Unified Analysis of Cooperative Spectrum Sensing over Generalized Multipath Fading Channels

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Abstract—The present work is devoted to the analytic performance evaluation of cooperative spectrum sensing (CSS) over generalized fading channels. The proposed analysis is based on the efficient Gaussian-Finite-Mixture (GFM) that allows the derivation of a simple and accurate closed-form expression for the average probability of energy detection (ED) under different fading environments. Capitalizing on this, we derive generalized closed-form expressions for the global probabilities of detection for the CSS with two main hard centralized fusion rules, namely, the AND and the OR rules. The efficiency and usefulness of the proposed expressions is justified by comparing the corresponding complementary receiver operating characteristic (ROC) curves for both multipath and composite multipath/shadowing fading channels, which are otherwise particularly difficult to obtain. The offered analytic results are corroborated by respective results from computer simulations and it is shown that the corresponding performance depends significantly on both the severity of fading and the involved number of users in the collaborative network.

I. INTRODUCTION

The last decades witnessed a rapid growth of data-intensive multimedia and interactive services along with a widespread deployment of wireless communications in myriad applications. However, the remarkably high data rates and overall increased performance offered by such broadband services have been mainly accomplished at the expense of dramatically increased bandwidth requirements which lead to a significant scarcity of spectrum resources. In addition, the high variability of the traffic statistics across time, space, and frequency has rendered the utilization of the licensed spectrum, assigned by the rigid static spectrum allocation scheme, rather inefficient [1]. As a result, investigating alternative and more efficient spectrum management policies became inevitable and to this end, cognitive radio (CR) has been flagged as the most promising solution thanks to its capability to increase substantially the utilization of the available spectrum resources [2], [3].

The distinct improvement in spectral utilization, motivated by the advancement of CR technology, is based on allowing unlicensed users - also known as secondary users (SU) - to opportunistically access the spectrum allocated to the licensees - also known as primary users (PU) - when the latter are idle. As a result, it is essential that SUs are equipped with the capability to detect whether a frequency band is utilized by a PU or not. This procedure is crucial in enabling the opportunistic spectrum access paradigm, and is widely known as spectrum sensing (SS) [4]. Various SS methods have been proposed over the past years with energy detection (ED) constituting the most widely adopted detection method due to its blind structure and low implementation complexity [5]. The principle of operation of ED is based on the deployment of a radiometer where its output determines the corresponding decision by comparing the received energy level with a predefined energy threshold.

It is also recalled that the operation of cognitive radio networks (CRN) is highly dependent upon the severity of fading which imposes nontrivial challenges on a single SU. As a result, cooperative spectrum sensing (CSS) strategies have been shown to provide an effective solution to sensing errors and performance degradation due to the detrimental effects of fading phenomena and to overcome the hidden terminal problem, which is typically encountered due to shadowing effects [20]. The CSS is technically performed by reporting sensing detection results by individual SUs to a central unit, called the fusion center (FC). Importantly, this process is carried out following either the soft or hard combining schemes.

The detection performance of a CR system is typically evaluated on the basis of two criteria, namely the probability of detection, $P_d$, and the probability of false alarm, $P_f$. Furthermore, the underlying ED based SS is crucial to the performance of both primary and CR networks. In this context, it is useful to quantify the detrimental effects of fading on the detection performance [6]–[19] and the references therein. Based on this, several investigations have been devoted to analyzing the average probability of detection, $P_d$, of unknown
deterministic signals over various fading channels, such as Rayleigh, Rician, Weibull and Nakagami-m fading channels [21], [22], [26], [27]. Likewise, the authors in [28], [29] analyzed ED over generalized fading conditions. However, in spite of the usefulness of these analyses, the derived expressions are typically complex and rather inconvenient, which raises the need for the derivation of more tractable algebraic representation [30]–[38] and the references therein. In addition, the authors in [20], [39]–[41] analyzed the performance over Rayleigh fading and shadowing conditions while the case of Nakagami-m frequency-selective fading channels was recently evaluated in [23]. Moreover, a comparison on the impact of Rayleigh, Nakagami-m and Ricean fading conditions on CSS was reported in [42].

Nevertheless, the aforementioned analyses assume fading conditions that are modeled by a single fading model, which is not a generic method since it limits the generality and usefulness of these expressions. Motivated by this, the present work is devoted to the derivation of an accurate and generalized closed-form approximation for the probability of detection in CSS that can be applied in the analysis of CR systems under different fading channels. This is realized with the aid of Gaussian-Finite-Mixture (GFM), which has been shown to provide accurate modeling of generalized fading conditions.

II. LOCAL SPECTRUM SENSING MODEL

We firstly revisit the methodology of sensing the presence of PU signals in the context of ED. It is recalled that ED is a blind scheme that does not require any priori information of the signal. Hence, the primary signal received at the jth SU, \( S_j(t) \), is assumed to be an unknown deterministic signal while the signal received by the SU, \( r_j(t) \), can be formulated as the following binary hypothesis testing problem

\[
r_j(t) = \begin{cases} w_j(t) & \text{if } H_0_j \text{ holds} \\ h_j S_j(t) + w_j(t) & \text{if } H_1_j \text{ holds} \end{cases}
\]

where both \( H_0_j \) and \( H_1_j \) refer to the two hypotheses of signal absence and signal presence, respectively. Given that SUs are geographically distributed, it is reasonable to assume that they experience independent fading effects with channel gain \( h_j = \alpha_j e^{j\phi_j} \) assumed to be constant during the period of observation. Also, \( w_j(t) \) is modeled as additive white gaussian noise (AWGN) with zero mean and variance \( \sigma^2 \) while the noise samples are assumed to be independent and identically distributed (i.i.d) as well as independent of the signal samples.

During the observation process, the received signal at the jth SU is firstly filtered and squared and then the energy of N samples are combined to produce the local test statistic, \( y_j \). Following that, the test statistic \( y_j \) is compared to a predefined energy threshold, \( \lambda \), which is assumed to be a sum of \( \gamma \) Gaussian random variables (RVs), each with zero mean and unit variance. The test statistic \( y_j \) under hypothesis \( H_0_j \) and \( H_1_j \) is modeled as additive white gaussian noise (AWGN) with zero mean and variance \( \sigma^2 \) while the noise samples are assumed to be independent and identically distributed (i.i.d) as well as independent of the signal samples.

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\]

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III. SPECTRUM SENSING OVER FADING CHANNELS

A. Single User Energy Detection

Evaluating the average \( P_d \) over various fading channels requires averaging of the unconditional \( P_d \) in (3) over the statistics of the corresponding fading process yielding

\[
P_d = \int_0^\infty Q_u(\sqrt{2\gamma}, \sqrt{\lambda'}) f(\gamma) d\gamma.
\]

It is recalled that the GFM approach constitutes a generic and rather useful distribution thanks to its capability to account for several fading environments. Its SNR PDF is given by

\[
p(\gamma) \simeq \frac{e^{-\frac{\gamma^2}{2\sigma^2}}}{\sqrt{2\pi}} \sum_{i=1}^{\gamma} \frac{\omega_i}{\gamma \sigma^2} e^{-\frac{\gamma^2}{2\sigma^2}},
\]

where \( \gamma = E[\alpha^2] E_s / \sigma^2 \) denotes the corresponding average SNR, \( G \) is the number of Gaussian components required to approximate \( p(\gamma) \) within a predetermined accuracy whereas

Fig. 1. Cooperative Spectrum Sensing System Model

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TABLE I

<table>
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<tr>
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<th>$\omega_i$</th>
<th>$\mu_i$</th>
<th>$\sigma_i$</th>
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<td>10</td>
<td>0.0082675</td>
<td>3.4304</td>
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</tr>
</tbody>
</table>

$\omega_i$, $\sigma_i^2$ and $\mu_i$ denote the weight, variance, and mean of the $i$th weighted PDF and are obtained by the well-known expectation maximization (EM) technique [45]. Thus, by substituting (5) into (4), the corresponding average $P_d$ is expressed as follows:

$$P_d = \frac{G}{\sqrt{2\pi} \sigma_i} e^{-\frac{\omega_i^2}{2\sigma_i^2}} \int_0^\infty Q_u(\sqrt{\frac{\omega_i}{\gamma}}, \sqrt{\lambda}) e^{-\frac{\gamma^2}{2\sigma_i^2}} \frac{e^\gamma}{\sqrt{2\pi}} d\gamma.$$  \hspace{1cm} (6)

The $Q_u(a, b)$ can be also expressed according to [44], namely

$$Q_u(\sqrt{\frac{\omega_i}{\gamma}}, \sqrt{\lambda}) = 1 - \sqrt{\frac{\omega_i}{\gamma}} e^{-\frac{\gamma^2}{2\sigma_i^2}} \Gamma(\frac{u}{2}) \left( e^{\frac{\gamma}{\sigma_i^2}} - 1 \right).$$  \hspace{1cm} (7)

Therefore, by substituting (7) into (6) and recalling that

$$Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{y^2}{2}} dy$$  \hspace{1cm} (8)

it immediately follows that

$$P_d \approx \sum_{i=1}^G \frac{\omega_i}{\sigma_i \sqrt{2\pi}} \int_0^\infty e^{-\frac{\omega_i^2}{2\sigma_i^2}} d\gamma$$

$$- \sum_{i=1}^G \frac{\omega_i \sqrt{\frac{\omega_i}{\gamma}}}{\sqrt{\pi} \sigma_i \sqrt{2\pi} \sigma_i} e^{-\frac{\gamma^2}{2\sigma_i^2}} \Gamma(\frac{u}{2}) \int_0^\infty e^{-\frac{\gamma^2}{2\sigma_i^2}} d\gamma \Biggr\}$$

$$\equiv \frac{T_1}{T_2}.$$  \hspace{1cm} (9)

Importantly, the above two terms can be expressed as

$$T_1 = \sum_{i=1}^G \omega_i Q \left( -\frac{\mu_i}{\sigma_i} \right)$$  \hspace{1cm} (10)

and

$$T_2 = \sum_{i=1}^G \omega_i B_i Q \left( \frac{\sigma_i^2 \gamma}{2} - \frac{\mu_i}{\sigma_i} \right)$$  \hspace{1cm} (11)

where

$$B_i = \frac{\lambda^{u-1}}{\Gamma(u) 2^{u-1}} \left( e^{\frac{\lambda}{\sigma_i^2}} - 1 \right) e^{\frac{\sigma_i^2 \gamma}{2} - \mu_i \gamma}.$$  \hspace{1cm} (12)

To this effect, it immediately follows that the average probability of detection in the case of GFM can be expressed as

$$P_d = 1 - \frac{T_1}{T_2}.$$  \hspace{1cm} (13)

It is noted that the advantage of (13) is the relative simplicity to evaluate the $P_d$ by using the weighting coefficients $\omega_i$, $\mu_i$, and $\sigma_i$, which also correspond to particular fading conditions.

B. Cooperative Energy Detection

As already mentioned, it has been shown extensively that cooperative SS increases substantially the corresponding probability of detection which is core principle in ensuring reliable and robust CR communications. The underlying CSS system model is illustrated in Fig. 1 where $n$ denotes the number of collaborating SUs experiencing fading conditions. Based on this, the global decision is made at the FC in order to determine the presence or absence of PU signals according to the following rule

$$\tilde{H} = \sum_{j=1}^n D_j \begin{cases} < k & \tilde{H}_0 \\ \geq k & \tilde{H}_1 \end{cases}$$  \hspace{1cm} (14)

where $\tilde{H}$ is the sum of all locally sensed decisions from the collaborating SUs, denoted by $D_j$. However, although the soft decision may lead to a better sensing performance, the hard decision requires less signaling overhead, since the local sensing users only forward one-bit decision to the FC. Therefore, $D_j$ may take the values 0 or 1. Furthermore, $\tilde{H}_1$ and $\tilde{H}_0$ denote the inferences drawn by the FC. The threshold $k$ is an integer, representing the "$k$-out-of-$n$" voting rule. It can be seen in (14) that the AND-rule corresponds to the case of $k = n$ while the OR-rule corresponds to the case of $k = 1$.
\( \lambda_1 = \cdots = \lambda_n = \lambda \), which yields [20]

\[
\bar{P}_{D,\text{AND}} = \prod_{j=1}^{n} \left\{ \bar{P}_{d_j}(\gamma_j) \right\} \quad (15)
\]

and

\[
\bar{P}_{F,\text{AND}} = \prod_{j=1}^{n} \left\{ 1 - P_{f_j}(\gamma_j) \right\} \quad (16)
\]

where \( P_{d_j} \) and \( P_{f_j} \) are the individual probabilities of detection and false-alarm, respectively. On the contrary, the \( \bar{P}_D \) and \( \bar{P}_F \) using an OR-rule based central CSS can be determined by

\[
\bar{P}_{D,\text{OR}} = 1 - \prod_{j=1}^{n} \left\{ 1 - \bar{P}_{d_j}(\gamma_j) \right\} \quad (17)
\]

and

\[
\bar{P}_{F,\text{OR}} = 1 - \prod_{j=1}^{n} \left\{ P_{f_j}(\gamma_j) \right\} . \quad (18)
\]

To this effect, the \( \bar{P}_{D,\text{AND}} \) over generalized fading channels for the AND rule and CSS can be readily deduced by substituting (13) in (15) yielding

\[
\bar{P}_{D,\text{AND}} = \prod_{j=1}^{n} \left\{ 1 - \sum_{i=1}^{G} \omega_i B_i Q \left( \frac{\sigma_i \tau_j}{2} - \frac{\mu_i}{\sigma_i} \right) \right\} . \quad (19)
\]

Likewise, the \( \bar{P}_{D,\text{OR}} \) in CSS can be obtained by substituting (13) in (17), resulting in

\[
\bar{P}_{D,\text{OR}} = 1 - \prod_{j=1}^{n} \left\{ \sum_{i=1}^{G} \omega_i B_i Q \left( \frac{\sigma_i \tau_j}{2} - \frac{\mu_i}{\sigma_i} \right) \right\} . \quad (20)
\]

For both types of the central fusion rules, \( P_M = 1 - \bar{P}_D \).

IV. NUMERICAL RESULTS

The effectiveness of the derived unified probability expressions is demonstrated with the aid of complimentary ROC curves which are generated by simply estimating the weighting coefficients \( \omega_i, \mu_i, \) and \( \sigma_i \) for the considered fading distributions. For example, the weights for the Nakagami-m with \( m = 3 \) and Weibull with \( \beta = 4 \) models are depicted in Tables I and II, respectively, where \( m \) and \( \beta \) are parameters relating to the severity of fading. These values were obtained with the aid of the EM algorithm with tolerance, \( \varepsilon = 10^{-6} \).

In the same context, Fig. 2 and Fig. 3 illustrate the ROC curves for the case of OR-rule and the AND-rule for a different number of cooperating users \( n \). As expected, both figures show a significant improvement in the performance of the ED in CSS as the number of cooperative users increases. This can be readily observed by the degradation in \( \bar{P}_M \), compared to the single user case i.e., \( n = 1 \). Furthermore, it can be clearly seen that the centralized CSS using OR-rule clearly outperforms the centralized CSS using AND-rule.

Fig. 4 depicts the probability of missed detection versus SNR to further illustrate the system performance by evaluating eq. (20) under Rayleigh and Nakagami-m fading channels. As expected, the performance of the ED in CSS for both types of fading improves as the value of the average SNR increases. Moreover, the performance gain becomes much

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**TABLE II**

<table>
<thead>
<tr>
<th>( n )</th>
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Fig. 5. Complimentary ROC curves of OR-rule under RL and NL composite fading for 1, 2 and 3 cooperative CR users ($\gamma = 10$ dB, N =10, $m = 3$, and $\zeta = 1$ dB).

Fig. 6. Complimentary ROC curves of AND-rule under RL and NL composite fading for 1, 2 and 3 cooperative CR users ($\gamma = 10$ dB, N =10, $m = 3$, and $\zeta = 1$ dB).

Specifically, in the OR-rule, the performance improvement at increasing the number of cognitive users is more evident in the presence of non-severe fading conditions.

Finally, the derived expressions are used to study the effect of the average SNR on the performance of CSS. Indicatively, in the case of AND-rule, an increase of the number of sensing users do not significantly enhance the corresponding performance. On the contrary, the resulting performance is substantially improved as the number of sensing users increases, when employing the OR-rule. Also, the involved performance gain increases as the average SNR increases. Thus, it is evident that (20) and (19) are useful analytical tools in analyzing the performance of ED based SS over various fading conditions.

V. CONCLUSIONS

We provided an extensive analysis of energy detection spectrum sensing over generalized multipath fading channels. The corresponding probability of detection of cooperative spectrum sensing was determined by deriving unified expressions for the fusion center detection probabilities for both the AND-rule and the OR-rule strategies. The derived expressions are rather generic and relatively simple since they can account for various fading conditions. For example, they were employed in analyzing the spectrum sensing performance over Weibull and Nakagami-$m$ fading channels. Thus, they can be useful tools in future design and analysis of cognitive radio systems.

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larger in the higher SNR regime as the number of cooperating users increases. It is also shown that for small values of $n$, all presented curves are almost parallel, indicating that they have the same diversity orders. However, for large values of $n$ the $P_M$ decreases slowly for low average SNR values and then decreases rapidly for high average SNR values, which implies a higher diversity order.

Furthermore, the performance of the proposed expression for CSS under composite multipath/shadowing fading channels is analyzed by means of complimentary ROC curves for the Rayleigh/Lognormal (RL) and Nakagami-$m$/lognormal (NL) fading conditions for the case of OR-rule and the AND-rule. These scenarios are presented in Fig. 5 and Fig. 6, respectively, for $\zeta$, that essentially defines the Lognormal distributions, set to 1 dB. It is clearly observed that the performance of the CSS with the OR-rule outperforms its AND-rule counterpart.

We provided an extensive analysis of energy detection spectrum sensing over generalized multipath fading channels. The corresponding probability of detection of cooperative spectrum sensing was determined by deriving unified expressions for the fusion center detection probabilities for both the AND-rule and the OR-rule strategies. The derived expressions are rather generic and relatively simple since they can account for various fading conditions. For example, they were employed in analyzing the spectrum sensing performance over Weibull and Nakagami-$m$ fading channels. Thus, they can be useful tools in future design and analysis of cognitive radio systems.

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