Game Theoretic Approach to Demand Side Management in Smart Grid with User-Dependent Acceptance Prices

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***Abstract*—Efficient demand side management through dynamic power pricing is an important application in the smart grids. However, in the absence of a detailed user consumption model, it is difficult to set an optimal power price. In this paper, we propose to efficiently capture the user consumption behavior through a user-dependent acceptance price. Each rational user will decide its own acceptance price based on its desire to get served. Then, we model the selfish interaction between operator and users as a Stackelberg game, where the operator aims to maximize its profit, while the individual users try to pay the lowest price and be served in time. After each user selfishly declares its own acceptance price, the operator sets an optimal power price, based on the user feedback and taking into account the random output of the renewable power sources. Simulation results confirm that the operator can maximize its profit and the users get served in time, while the proposed scheme leads to the optimal usage of the renewable power production.**

I. Introduction

Since its invention in the 19th century, electricity has greatly affected human society. As a country modernizes and its economy develops, the dependency on electricity increases. This is partly due to the strategy of driving rapid economy growth by producing cheap electricity. Unfortunately, cheap and conveniently available electricity is provided at the expense of the environment. This is because electricity is mostly generated by burning fossil fuel that releases green house gases into the atmosphere. Furthermore, when there is disruption in the economy, which leads to a sudden spike in electricity price, daily life and market operation can be significantly impacted. A big increase in energy cost can lead to the usage of biomass (e.g., wood, pellet, etc.) for residential heating and consequently, causing cities be buried under thick smog. This is a wake up call to all modern societies to reduce dependencies on the environmentally harmful energy sources. One critical step in this direction is to encourage energy conservation by performing demand side management (DSM) to discourage usage, when high power demand requires burning more fossil fuel.

DSM has been made possible with the development of smart grids [1], and it can be realized in three ways, namely direct load control (DLC) [2], autonomous demand response (ADR) [3], and dynamic pricing [4]. Among them, DLC is not a proper scheme for residential electrical load control because of the users' demand for privacy. ADR is a very important mechanism for the future smart grids, since it enables the automatic scheduling of the energy consumption. Also, if ADR is combined with an incentive-based consumption scheduling scheme, it may lead to promising results on reducing the energy costs and the peak to average power ratio. However, ADR may need users to declare their power demand far ahead in time. On the other hand, dynamic pricing does not require users to allow direct access of the operator to their electrical appliances nor require users to declare their usage hours before turning on the switch [5]–[7]. One major problem in dynamic power pricing is load synchronization, especially when there are limitations on the exchange of information. Since the power provider sets the power price selfishly without a proper contract on time-of-use and prices between operator and users, it is difficult for the operator to accurately predict and set an appropriate power price. In face of this problem, the operator needs to collect information about instantaneous user demand and availability of alternative renewable energy sources. This is partially studied in [10], [11], where the loads interact with the smart grid in the context of a Stackelberg game and via a number iterations before the optimum power price is decided. However, this model is not appropriate for individual users and appliances, because the consumption of which is not a continuous function with respect to the price, and there is no criterion to ensure the reliability of the power delivery for the inelastic users. Also, the renewable generation of the users has not been taken into account, while the specifications of the required communication network have not been considered.

When designing a DSM scheme, the capacity of the required communication network is critical, in order to avoid a bottleneck. In the literature, communications in smart grids are commonly assumed error-free. However, this assumption is not always true in practice, especially when there is a need for high transmission rates and increased information exchange. Communication impairments in smart grids and their effect on the dynamic pricing update interval and step size have been studied in [8]. Furthermore, a realistic communication network with transmission errors was considered in [9] and a scenario was presented, where the operator forecasts the impact of the real-time selected price to its total profit, after considering a probabilistic model for the power consumption of each user. However, the proposed user consumption model is too simplistic, while in practice, a user consumption model is complex and unknown. In the absence of an accurate user consumption model, it is difficult to determine an optimal power price in one step. On the other hand, acquiring such a detailed consumption model from a group of individual users may apply excessive pressure on communication networks.

In view of the challenge, this paper proposes to capture the user consumption behavior through a dynamic user acceptance price which is a single value that can be communicated efficiently through a wireless network. In short, each user determines his own acceptance price from time to time to reflect the user's urgency to consume electricity. When his acceptance price is higher (lower) than a power price, he will (not) consume the power. We model this price assignment dynamics via a Stackelberg game, where the operator is the leader and the consumers are the followers. This model considers the non-cooperative interactions among operator and consumers, while it takes into account the time-varying availability of alternative renewable power sources. Emphasis is given on the limitation of the information gathering, making our model appropriate for the practical case.

The contributions of this paper are summarized as follows:

1) A practical system model is proposed, where the operator does not have the detailed user consumption model. Instead, users provide feedback to the control center of their instantaneous acceptance prices and demands.
2) A Stackelberg game for power price setting, which captures non-cooperative interactions among operator and users, while taking into account the time-varying availability of the renewable power.
3) A simple algorithm to solve the Stackelberg game, appropriate for real-time use, which can guarantee that the operator will choose the optimal power price, in order to maximize its profit.
4) A model for the rational reaction of users is presented, based on which, the users choose their acceptance prices without any cooperation or information exchange, with the aim to guarantee that their loads will be served in time at reasonable prices.

II. SYSTEM MODEL

We consider a system model as illustrated in Fig. 1, where the distribution power network connects houses to a traditional power source. Here, each house has a residential renewable energy generator in the form of a roof-top photo-voltaic panel or a small wind turbine. Within each house, there are several electrical appliances with individual smart meters and a residential control unit (RCU). We will use the terms "appliance" and "smart meter" interchangeably hereafter. These smart meters and RCU are interconnected through a home area network (HAN), where the RCU is the communication gateway between the HAN and the control center (CC). The RCUs are connected to the CC wirelessley through the cellular communication network.

In the model, the CC collects information about the instantaneous power supply and demand, before deciding the

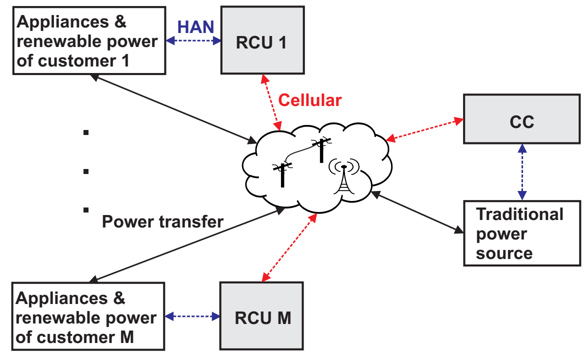


Fig. 1. System model composed of a traditional power source, users with residential control units, cellular communication network and power distribution network.

power price using the scheme proposed in this paper. The price decision will be made on a regular interval of τ. Considering a real-time pricing paradigm, τ does not exceed a few, says 5 minutes. We further assume the price that is decided at the beginning of an interval will be effective for the entire interval, before the next price decision.

To facilitate the regular decision making process, we assume the cellular communication network will allocate sufficient transmission time slots to RCUs and CC such that a desired reliability and latency can be achieved in supporting the real-time pricing [12]. In practice, these time slots are organized into regular transmission frames with the support of MAC protocol such as [13]. Assuming a proper mapping between decision making intervals and transmission frames, we will use the terms "interval" and "frame" interchangeably hereafter.

III. NON-COOPERATIVE STACKELBERG GAME

The operator's objective is to decide the optimal power price $u[j]$ at the beginning of each interval j so that its profit can be maximized. This objective can be formulated as follows:

$$\begin{aligned} \max_{u[j]} \quad & R[j] \\ \textbf{s.t.} \quad & u_l \le u[j] \le u_h. \end{aligned} \tag{1}$$

In (1), $R[j]$ is the operator's profit in the j-th interval and it is determined as the difference between revenue and cost as follows

$$R[j] = D[j]u[j] - a_t \left(D[j]\right)^{\gamma_t}, \tag{2}$$

where γ_t is the power cost exponent for traditional power, α_t is the power cost coefficient and $D[j]$ is the power demand in the j-th interval. The dependency of $D[j]$ on power price and acceptance price will be described later. In the constraint of (1), u_h is the highest acceptable prices and u_l is the lowest acceptable prices. In practice, u_h is the peak power price and its existence guarantees the reliability of the power delivery, i.e. an inelastic appliance, e.g. a refrigerator, must know u_h and set its acceptance price equal to it. The other reading of the constraint $u[j] \le u_h$ is that an upper barrier on the electricity price could moderate in some cases the environmental impact.

For example, carbon emissions can be reduced by upper-bounding the power price, since a high price will encourage biomass usage instead of electricity consumption, which is a cleaner solution.

For simplicity, we assume that neither the operator of the power distribution network, nor the renewable generators can sell power to a neighboring power network. Let $\mathcal{N}$ be the set of all smart meters in the distribution network, and $N = |\mathcal{N}|$. As such, the power demand $D[j]$ as observed by the operator for a the j-th interval is given as

$$D[j] = \left[\sum_{n=1}^{N} \beta_n[j] I(V_n[j] \geq u[j]) - S[j] \right]^+, \quad (3)$$

where $[A]^+ = \max(A, 0)$, $\beta_n[j]$ denotes the amount of power required by the appliance n in the j-th interval, $V_n[j]$ denotes the acceptance price of smart meter n in the j-th interval, and $S[j]$ denotes the total renewable power output in the j-th interval. In (3), $I(A)$ is the indicator function, which is equal to 1, if A is true and 0 otherwise. Thus, it is implied that the appliances with acceptance price V_n, which is lower than u, will prefer not to consume power, and therefore their consumption is excluded from the total consumption in the j-th interval.

With the description above, the operator wants to maximize its profit subject to the customers' decisions on their acceptance prices. There is an interaction between operators and customers in a non-cooperative manner, since each of them wants to selfishly maximize their own interest. While the operator wants to increase its profit, the customers try to cover their power demand at the lowest price possible. Therefore, we further model the optimization problem (1) as part of a Stackelberg game, where the dominant player (the operator) maximizes its profit in response to all other players (the customers) in a competitive equilibrium.

Definition 1: (Players of the game) The N appliances are the followers of the game, while the operator is the leader. At each decision point, the leader has knowledge of the acceptance prices chosen by the followers. The above constitutes the *information* available at each decision point. The *action* available to the follower is setting an appropriate acceptance price, and subsequently deciding to accept or reject the operator's price. Since the game is non-cooperative, all players try to maximize their individual *payoffs*. Specifically, the followers try to buy cheap power, while the leader tries to maximize its profit.

Definition 2: (Stages of the game) The game consists of two levels (stages). In the first stage, the leader (operator) chooses the power price $u[j]$, taking the acceptance prices $\mathbf{V} = [V_1 ... V_N]$ and the respective power demands $\mathbf{B} = [\beta_1 ... \beta_n]$ of the customers into account, with aim to maximize its profit, as given in (2), which is the payoff of the leader. In the second stage of the game, after the power price is announced, the followers choose whether they will buy power from the leader or not, according to their respective acceptance prices $\mathbf{V}$.

Remark 1: We consider that the consumption of each appliance is not infinitesimally small and can have an impact on the operator's profit. Moreover, we assume that the operator cannot strategically influence the acceptance prices.

A. User utility function

The Stackelberg game can be solved by backward induction, such that the subgame equilibrium in the first stage can be determined given the equilibrium of the second stage. In the first stage, the leader can determine the best power price after establishing the different equilibria of the second stage among the followers, in response to different power prices. For this backward induction mechanism to work, proper utility function for the followers must first be defined and known to the leader.

After the announcement of the power price u by the operator, the appliance n has two options, either to accept the price and consume β_n, or reject the price and consume no power. Of course, if the consumer finally buys power at a lower price than the acceptance price they proposed, they will be even more happy. Therefore, for the final consumption of the appliance, it holds $x_n \in \{0, \beta_n\}$, while the appliance will opt for the strategy that maximizes the utility function given below

$$U_n(x_n, u) = x_n(V_n - u + \epsilon), \quad (4)$$

where $\epsilon > 0 \to 0$, $\epsilon \to 0$ which is needed for the case that $V_n = u$, since in this case, the appliance will prefer to consume power. It is clear from the above that, after u is announced by the leader, the followers will choose their best response x_n^* as follows

$$x_n^* = \begin{cases} \beta_n, & V_n \geq u \\ 0, & V_n < u \end{cases} \quad (5)$$

To better explain that, an appliance n will refuse to buy power in a price higher than V_n, because in this case it holds

$$U_n(\beta_n, u) < U_n(0, u) \quad (6)$$

and therefore it will prefer to wait for the next frame.

Remark 2: At the end of the game, all served appliances will pay a price u and not V_n for the power they consume from the operator. The appliances for which it holds $V_n > u$ were willing to pay up to V_n, therefore paying u, makes them "happier". That's why we consider that the utility function is proportional to the term $(V_n - u + \epsilon)$.

B. Acceptance price of rational users

Generally, users want to consume power at the lowest prices, while they also want to get their tasks done not later than a specific moment. Let $t_{d,n}$ denote the duration of a task, e.g. the time that an electrical vehicle needs to be fully charged, and let $t_{r,n}$ denote the remaining time until the moment that the task must be completed. We consider t_r and t_d as integer multiples of the duration of an interval τ, such that $t_{r,n} = r_n\tau$ and $t_{d,n} = d_n\tau$, respectively, where $r_n, d_n \in \mathbb{Z}^+$. When $t_{r,n} - t_{d,n} = \tau$, the probability for an appliance to be served at the next frame, should be strictly equal to 1 so that the specific task can be completed in time. This is ensured only when the acceptance price for this frame is $V_n = u_h$.

To capture the rational reactions of consumers, we have two considerations: i) the desire to be served at a specific time interval and ii) the desire to be served at a low price. For the first consideration, the probability to be served depends on the acceptance prices of other users. Therefore, a higher proposed acceptance price will lead to a higher probability for the appliance to be served. In the literature, e.g. in [8] and [14] the user acceptance probability to the policy price u is often modeled as $P[V > u] = \alpha u^{\gamma}$, where V is a random variable representing the price acceptance level of a user, $\gamma \leq 0$ and $\alpha \geq 0$ is a normalization factor. Inspired by this, we use the following function to capture the desire of the appliance n to be served within the current frame

$$f_n = 1 - \alpha V_n^{\gamma}. \tag{7}$$

For the second consideration, a function g_n is introduced to captures the unwillingness of the consumer n to pay a high power price. Function g_n must be increasing with u as follows

$$\frac{dg_n}{du} \geq 0. \tag{8}$$

Furthermore, while the time passes, the consumers care more for the probability to be served, so the unwillingness described by g_n should be decreasing as the time passes. The function g_n is time dependent and depicts the remaining time sensitivity, i.e. g_n is a function of $t_{r,n}$, $t_{d,n}$ and u, while it is increasing with respect to the remaining time $t_{r,n}$. For obtaining specific results, we will consider that $g_n = (r_n - 1 - d_n)u$. Note that the consumer wants to minimize the unwillingness described by g_n.

According to the above, the objective that the appliance wants to maximize is given by

$$\phi_n(V_n, u, r_n, d_n) = w_{n,1}\, f_n - w_{n,2}\, g_n, \tag{9}$$

where the $w_{n,1}$ and $w_{n,2}$ are the weight factors. In order for the user to choose a proper acceptance price, we should consider the worst case scenario, in which the appliance buys power exactly at the acceptance price it proposed, i.e. $u = V_n$. In this case, its objective can be written as

$$\phi_n(V_n, r_n, d_n) = w_{n,1}(1 - \alpha V_n^{\gamma}) - w_{n,2}(r_n - 1 - d_n)V_n. \tag{10}$$

Thus, in the worst case, the satisfaction of any appliance n is maximized as

$$\begin{aligned} \max_{V_n} \quad & \phi_n(V_n, r_n, d_n) \\ \text{s.t.} \quad & u_l \leq V_n \leq u_h \end{aligned} \tag{11}$$

The acceptance price that maximizes the multi-objective convex optimization problem (11) can be easily found in the RCU using the method of the Karush-Kuhn-Tucker conditions [15].

C. Stackelberg equilibrium and real-time algorithm

The suitable solution for the formulated game is the equilibrium in which the leader reaches its optimal price, given the followers' expected actions. Since the leader has the acceptance prices $\mathbf{V}$ and the power demands $\mathbf{B}$ as information, the actions of the followers can be predicted. Thus, in the first stage of the game, the equilibrium is given by the solution of the following set of inequalities:

$$U_n(x_n^*, u) \geq U_n(x_n, u),\ \forall n \in \mathcal{N}, \tag{12a}$$

$$R(u^*, \mathbf{X}^*) \geq R(u, \mathbf{X}^*), \tag{12b}$$

where $\mathbf{X}^*$ denotes the set of all the appliances expected best responses. In (12b), $R(\cdot)$ is the operator's profit as defined earlier in (1), and the operator attempts to increase its profit.

Remark 3: The final price u^* will be equal to one of the acceptance prices V_n. For every other u, the appliances for which it holds $V_n \geq u$, will buy power from the operator at the price u. However, u will be larger than the highest acceptance price V_n among the appliances that do not buy power. Thus, the operator would still be able to increase its profit, by choosing this specific acceptance price as the final price u^*, since the served load would stay the same.

From the remark above, it is evident that the profit function is defined in the interval $u_l \leq u \leq u_h$ and it is discontinuous for the values of u for which $u = V_n$ for some n, since at that values, the served load also changes in discrete amounts. However, the best value u^* is known to belong to the finite solution set $\mathbf{V}$ beforehand. Thus, the operator does not need to sweep through all possible values of u, but only the set $\{u : u \in \mathbf{V}\}$. Therefore, we propose the following real time algorithm.

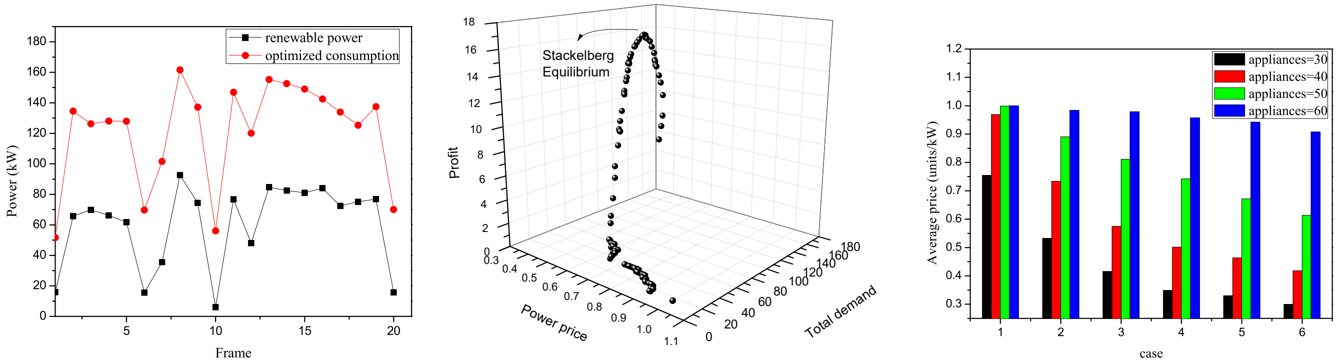
Algorithm 1 Executed by the operator in each frame

1: Receive the information about the acceptance prices, $\mathbf{V}$, the possible demand of each appliance $\mathbf{B}$ and the renewable power.
2: Compute the finite set $R(u, \mathbf{X}^*)$ for the value set $\{u : u \in \mathbf{V}\}$. Find the maximum value of this set, which is the Stackelberg equilibrium for the leader, and set the power price u^*.
3: Announce the power price to the consumers.

In case of a tie, the tie-breaking rule is to choose the price u^*, among those that maximize the profit, which serves the highest demand, as a social welfare measure. In the second stage of the game, after the power price is set, each follower makes its own decision according to (5), maximizing its own utility function.

IV. Evaluation Results

We have evaluated the proposed DSM scheme through simulations. In the simulations, the following typical values for various parameters must be assumed unless stated otherwise: $u_l = 0.3$ unit of money per kW, $u_h = 1$ unit of money per kW, $\gamma = -1$, $\alpha_t = 0.003$, $\gamma_t = 2$, $w_{n,2} = 2$, $N = 50$, and $d_n = 1$. Also, $S[j]$ is a random variable within range $[0, 5]$ kW, and r_n is a random variable from the set $\{2, 3, 4, 5, 6\}$. For each simulation, $w_{n,1}$ is randomly selected from range $[0.3, 0.5]$ and stays the same.



(a) Power supply and demand.

(b) Optimal profit.

(c) Average power price.

Fig. 2. Performance of the proposed DSM schemes in demand curtailment, operator's profit and power price.

Fig. 2(a) shows the effectiveness of the proposed scheme in managing demand in response to the time-varying supply of renewable power. More specifically, as the renewable power output increases, the operator reduces the price in order to increase the total demand and finally increase its profit. Therefore, with our proposed scheme, the operator maximizes its profit, while the renewable generators are optimally exploited.

The choice of the power price by the operator is illustrated in Fig. 2(b), which is a zoom-in at the 5-th frame of Fig. 2(a). In Fig. 2(b), the total demand and the operator's profit is shown, according to the different choices for power price. The operator chooses the price that maximizes its profit, which is the Stackelberg equilibrium for this frame, since no player has the incentive to select another solution.

Fig. 2(c) shows the average power price considering 20 consecutive frames over 1000 independent simulation runs. Various different cases of elasticity among the users are considered. Specifically, we examine six different cases for r_n: In case 1, all the loads are inelastic and therefore $r_n = 2$, while in case i, $i = 3, \ldots, 6$, r_n takes values randomly from the set $\{2, \ldots, i+1\}$. It is clearly illustrated that the number of appliances requesting power in each frame strongly affects the power price, due to load congestion. As the number of appliances inserted into the system increases the demand, there are more appliances willing to consume power at a higher price and the operator needs to set a higher price to maintain his profit as the power cost takes a toll. Also, it is evident that, as the elasticity of the loads increases, the average power price decreases. This gives the incentive to the users to be as elastic as possible, and thus to propose lower acceptance prices, which in most cases leads to a lower selected price by the operator.

V. Conclusions

In this paper, an innovative concept has been introduced to perform DSM over a smart grid where the operator has full feedback of the acceptance prices of the appliances. The DSM scheme is modeled as a Stackelberg game, in which the consumers attempt to be served in time, while the operator wants to maximize his profit. The proposed scheme considers both the traditional power source and the distributed renewable power generation. It is remarkable that, with a simple algorithmic approach, both the operator and the appliances can reach their equilibrium points. The simulation results demonstrate that, with the proposed scheme, the operator maximizes its profit, while the optimal utilization of the renewable power leads to reduction of the power price for the consumers, which strongly depends on the users elasticity. The proposed scheme is practical and easy to implement in smart grids.

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