OFDM-IM vs FQAM: A Comparative Analysis

Stavros G. Domouchtsidis, Georgia D. Ntouni, Vasileios M. Kapinas, and George K. Karagiannidis
Department of Electrical and Computer Engineering,
Aristotle University of Thessaloniki, 54636 Thessaloniki, Greece
e-mail: {stavrosd, gntouni, kapinas, geokarag}@auth.gr

***Abstract*—Recently, orthogonal frequency division multiplexing (OFDM) with index modulation (OFDM-IM) as well as frequency and quadrature amplitude modulation (FQAM) have been proven to be promising techniques for the multicarrier transmission over frequency selective fading channels by successfully mitigating the induced interference due to multipath, while at the same time boosting the overall spectral efficiency. In this paper, we present a thorough study of OFDM-IM and FQAM schemes and conduct a comparison between them and the classic OFDM system. Different configurations are considered and evaluated in terms of their spectral efficiency and error performance, capitalizing on recent results from the literature.**

***Index Terms*—Frequency and quadrature amplitude modulation (FQAM), orthogonal frequency division multiplexing with index modulation (OFDM-IM), subcarrier selection.**

I. Introduction

Multicarrier systems have been extensively utilized in high-speed wireless communications thanks to their robustness against multipath fading and interference [1]. Although new multicarrier techniques, such as generalized frequency division multiplexing and filter bank multicarrier modulation, have been recently envisaged as fifth generation (5G) candidate technologies, orthogonal frequency division multiplexing (OFDM) is the dominant and well-established multicarrier modulation technique that forms the basis of many existing wireless standards, such as the IEEE 802.11 (Wi-Fi) and the Long Term Evolution-Advanced (LTE-A) [2], [3].

A multicarrier transmission scheme, called OFDM with index modulation (OFDM-IM), that combines OFDM with the spatial modulation concept has been proposed in [4]. In more detail, OFDM-IM transmits information utilizing not only the constellation symbols but also the indices of the subcarriers that carry data symbols. According to [5], OFDM-IM turns frequency selectivity into a benefit, thus enhancing the system performance in time/doubly-dispersive fading channels. The performance of OFDM-IM, along with some extra modulation and grouping strategies, were further examined in [6]. Finally, novel detectors and generalization schemes for OFDM-IM were presented in [7], [8] and [9], respectively.

Frequency and quadrature amplitude modulation (FQAM) is a modulation technique that combines quadrature amplitude modulation (QAM) with frequency shift keying (FSK) [10], thus providing a desired tradeoff between power and spectral efficiency in mobile communication applications. FQAM was firstly proposed in [11] and further examined in [12], [13], while, in [14], it was employed in a downlink orthogonal frequency division multiple access (OFDMA) system to increase the channel capacity in environments dominated by interference. At last, multitone FQAM, a variation of standard FQAM, was recently proposed and evaluated in [15].

In this paper, we present and study the OFDM-IM and FQAM schemes, and compare their performance with that of the classic OFDM. Firstly, we give the system model and block diagram of each multicarrier system, emphasizing on their implementation details. We also investigate the spectral efficiency achieved by each configuration and we capitalize on recent results from the literature concerning the evaluation of their bit error performance. In the sequel, $\Pr\{\cdot\}$ denotes probability of an event and $\mathrm{E}\{\cdot\}$ expectation. Bold lowercase and uppercase letters stand for column vectors and matrices, respectively. Additionally, $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ denote transposition and Hermitian transposition, respectively. Finally, $\lfloor\cdot\rfloor$ is the floor function, while $\det(\cdot)$ gives the matrix determinant.

II. System Model

In this paper, we present multicarrier schemes that share basic characteristics with the classic OFDM. In our analysis, we assume that the available channel bandwidth W is divided into N subcarriers of width $\Delta f = W/N$ each and that Δf is small enough, so that the channel frequency response across each subchannel remains constant. We further assume that the information symbols are transmitted over a frequency-selective Rayleigh fading channel. The channel is modeled by circularly symmetric zero-mean complex Gaussian random variables, i.e., $h_T(l) \sim \mathcal{CN}(0, \frac{1}{v})$, $l = 1, \ldots, v$. In all cases, the length of multipath channel v is considered to be smaller than the cyclic prefix length L added to the constructed signal, so that the channel remains constant over an OFDM block.

A. Classic OFDM

In the classic OFDM, the sequence of m bits transmitted over an OFDM block is grouped into N sets of $\log_2 M$ bits. Then, each of these sets is mapped to an M-ary QAM symbol $x(j)$, $j = 1, \ldots, N$, which is transmitted over one subcarrier. Thus, the frequency domain OFDM block is formed as

$$\mathbf{x}_F = \begin{bmatrix} x(1) & x(2) & \cdots & x(N) \end{bmatrix}^{\mathrm{T}}. \tag{1}$$

We derive the OFDM block in the time domain $\mathbf{x}_T$ from the N point IFFT of the frequency domain OFDM block $\mathbf{x}_F$ using a proper factor to normalize (to unity) the power of the transmitted block. Finally, a CP of proper length L is added

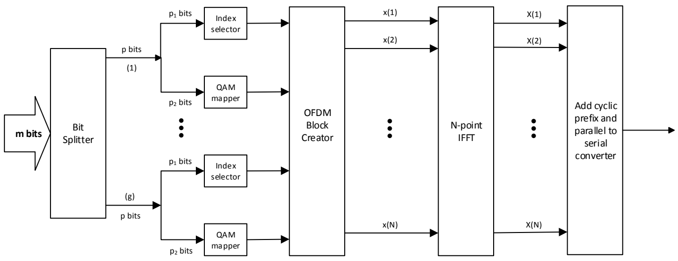

Fig. 1. Block diagram of the OFDM-IM transmitter.

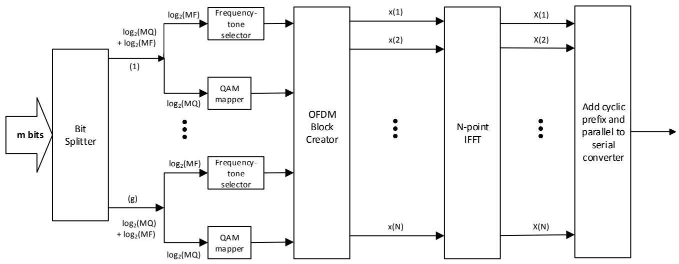

Fig. 2. Block diagram of the FQAM transmitter.

to the OFDM block and after parallel-to-serial and digital-to-analog conversions, the signal is transmitted.

At the receiver side, after the analog-to-digital and the serial-to-parallel conversions, the CP is removed and the FFT of the received signal is derived as

$$y_F(i) = x(i)h_F(i) + n_F(i), \quad i = 1, \ldots, N, \tag{2}$$

where $h_F(i) \sim \mathcal{CN}(0, 1)$ are the channel fading coefficients in the frequency domain and $n_F(i) \sim \mathcal{CN}(0, N_0)$ are the noise samples. Finally, the received signal is detected utilizing the maximum likelihood (ML) detector.

B. OFDM-IM

In the OFDM-IM scheme, information is transmitted both by the M_Q-ary symbols and the subcarriers' indices that carry symbols [5]. Thus, the subcarriers of the OFDM block should be activated properly.

The block diagram of the OFDM-IM transmitter in Fig. 1 indicates that the bit sequence of length m that enters the transmitter is first split into g groups of p bits ($m = pg$) and, then, each group is mapped to an OFDM sub-block of n subcarriers, where $n = N/g$.

As has been already mentioned, in OFDM-IM, not all the bits are mapped onto an M_Q-ary signal constellation because some of them are transimitted via the active subcarriers indices. The selection of the k subcarriers that will be activated out of the n subcarriers of the OFDM sub-block is based on the first p_1 bits of the p bits. The mapping is conducted using the combinatorial method [16], whereas other methods, such as a look-up table, can also be employed. After denoting $C(n, k)$ as the binomial coefficient, we have $p_1 = \lfloor \log_2 C(n, k) \rfloor$. The rest $p_2 = k \log_2 M$ bits are mapped onto the M_Q-ary signal constellation in order to determine the symbols that will be transmitted by the active subcarriers.

Regarding the detection, the simple ML detector of the classic OFDM is not suitable, due to the utilization of the index modulation. Instead, the receiver employs the following log-likelihood ratio (LLR) detector, as described in [5]

$$\lambda(i) = \frac{\sum_{x=1}^{M} \Pr\{x(i) = s_x | y_F(i)\}}{\Pr\{x(i) = 0 | y_F(i)\}}, \quad i = 1, \ldots, N, \tag{3}$$

where s_x represents a symbol drawn from the M_Q-ary signal constellation. The larger the value of the ratio for the index i, the more probable it is that this index has been selected to be active by the transmitter. After calculating the LLRs for all the indices, the receiver decides which are the active indices and, then, the demodulation continues with the ML criterion like in the classic OFDM scheme.

C. FQAM

FQAM combines M_F-FSK modulation and M_Q-QAM modulation and is referred to as M-ary FQAM or (M_F, M_Q)-FQAM, where $M = M_F M_Q$ [10]. It should be noted that FQAM has many similarities with OFDM-IM, as in both cases the information is not only conveyed through the modulated symbols but also via the indices of the active subcarriers.

Fig. 2 illustrates the block diagram of the FQAM transmitter. It can be seen that the N subcarriers of the OFDM block are divided into g sub-blocks of M_F subcarriers each ($g = N/M_F$). Again, a bit splitter splits the m bits into g groups. The first $\log_2 M_F$ bits of each group are used to select that subcarrier out of the M_F available that will carry the symbol of the M_Q-ary signal constellation to which the next $\log_2 M_Q$ bits of the group are mapped. Thus, each sub-block transmits $\log_2 M_F + \log_2 M_Q$ bits. The following parts of transmitter are the same as in the classic OFDM.

At the receiver side, the detection process is similar to that of the OFDM-IM. The receiver employs the LLR detector given by (3) in order to determine the active subcarrier out of the M_F in each sub-block and, afterwards, estimates the received symbols using an ML detector.

III. Performance Comparison

In this section, the performance of the two schemes described in Section II is analysed in terms of the average bit error probability (BER). The spectral efficiency of the considered schemes is given by $m/(N + L)$ bits/s/Hz, where m is the number of bits transmitted per OFDM block as mentioned before. The signal-to-noise ratio (SNR) is defined as $E_b/N_{0,T}$, where $E_b = m/(N + L)$ is the average transmitted energy per bit and $N_{0,T}$ is the noise variance in the time domain. The relationship between the noise variance in the time and frequency domains is $N_{0,F} = (K/N)N_{0,T}$, where K is the number of active subcarriers per OFDM block.

A. Classic OFDM

For comparison purposes, we recall here that the average BER of an OFDM system employing BPSK is given by [17,

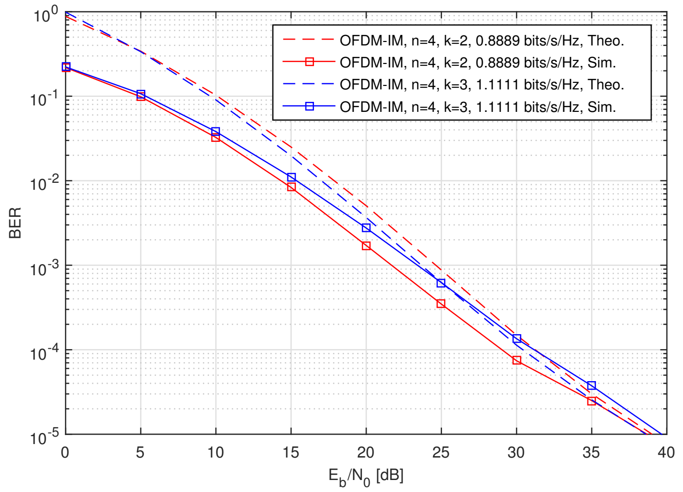

Fig. 3. Theoretical and simulation results for the BER of OFDM-IM.

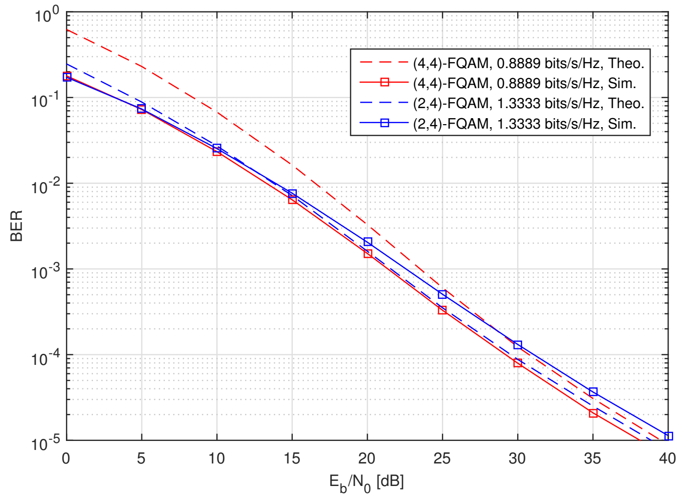

Fig. 4. Theoretical and simulation results for the BER of (M_F,M_Q)-FQAM.

eq. (6.156)]

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{E_b/N_{0,T}}{E_b/N_{0,T}+1}}\right). \tag{4}$$

B. OFDM-IM

The average bit error probability of the OFDM-IM is approximated as [5, eq. (32)]

$$P_b \approx \frac{1}{pn_{\mathbf{x}}} \sum_{\mathbf{X}} \sum_{\widehat{\mathbf{X}}} \Pr\{\mathbf{X} \to \widehat{\mathbf{X}}\} e(\mathbf{X}, \widehat{\mathbf{X}}), \tag{5}$$

where $\mathbf{X}$ is an $n \times n$ all-zero matrix except from its main diagonal elements denoted by $x(1), \ldots, x(n)$, $n_{\mathbf{x}}$ is the number of the possible realizations of $\mathbf{X}$, and $e(\mathbf{X}, \widehat{\mathbf{X}})$ is the number of bit errors in the event that $\widehat{\mathbf{X}}$ is detected when $\mathbf{X}$ is transmitted. Finally, $\Pr\{\mathbf{X} \to \widehat{\mathbf{X}}\}$ is the unconditional pairwise error probability that is calculated as [5, eq. (28)]

$$\Pr\{\mathbf{X} \to \widehat{\mathbf{X}}\} = \frac{1/12}{\det(\mathbf{I}_n + q_1\mathbf{K}_n\mathbf{A})} + \frac{1/4}{\det(\mathbf{I}_n + q_2\mathbf{K}_n\mathbf{A})}, \tag{6}$$

where $\mathbf{A} = (\mathbf{X} - \widehat{\mathbf{X}})^{\mathrm{H}}(\mathbf{X} - \widehat{\mathbf{X}})$, $\mathbf{K}_n = \mathrm{E}\{\mathbf{h_F}\mathbf{h_F}^{\mathrm{H}}\}$, and $q_1 = 1/(4N_{0,F})$, $q_2 = 1/(3N_{0,F})$.

C. FQAM

(M_F,M_Q)-FQAM can be considered as a special case of the OFDM-IM with $n = M_F$ and $k = 1$ employing an M_Q-ary QAM modulation. Therefore, its average bit error probability can be computed from (5) such as in the case of OFDM-IM.

IV. Numerical Results and Discussion

In this section, the performance of both OFDM-IM and FQAM schemes in terms of the bit error rate (BER) is evaluated, considering different configurations and transmission over a frequency selective Rayleigh fading channel. A comparison between these schemes is also conducted, followed by discussion. Besides, we investigate the spectral efficiency that can be achieved by both schemes when tuning their parameters. We note that the performance of the considered systems is evaluated via Monte Carlo simulations. In all cases, the following assumptions are made: i) perfect channel estimation at the receiver, ii) $N = 128$ subcarriers per OFDM block, iii) $v = 10$ channel taps, iv) CP length $L = 16$, and v) BPSK modulation for all the OFDM-IM schemes.

First of all, in Fig. 3, the BER approximation using (5) is compared with simulation results for OFDM-IM schemes achieving spectral efficiencies equal to 0.8889 and 1.1111 bits/s/Hz. It can be observed that the approximations become more accurate with increasing SNR values and for higher spectral efficiencies. Same conclusions can be drawn for the FQAM schemes illustrated in Fig. 4 with the additional observation that this approximation acts as an upper bound on the system's performance for lower spectral efficiencies, while at higher spectral efficiencies it behaves as a lower bound after an SNR value. This reveals the need for a better approximation for the error performance of these schemes.

The spectral efficiencies that can be achieved by OFDM-IM are illustrated in Fig. 5, where the red dotted line represents the spectral efficiency (0.8889 bits/s/Hz) achieved by the classic OFDM scheme with the same parameters. Higher spectral efficiencies can be achieved with increasing n, namely the number of subcarriers that are grouped. More importantly, the highest spectral efficiency occurs for $n = 128$, i.e., with no grouping at all, which though is a computationally inefficient scheme. Additionally, the spectral efficiency curves are all concave, starting from values lower than 0.8889 bits/s/Hz, reaching an upper value, and then decreasing again to 0.8889 bits/s/Hz as expected. Thus, there exists a limit in the number k of the subcarriers that should be considered active in a group so as to obtain a gain in spectral efficiency. Obviously, we can reach a specific spectral efficiency with many configurations, which renders the error performance evaluation of these schemes an interesting task. From Fig. 6, which illustrates the

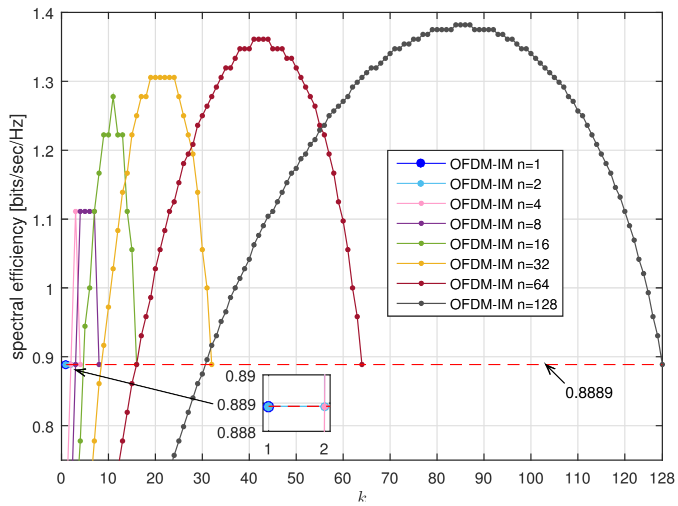

Fig. 5. Spectral efficiency of OFDM-IM.

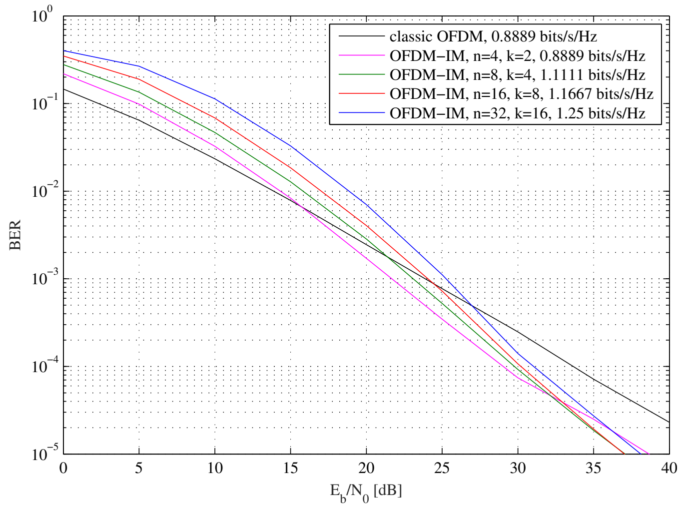

Fig. 7. BER performance of OFDM-IM with different configurations.

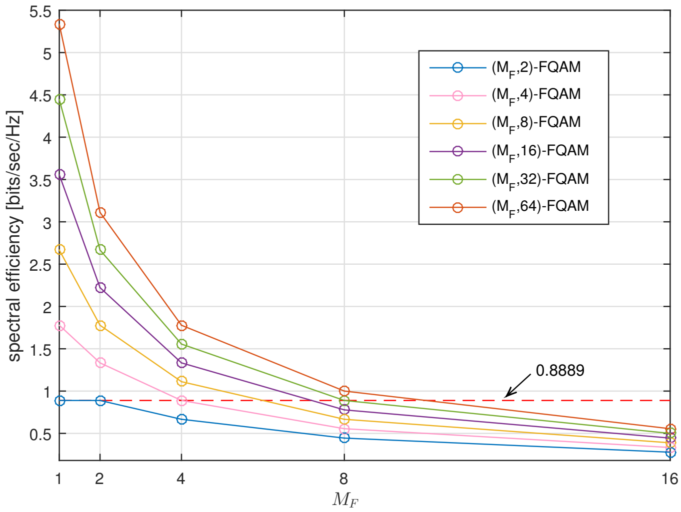

Fig. 6. Spectral efficiency of (M_F,M_Q)-FQAM.

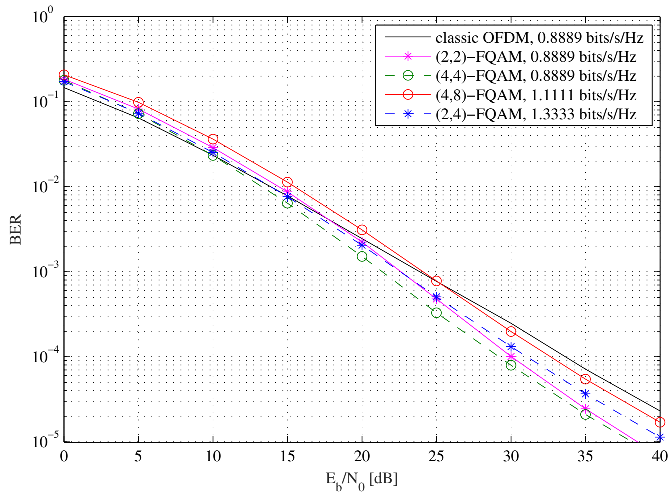

Fig. 8. BER performance of (M_F,M_Q)-FQAM.

spectral efficiency of (M_F,M_Q)-FQAM, we first observe that an increase in the QAM modulation order M_Q leads to higher spectral efficiency with the subsequent decrease in the error performance. It is also evident that there are not so many different FQAM configurations that can be considered, while the curves are monotonically decreasing with M_F, revealing that only small values of M_F (e.g., 2, 4) lead to spectrally efficient schemes.

In Figs. 7 and 8, the BER performance of OFDM-IM and FQAM with different configurations is compared over the classic OFDM scheme employing BPSK modulation. Interestingly, both schemes outperform the classic OFDM scheme for high SNR values, which is not true for low SNR values. This observation verifies that it is spectrally efficient to transmit information bits via the indices of the subcarriers at the high SNR region. Fig. 7 also illustrates that, although any increase in the spectral efficiency leads to worse performance for low SNR values, as the SNR increases, schemes with higher spectral efficiency show an improved error performance. On the other hand, the performance of FQAM with high spectral efficiency does not enhance for high SNR values when compared to FQAM schemes of the same spectral efficiency as the classic OFDM, i.e., 0.8889 bits/s/Hz.

Finally, a comparison between OFDM-IM and FQAM schemes with the same spectral efficiency is shown in Figs. 9 and 10. In both cases, FQAM is advantageous in the low SNR region and becomes worse than OFDM-IM with increasing SNR. Besides, the performance gap of the two schemes is smaller for lower spectral efficiency values. In Fig. 10, (4,8)-FQAM utilizes 8-QAM symbols and OFDM-IM employs BPSK, which partially justifies the behaviour described above. It should also be mentioned that OFDM-IM schemes with higher n, i.e., with more subcarriers per group, perform better than all the other schemes at high SNR.

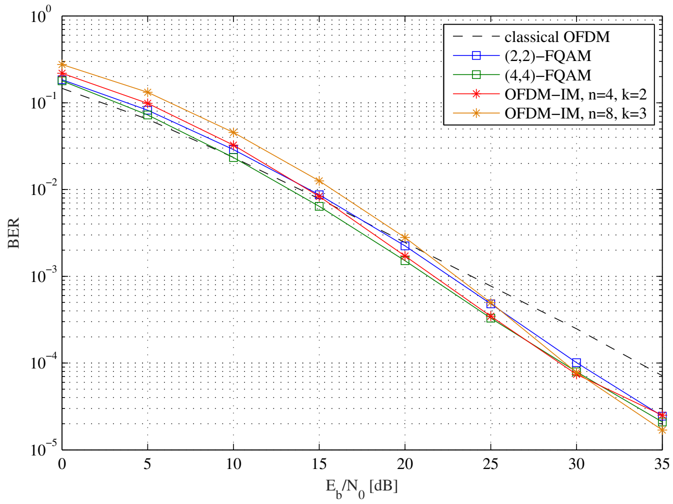

Fig. 9. BER of schemes with spectral efficiency 0.8889 bits/s/Hz.

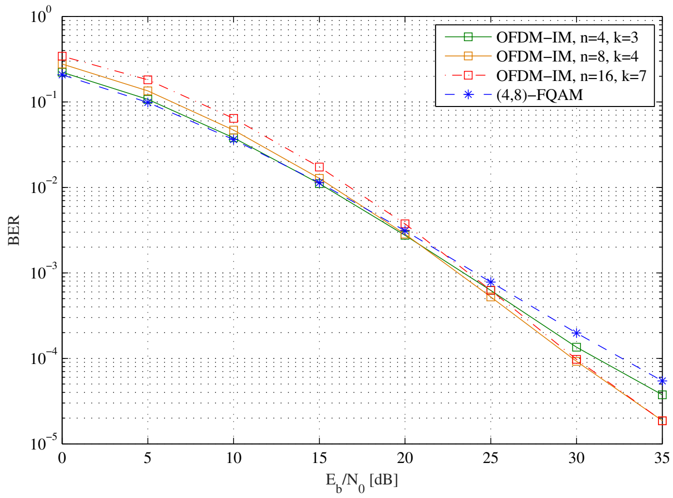

Fig. 10. BER of schemes with spectral efficiency 1.1111 bits/s/Hz.

V. Conclusion

In this paper, OFDM-IM and FQAM have been thoroughly investigated and compared under a variety of configuration scenarios. These schemes have been proven to be capable of achieving significant gains in terms of SNR and spectral efficiency with respect to the classic OFDM technique. Moreover, it was shown that OFDM-IM is more flexible and outperforms FQAM in the high SNR region under the same spectral efficiency requirement. Our current studies indicate that different grouping and activation strategies of the OFDM-IM subcarriers may further improve its performance. To this end, the derivation of novel closed-form expressions for the exact, approximate or even asymptotic performance evaluation of OFDM-IM and FQAM systems is of vital importance.

References

[1] A. Sahin, I. Guvenc, and H. Arslan,"A survey on multicarrier communications: Prototype filters, lattice structures, and implementation aspects," *IEEE Commun. Surveys Tuts.*, vol. 16, no. 3, pp. 1312–1338, Third Quarter 2014.

[2] P. Banelli, S. Buzzi, G. Colavolpe, A. Modenini, F. Rusek, and A. Ugolini,"Modulation formats and waveforms for 5G networks: Who will be the heir of OFDM?: An overview of alternative modulation schemes for improved spectral efficiency," *IEEE Signal Process. Mag.*, vol. 31, no. 6, pp. 80–93, Nov. 2014.

[3] J. G. Andrews, S. Buzzi,S. V. Hanly, A. Lozano, A. C. K. Soong, and J. C. Zhang,"What will 5G be?," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1065–1082, Jun. 2014.

[4] E. Basar, U. Aygolu, E. Panayirci, and H. V. Poor, "Orthogonal frequency division multiplexing with index modulation," in *Proc. IEEE Global Communications Conference (GLOBECOM)*, Anaheim, CA, USA, Dec. 2012, pp. 4741–4746.

[5] E. Basar, U. Aygolu, E. Panayirci, and H. V. Poor, "Orthogonal frequency division multiplexing with index modulation," *IEEE Trans. Signal Process.*, vol. 61, no. 22, pp. 5536–5549, Nov. 2013.

[6] M. Wen, X. Cheng, M. Ma, B. Jiao, and H. V. Poor, "On the achievable rate of OFDM with index modulation," *IEEE Trans. Signal Process.*, vol. PP, no. 99, 2015.

[7] A. I. Siddiq, "Low complexity OFDM-IM detector by encoding all possible subcarrier activation patterns," *IEEE Commun. Lett.*, vol. PP, no. 99, 2016.

[8] B. Zheng, F. Chen, M. Wen, F. Ji, H. Yu, and Y .Liu, "Low-complexity ML detector and performance analysis for OFDM with in-phase/quadrature index modulation," *IEEE Commun. Lett.*, vol. 19, no. 11, pp. 1893–1896, Nov. 2015.

[9] R. Fan, Y. J. Yu, and Y .L. Guan, "Generalization of orthogonal frequency division multiplexing with index modulation," *IEEE Trans. Wireless Commun.*, vol. 14, no. 10, pp. 5350–5359, Oct. 2015

[10] S. Hong, M. Sagong, C. Lim, K. Cheun, and S. Cho, "FQAM: A modulation scheme for beyond 4G cellular wireless communication systems," in *Proc. IEEE Globecom Workshops (GC Wkshps)*, Atlanta, GA, USA, Dec. 2013, pp. 25–30.

[11] R. Padovani and J. Wolf, "Coded phase/frequency modulation," *IEEE Trans. Commun.*, vol. COM-34, no. 5, May 1986.

[12] S. S. Periyalwar and S. M. Fleiser, "Multiple trellis coded frequency and phase modulation," *IEEE Trans. Commun.*, vol. 40, no. 6, Jun. 1992.

[13] A. Larif and N. D. Gohar, "Performance of hybrid MQAM-LFSK (HQFM) OFDM transceiver in Rayleigh fading channels," in *Proc. IEEE Multitopic Conference)*, Islamabad, Pakistan, Dec. 2006, pp. 52–55.

[14] S. Hong, M. Sagong, C. Lim, S. Cho, K. Cheun, and K. Yang, "Frequency and quadrature-amplitude modulation for downlink cellular OFDMA networks," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1256–1267, Jun. 2014.

[15] J. Son, S. Hong, K. Cheun, and K. Yang, "Performance of multitone-frequency and quadrature-amplitude modulation over Rayleigh fading channels," *IET Commun.*, vol. 9, no. 14, pp. 1774–1780, Sep. 2015.

[16] J. D. McCaffrey (2004, Jul.), "Generating the mth lexicographical element of a mathematical combination," *text MSDN Library*. Available: http://msdn.microsoft.com/en-us/library/aa289166(VS.71).aspx

[17] T. S. Rappaport, *Wireless Communications: Principles and Practice*, 2nd ed., Prentice Hall, 2002.