

# Analytic Symbol Error Rate Evaluation of $M$ -PSK Based Regenerative Cooperative Networks Over Generalized Fading Channels

Mulugeta K. Fikadu<sup>1</sup>, Paschal C. Sofotasios<sup>1,2</sup>, Mikko Valkama<sup>1</sup>, Qimei Cui<sup>3</sup>, Sami Muhaidat<sup>4,5</sup>, and George K. Karagiannidis<sup>2,4</sup>

<sup>1</sup>Department of Electronics and Communications Engineering, Tampere University of Technology, 33101 Tampere, Finland.

e-mail: {mulugeta.fikadu; paschal.sofotasios; mikko.e.valkama} @tut.fi

<sup>2</sup>Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece.

e-mail: {sofotasios; geokarag} @auth.gr

<sup>3</sup>Wireless Technology Innovation Institute, Beijing University of Posts and Telecommunications, 100876 Beijing, China.

e-mail: cuiqimei@bupt.edu.cn

<sup>4</sup>Department of Electrical and Computer Engineering, Khalifa University, PO Box 127788, Abu Dhabi, UAE.

<sup>5</sup>Department of Electronic Engineering, University of Surrey, GU2 7XH, Guildford, United Kingdom.

e-mail: muhaidat@ieee.org

**Abstract**—This paper is devoted to the analytic investigation of a maximum-ratio-combining based regenerative multi-relay cooperative wireless network over non-homogeneous scattering environments. Such propagation conditions are rather realistic as they are encountered often in practical wireless transmission scenarios. Novel analytic expressions are derived for the symbol-error-rate of  $M$ -ary phase shift keying ( $M$ -PSK) over independently and non-identically distributed fading channels. The derived expressions are based on the moment-generating-function (MGF) approach and are given in closed-form in terms of the generalized Lauricella series. A simple algorithm for computing this special function is also proposed while the offered results are validated extensively through comparisons with respective results from computer simulations. Based on this, they are particularly useful in the analytic performance evaluation of such cooperative systems. To this end, it is shown that the performance of the cooperative system is significantly affected, as expected, by the number of employed relays as well as by the value of the involved fading parameters  $\eta$  and  $\mu$ .

## I. INTRODUCTION

Cooperative transmission methods have attracted significant interest over the past decade due to their applicability in size, power, hardware and price constrained devices such as cellular mobile devices, wireless sensors, ad-hoc networks and military communications [1]–[20] and the references therein. Such systems exploit the broadcast nature and the inherent spatial diversity of wireless paths and are typically distinguished between regenerative (decode-and-forward) or non-regenerative (amplify-and-forward) relaying schemes. In general, the digital processing nature of regenerative relaying is considered more efficient than non-regenerative relaying, which requires costly RF transceivers to scale up the analog signal in order to avoid forwarding a noisy version of the signal [21]–[24] and the

references therein.

It is also widely known that fading phenomena create non-negligible detrimental effects on the performance of both conventional and cooperative communication systems [25]–[36] and the references therein. As a result, the limits of various communication scenarios have been analyzed by several researchers over the most basic multipath fading conditions. In this context, Huang *et al.* [37] derived upper and lower bounds for the outage probability (OP) of multi-relay decode-and-forward (DF) networks over independent but non-identically distributed (i.n.i.d) Nakagami- $m$  fading channels. Likewise, Duong *et al.* [38] analyzed the symbol error rate (SER) and OP of DF systems with relay selection over i.n.i.d Nakagami- $m$  fading channels, with integer values of  $m$ . A comprehensive analytical framework of a dual-hop multi-antenna DF system under multipath fading was derived in [39]. In the same context, the performance of DF systems over different fading conditions was addressed in [40]–[42], whereas analysis for the SER of dual-hop DF relaying for  $M$ -ary shift keying ( $M$ -PSK) and  $M$ -ary quadrature amplitude modulation ( $M$ -QAM) over Nakagami- $m$  fading channels was reported in [43].

However, in spite of the numerous investigations on relay communications over fading channels, the majority of investigations assume that multipath fading effects follow either the Rayleigh or the Nakagami- $m$  distributions. This is mainly due to the presence of cumbersome integrals that involve elementary and/or special functions [44]–[54] and the references therein. Nevertheless, these conventional fading models are based on the underlying concept of homogeneous scattering environments, which is not practically realistic since surfaces

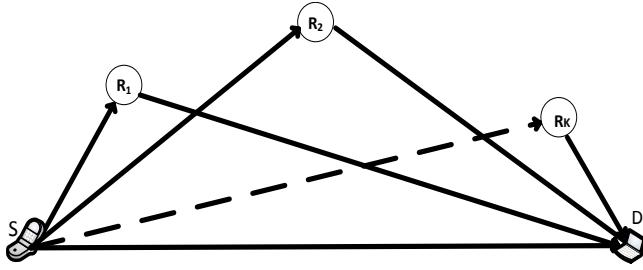


Fig. 1. Multi-node dual hop cooperative relay network.

in most practical radio propagation environments are spatially correlated [55]. This issue was addressed in [56] by proposing the  $\eta-\mu$  distribution, which is a generalized fading model that has been shown to provide particularly accurate fitting to realistic measurement results while it includes as special cases the well known Rayleigh, Nakagami- $m$  and Hoyt distributions [56]–[58]. Based on this, several contributions have been devoted to the analysis of various communication scenarios over generalized fading channels that follow the  $\eta-\mu$  distribution [58]–[62] and the references therein. Motivated by this, the present work is devoted to the evaluation of SER in regenerative relaying for  $M$ -PSK Modulations over generalized fading conditions. Based on this, useful insights are extracted on the effects of fading on the system performance that can be useful in future design and deployments of regenerative cooperative communications.

The reminder of this paper is organized as follows: Section II presents the considered system and channel models. The exact SER analysis over generalized multipath fading conditions is derived in Section III while Section IV presents the corresponding numerical results along with related discussions. Closing remarks are finally provided in Section V.

## II. SYSTEM AND CHANNEL MODELS

### A. System Model

We consider a multi-node cooperative radio access network consisting of a source node  $S$ , intermediate relay nodes  $R_k$ , with  $k = \{1, 2, \dots, K\}$ , and a destination node  $D$ , as depicted in Fig.1. Each node in the system is equipped with a single antenna and a half-duplex decode-and-forward protocol is adopted. Furthermore, the nodes in the system transmit signals through orthogonal channels for avoiding inter-relay interference using for example time division multiple access (TDMA). Based on this, In phase I, the source broadcasts the signal to the destination and to all relay nodes in the network yielding

$$y_{S,D} = \sqrt{P_0} \alpha_{S,D} x + n_{S,D} \quad (1)$$

$$y_{S,R_k} = \sqrt{P_0} \alpha_{S,R_k} x + n_{S,R_k} \quad (2)$$

where  $P_0$  is the transmit source power,  $x$  is the transmitted symbol with normalized unit energy in the first transmission phase,  $\alpha_{S,D}$  and  $\alpha_{S,R_k}$  are the complex fading coefficients

from the source to the destination and from the source to the  $k^{\text{th}}$  relay, respectively, whereas  $n_{S,D}$  and  $n_{S,R_k}$  represent the corresponding additive white Gaussian noise (AWGN) with zero mean and variance  $N_0$ . In the next  $k+1$  time slot, if the  $k^{\text{th}}$  relay decodes correctly, then it forwards the decoded and re-encoded signal to the destination with power  $\bar{P}_{R_k} = P_{R_k}$ ; otherwise, if the decoding is unsuccessful the relay remains silent i.e.,  $\bar{P}_R = 0$ . Based on this, the received signal at the destination terminal can be represented as follows:

$$y_{R_k,D} = \sqrt{\bar{P}_{R_k}} \alpha_{R_k,D} x + n_{R_k,D} \quad (3)$$

where  $\alpha_{R_k,D}$  is the complex fading coefficient from the  $k^{\text{th}}$  relay to the destination and  $n_{R_k,D}$  is the corresponding AWGN. It is assumed that each path experiences narrowband multipath fading that follows the  $\eta-\mu$  distribution and that maximum-ratio-combining (MRC) diversity is employed at the destination. Based on this, the output received signal can be expressed as follows:

$$y_D = w_0 y_{S,D} + \sum_{k=1}^K w_k y_{R_k,D} \quad (4)$$

where  $w_0 = \sqrt{P_0} \alpha_{S,D}^*/N_0$  and  $w_k = \sqrt{\bar{P}_{R_k}} \alpha_{R_k,D}^*/N_0$  denote the optimal MRC coefficients for  $y_{S,D}$  and  $y_{R_k,D}$ , respectively with  $(\cdot)^*$  representing the complex conjugate operator.

### B. Generalized Multipath Fading

It is recalled that  $\eta-\mu$  distribution has been shown to account accurately for small-scale variations of the signal in non-line-of-sight communication scenarios. This fading model is described by the two named parameters,  $\eta$  and  $\mu$ , and it is valid for two different formats that correspond to two physical models [56]. The probability density function (PDF) of the instantaneous SNR  $\gamma = |\alpha|^2 P/N_0$  is given by [56], [59]

$$f_\gamma(\gamma) = \frac{2\sqrt{\pi} \mu^{\mu+\frac{1}{2}} h^\mu \gamma^{\mu-\frac{1}{2}} I_{\mu-\frac{1}{2}}\left(\frac{2\mu H\gamma}{\bar{\gamma}}\right)}{\Gamma(\mu) H^{\mu-\frac{1}{2}} \bar{\gamma}^{\mu+\frac{1}{2}} \exp\left(\frac{2\mu H\gamma}{\bar{\gamma}}\right)} \quad (5)$$

where  $\bar{\gamma} = E[\gamma] = (P/N_0)\Omega$  is the average SNR per symbol with transmit power  $P$  and  $\Omega = E[|\alpha|^2]$  channel variance, whereas  $\Gamma(\cdot)$  and  $I_\alpha(\cdot)$  denote the Euler gamma function and the modified Bessel function of the first kind, respectively [67]. The parameters  $h$  and  $H$  are given in terms of  $\eta$  in two formats: In Format-1,  $h = (2 + \eta^{-1} + \eta)/4$  and  $H = (\eta^{-1} - \eta)/4$  where  $0 < \eta < \infty$  is the scattered-waves power ratio between the in-phase and quadrature components of each multipath cluster. In Format-2,  $h = 1/(1 - \eta^2)$  and  $H = \eta/(1 - \eta^2)$  where  $-1 < \eta < 1$  is the correlation coefficient between the in-phase and quadrature components of the scattered waves in each multipath cluster. In both formats,  $\mu = E^2(\gamma)(1 + (H/h)^2)/2V(\gamma)$  is related to multipath clustering, with  $E(\cdot)$  and  $V(\cdot)$  denoting statistical expectation and variance operations, respectively [56]. The  $\eta-\mu$  distribution includes other important fading models such as: i)

Nakagami- $m$  for  $\mu = m$  and  $\eta \rightarrow 0$  or  $\eta \rightarrow \infty$  in Format-1 or  $\eta \rightarrow \pm 1$  in Format-2, as well as for  $\mu = m/2$  and  $\eta \rightarrow 1$  in Format-1 or  $\eta \rightarrow 0$  in Format-2; ii) Nakagami- $q$  (Hoyt) for  $\mu = 0.5$  and  $\eta = q^2$  in Format-1 or  $q^2 = (1 - \eta)/(1 + \eta)$  in Format-2; iii) Rayleigh for  $\mu = 0.5$  and  $\eta = 1$  in Format-1 or  $\mu = 0.5$  and  $\eta = 0$  in Format-2 [56].

### III. EXACT SYMBOL ERROR RATE ANALYSIS

It is recalled that the end-to-end SER for the considered cooperative system can be expressed as follows [66]

$$P_{\text{SER}}^D = \sum_{z=0}^{2^K-1} P(e|\mathbf{A} = \mathbf{C}_z)P(\mathbf{A} = \mathbf{C}_z) \quad (6)$$

where the binary vector space in the above expression is  $\mathbf{A} = [A(1), A(2), A(3), \dots, A(K)]$  of dimension  $(1 \times K)$  and denotes the state of the relay nodes in the system, with  $A(k)$  taking the binary values of 1 and 0 for successful and unsuccessful decoding, respectively. For the case of statistically independent channels the joint probability of the possible state outcomes can be expressed as

$$\begin{aligned} P(\mathbf{A}) &= P(A(1))P(A(2))P(A(3)) \cdots P(A(K)) \\ &= \prod_{k=1}^K P(A(k)). \end{aligned} \quad (7)$$

Furthermore,  $\mathbf{C}_z = [C(1), C(2), C(3), \dots, C(K)]$  denotes different possible decoding combination of the relays with  $z \in \{0, 2^K - 1\}$ . The conditional error probability  $P(e|\mathbf{A} = \mathbf{C}_z)$  is the error probability conditioned on particular decoding results at relays while  $P(\mathbf{A} = \mathbf{C}_z)$  is the corresponding probability of the decoding outcomes. Based on the MRC method, the instantaneous SNR at the destination for decoding combination,  $\mathbf{C}_z$ , can be expressed as [66]

$$\gamma_{\text{MRC}}(\mathbf{C}_z) = |\alpha_{S,D}|^2 \frac{P_0}{N_0} + \sum_{k=1}^K C(k) |\alpha_{R_k,D}|^2 \frac{P_{R_k}}{N_0}. \quad (8)$$

It is also recalled that the MGF for independent fading channels in DF scheme is given by [63]

$$M_{\gamma_{\text{MRC}}}(s) = M_{\gamma_{S,D}}(s) \prod_{k=1}^K C(k) M_{\gamma_{R_k,D}}(s) \quad (9)$$

which in the present analysis can be expressed according to [59, eq. (6)], namely

$$M_{\gamma_{\eta-\mu}}\left(\frac{g}{\sin^2 \theta}\right) = \left(\frac{4\mu^2 h(2(h-H)\mu + \frac{g}{\sin^2 \theta} \bar{\gamma})^{-1}}{(2(h+H)\mu + \frac{g}{\sin^2 \theta} \bar{\gamma})}\right)^\mu. \quad (10)$$

Based on the MGF expression, the end-to-end error probability for  $M$ -PSK constellations over individual  $\eta - \mu$  fading link when  $\eta$ ,  $\mu$  and  $\bar{\gamma}$  in each path are not necessarily equal can be expressed as [65, eq. (5.78)]

$$\begin{aligned} \bar{P}_{\text{M-PSK}} &= \underbrace{\frac{1}{\pi} \int_0^{\pi/2} M_\gamma\left(\frac{g_{\text{PSK}}}{\sin^2 \theta}\right) d\theta}_{\triangleq \mathcal{I}_1} \\ &\quad + \underbrace{\frac{1}{\pi} \int_{\pi/2}^{\frac{(M-1)\pi}{M}} M_\gamma\left(\frac{g_{\text{PSK}}}{\sin^2 \theta}\right) d\theta}_{\triangleq \mathcal{I}_{11}} \end{aligned} \quad (11)$$

where  $g_{\text{PSK}} = \sin^2(\pi/M)$ . It is noted here that in order to evaluate (6), we firstly need to determine the error probability for decoding at the destination terminal, using MRC, under given decoding outcomes at nodes i.e., for a given  $\mathbf{C}_z$ . To this end and based on the MGF approach the probability  $P(e|\mathbf{A} = \mathbf{C}_z)$  can be expressed as (12), given at the top of the next page. Evidently, the analytic solution of (12) is subject to analytic evaluation of the integrals  $\mathcal{I}_1$  and  $\mathcal{I}_{11}$ . To this end, for the case of non-identical fading parameters, i.e.,  $\mu_{S,D} \neq \mu_{R_1,D} \neq \dots \neq \mu_{R_K,D}$ ,  $\eta_{S,D} \neq \eta_{R_1,D} \neq \dots \neq \eta_{R_K,D}$  and  $\bar{\gamma}_{S,D} \neq \bar{\gamma}_{R_1,D} \neq \dots \neq \bar{\gamma}_{R_K,D}$ , the  $\mathcal{I}_1$  term can be re-written as

$$\begin{aligned} \mathcal{I}_1 &= \frac{1}{\pi} \int_0^{\pi/2} \frac{1}{\left(1 + \frac{A_1}{\sin^2 \theta}\right)^{\mu_{S,D}} \left(1 + \frac{A_2}{\sin^2 \theta}\right)^{\mu_{S,D}}} \\ &\quad \times \prod_{k=1}^K C(k) \left( \frac{1}{\left(1 + \frac{B_{1k}}{\sin^2 \theta}\right)^{\mu_{R_k,D}} \left(1 + \frac{B_{2k}}{\sin^2 \theta}\right)^{\mu_{R_k,D}}} \right) d\theta \end{aligned} \quad (13)$$

where

$$\left\{ \begin{array}{c} A_1 \\ A_2 \end{array} \right\} = \frac{\bar{\gamma}_{S,D} g_{\text{PSK}}}{2(h_{S,D} \{\mp\} H_{S,D}) \mu_{S,D}} \quad (14)$$

and

$$\left\{ \begin{array}{c} B_{1k} \\ B_{2k} \end{array} \right\} = \frac{\bar{\gamma}_{R_k,D} g_{\text{PSK}}}{2(h_{R_k,D} \{\mp\} H_{R_k,D}) \mu_{R_k,D}}. \quad (15)$$

By also setting  $u = \sin^2(\theta)$  and carrying out long but basic algebraic manipulations one obtains

$$\begin{aligned} \mathcal{I}_1 &= \frac{\beta_{\gamma_{\text{MRC}}}(g_{\text{PSK}})}{2\pi} \int_0^1 \frac{(1-u)^{-\frac{1}{2}} u^{2(\mu_{S,D} + \sum_{k=1}^K C(k)\mu_{R_k,D})-\frac{1}{2}}}{\left(1 + \frac{u}{A_1}\right)^{\mu_{S,D}} \left(1 + \frac{u}{A_2}\right)^{\mu_{S,D}}} \\ &\quad \times \prod_{k=1}^K \frac{C(k) du}{\left(1 + \frac{u}{B_{1k}}\right)^{\mu_{R_k,D}} \left(1 + \frac{u}{B_{2k}}\right)^{\mu_{R_k,D}}} \end{aligned} \quad (16)$$

where

$$\begin{aligned} \beta_{\gamma_{\text{MRC}}}(g_{\text{PSK}}) &= \left( \frac{4\mu_{S,D}^2 (h_{S,D}^2 - H_{S,D}^2)}{\bar{\gamma}_{S,D}^2 g_{\text{PSK}}^2} \right)^{\mu_{S,D}} \\ &\quad \times \prod_{k=1}^K C(k) \left( \frac{4\mu_{R_k,D}^2 (h_{R_k,D}^2 - H_{R_k,D}^2)}{\bar{\gamma}_{R_k,D}^2 g_{\text{PSK}}^2} \right)^{\mu_{R_k,D}}. \end{aligned} \quad (17)$$

$$\begin{aligned}
P(e|\mathbf{A} = \mathbf{C}_z) &= \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{4\mu_{S,D}^2 h_{S,D} (2(h_{S,D} - H_{S,D})\mu_{S,D} + \frac{g_{PSK}}{\sin^2 \theta} \bar{\gamma}_{S,D})^{-1}}{(2(h_{S,D} + H_{S,D})\mu_{S,D} + \frac{g_{PSK}}{\sin^2 \theta} \bar{\gamma}_{S,D})} \right)^{\mu_{S,D}} \\
&\quad \times \prod_{k=1}^K C(k) \left( \frac{4\mu_{R_k,D}^2 h_{R_k,D} (2(h_{R_k,D} - H_{R_k,D})\mu_{R_k,D} + \frac{g_{PSK}}{\sin^2 \theta} \bar{\gamma}_{R_k,D})^{-1}}{(2(h_{R_k,D} + H_{R_k,D})\mu_{R_k,D} + \frac{g_{PSK}}{\sin^2 \theta} \bar{\gamma}_{R_k,D})} \right)^{\mu_{R_k,D}} d\theta \\
&\quad + \frac{1}{\pi} \int_{\pi/2}^{\frac{(M-1)\pi}{M}} \left( \frac{4\mu_{S,D}^2 h_{S,D} (2(h_{S,D} - H_{S,D})\mu_{S,D} + \frac{g_{PSK}}{\sin^2 \theta} \bar{\gamma}_{S,D})^{-1}}{(2(h_{S,D} + H_{S,D})\mu_{S,D} + \frac{g_{PSK}}{\sin^2 \theta} \bar{\gamma}_{S,D})} \right)^{\mu_{S,D}} \\
&\quad \times \prod_{k=1}^K C(k) \left( \frac{4\mu_{R_k,D}^2 h_{R_k,D} (2(h_{R_k,D} - H_{R_k,D})\mu_{R_k,D} + \frac{g_{PSK}}{\sin^2 \theta} \bar{\gamma}_{R_k,D})^{-1}}{(2(h_{R_k,D} + H_{R_k,D})\mu_{R_k,D} + \frac{g_{PSK}}{\sin^2 \theta} \bar{\gamma}_{R_k,D})} \right)^{\mu_{R_k,D}} d\theta.
\end{aligned} \tag{12}$$

Importantly, eq. (16) can be expressed in closed-form in terms of [64, eq. (7.2.4.57)] yielding

$$\begin{aligned}
\mathcal{I}_1 &= \frac{\beta_{\gamma_{MRC}}(g_{PSK})\Gamma(2\mu_{S,D} + 2\sum_{k=1}^K C(k)\mu_{R_k,D} + \frac{1}{2})}{2\sqrt{\pi}\Gamma(2\mu_{S,D} + 2\sum_{k=1}^K C(k)\mu_{R_k,D} + 1)} \\
&\quad \times F_D^{(2K+2)} \left( 2\mu_{S,D} + 2\sum_{k=1}^K C(k)\mu_{R_k,D} + \frac{1}{2}; \mu_{S,D}, \mu_{S,D}, \right. \\
&\quad \left. \mu_{R_1,D}, \dots, \mu_{R_K,D}, \mu_{R_1,D}, \dots, \mu_{R_K,D}; \right. \\
&\quad \left. 2\mu_{S,D} + 2\sum_{k=1}^K C(k)\mu_{R_k,D} + 1; -\frac{1}{A_1}, -\frac{1}{A_2}, \right. \\
&\quad \left. -\frac{1}{B_{11}}, \dots, -\frac{1}{B_{1K}}, -\frac{1}{B_{21}}, \dots, -\frac{1}{B_{2K}} \right)
\end{aligned} \tag{18}$$

where  $F_D^{(n)}(\cdot)$  denotes the generalized Lauricella hypergeometric function of  $n$  variables [64].

In the same context, for the  $\mathcal{I}_{11}$  integral we set  $u = \cos^2(\theta)/\cos^2(\pi/M)$  in (12), which yields

$$\begin{aligned}
\mathcal{I}_{11} &= \frac{M_{\gamma_{MRC}}(g_{PSK}) \cos(\pi/M)}{2\pi} \\
&\quad \times \int_0^1 \frac{u^{-\frac{1}{2}} (1 - \cos^2(\pi/M)u)^{2(\mu_{S,D} + \sum_{k=1}^K C(k)\mu_{R_k,D}) - \frac{1}{2}}}{\left(1 - \frac{\cos^2(\pi/M)}{1+A_1} u\right)^{\mu_{S,D}} \left(1 - \frac{\cos^2(\pi/M)}{1+A_2} u\right)^{\mu_{S,D}}} \\
&\quad \times \prod_{k=1}^K \frac{C(k)du}{\left(1 - \frac{\cos^2(\pi/M)}{1+B_{1k}} u\right)^{\mu_{R_k,D}} \left(1 - \frac{\cos^2(\pi/M)}{1+B_{2k}} u\right)^{\mu_{R_k,D}}}.
\end{aligned} \tag{19}$$

Evidently, the above integral can be also expressed in closed-form in terms of the  $F_D^{(n)}(\cdot)$  function. As a result, by performing the necessary change of variables and substituting in (19), one obtains

$$\begin{aligned}
\mathcal{I}_{11} &= \frac{M_{\gamma_{MRC}}(g_{PSK})}{\pi} F_D^{(2K+3)} \left( \frac{1}{2}; \mu_{S,D}, \mu_{S,D}, \mu_{R_1,D}, \dots, \right. \\
&\quad \left. \mu_{R_K,D}, \mu_{R_1,D}, \dots, \mu_{R_K,D}, \frac{1}{2} - 2\mu_{S,D} + 2\sum_{k=1}^K C(k)\mu_{R_k,D}; \right. \\
&\quad \left. \frac{3}{2}; \frac{\cos^2(\pi/M)}{1+A_1}, \frac{\cos^2(\pi/M)}{1+A_2}, \frac{\cos^2(\pi/M)}{1+B_{11}}, \dots, \right. \\
&\quad \left. \frac{\cos^2(\pi/M)}{1+B_{1K}}, \frac{\cos^2(\pi/M)}{1+B_{21}}, \dots, \frac{\cos^2(\pi/M)}{1+B_{2K}}, \cos^2(\pi/M) \right).
\end{aligned} \tag{20}$$

It is evident that with the aid of the derived closed-form expressions for  $\mathcal{I}_1$  and  $\mathcal{I}_{11}$ , the corresponding error probability term for  $M$ -PSK Modulation can be straightforwardly determined by,  $P(e|\mathbf{A} = \mathbf{C}_z) = \mathcal{I}_1 + \mathcal{I}_{11}$ .

In order to derive a closed-form expression for the overall average SER of the considered system, we additionally need to determine the decoding probability of the relay nodes  $P(\mathbf{A} = \mathbf{C}_z)$  which is a direct product of the element terms  $P(\bar{\gamma}_{S,R_k}) = P(A(k) = C(k) = 0)$  i.e. decoding error at the relays and  $(1 - P(\bar{\gamma}_{S,R_k})) = P(A(k) = C(k) = 1)$  i.e. successful decoding at the relays. This can be also evaluated using the MGF approach, namely

$$\begin{aligned}
P(\bar{\gamma}_{S,R_k}) &= \underbrace{\frac{1}{\pi} \int_0^{\pi/2} M_{\gamma_{S,R_k}} \left( \frac{g_{PSK}}{\sin^2 \theta} \right) d\theta}_{\triangleq \mathcal{I}_2} \\
&\quad + \underbrace{\frac{1}{\pi} \int_{\pi/2}^{\frac{(M-1)\pi}{M}} M_{\gamma_{S,R_k}} \left( \frac{g_{PSK}}{\sin^2 \theta} \right) d\theta}_{\triangleq \mathcal{I}_{22}}.
\end{aligned} \tag{21}$$

In order to evaluate  $\mathcal{I}_2$  and  $\mathcal{I}_{22}$  in closed-form, we follow the same procedure as in the derivation of the closed-form expressions for  $\mathcal{I}_1$  and  $\mathcal{I}_{11}$  as the difference is that there is no involvement of  $\mu_{R_k,D}$  term in this case. Based on this, and after long but basic algebraic manipulations, the closed-form expressions for  $\mathcal{I}_2$  and  $\mathcal{I}_{22}$  can be expressed as,

$$\begin{aligned} \mathcal{I}_2 &= \frac{\beta_{\gamma_{S,R_k}}(g_{\text{PSK}})\Gamma(2\mu_{S,R_k} + \frac{1}{2})}{2\sqrt{\pi}\Gamma(2\mu_{S,R_k} + 1)} \\ &\times F_D^{(2)}\left(2\mu_{S,R_k} + \frac{1}{2}; \mu_{S,R_k}, \mu_{S,R_k}; 2\mu_{S,k} + 1; -\frac{1}{C_1}, -\frac{1}{C_2}\right) \end{aligned} \quad (22)$$

$$\begin{aligned} \mathcal{I}_{22} &= \frac{M_{\gamma_{S,R_k}}(g_{\text{PSK}})}{\pi} F_D^{(3)}\left(\frac{1}{2}; \mu_{S,R_k}, \mu_{S,R_k}, \frac{1}{2} - 2\mu_{S,R_k}; \right. \\ &\left. \frac{3}{2}; \frac{\cos^2(\pi/M)}{1+C_1}, \frac{\cos^2(\pi/M)}{1+C_2}, \cos^2(\pi/M)\right) \end{aligned} \quad (23)$$

where

$$\begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \frac{\bar{\gamma}_{S,R_k} g_{\text{PSK}}}{2(h_{S,R_k} \{\mp\} H_{S,R_k})} \quad (24)$$

$$\beta_{S,R_k}(g_{\text{PSK}}) = \left( \frac{4\mu_{S,R_k}^2(h_{S,R_k}^2 - H_{S,R_k}^2)}{\bar{\gamma}_{S,R_k}^2 g_{\text{PSK}}^2} \right)^{\mu_{S,R_k}}. \quad (25)$$

To this effect, the decoding error probability of the relay nodes can be readily obtained by  $P(\mathbf{A} = \mathbf{C}_z = 0) = \mathcal{I}_2 + \mathcal{I}_{22}$  whereas the  $P_{\text{SER}}^D$  for  $M$ -PSK is deduced by inserting  $P(e|\mathbf{A} = \mathbf{C}_z)$  and  $P(\mathbf{A} = \mathbf{C}_z)$  in (6).

To the best of the authors' knowledge, the derived analytic expressions are novel. Furthermore, it is noted that the  $F_D^{(n)}(\cdot)$  function has been studied extensively over the past decades. However, it is not included as built-in function in popular software packages such as MATLAB, MATHEMATICA and MAPLE. Based on this, a simple MATLAB algorithm for computing this function straightforwardly is proposed in the Appendix.

#### IV. NUMERICAL RESULTS

In this Section, the offered analytic results are employed in evaluating the performance of the considered regenerative system for different communications scenarios. Respective results from computer simulations are also provided for verifying the validity of the analytic results. The variance of the noise is assumed to be  $N_0 = 1$  while  $M$ -PSK constellation is employed assuming equally allocated transmit powers to the source and the relay nodes. It is noted here that the presented results are limited to Format-1 of the  $\eta - \mu$  distribution but they can be also readily extended to scenarios that correspond to Format-2.

We firstly plot the corresponding SER as a function of SNR employing one, two and three relay nodes using equal power allocation, i.e.,  $P_0 = P_{R_k} = P/(K+1)$  over symmetric and balanced  $\eta - \mu$  fading channels for 4-PSK constellations. In Fig. 2, the  $\eta - \mu$  fading parameters are set to  $\mu = 0.5$  and  $\eta = 1$  while  $\Omega$  parameters are equal to 0dB. Also, the performance of direct transmission mode with  $P_0 = P$  is included as a reference for demonstrating the benefits of cooperative transmissions in wireless network designs. It is

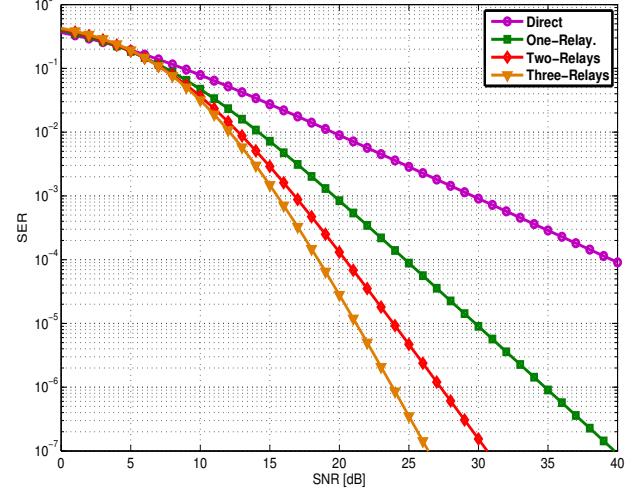


Fig. 2. SER in  $\eta - \mu$  fading with  $\mu = 0.5$ ,  $\eta = 1$  and  $\Omega_{S,D} = \Omega_{S,R_k} = \Omega_{R_k,D} = 0$ dB for 4-PSK Modulation with different number of relays.

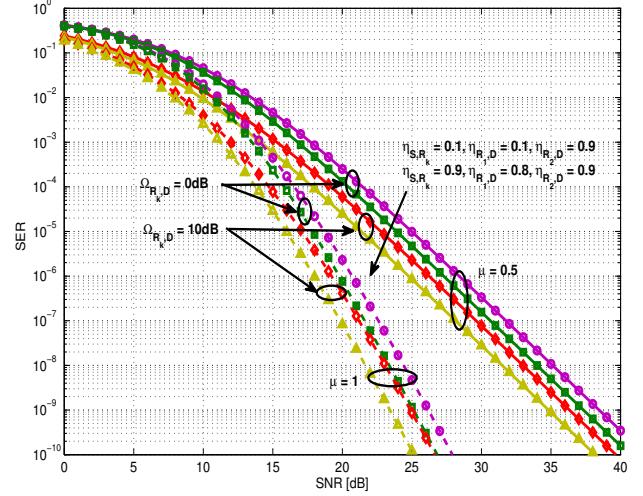


Fig. 3. SER in  $\eta - \mu$  fading with  $\mu = \{0.5, 1\}$ ,  $\eta_{S,D} = 0.9$  and  $\Omega_{S,D} = \Omega_{S,R_k} = 0$ dB,  $\Omega_{R_k,D} = \{0, 10\}$ dB with different  $\eta_{S,R_k}$  and  $\eta_{R1,D}$  for 4-PSK Modulation for  $K = 2$ .

observed that at a target SER of  $10^{-4}$  the single relay system exhibits a gain of 15dB over the direct transmission, whereas the two and three relay systems outperform the direct scenario by about 19.5dB and 21.5dB, respectively.

Likewise, Fig. 3, shows the SER performance in non-identical  $\eta - \mu$  fading condition for 4-PSK modulations for the case of two relays with equal power allocation. It is assumed that  $\eta_{S,D} = 0.9$ ,  $\Omega_{S,D} = \Omega_{S,R_k} = 0$ dB,  $\Omega_{R_k,D} = \{0, 10\}$ dB,  $\mu_{S,D} = \mu_{S,R_k} = \mu_{R_k,D} = \mu = \{0.5, 1\}$  with different values of  $\eta_{S,R_k}$  and  $\eta_{R1,D}$ . The figure reveals that when  $\mu$  is maintained fixed, the fading becomes less severe as  $\{\eta_{S,R_k}, \eta_{R1,D}\}$  increases from  $\{0.1, 0.1\}$  to  $\{0.9, 0.8\}$  for  $\eta_{R2,D} = 0.9$ . For example, at a SER of  $10^{-4}$  almost 1.25dB and 1.75dB

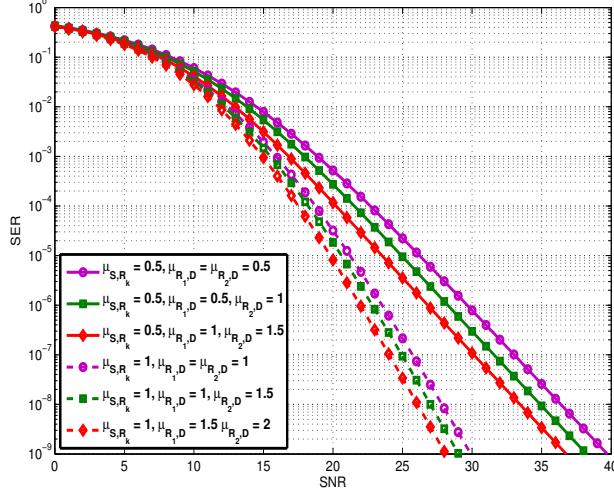


Fig. 4. SER in  $\eta - \mu$  fading with  $\mu_{S,D} = 0.5$  and  $\eta = 0.1$ ,  $\Omega_{S,D} = \Omega_{S,R_k} = \Omega_{R_k,D} = 0\text{dB}$  for different  $\mu_{S,R_k}$  and  $\mu_{R_k,D}$  for 4-PSK Modulation and  $K = 2$ .

gains are achieved for  $\Omega_{R_k,D} = 0\text{dB}$  and  $\Omega_{R_k,D} = 10\text{dB}$ , respectively, for the considered values of  $\mu$ .

Finally, Fig. 4 shows the SER performance in  $\eta - \mu$  fading scenario with  $\mu_{S,D} = 0.5$  and  $\eta = 0.1$  for 4-PSK modulation and balanced links of channel variance 0dB employing two relays with equal power allocation for different values of  $\mu_{S,R_k}$  and  $\mu_{R_k,D}$ . It is noticed that increasing at least one of  $\mu_{R_k,D}$ 's value at a fixed  $\mu_{S,R_k}$  or increasing both  $\mu_{S,R_k}$  and  $\mu_{R_k,D}$  simultaneously can improve the cooperation performance of the system. Indicatively, at a SER of  $10^{-4}$ , almost 1.25dB and 1.75dB gains are observed when  $\{\mu_{S,R_k}, \mu_{R_1}, \mu_{R_2}\}$  changes from  $\{0.5, 0.5, 0.5\}$  to  $\{0.5, 0.5, 1\}$  and from  $\{0.5, 0.5, 1\}$  to  $\{0.5, 1, 1.5\}$ , respectively. Also, nearly 0.75dB and 1dB gains are achieved when  $\{\mu_{R_1}, \mu_{R_2}\}$  varies from  $\{1, 1\}$  to  $\{1, 1.5\}$  and then to the less-severe fading conditions of  $\{1.5, 2\}$  when  $\mu_{S,R_k} = 1$ . On the other hand, about 4dB is gained when both  $\mu_{S,R_k}$  and  $\mu_{R_k,D}$  increase at the same time.

## V. CONCLUSION

In this paper, we analyzed the performance of regenerative cooperative systems over generalized fading conditions. Novel exact closed-form expressions for the end-to-end average SER for  $M$ -PSK modulated signals were derived over independently and non-identically distributed channels. The derived analytic expressions were subsequently employed to draw insights of the different fading parameters in the generalized  $\eta - \mu$  fading conditions and their impact on the end-to-end system performance. Indicatively, it was shown that the system performance is highly affected by the  $\eta - \mu$  fading parameters regardless of the number of employed relay nodes.

## APPENDIX

## A MATLAB ALGORITHM FOR COMPUTING THE GENERALIZED LAURICELLA FUNCTION

The Generalized Lauricella function is defined by the following non-infinite single integral,

$$F_D^{(n)}(a, b_1, \dots, b_n, c; x_1, \dots, x_n) \triangleq \int_0^1 \frac{\Gamma(c)^{a-1}(1-x_1t)^{-b_1} \cdots (1-x_nt)^{-b_n}}{\Gamma(a)\Gamma(c-a)(1-t)^{a-c+1}} dt \quad (26)$$

The above representation can be straightforwardly evaluated with the aid of the following MATLAB algorithm,

```
Function FD = Lauricella(a, b1, ..., bn, c, x1, ..., xn);
f = gamma(c). / gamma(a). * gamma(c - a);
Q = @(t) f. * t.^{a-1}. * (1 - t).^(c - a - 1). * ...
(1 - x1. * t).^( -b1) ... (1 - xn. * t).^( -bn);
FD = quad(Q, 0, 1);
```

## ACKNOWLEDGEMENTS

This work was supported by the Finnish Funding Agency for Technology and Innovation (Tekes) under the project entitled “Energy-Efficient Wireless Networks and Connectivity of Devices-Systems (EWINE-S)”, by the Academy of Finland under the projects No. 251138 “Digitally-Enhanced RF for Cognitive Radio Devices and No. 284694 “Fundamentals of Ultra Dense 5G Networks with Application to Machine Type Communication” and by the National Nature Science Foundation of China Project “Grant No. 61471058”.

## REFERENCES

- [1] A. Nosratinia, T. E. Hunter, and A. Hedayat, “Cooperative communication in wireless networks,” *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 74–80, Oct. 2004.
- [2] H. A. Suraweera, P. J. Smith and J. Armstrong, “Outage probability of cooperative relay networks in Nakagami- $m$  fading channels,” *IEEE Commun. Lett.*, vol. 10, no. 12, pp. 834–836, Dec. 2006.
- [3] K. Ho-Van, P. C. Sofotasios, and S. Freear, “Underlay cooperative cognitive networks with imperfect Nakagami- $m$  fading channel information and strict transmit power constraint: interference statistics and outage probability analysis,” *IEEE/KICS J. Commun. Networks*, vol. 16, no. 1, pp. 10–17, Feb. 2014.
- [4] Z. Ding, M. Peng and H. V. Poor, “Cooperative Non-Orthogonal Multiple Access in 5G Systems,” *IEEE Commun. Lett.*, to appear in 2015.
- [5] K. Ho-Van, P. C. Sofotasios, G. C. Alexandropoulos, and S. Freear, “Bit error rate of underlay decode-and-forward cognitive networks with best relay selection,” *IEEE/KICS J. Commun. Networks*, vol. 17, no. 2, pp. 162–171, Apr. 2015.
- [6] T. T. Duy, T. Q. Duong, D. B. da Costa, V. N. Q. Bao, and M. Elkashlan, “Proactive relay selection with joint impact of hardware impairment and co-channel interference,” *IEEE Trans. Commun.*, vol. 63, no. 5, pp. 1594–1606, May 2015.
- [7] M. K. Fikadu, P. C. Sofotasios, Q. Cui, M. Valkama, and G. K. Karagiannidis, “Exact error analysis and energy-efficiency optimization of regenerative relay systems,” *IEEE Trans. Veh. Technol.*, Accepted for Publication.
- [8] K. Ho-Van, and P. C. Sofotasios, “Exact BER analysis of underlay decode-and-forward multi-hop cognitive networks with estimation errors,” *IET Commun.*, vol. 7, no. 18, pp. 2122–2132, Dec. 2013.
- [9] G. C. Alexandropoulos, P. C. Sofotasios, K. Ho-Van, and S. Freear, “Symbol error probability of DF relay selection over arbitrary Nakagami- $m$  fading channels,” *HINDAWI Journal of Engineering*, vol. 2013, Article ID 325045, 2013.

- [10] M. K. Fikadu, P. C. Sofotasios, M. Valkama, Q. Cui, S. Muhaidat, and G. K. Karagiannidis, "Outage probability analysis of full-duplex regenerative relaying over generalized asymmetric fading channels," in *IEEE Globecom '15*, San Diego, CA, USA, Dec. 2015.
- [11] G. C. Alexandropoulos, A. Papadogiannis, and P. C. Sofotasios, "A comparative study of relaying schemes with decode-and-forward over Nakagami- $m$  fading channels," *Journal of Computer Networks and Communications*, vol. 2011, Article ID 560528, Dec. 2011.
- [12] M. K. Fikadu, P. C. Sofotasios, M. Valkama, S. Muhaidat, Q. Cui, and G. K. Karagiannidis, "Outage probability analysis of dual-hop full-duplex decode-and-forward relaying over generalized multipath fading conditions," in *IEEE WiMob '15*, Abu Dhabi, UAE, Oct. 2015.
- [13] K. Ho-Van, and P. C. Sofotasios, "Bit error rate of underlay multi-hop cognitive networks in the presence of multipath fading," in *IEEE ICUFN '13*, Da Nang, Vietnam, July 2013.
- [14] M. K. Fikadu, P. C. Sofotasios, M. Valkama, Q. Cui, S. Muhaidat, and G. K. Karagiannidis, "Analytic symbol error rate evaluation of  $M$ -PSK based regenerative cooperative networks over generalized fading channels," in *IEEE WiMob '15*, Abu Dhabi, UAE, Oct. 2015.
- [15] K. Ho-Van, P. C. Sofotasios, V. Que Son, L. Thanh Tra, and P. Hong Lien, "Analysis of cognitive cooperative networks with best relay selection and diversity reception," in *IEEE ATC '15*, HoChiMinh City, Vietnam, Oct. 2015.
- [16] L. Mohjazi, D. Dawoud, P. C. Sofotasios, S. Muhaidat, M. Dianati, M. Valkama, and G. K. Karagiannidis, "Unified analysis of cooperative spectrum sensing over generalized multipath fading channels," in *IEEE PIMRC '15*, Hong Kong, Aug/Sep. 2015.
- [17] M. K. Fikadu, P. C. Sofotasios, M. Valkama, Q. Cui, and G. K. Karagiannidis, "Energy-efficiency analysis of regenerative cooperative systems under spatial correlation," in *IEEE PIMRC '15*, Hong Kong, Aug/Sep. 2015.
- [18] K. Ho-Van, P. C. Sofotasios, S. V. Que, T. D. Anh, T. P. Quang, and L. P. Hong, "Analytic performance evaluation of underlay relay cognitive networks with channel estimation errors," in *IEEE ATC '13*, HoChiMinh City, Vietnam, Oct. 2013.
- [19] M. K. Fikadu, P. C. Sofotasios, M. Valkama, and Q. Cui, "Analytic performance evaluation of  $M$ -QAM based decode-and-forward relay networks over enriched multipath fading channels," in *IEEE WiMob '14*, Larnaca, Cyprus, Oct. 2014.
- [20] K. Ho-Van, and P. C. Sofotasios, "Outage behaviour of cooperative underlay cognitive networks with inaccurate channel estimation," in *IEEE ICUFN '13*, Da Nang, Vietnam, July 2013.
- [21] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity part I: System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.
- [22] J. N. Laneman, D. N. C. Tse, and G.W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. on Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
- [23] T. Q. Duong, D. B. da Costa, M. Elkashlan, and V. N. Q. Bao, "Cognitive amplify-and-forward relay networks over Nakagami- $m$  fading," *IEEE Trans. Veh. Technol.*, vol. 61, no. 5, pp. 2368–2374, June 2012.
- [24] C. Zhong, H. A. Suraweera, A. Huang, Z. Zhang and C. Yuen, "Outage probability of dual-hop multiple antenna AF relaying systems with interference,"
- [25] S. Ki Yoo, S. L. Cotton, P. C. Sofotasios, M. Matthaiou, M. Valkama, and G. K. Karagiannidis, "The  $\kappa - \mu$  / Inverse gamma fading model," in *IEEE PIMRC '15*, Hong Kong, Aug/Sep. 2015.
- [26] S. Ki Yoo, P. C. Sofotasios, S. L. Cotton, M. Matthaiou, M. Valkama, and G. K. Karagiannidis, "The  $\eta - \mu$  / Inverse gamma composite fading model," in *IEEE PIMRC '15*, Hong Kong, Aug/Sep. 2015.
- [27] P. C. Sofotasios, T. A. Tsiftsis, K. Ho-Van, S. Freear, L. R. Wilhelmsson, and M. Valkama, "The  $\kappa - \mu$ /inverse-Gaussian composite statistical distribution in RF and FSO wireless channels," in *IEEE VTC '13 - Fall*, Las Vegas, USA, Sep. 2013.
- [28] P. C. Sofotasios, T. A. Tsiftsis, M. Ghogho, L. R. Wilhelmsson, and M. Valkama, "The  $\eta - \mu$ /inverse Gaussian distribution: A novel physical multipath/shadowing fading model," in *IEEE ICC'13*, Budapest, Hungary, June 2013.
- [29] P. C. Sofotasios, and S. Freear, "The  $\alpha - \kappa - \mu$ /gamma composite distribution: A generalized non-linear multipath/shadowing fading model," in *IEEE INDICON '11*, Hyderabad, India, Dec. 2011.
- [30] P. C. Sofotasios, and S. Freear, "The  $\alpha - \kappa - \mu$  extreme distribution: characterizing non linear severe fading conditions," in *ATNAC '11*, Melbourne, Australia, Nov. 2011.
- [31] P. C. Sofotasios, and S. Freear, "The  $\eta - \mu$ /gamma and the  $\lambda - \mu$ /gamma multipath/shadowing distributions," in *ATNAC '11*, Melbourne, Australia, Nov. 2011.
- [32] P. C. Sofotasios, and S. Freear, "On the  $\kappa - \mu$ /gamma composite distribution: A generalized multipath/shadowing fading model," in *IEEE IMOC '11*, Natal, Brazil, Oct. 2011.
- [33] S. Harput, P. C. Sofotasios, and S. Freear, "A novel composite statistical model for ultrasound applications," in *IEEE IUS '11*, Orlando, FL, USA, Oct. 2011.
- [34] P. C. Sofotasios, and S. Freear, "The  $\kappa - \mu$ /gamma extreme composite distribution: A physical composite fading model," in *IEEE WCNC '11*, Cancun, Mexico, Mar. 2011.
- [35] P. C. Sofotasios, and S. Freear, "The  $\kappa - \mu$ /gamma composite fading model," in *IEEE ICWITS '10*, Honolulu, HI, USA, Aug/Sep. 2010.
- [36] P. C. Sofotasios, and S. Freear, "The  $\eta - \mu$ /gamma composite fading model," in *IEEE ICWITS '10*, Honolulu, HI, USA, Aug/Sep. 2010.
- [37] S.-Q. Huang, H.-H. Chen, and M.-Y. Lee, "Performance bounds of multi-relay decode-and-forward cooperative networks over Nakagami- $m$  fading channels," in *Proc. IEEE ICC '11*, 2011, Kyoto, Japan, 5–9 June, pp. 1–5.
- [38] T. Duong, V. N. Q. Bao, and H. J. Zepernick, "On the performance of selection decode-and-forward relay networks over Nakagami- $m$  fading channels," *IEEE Commun. Lett.*, vol. 13, no. 3, pp. 172–174, Mar. 2009.
- [39] S. N. Datta, S. Chakrabarti, and R. Roy, "Comprehensive error analysis of multi-antenna decode-and-forward relay in fading channels," *IEEE Commun. Lett.*, vol. 16, no. 1, pp. 47–49, Jan. 2012.
- [40] S. N. Datta and S. Chakrabarti, "Unified error analysis of dual-hop relay link in Nakagami- $m$  fading channels," *IEEE Commun. Lett.*, vol. 14, no. 10, pp. 897–899, Oct. 2010.
- [41] S.S. Ikki and M.H. Ahmed, "Multi-branch decode-and-forward cooperative diversity networks performance analysis over Nakagami- $m$  fading channels," *IET Communications*, vol. 5, no. 6, pp. 872–878, June 2011.
- [42] S.S. Ikki and M.H. Ahmed, "Performance analysis of adaptive decode-and-forward cooperative diversity networks with best-relay selection," *IEEE Trans. on Commun.*, vol. 58, no. 1, pp. 68–72, Jan. 2010.
- [43] Y. Lee and M.-H. Tsai, "Performance of decode-and-forward cooperative communications over Nakagami- $m$  fading channels," *IEEE Trans. Veh. Technol.*, vol. 58, no. 3, pp. 1218–1228, Mar. 2009.
- [44] P. C. Sofotasios, S. Muhaidat, G. K. Karagiannidis, and B. S. Sharif, "Solutions to integrals involving the Marcum  $Q$ -function and applications," *IEEE Signal Process. Lett.*, vol. 22, no. 10, pp. 1752–1756, Oct. 2015.
- [45] P. C. Sofotasios, T. A. Tsiftsis, Yu. A. Brychkov, S. Freear, M. Valkama, and G. K. Karagiannidis, "Analytic expressions and bounds for special functions and applications in communication theory," *IEEE Trans. Inf. Theory*, vol. 60, no. 12, pp. 7798–7823, Dec. 2014.
- [46] P. C. Sofotasios, M. Valkama, Yu. A. Brychkov, T. A. Tsiftsis, S. Freear, and G. K. Karagiannidis, "Analytic solutions to a Marcum  $Q$ -function-based integral and application in energy detection," in *CROWNCOM '14*, Oulu, Finland, June 2014.
- [47] P. C. Sofotasios, K. Ho-Van, T. D. Anh, and H. D. Quoc, "Analytic results for efficient computation of the Nuttall- $Q$  and incomplete Toronto functions," in *IEEE ATC '13*, HoChiMinh City, Vietnam, Oct. 2013.
- [48] P. C. Sofotasios, and S. Freear, "New analytic expressions for the Rice Function and the Incomplete Lipschitz-Hankel Integrals," in *IEEE INDICON '11*, Hyderabad, India, Dec. 2011.
- [49] P. C. Sofotasios, and S. Freear, "Upper and lower bounds for the Rice  $Ie$ -function," in *ATNAC '11*, Melbourne, Australia, Nov. 2011.
- [50] P. C. Sofotasios, and S. Freear, "Novel results for the incomplete Toronto function and incomplete Lipschitz-Hankel integrals," in *IEEE IMOC '11*, Natal, Brazil, Oct. 2011.
- [51] P. C. Sofotasios, and S. Freear, "Simple and accurate approximations for the two dimensional Gaussian  $Q$ -function," in *IEEE VTC-Spring '11*, Budapest, Hungary, May 2011.
- [52] P. C. Sofotasios, and S. Freear, "Novel expressions for the Marcum and one dimensional  $Q$ -functions," in *7th ISWCS '10*, York, UK, Sep. 2010.
- [53] P. C. Sofotasios, and S. Freear, "Novel expressions for the one and two dimensional Gaussian  $Q$ -functions," in *IEEE ICWITS '10*, Honolulu, HI, USA, Aug/Sep. 2010.
- [54] P. C. Sofotasios, and S. Freear, "A novel representation for the Nuttall  $Q$ -function," in *IEEE ICWITS '10*, Honolulu, HI, USA, Aug/Sep. 2010.
- [55] W. Braun and U. Dersch, "A physical mobile radio channel model," *IEEE Trans. Veh. Technol.*, vol. 40, no. 2, pp. 472–482, May 1991.
- [56] M. D. Yacoub, "The  $\kappa - \mu$  distribution and the  $\eta - \mu$  distribution," *IEEE Ant. Propag. Mag.*, vol. 49, no. 1, pp. 68–81, Feb. 2007.

- [57] J. C. Silveira Santos Filho and M. D. Yacoub, "Highly accurate  $\eta-\mu$  approximation to sum of M independent non-identical Hoyt variates," *IEEE Ant. Wireless Propag. Lett.*, vol. 4, pp. 436–438, 2005.
- [58] K. Peppas, F. Lazarakis, A. Alexandridis, and K. Dangakis, "Error performance of digital modulation schemes with MRC diversity reception over  $\eta-\mu$  fading channels," *IEEE Trans. on Wireless Commun.*, vol. 8, no. 10, pp. 4974–4980, Oct. 2009.
- [59] N. Y. Ermolova, "Moment generating functions of the generalized  $\eta-\mu$  and  $\kappa-\mu$  distributions and their applications to performance evaluations of communication systems," *IEEE Commun. Lett.*, vol. 12, no. 7, pp. 502–504, Jul. 2008.
- [60] K. P. Peppas, "Dual-hop relaying communications with co-channel interference over  $\eta-\mu$  fading channels," *IEEE Trans. Veh. Technol.*, vol. 62, no. 8, pp. 4110–4116, Oct. 2013.
- [61] R. Mesleh, O.S. Badarneh, A. Younis, and H. Haas, "Performance analysis of spatial modulation and space-shift keying with imperfect channel estimation over generalized  $\eta-\mu$  fading channels," *IEEE Trans. Veh. Technol.*, vol. 64, no. 1, pp. 88–96, Jan. 2015.
- [62] J. P. P.-Martin, J.M. R.-Jerez, and C. T.-Labao, "Performance of selection combining diversity in  $\eta-\mu$  fading channels with integer values of  $\mu$ ," *IEEE Trans. Veh. Technol.*, vol. 64, no. 2, pp. 834–839, Feb. 2015.
- [63] S. Amara, H. Boujemaa, and N. Hamdi, "SEP of cooperative systems using amplify and forward or decode and forward relaying over Nakagami- $m$  fading channels," in *Proc. IEEE ICCS '09*, Medenine, 2009.
- [64] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integrals and Series Volume 3: More Special Functions*, 1<sup>st</sup> edn., Gordon and Breach Science Publishers, 1986.
- [65] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2<sup>nd</sup> edn., Wiley, New York, 2005.
- [66] Y. Lee, M.-H. Tsai, and S.-I. Sou, "Performance of decode-and-forward cooperative communications with multi dual-hop relays over Nakagami- $m$  fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2853–2859, June 2009.
- [67] I. S. Gradshteyn, and I. M. Ryzhik, *Tables of Integrals, Series, and Products* -7<sup>th</sup> edn. Academic Press, 2007.