresponding instantaneous signal-to-noise (SNR) PDF of $\kappa-\mu$ fading model is given by [2]

$$f_\gamma(\gamma)=\frac{\mu(\kappa+1)^{\frac{\mu+1}{2}}\gamma^{\frac{\mu-1}{2}}I_{\mu-1}\left(2\mu\sqrt{\frac{\kappa(\kappa+1)\gamma}{\bar{\gamma}}}\right)}{\kappa^{\frac{\mu-1}{2}}\bar{\gamma}^{\frac{\mu+1}{2}}\exp\left(\mu\kappa+\frac{\mu(\kappa+1)\gamma}{\bar{\gamma}}\right)} \tag{2}$$

where γ is the instantaneous SNR and $\overline{\gamma}=E(\gamma)$ is the average SNR per symbol with $E(\cdot)$ denoting statistical expectation. In terms of physical interpretation $\kappa>0$ is the ratio of the total power of the dominant components (d^2) to the total power of the scattered waves ($2\mu\sigma^2$) whereas

$$\mu=\frac{[\mathrm{E}(\gamma)]^2(1+2\kappa)}{\mathrm{V}(\gamma)(1+\kappa)^2} \tag{3}$$

is related to the multipath clustering, with $2\sigma^2$ denoting the power of the scattered waves in each of the clusters and $V(\cdot)$ representing statistical variance. It is noted that the κ-μ fading model includes as special cases the widely known and utilized Rayleigh, Rice and Nakagami-m fading models.

B. The Inverse Gamma Distribution

Consider a gamma random variable, X with shape parameter α and scale parameter b, where $\alpha>0$ and $b>0$ and the following probability density function

$$f_X(x)=\frac{b^{-\alpha}}{\Gamma(\alpha)}x^{\alpha-1}\exp\left(-\frac{x}{b}\right) \tag{4}$$

where $x>0$ and $\Gamma(\cdot)$ denotes the gamma function. It is noted that the inverse gamma PDF is closely related to the PDF of gamma distribution since it readily follows that when $X\sim\mathrm{Gamma}(\alpha,b)$, then $1/X\sim\mathrm{Inv\text{-}Gamma}(\alpha,b^{-1})$. By letting $\Omega=1/X$ and $\beta=b^{-1}$, the PDF of the inverse of the random variable X can be expressed as follows [20]

$$f_\Omega(\omega)=\frac{\beta^\alpha}{\Gamma(\alpha)}\frac{1}{\omega^{\alpha+1}}\exp\left(-\frac{\beta}{\omega}\right) \tag{5}$$

where $\omega>0$. In what follows, we exploit this straightforward relationship between the two distributions along with the semi heavy-tailed characteristics of the inverse gamma PDF to accurately model the fading behavior of the mean signal power in composite $\kappa-\mu$ shadowed fading channels.

C. The $\kappa-\mu$ / Inverse Gamma Distribution

The PDF of the received signal envelope in a $\kappa-\mu$ / inverse gamma composite fading channel, R, can be obtained by the superposition of the multipath distribution and shadowing distribution. This is mathematically represented by the averaging of the conditional probability density of the $\kappa-\mu$ fading process over the statistics of the random variation of the mean signal power, Ω (where $\Omega=\hat{r}^2$), namely

$$f_R(r)=\int_0^\infty f_{R|\Omega}(r|\omega)f_\Omega(\omega)\mathrm{d}\omega \tag{6}$$

Based on this and by initially assuming that the mean signal power is kept constant along with carrying out some basic algebraic manipulations, the PDF of the proposed $\kappa-\mu$ composite fading model can be expressed as follows:

$$f_{R|\Omega}(r|\omega)=\frac{2\mu(1+\kappa)^{\frac{\mu+1}{2}}r^\mu\exp\left(-\mu(1+\kappa)\left(\frac{r}{\sqrt{\omega}}\right)^2\right)}{\kappa^{\frac{\mu-1}{2}}\exp(\mu\kappa)\omega^{\frac{\mu+1}{2}}} \times I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)}\left(\frac{r}{\sqrt{\omega}}\right)\right) \tag{7}$$

To this effect, the received signal envelope R is deduced by substituting (5) and (7) into (6) which yields

$$f_R(r)=\frac{2\mu(1+\kappa)^{\frac{\mu+1}{2}}\beta^\alpha r^\mu}{\kappa^{\frac{\mu-1}{2}}\exp(\mu\kappa)\Gamma(\alpha)} \times\int_0^\infty\left(\frac{1}{\sqrt{\omega}}\right)^{2\alpha+\mu+3}\frac{I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)}r\left(\frac{1}{\sqrt{\omega}}\right)\right)}{\exp\left(\mu(1+\kappa)r^2\left(\frac{1}{\sqrt{\omega}}\right)^2+\frac{\beta}{\omega}\right)}\mathrm{d}\omega \tag{8}$$

Notably, by also setting $z=1/\omega$, the resulting representation can be expressed in exact closed-form as follows [21]

$$f_R(r)=\frac{2\mu^\mu(1+\kappa)^\mu r^{2\mu-1}}{\exp(\mu\kappa)(\mu(1+\kappa)r^2+\beta)^{\alpha+\mu}}\frac{\beta^\alpha}{\mathrm{B}(\alpha,\mu)} \times{}_1F_1\left(\alpha+\mu;\mu;\frac{\mu^2\kappa(1+\kappa)r^2}{\mu(1+\kappa)r^2+\beta}\right) \tag{9}$$

where $\mathrm{B}(\cdot,\cdot)$ and ${}_1F_1(\cdot;\cdot;\cdot)$ are the beta function and the confluent hypergeometric function, respectively [21]. Using [3, eq. (1)] and [4, eq. (2.3)] for the normalized envelope yields

$$f_\gamma(\gamma)=\frac{\mu^\mu(1+\kappa)^\mu\beta^\alpha\overline{\gamma}^\alpha\gamma^{\mu-1}}{B(\alpha,\mu)e^{\mu\kappa}(\mu(1+\kappa)\gamma+\beta\overline{\gamma})^{\alpha+\mu}} \times{}_1F_1\left(\alpha+\mu;\mu;\frac{\kappa\mu^2(1+\kappa)\gamma}{\mu(1+\kappa)\gamma+\beta\overline{\gamma}}\right) \tag{10}$$

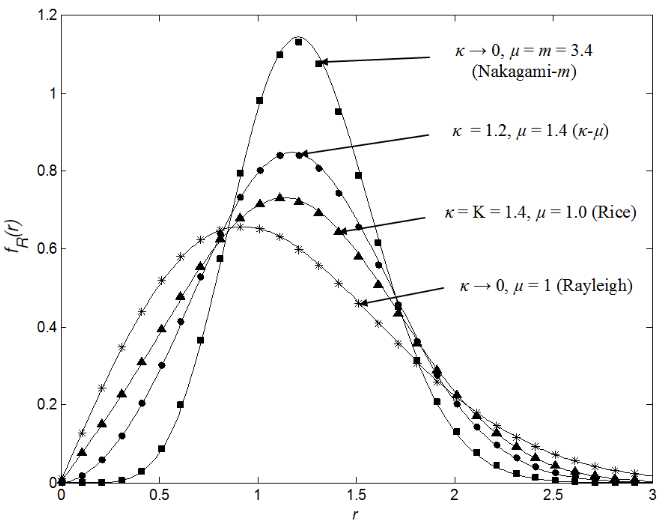

Fig. 1. PDF of the $\kappa-\mu$ / inverse gamma composite fading model (continuous lines) for special cases: $\kappa-\mu$ (circles), Nakagami-m (squares), Rice (triangles) and Rayleigh (asterisks) PDFs.

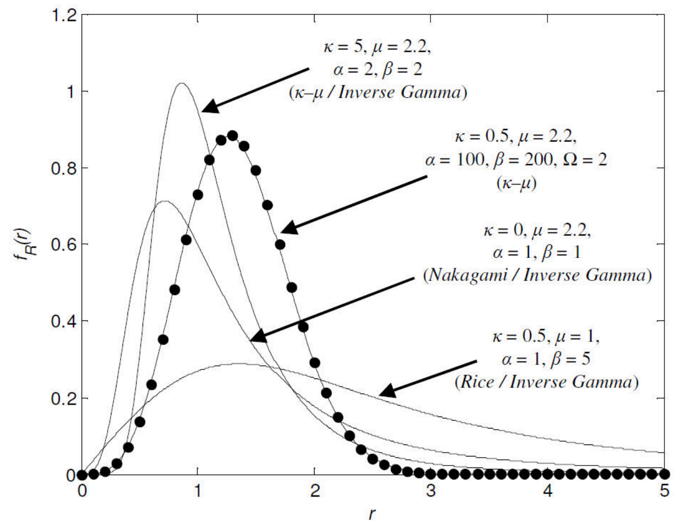

Fig. 2. PDF of the $\kappa-\mu$ / inverse gamma composite fading model (continuous lines) for various values of κ, μ, α and β. The circle symbols represent the special case when the $\kappa - \mu$ / inverse gamma PDF degenerates to the $\kappa - \mu$ PDF for large α.

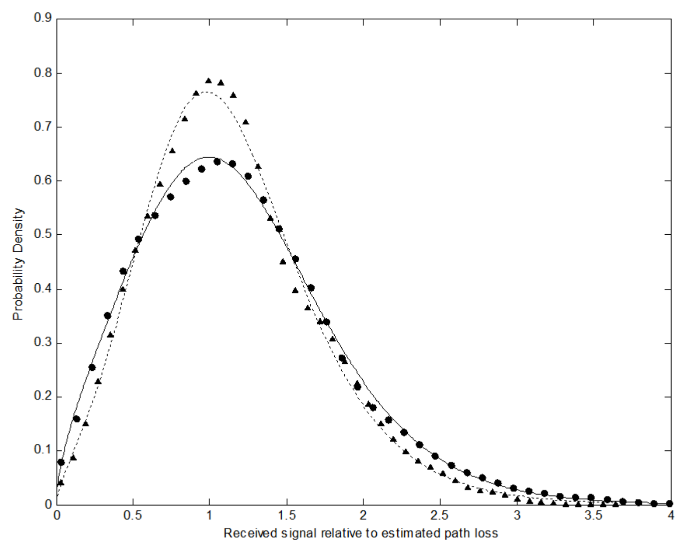

Fig. 4. Empirical (symbols) and theoretical probability densities for the LOS (continuous lines) and NLOS (dashed lines) measurements. The circle and triangle symbols correspond to the empirical probability densities for the LOS and NLOS movements respectively. All parameter estimates are given in Table I.

To the best of the authors' knowledge, the derived expressions have not been previously reported in the open technical literature. Furthermore, it is evident that their algebraic manipulation is relatively tractable, which renders them convenient to use both analytically and numerically. Furthermore, the offered results possess all inherent advantages of the $\kappa - \mu$ distribution and thus, it includes as special cases the Rice (Nakagami-n), Nakagami-m and Rayleigh distributions. This is also illustrated in Fig. 1, where it is shown that in the absence of shadowing of the mean signal power ($\alpha \to \infty$), the $\kappa - \mu$ / inverse gamma composite model coincides with the conventional $\kappa - \mu$ fading model. Likewise, by setting $\mu = 1$ and again letting $\alpha \to \infty$, the Rice PDF is deduced, where κ becomes equivalent to the Rice K factor. Based on this, the Rayleigh distribution can be readily deduced by additionally setting $\kappa = K = 0$. Similarly, the Nakagami$-m$ distribution can be obtained by letting $\kappa \to 0$ with the μ parameter becoming equivalent to the m parameter of Nakagami-m distribution. Moreover, the proposed model includes as special cases the Nakagami-m / inverse gamma composite PDF ($\kappa = 0$), Rice / inverse gamma composite PDF ($\mu = 1$) and Rayleigh / inverse gamma composite PDF ($\kappa = 0$ and $\mu = 1$). Fig. 2 illustrates the behavior of the $\kappa - \mu$ / inverse gamma PDF for various values of κ, μ, α and β and it is noted that as $\alpha >> 0$, the mean signal power quickly becomes deterministic and thus, the PDF in (9) coincides with the $\kappa - \mu$ PDF in [3] with $\Omega = \beta/(\alpha - 1)$.

III. A Comparison with Field Measurements

This section is devoted to the application of the derived analytic results in the context of body area networks. To this end, the usefulness of the $\kappa-\mu$ / inverse gamma fading model is demonstrated in modeling signal reception in body centric communications channels which are known to be susceptible to shadowed fading. As shown in Fig. 3, the measurements were conducted at 5.8 GHz in an indoor open office area (red rectangle: 10.62 m $\times$ 12.23 m) situated on the 1st floor of the the Institute of Electronics, Communications and Information Technology (ECIT) at Queen's University Belfast in the United Kingdom. Both the transmitter and receiver utilized identical omnidirectional sleeve dipole antennas with +2.3 dBi gain (Mobile Mark model PSKN3-24/55S). The transmitter was positioned on the right wrist of an adult male of height 1.83 m and weight 73 kg, which is a possible location for a smart watch, and configured to transmit a continuous wave signal with an output power of +21 dBm. The receive antenna was

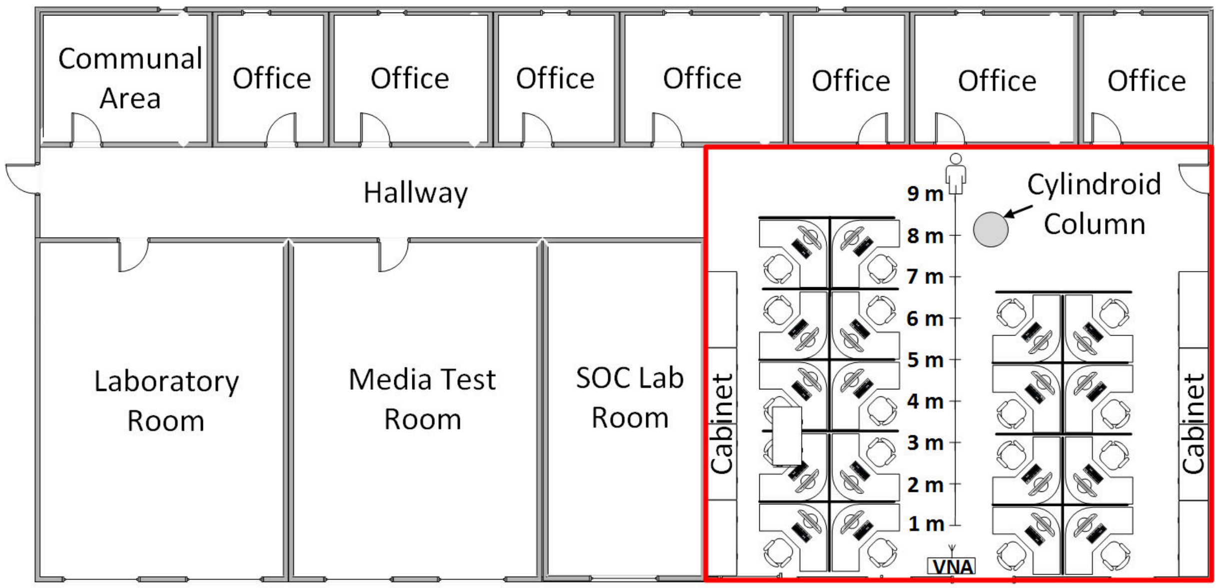

Fig. 3. Measurement environment, Open office area, red rectangle: 129.88 m^2.

positioned on a non-conductive pole at a height of 1.10 m so that it was vertically polarized. It was then attached using a low-loss coaxial cable to port 1 of a Rohde & Schwarz ZVB-8 vector network analyzer (VNA) which was pre-calibrated and configured to record the b_1 wave quantity with a sample rate of 56 Hz. A pre-measurement calibration was conducted to eliminate the effects of the power amplifier and cable loss using a Rohde & Schwarz ZV-Z51 calibration unit.

In this context, four scenarios of interest were considered, namely: *walking movements* in LOS and NLOS conditions, where the test subject walked towards and then away from the receiver in a straight line between 1 m point and 9 m point as shown in Fig. 3; a *rotational movement*, where the test subject rotated 360 degrees in a clockwise direction from LOS through to the maximum shadowing condition before returning back to LOS at a separation distance of 1 m from the receiver; *random movements*, where the test subject walked randomly within a circle area with a radius of 1 m at a distance of 9 m from the receiver.

To abstract the shadowed fading signal for each movement scenarios, the path loss based upon an estimate of the test subject's velocity and the corresponding elapsed time was removed from the measurement data for both the LOS and NLOS scenarios. On the contrary, the path loss effects were ignored and the global mean signal power was removed from the measurement data for rotational and random movement scenarios as the test subject only moved over very small distances. Parameter estimates for κ, μ, α and β of the $\kappa-\mu$ / inverse gamma fading model were obtained using a non-linear least squares routine programmed in MATLAB to fit (9) to the field measurements. The smallest data set considered for model fitting consisted of 1000 samples.

Figs 4 and 5 illustrate the PDFs of the $\kappa-\mu$ / inverse gamma composite fading model fitted to the measurement data. It is clearly noticed that the proposed composite model provides an excellent fit to all sets of considered data. Furthermore, the estimated shadowing of the corresponding mean signal power is shown inset in Fig. 5. Significant shadowed fading of the mean signal power was observed for both the rotation and random movement scenarios. This is practically expected since as the human body moves, it acts as a shadowing object in the channel. For convenience, Table I provides the parameter

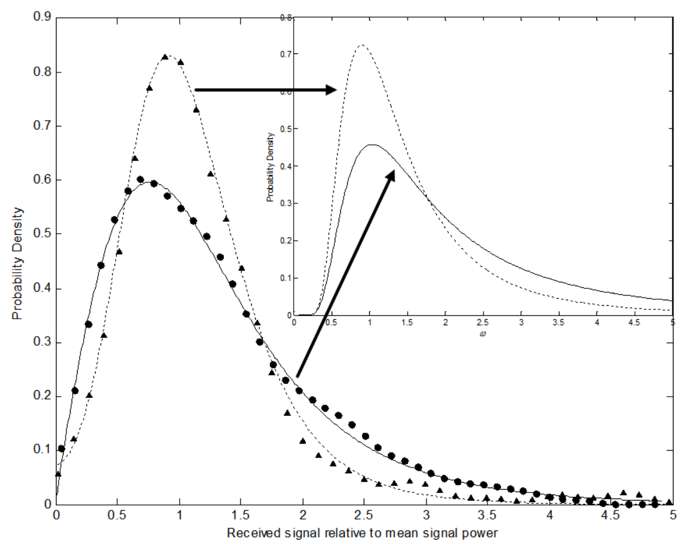

Fig. 5. Empirical (symbols) and theoretical probability densities for the rotational (dashed lines) and random (continuous lines) measurements. The triangle and circle symbols correspond to the empirical probability densities for the rotational and random movements respectively. Also shown inset is the estimated shadowing of the mean signal power. All parameter estimates are given in Table I.

TABLE I
PARAMETER ESTIMATES FOR BOTH THE $\kappa - \mu$ / GAMMA COMPOSITE FADING MODEL AND $\kappa - \mu$ / INVERSE GAMMA COMPOSITE FADING MODEL FOR ALL OF THE MOVEMENT SCENARIOS

	$\kappa - \mu$ / gamma PDF				**$\kappa - \mu$ / inverse gamma PDF**			
	$\hat{\kappa}$	$\hat{\mu}$	$\hat{\alpha}$	$\hat{\beta}$	κ	μ	α	β
LOS Walking	1.38	0.83	7.63	0.23	1.46	0.82	6.40	10.00
NLOS Walking	2.63	0.83	4.96	0.32	2.26	0.88	5.20	6.98
Rotation	0.01	1.08	1.96	1.16	0.01	0.96	2.17	3.28
Random	6.97	0.48	4.02	0.36	6.28	0.51	3.54	4.08

estimates for all considered scenarios to allow immediate reproduction of the estimated probability densities.

In addition, the respective parameter estimates for the $\kappa - \mu$ / gamma fading model proposed in [11] were also produced for comparative purposes. These are depicted in Table I and are denoted as $\hat{\kappa}$, $\hat{\mu}$, $\hat{\alpha}$ and $\hat{\beta}$ to avoid any confusion with the $\kappa - \mu$ / inverse gamma estimates. It is clear that there is no substantial difference in the parameters of interest, i.e. κ, μ and α, between the $\kappa - \mu$ / inverse gamma and $\kappa - \mu$ / gamma fading models. Nevertheless, there is a distinct difference between β parameter, which is due to the difference of the two distributions. Because the κ, μ and α parameter estimates are virtually identical across the $\kappa - \mu$ gamma and $\kappa - \mu$ / inverse gamma models, it is inferred that the latter can be considered as an effective alternative tool to account for such fading environments. Furthermore, this can be achieved in a straightforward manner thanks to the particularly simple algebraic representation of its PDF.

IV. CONCLUSION

In this paper we have proposed an alternative composite model for characterizing fading channels which are susceptible to shadowing of the mean signal power. In particular, this new model considers a $\kappa - \mu$ signal envelope in which the mean signal power follows an inverse gamma distribution. Novel analytic expressions were derived for the probability density function of this composite model which were subsequently employed in the context of body centric communications. An excellent agreement was shown between theoretical and measurement results, which justifies the validity and usefulness of the proposed composite fading model. As part of our future work, we will seek to derive analytic expressions for other performance measures, such as outage probability and ergodic capacity.

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