Outage Probability Analysis of Dual-hop Full-Duplex Decode-and-Forward Relaying over Generalized Multipath Fading Conditions

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Abstract—The present paper analyzes the outage probability of full-duplex (FD) regenerative relay systems over multipath fading channels. Unlike the majority of investigations that assume basic symmetric fading conditions, this analysis considers asymmetric generalized fading conditions, which are more realistic as they are encountered more often in practical wireless transmissions. To this end, it is assumed that the source-relay and source-destination links are subject to $\kappa - \mu$ multipath fading conditions, which can represent generalized line-of-sight communication scenarios; on the contrary, the relay-to-destination link is subject to $\eta - \mu$ fading conditions that typically represent generalized non-line-of-sight communication scenarios. Novel analytic expressions are derived for the outage probability (OP) of the considered FD relay system. These expressions are given in closed-form and have a relatively tractable algebraic form which renders them convenient to handle both analytically and numerically. To this effect, they are subsequently employed in analyzing the corresponding performance for various communication scenarios. It is shown that the OP of the FD relay system is, as expected, highly dependent upon the severity of fading, the relay self-interference and the interference from the direct link. Furthermore, it is shown that at relatively high average signal-to-noise ratio values, the outage probability at low fading severity and at high relay self-interference outperforms the respective performance for the case of high fading severity, but with low relay self-interference. Based on this, the offered results can be useful in the design and deployment of future full-duplex based cooperative communication systems.

I. INTRODUCTION

Cooperative communications have experienced significant advancements over the past decade and constitute considerable candidates for the next generation of wireless communication systems [1]–[7]. This is largely based on their distinct ability to extend network coverage, improve transmission reliability, enhance channel capacity and reduce power consumption, e.g. see [8]–[27] and the references therein. As a result, such technologies have been adopted in recent communication standards such as, IEEE 802.16 WiMAX and 3GPP Long Term Evolution (LTE) - Advanced standard [28]. It is recalled that conventional relay communication systems are typically based on half-duplex (HD) transmission which practically constitutes a non-negligible constraint as the involved relay nodes receive and transmit signals on orthogonal channels [29]–[32], which results to inefficient utilization of the system resources that ultimately incur considerable performance losses.

Nevertheless, it was recently shown that the above issue can be effectively overcome by means of full-duplex (FD) relaying, which has been proposed as an effective alternative architecture that allows relay(s) to receive and transmit at the same frequency band and time. Yet, this method is exploited at a cost of relay self-interference (RSI) that leaks between the transmit and receiver input. However, recent studies have extensively shown that FD relaying strategies are still feasible and can perform effectively even in the presence of high self-interference levels, see e.g. [33]–[38] and the reference therein. Motivated by this, Baranwal et al. [39] derived the outage probability (OP) of a multihop full-duplex relaying (FDR) system by considering the RSI and interference from adjacent terminals over Rayleigh fading channels. In the same context, Day et al. [40] derived tight upper and lower bounds on the end-to-end achievable rate of decode-and-forward (DF) based FD multiple-input multiple-output (MIMO) relay systems. Likewise, Altieri et al. [41] analyzed the outage probability performance of FD interference-limited relaying for DF and compress-and-forward (CF) schemes for the case of conven-
tional multipath fading and path loss effects.

It is also recalled that fading phenomena affects significantly the performance of both conventional and emerging communication systems, which has led to the proposition of several fading models [42]–[53]. However, considering advanced fading models is typically cumbersome due to the presence of complex integrals that involve elementary and/or special functions [54]–[64] and the references therein. Based on this, all reported analyses and investigations on FD relaying assume signal transmission over symmetric multipath fading channels, i.e., that the source-relay and relay-destination links undergo the same fading conditions. Nevertheless, this is encountered rarely in realistic fading conditions as wireless radio propagation links are typically subject to asymmetric fading conditions. Indicative examples of such communication scenarios can be found in cases that one path corresponds to line of sight (LOS) communications in the presence of a dominant component, while another path might experience severe non-line of sight (NLOS) conditions. Based on this, Suraweera et al. [65] investigated the end-to-end performance of a dual-hop fixed gain relaying system when the source-relay and the relay-destination channels experience Rayleigh/Rician and Rician/Rayleigh fading conditions respectively. In the same context, Jayasinghe et al. [66] analyzed the performance of dual-hop transmissions for optimal beamforming in fixed gain amplify-and-forward (AF) MIMO relaying over asymmetric fading conditions. Likewise, Peppas et al. [67] investigated the end-to-end performance over generalized small scale NLOS and LOS asymmetric fading channels.

It is noted here that the reported investigations over asymmetric fading conditions are analyzed solely in the context of HD relaying systems, which, as already mentioned, constitues a suboptimal communication principle with non-negligible performance degradation. Motivated by this, the present work analyzes the OP in FD relay systems over generalized LOS and NLOS asymmetric fading conditions. To this end, novel analytic expressions are derived for the case that the source-relay and source-destination paths are subject to $\kappa-\mu$ fading conditions while the relay-destination link is subject to $\eta-\mu$ fading conditions. The offered analytic expressions are represented in closed-form and their validity is justified through comparisons with respective results from computer simulations. The derived expressions are subsequently employed in analyzing the OP for different communication scenarios and as expected, it is shown that the performance of the considered FD relaying is highly dependent upon the severity of the fading conditions and the levels of the involved self-interference at the relay.

The reminder of this paper is organized as follows: Section II presents the considered system and channel models, whereas Section III is devoted to the derivation of novel analytic expressions of the OP over asymmetric generalized multipath fading conditions. The corresponding numerical results are presented in Section IV while closing remarks are provided in Section V.

II. SYSTEM AND CHANNEL MODEL

A. System model of Full-Duplex Relay

We consider a three-node relay network consisting of one source, $S$, a single relay, $R$, and a destination, $D$. The source and destination nodes are equipped with a single antenna while the relay has two antennas, one for receiving and one for transmitting, that implement FD mode operation as illustrated in Fig. 1. It is assumed that the relay uses DF relaying protocol to re-transmit the received signal to the destination and that reception and re-transmission occur simultaneously at the same frequency band, which ultimately induces a certain amount of relay self-interference. Based on this, the received signals at the relay and destination nodes can be expressed as

$$y_R = \sqrt{P_S} \alpha_{SR} x_S + \sqrt{P_R} \alpha_{R,R} x_R + n_R$$

and

$$y_D = \sqrt{P_R} \alpha_{RD} x_R + \sqrt{P_S} \alpha_{SR} x_S + n_D$$

respectively, where $P_S$ and $P_R$ are the transmission powers at the source and relay nodes, respectively, $x_S$ and $x_R$ denote the transmitted signals from the source and relay nodes with normalized unit energy, whereas $\alpha_{SR}, \alpha_{R,R}, \alpha_{SR}, \alpha_{RD}$ and $\alpha_{D,R}$ are the fading coefficients of the $S \rightarrow R$, $R \rightarrow D$, $S \rightarrow D$ and $R \rightarrow R$ links, respectively. Also, $n_R$ and $n_D$ denote the additive white Gaussian noise (AWGN) with zero mean and variance $\sigma^2_0$ at the relay and destination nodes, respectively, while each path is assumed to experience narrow band multipath fading conditions.

B. Asymmetric Channel Model

In the considered asymmetric fading channel model, we assume that the $S \rightarrow R$ and $S \rightarrow D$ paths are subject to $\kappa-\mu$ fading conditions, whereas the $R \rightarrow D$ path experiences $\eta-\mu$ fading conditions. It is recalled that these fading models have been shown to characterize accurately multipath fading effects in LOS and NLOS communication scenarios, respectively [68]. In addition, the relay self-interference channel is assumed to be Rayleigh distributed, which corresponds to relatively severe multipath fading conditions.

1) The $\eta-\mu$ fading model: As already mentioned, the $\eta-\mu$ distribution is a generalized fading model that provides remarkable characterization of generalized multipath fading conditions in NLOS communications. This model consists of two different formats and its signal-to-noise ratio (SNR) probability density function (PDF) is expressed as [68]
\[ f_{\gamma}(\gamma) = \frac{2\sqrt{\pi} \mu^{\frac{1}{2}} h^\mu \gamma^{\mu-\frac{1}{2}} I_{\mu-\frac{1}{2}} \left( \frac{2\mu H\gamma}{\sqrt{\pi}} \right)}{\Gamma(\mu) H^{\mu-\frac{1}{2}} \gamma^{\mu+\frac{1}{2}} \exp \left( \frac{2\mu h}{\sqrt{\pi}} \right)} \]  

where \( \gamma = \left| \alpha \right|^2 P / N_0 \) denotes the instantaneous SNR, \( \tau = E|\gamma| \) is the average SNR per symbol, whereas \( \Gamma(\cdot) \) and \( I_\cdot(\cdot) \) denote the gamma function and the modified Bessel function of the first kind, respectively [69]. The parameters \( h \) and \( H \) are functions of \( \eta \), which varies according to the referring format. More specifically, in Format-1, \( h = (2 + \eta^{-1} + \eta)/4 \) and \( H = \eta/(1 - \eta^2) \) where \( 0 < \eta < \infty \) is the scattered-waves power ratio between the in-phase and quadrature components of each multipath cluster. On the contrary, in Format-2, \( h = 1/(1 - \eta^2) \) and \( H = \eta/(1 - \eta^2) \) where \( -1 < \eta < 1 \) represents the correlation coefficient between the in-phase and quadrature components of the scattered waves in each multipath cluster. In both formats, the parameter \( \mu > 0 \) denotes the number of multipath clusters. Furthermore, it is recalled that the \( \eta - \mu \) fading model includes as special cases the well-known Nakagami-\( m \), Nakagami-\( q \) (Hoyt) and Rayleigh fading distributions [68].

2) The \( \kappa - \mu \) fading model: The \( \kappa - \mu \) fading distribution is also a distinct fading model that differs from \( \eta - \mu \) in that it accounts for generalized multipath fading conditions in LOS communications. The SNR PDF of the \( \kappa - \mu \) distribution is expressed as [68]

\[ f_{\gamma}(\gamma) = \frac{\mu(\kappa + 1)^{\frac{\mu(\kappa+1)}{\gamma}} \gamma^{\mu(\kappa+1) - 1} I_{\mu-1} \left( 2\mu \gamma^{\kappa(\kappa+1)} \right)}{\kappa^{\frac{\mu(\kappa+1)}{\gamma}} \gamma^{\mu(\kappa+1) - \frac{1}{2}}} \exp \left( \mu \kappa + \frac{\mu(\kappa+1)\gamma}{\gamma} \right) \]  

where \( \kappa > 0 \) denotes the ratio between the total power of the dominant components to that of the scattered waves whereas \( \mu > 0 \) denotes the number of multipath clusters. The \( \kappa - \mu \) fading model includes the Nakagami-\( m \), the Nakagami-\( n \) (Rice) and Rayleigh fading models as special cases while its corresponding CDF is computed with the aid of [68, eq. (3)].

III. OUTAGE PROBABILITY ANALYSIS

In the considered DF-FD relay system, the relay is subject to self-interference while the destination is subject to interference from the signal transmission along the S-D path. To this effect, the instantaneous signal-to-interference-and-noise-ratios (SINRs) at the relay and destination nodes are expressed as follows [70]

\[ \Gamma_R = \frac{|h_{S,R}|^2 P_S}{|h_{R,R}|^2 P_R + N_0} \]  

and

\[ \Gamma_D = \frac{|h_{R,D}|^2 P_R}{|h_{S,D}|^2 P_S + N_0} \]  

respectively. It is also noted that the overall outage probability in terms of the instantaneous SINR is expressed as

\[ P_{\text{out}} = \Pr(\Gamma_R < \Gamma_T) + (1 - \Pr(\Gamma_R < \Gamma_T))\Pr(\Gamma_D < \Gamma_T) \]  

where \( \Gamma_T = 2^R - 1 \) is the required SNR with spectral efficiency \( R \) in bits/sec/Hz.

In order to derive an analytic expression for the OP of the considered system, we firstly represent (5) and (6) in terms of the equivalent SNRs, namely

\[ \Gamma_R = \frac{\gamma_{S,R}}{\gamma_{R,R} + 1} \]  

and

\[ \Gamma_D = \frac{\gamma_{R,D}}{\gamma_{S,D} + 1} \]  

respectively, where \( \gamma_{R,R} \) is exponentially distributed with mean value \( \mu_{R,R} \). Furthermore, \( \Gamma_R \) and \( \Gamma_D \) can be equivalently expressed in terms of the common RVs \( X \) and \( Y \) according to \( Z = X/(Y + 1) \). To this effect and based on the foundations of probability theory, the CDF of \( Z \) can be represented as follows [71]

\[ F_Z(z) = F_X(z(y + 1))f_Y(y)dy. \]  

Therefore, for the case of the \( S \rightarrow R \) we substitute in (10) the CDF of the \( \kappa - \mu \) distribution along with the PDF of Rayleigh distribution for \( \gamma_{R,R} \). To this end and with the aid of [72, eq. (13)] along with long but basic algebraic manipulations, the CDF of \( \Gamma_R \) can be expressed as follows

\[ F_Z(z) = 1 - Q_{\mu_{S,R}} \left( \sqrt{b}, \sqrt{a} \right) + Q_{\mu_{S,R}} \left( \frac{a_{\frac{\tau_{S,R}}{\tau_{S,R} + 2}}}{e^{\frac{1}{\mu_{S,R}}} + 1} \right) \times \frac{1}{\sqrt{\gamma_{S,R}}} \sqrt{\frac{2}{\gamma_{S,R}} + a} \]  

where

\[ a = \frac{2\mu_{S,R}(1 + \kappa)z}{\gamma_{S,R}} \]  

and \( b = 2\kappa_{S,R} \).

In order to determine the CDF of \( \Gamma_D \), it is firstly necessary to express the CDF of the \( \eta - \mu \) distribution in terms of a simpler representation with the aid of \( F_{\gamma}(z) \). This effect, by expressing the \( I_n(\cdot) \) term in (3) according to [69, eq. (8.467)], using [69, eq. (8.350.1)] along with [69, eq. (8.352.6)] and carrying out long but basic algebraic manipulations, the SNR CDF of \( \eta - \mu \) distribution for integer values of \( \mu \) can be expressed as follows:

\[ \frac{2\mu_{S,R}(1 + \kappa)z}{\gamma_{S,R}} \]  

and \( b = 2\kappa_{S,R} \).
By substituting the CDF of the \( \eta-\mu \) distribution in (13) and the PDF of the \( \kappa-\mu \) distribution in (4) into (10), the CDF of \( \Gamma_D \), \( F_Z(z) \) can be expressed as

\[
F_Z(z) = \int_0^\infty \left( \sum_{l=0}^{\mu-1} \frac{l!}{l!(\mu+l)!} \Gamma(\mu+l)H^l\mu^l + \sum_{l=0}^{\mu-l-1} \frac{l!}{l!(\mu+l)!} \Gamma(\mu+l)H^l\mu^l \right) \left( \frac{(-1)^l (h-H)^{l+i-\mu}}{\Gamma(\mu+l)H^l\mu^l} + \frac{(-1)^l (h+H)^{l+i-\mu}}{\Gamma(\mu+l)H^l\mu^l} \right) dx
\]

where

\[
A = \mu S_D (1 + \kappa S_D)^{\mu S_D + 1} \frac{\mu S_D + 1}{\kappa S_D}.
\]

By also applying [69, eq. (1.111)] and after long but basic algebraic manipulations, equation (14) can be expressed according to (16), at the top of the next page, where

\[
B = \frac{h_R D \mu S_D (1 + \kappa S_D)^{\mu S_D + 1}}{\Gamma(\mu R D) \kappa S_D} \frac{\mu S_D + 1}{\kappa S_D}.
\]

Importantly, the integral involved in (16) can be expressed in a closed form with the aid of [69, eq. (6.621.1)]. Therefore, by performing the necessary variable transformations and carrying out long but basic algebraic operations, the CDF of \( \Gamma_D \) can be expressed according to (18), at the top of the next page. It is noted here that the derived expression can be readily substituted in (7) for deriving a novel closed-form expression for the OP of the considered FD DF relay system over generalized asymmetric fading conditions. This expression is long but relatively convenient to handle both analytically and numerically as it involves elementary and special functions that are built-in in most popular scientific software packages such as MATLAB, MAPLE and MATHEMATICA. In addition, given that the derived closed-form expression is exact, it can form the basis for the derivation of simpler asymptotic and/or approximative expressions that can provide additional insights on the performance of the considered system and the effects of the involved parameters.

![Fig. 2. Outage probability of full-duplex relaying vs. average SNR over asymmetric \( \kappa-\mu \) and \( \eta-\mu \) fading channels for different fading and interference parameters.](image)

**IV. NUMERICAL RESULTS AND DISCUSSION**

In this section, we employ the derived analytic results for evaluating the performance of the considered DF-FDR system over asymmetric generalized fading conditions for different fading and interference scenarios. Furthermore, it is assumed that the transmit power of the system is allocated equally to the source and relay nodes i.e. \( \tau = \tau S, R = \tau R, D \).

Fig.2 illustrates the OP behavior of the FD system where the \( S \to R \) and \( S \to D \) links are subject to \( \kappa-\mu \) fading model with fading parameters \( \mu S, R = \mu R, D = \mu S, D = 1, \eta S, D = 1, \kappa S, D = 2dB, \tau S, R = -5dB, \tau S, D = -10dB \) and different values of \( \kappa S, R \) and \( \Gamma T \).
This is because, the system remains in complete outage until the target SINR is achieved for successful transmission.

Fig.3 depicts the outage probability of the FD system as a function of the average SNR with target SINRs of $\Gamma_T = 0$dB and $\Gamma_T = 5$dB for $\kappa_{S,R} = 4$dB, $\mu_{S,D} = 1$, $\kappa_{S,D} = 2$dB, $\eta_{R,D} = 1$, $\tau_{R,R} = -10$dB with different values of $\mu_{S,R}$, $\mu_{R,D}$ and relay self-interference $\gamma_{R,R}$. It is observed that the outage performance improves when the required target $\Gamma_T$ decreases. Furthermore, it is noticed that, at the lower portion of the high-SNR regime, the RSI degrades the outage probability of the system even for low fading severity. However, this changes dramatically in the case of the higher portion of the high SNR regime. For example, the OP for $\Gamma_T = 5$dB with fading parameters of $\mu_{S,R} = \mu_{R,D} = 3$ with $\tau_{R,R} = 4$dB performs better than in the case of $\mu_{S,R} = \mu_{R,D} = 2$ and $\tau_{R,R} = 0$dB beyond 25dB. This is also the case for average SNRs beyond 20dB and target SINR at 0dB.

Finally, Fig. 4 demonstrates the effect of the direct $S \rightarrow D$ interference on the behavior and performance of the outage probability for fading parameters of $\mu_{S,R} = 2$, $\kappa_{S,R} = 4$dB, $\mu_{S,D} = 1$, $\kappa_{S,D} = 2$dB, $\mu_{R,D} = 1$, $\eta_{R,D} = 1$, $\tau_{R,R} = -10$dB and different values of $\tau_{S,D}$ with target SINRs of $\Gamma_T = 0$dB and 5dB, respectively. It is observed that the outage performance of the system is, as expected, significantly affected by the interference from the S-D link. This is because as the direct interference increases, the equivalent SINR at the destination is reduced, which ultimately degrades the corresponding outage probability.

V. CONCLUSION

This work analyzed the outage probability of a dual-hop full-duplex regenerative system with relay and direct interference over asymmetric generalized multipath fading channels. Novel analytic expressions were derived for the outage probability for the case that the source-to-relay and source-to-destination paths experience $\kappa - \mu$ fading conditions whereas the relay-to-destination path experiences $\eta - \mu$ fading conditions. It was demonstrated that the performance of the considered cooperative system is affected significantly by the fading conditions in each communication path as well as by...
Fig. 3. Outage probability of full-duplex relaying vs. average SNR $\gamma$ over asymmetric $\kappa-\mu$ and $\eta-\mu$ fading channels for $\kappa_{S,R} = 4\text{dB}, \kappa_{S,D} = 2\text{dB}$, $\mu_{S,R} = 1$, $\mu_{S,D} = 1$, $\eta_{S,R} = 4\text{dB}$, $\eta_{S,D} = 2\text{dB}$, $\tau_{R,R} = -10\text{dB}$ and different values of $\tau_{S,D} = \tau_{S,D}$ and $\Gamma_T$.

Fig. 4. Outage probability of full-duplex relaying vs. average SNR $\gamma$ over asymmetric $\kappa-\mu$ and $\eta-\mu$ fading channels for $\mu_{S,R} = 2$, $\mu_{S,D} = \mu_{S,D} = 1$, $\eta_{S,R} = 4\text{dB}$, $\eta_{S,D} = 2\text{dB}$, $\tau_{R,R} = -10\text{dB}$ and different values of $\tau_{S,D} = \tau_{S,D}$ and $\Gamma_T$.

the relay and direct interference. In addition, it is shown that in the high-SNR regime, the outage probability at low fading severity and at high relay self interference outperforms the respective performance for the case of high fading severity, but with low relay self-interference levels.

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