Performance of Differential Modulation Under RF Impairments

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***Abstract*—Coherent detection requires exact knowledge of the channel state information, which is often a challenging task in demanding practical applications. Based on this, non-coherent detection of differentially modulated signals can be considered as an alternative method. The present paper investigates the effects of in-phase/quadrature-phase imbalance (IQI), which are known to degrade the performance of wireless communication systems. Specifically, we evaluate the effects of IQI on the bit error rate (BER) performance of differential quadrature phase shift keying (DQPSK) for ideal receiver (RX) with transmitter (TX) IQI, ideal TX with RX IQI and joint TX/RX IQI. Explicit analytic expressions are derived for the BER of both single-carrier and multi-carrier systems suffering from IQI at the TX and/or RX. Extensive Monte-Carlo simulation as well as offered analytic results show that realistic TX/RX IQI values can degrade the corresponding BER by over 30%. Likewise, it is shown that the detrimental effects of IQI are more considerable on DQPSK than on QPSK.**

I. Introduction

It is widely known that coherent information detection requires full knowledge of the channel state information (CSI) at the receiver site. However, this is a challenging task in practice as sophisticated and often complex channel estimation algorithms are required. In this context, differential modulation (DM) has been proposed as an efficient technique, particularly for low-power wireless systems, such as wireless sensor networks and relay networks [1]. The main advantage of this scheme stems from the fact that it simplifies the detection since it eliminates the need for channel estimation and tracking. Based on this, the authors in [2] investigated recently the performance of DM in simultaneous wireless information and power transfer cooperative amplify-and-forward networks. In [3], the pairwise error probability of differential space-time modulation (DSTM), which is an extension of differentially modulated quadrature phase shift keying (DQPSK) for multi-antenna systems, was analyzed for the case of Rayleigh distributed multipath fading. Likewise, the authors in [4] analyzed the effects of asymmetric multipath fading and shadowing effects on differentially modulated amplify-and-forward-relaying systems, while [5] analyzed the error performance of DM based decode-and-forward free space optical cooperative communication system over Gamma-Gamma fading channels. In the same context, the authors in [6] consider DM for the case of single input single output (SISO) LTE downlink transmission.

It is recalled that direct conversion receivers have attracted considerable attention thanks to their potential to reduce the cost and power consumption. However, these receivers also introduce radio frequency (RF) impairments. A critical example is the I/Q imbalance (IQI), which refers to the amplitude and phase mismatch between the in-phase (I) and quadrature-phase (Q) branches of a transceiver and leads to imperfect image rejection which results in considerable performance degradation of conventional and emerging communication systems [7]. In fact, in ideal scenarios, the I and Q branches of a mixer should have equal amplitude and a phase shift of $90°$; however, in practice, the direct-conversion transceivers are sensitive to certain analog front-end related impairments. Motivated by this practical concern, several recent works have been proposed to model, overcome or even exploit IQI [8]-and the references therein. For example, in [9], the signal-to-interference-plus-noise-ratio (SINR) is derived for orthogonal frequency division multiplexing (OFDM) systems, while analytical expressions for the symbol error probability (SEP) was derived in [10] for the case of Rayleigh fading channels.

Nevertheless, despite the importance of RF front-ends on the system performance, the detrimental effect of RF impairments are often neglected. To the best of the authors knowledge, this is also the case concerning the effects of IQI on DM based systems. Motivated by this, the present contribution analyzes the effects of IQI on the performance of DQPSK based single-carrier and multi-carrier systems over multipath fading channel. To this end, novel analytic expressions are derived for the corresponding exact average bit error rate (BER) for the cases of TX IQI and ideal receiver (RX), RX IQI and ideal TX and joint TX/RX IQI. In addition, simple and tight approximations are provided for the respective cases. The offered analytic results are subsequently employed in quantifying the detrimental effects on each considered scenario and provide meaningful insights that can be useful in the design of such systems.

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II. System and Signal Model

We assume a signal, s, is transmitted over a flat fading wireless channel, h, which follows a Rayleigh distribution and is subject to additive white Gaussian noise, n. We also assume that the channel is slowly varying and that h remains constant over two consecutive symbols. Considering the case where the TX and RX are equipped with a single antenna, the transmitter employs differentially encoded quadrature phase shift keying (QPSK) and the receiver performs differential detection of the received signal. That is, for the conventional detection of two symbol observations, the information phase is modulated on the carrier as the difference between two adjacent transmitted phases, and the receiver takes the difference of two adjacent phases to reach a decision on the information phase, which eliminates the need of knowledge of the carrier phase and channel state [11]. The information phases $\Delta\theta_l$ are first differentially encoded to a set of phases as follows

$$\theta_l = (\theta_{l-1} + \Delta\theta_l) \bmod 2\pi \tag{1}$$

where $\Delta\theta_l = 2m\pi/M,\ m = 0, 1, ..., M-1$ and M is the modulation order. The modulated symbol $s[l]$ is obtained by applying a phase offset to the previous symbol $s[l-1]$ as

$$s[l] = s[l-1]e^{j\theta_l} \tag{2}$$

where $s[1] = 1$.

Similarly, the estimated symbol is obtained from the phase difference between two consecutive received symbols as

$$\widehat{s}[l] = r^*[l-1]r[l]. \tag{3}$$

At the RF front end of the receiver, the received RF signal undergoes various processing stages including filtering, amplification, and analog I/Q demodulation (down-conversion) to baseband and sampling.

A. Single-carrier systems impaired by IQI

The baseband representation of the IQI impaired signal is given by [12]

$$g_{\text{IQI}} = \mu_{T/R}\, g_{\text{ideal}} + \nu_{T/R}\, g^*_{\text{ideal}} \tag{4}$$

where g_{ideal} is the baseband IQI-free signal and g^*_{ideal} results from the involved IQI. The subcripts T/R denote the up/down-conversion process at the TX/RX, respectively while the IQI coefficients $\mu_{T/R}$ and $\nu_{T/R}$ are given by

$$\mu_T = \frac{1}{2}\left(1 + \epsilon_T e^{j\phi_T}\right),\ \nu_T = \frac{1}{2}\left(1 - \epsilon_T e^{-j\phi_T}\right),$$
$$\mu_R = \frac{1}{2}\left(1 + \epsilon_R e^{-j\phi_R}\right),\ \nu_R = \frac{1}{2}\left(1 - \epsilon_R e^{j\phi_R}\right).$$

where $\epsilon_{T/R}$ and $\phi_{T/R}$ are the TX/RX amplitude and phase mismatch, respectively. Moreover, the TX/RX image rejection ratio (IRR) is given by

$$\text{IRR}_{T/R} = \frac{\left|\mu_{T/R}\right|^2}{\left|\nu_{T/R}\right|^2}. \tag{5}$$

1) TX impaired by IQI: This case assumes that the RF front-end of the RX is ideal, while the TX experiences IQI. To this effect, the instantaneous SINR per symbol at the input of the receiver is given by [13]

$$\gamma = \frac{\left|\mu_T\right|^2}{\left|\nu_T\right|^2 + \frac{1}{\gamma_{\text{ideal}}}} \tag{6}$$

where $\gamma_{\text{ideal}} = \frac{E_s|h|^2}{N_o}$ is the instantaneous signal to noise ratio (SNR) per symbol at the receiver input, assuming an ideal RF front end, E_s is the energy per transmitted symbol and N_0 is the single-sided AWGN power spectral density.

2) RX impaired by IQI: This case assumes that the RF front-end of the TX is ideal, while the RX is subject to IQI. Therefore, at the input of the RX, the instantaneous SINR per symbol is expressed as [13]

$$\gamma = \frac{\left|\mu_R\right|^2}{\left|\nu_R\right|^2 + \frac{|\mu_R|^2 + |\nu_R|^2}{\gamma_{\text{ideal}}}}. \tag{7}$$

3) Joint TX/RX impaired by IQI: This case assumes that both TX and RX are impaired by IQI. Based on this, the instantaneous SINR per symbol at the input of the RX is given by [13]

$$\gamma = \frac{\left|\xi_{11}\right|^2 + \left|\xi_{22}\right|^2}{\left|\xi_{12}\right|^2 + \left|\xi_{21}\right|^2 + \frac{|\mu_R|^2 + |\nu_R|^2}{\gamma_{\text{ideal}}}} \tag{8}$$

where $\xi_{11} = \mu_R\mu_T$, $\xi_{22} = \nu_R\nu_T^*$, $\xi_{12} = \mu_R\nu_T$, and $\xi_{21} = \nu_R\mu_T^*$.

B. Multi-carrier systems impaired by IQI

It is assumed that the RF carriers are down converted to the baseband by wideband direct conversion. We also denote the set of signals as $S = \{-K, ..., -1, 1, ...K\}$ and assume that there is a signal present at the image carrier.

1) TX impaired by IQI: Assuming that the RF front-end of the RX is ideal, while the TX experiences IQI, the baseband equivalent received signal is

$$s_{\text{IQI}} = \mu_T s\left(k\right) + \nu_T s^*\left(-k\right). \tag{9}$$

Hence, the instantaneous SINR per symbol at the input of the RX is given by [13]

$$\gamma = \frac{\left|\mu_T\right|^2}{\left|\nu_T\right|^2 + \frac{1}{\gamma_{\text{ideal}}(k)}}. \tag{10}$$

2) RX impaired by IQI: Assuming that the RF front-end of the TX is ideal, while the RX is impaired by IQI, the instantaneous SINR per symbol at the input of the RX is expressed as [13]

$$\gamma = \frac{\left|\mu_R\right|^2}{\left|\nu_R\right|^2 \frac{\gamma_{\text{ideal}}(-k)}{\gamma_{\text{ideal}}(k)} + \frac{|\mu_R|^2 + |\nu_R|^2}{\gamma_{\text{ideal}}(k)}} \tag{11}$$

where $\gamma_{\text{ideal}}\left(-k\right) = \frac{E_s}{N_o}\left|h\left(-k\right)\right|^2$.

$$P_b = \frac{1}{2}\left(1 - \sum_{k=0}^{\infty}\sum_{m=0}^{2k}\frac{(2k)!}{(2k-m)!m!}\frac{\sqrt{2}\left(-\alpha\right)^{2k-m}e^{-\left(2\frac{\alpha}{\beta}-\frac{A}{\overline{\gamma}\beta}\right)}}{2^k\left(k!\right)^2\beta^{2k+1}}\int_0^{\alpha}\left(x\right)^m e^{\frac{2x}{\beta}-\frac{A\alpha}{\overline{\gamma}\beta x}}dx\right) \tag{21}$$

$$P_b = \frac{1}{2}\left(1 - \sum_{k=0}^{\infty}\sum_{m=0}^{2k}\frac{(2k)!\left(-1\right)^{2k-m}}{(2k-m)!m!}\frac{\sqrt{2}\alpha^{2k+1}A^{m+1}e^{-\left(2\frac{\alpha}{\beta}-\frac{A}{\overline{\gamma}\beta}\right)}}{\overline{\gamma}^{m+1}2^k\left(k!\right)^2\beta^{2k+m+2}}\Gamma\left(-\left(m+1\right),\frac{A}{\overline{\gamma}\beta},-\frac{2\alpha A}{\overline{\gamma}\beta^2},1\right)\right) \tag{22}$$

3) Joint TX/RX impaired by IQI: Finally, assuming that both the TX and RX suffer from IQI, the instantaneous SINR per symbol at the input of the RX is given by

$$\gamma = \frac{\left|\xi_{11}\right|^2 + \left|\xi_{22}\right|^2\frac{\gamma_{\text{ideal}}(-k)}{\gamma_{\text{ideal}}(k)}}{\left|\xi_{12}\right|^2 + \left|\xi_{21}\right|^2\frac{\gamma_{\text{ideal}}(-k)}{\gamma_{\text{ideal}}(k)} + \frac{|\mu_R|^2+|\nu_R|^2}{\gamma_{\text{ideal}}(k)}}. \tag{12}$$

It is observed that in multi-carrier systems, IQI causes distortion to the transmitted signal at carrier k by its image signal at carrier $-k$.

III. Performance Analysis

When Gray coding is employed, the bit error probability for DQPSK over AWGN channel is given by [14]

$$P_b = Q\left(\sqrt{\gamma}a,\sqrt{\gamma}b\right) - \frac{1}{2}I_0\left(\sqrt{2}\gamma\right)e^{-2\gamma} \tag{13}$$

where $a=\sqrt{2-\sqrt{2}}$, $b=\sqrt{2+\sqrt{2}}$, $Q\left(\alpha,\beta\right)$ is the Marcum Q-function [15] and $I_n\left(z\right)$ is the modified Bessel function of the first kind [16].

In case of fading conditions, we average (13) over the statistics of the involved fading channel yielding

$$P_b = \int_0^{\infty}\left(Q\left(\sqrt{\gamma}a,\sqrt{\gamma}b\right) - \frac{1}{2}I_0\left(\sqrt{2}\gamma\right)e^{-2\gamma}\right)f_\gamma\left(\gamma\right)d\gamma \tag{14}$$

where $f_\gamma\left(\gamma\right)$ denotes the corresponding SNR probability density function (PDF). To this effect, it is also recalled that the SNR PDF and cumulative distribution function (CDF) for the case of Rayleigh fading conditions are given in [17], namely

$$f_\gamma\left(\gamma\right) = \frac{1}{\overline{\gamma}}e^{-\frac{\gamma}{\overline{\gamma}}} \tag{15}$$

and

$$F_\gamma\left(\gamma\right) = 1 - e^{-\frac{\gamma}{\overline{\gamma}}} \tag{16}$$

respectively.

A. Single-carrier systems impaired by IQI

In this subsection, we derive the BER performance of single-carrier transmission using DQPSK over Rayleigh fading channels in the presence of IQI. To this end, and based on (6)−(8), one obtains

$$\begin{aligned} F_{\gamma_{IQI}}\left(\gamma\right) &= F_{\gamma_{ideal}}\left(\frac{\alpha}{\beta+\frac{A}{\gamma}}\right) \\ &= 1 - e^{-\frac{A}{\overline{\gamma}\left(\frac{\alpha}{\gamma}-\beta\right)}}, \qquad 0 \le \gamma \le \frac{\alpha}{\beta} \end{aligned} \tag{17}$$

where $F_{\gamma_{ideal}}$ is given by (16) and the parameters α, β, and A are given by

TABLE I: Single-carrier systems impaired by IQI parameters

	α	β	A
TX IQI	$\left\|\mu_T\right\|^2$	$\left\|\nu_T\right\|^2$	1
RX IQI	$\left\|\mu_R\right\|^2$	$\left\|\nu_R\right\|^2$	$\left\|\mu_R\right\|^2+\left\|\nu_R\right\|^2$
Joint TX/RX IQI	$\left\|\xi_{11}\right\|^2+\left\|\xi_{22}\right\|^2$	$\left\|\xi_{12}\right\|^2+\left\|\xi_{21}\right\|^2$	$\left\|\mu_R\right\|^2+\left\|\nu_R\right\|^2$

Since $f_\gamma\left(\gamma\right)=\frac{d}{d\gamma}F_\gamma\left(\gamma\right)$, we have

$$f_{\gamma_{IQI}}\left(\gamma\right) = \frac{\alpha A e^{-\frac{A}{\overline{\gamma}\left(\frac{\alpha}{\gamma}-\beta\right)}}}{\overline{\gamma}\left(\alpha-\gamma\beta\right)^2} \tag{18}$$

Substituting (18) in (14), taking $u=f_{\gamma_{IQI}}\left(\gamma\right)$ and $dv=Q\left(\sqrt{\gamma}a,\sqrt{\gamma}b\right)-\frac{1}{2}I_0\left(\sqrt{2}\gamma\right)e^{-2\gamma}$, integrating by parts and after some mathematical manipulations, one obtains

$$P_b = \frac{1}{2}\left[1 - C\int_0^{\alpha}I_0\left(\sqrt{2}\frac{\alpha-x}{\beta}\right)e^{\frac{2x}{\beta}-\frac{A\alpha}{\overline{\gamma}\beta x}}dx\right] \tag{19}$$

where

$$C = \frac{\sqrt{2}e^{-\left(2\frac{|\mu_T|^2}{|\nu_T|^2}-\frac{A}{\overline{\gamma}|\nu_T|^2}\right)}}{\left|\nu_T\right|^2}. \tag{20}$$

By then substituting the involved Bessel function by its series representation in [18, Eq.(8.447)] and expanding the binomial, one obtains (21) at the top of the page.

By also setting $y=\frac{1}{x}$ and carrying out some manipulations yields (22), where $\Gamma\left(\alpha,x,b,\beta\right)$ is the extended incomplete Gamma function [19]. Moreover, if we realistically assume that $\beta\approx0$ in the exponential function argument, the BER can be expressed as [15]

$$P_b \approx \frac{1}{2} - \left(\frac{2}{s^2-2}\right)^{\frac{1}{2}}\left[\frac{1}{2}+\frac{e^{\frac{-s\alpha}{\beta}}}{2}I_0\left(\frac{\sqrt{2}\alpha}{\beta}\right)-Q(f,g)\right] \tag{23}$$

where

$$s = \frac{A}{\overline{\gamma}\alpha+2}, \tag{24}$$

$$f = \sqrt{\frac{\alpha}{\beta}\left(s-\sqrt{s^2-2}\right)}, \tag{25}$$

and

$$g = \sqrt{\frac{\alpha}{\beta}\left(s+\sqrt{s^2-2}\right)}. \tag{26}$$

It is noted here that since $\forall\gamma\in[0,\frac{\alpha}{\beta}]$, $e^{-2\gamma}I_0\left(\sqrt{2}\gamma\right)>0$ and $\frac{A}{\overline{\gamma}\left(\frac{\alpha}{\gamma}-\beta\right)}>\frac{A\gamma}{\overline{\gamma}\alpha}$, (23) is in fact a lower bound to (22).

$$f_{\gamma}\left(\gamma\right)=\frac{\exp\left(-\frac{x\left(|\mu_R|^2+|\nu_R|^2\right)}{\overline{\gamma}\left(|\xi_{11}|^2-x|\xi_{12}|^2\right)}\right)\left(\frac{|\xi_{11}|^2\left(|\mu_R|^2+|\nu_R|^2\right)}{\overline{\gamma}}+\frac{|\xi_{21}|^2|\xi_{11}|^2-|\xi_{12}|^2|\xi_{22}|^2}{1+\frac{x|\xi_{21}|^2-|\xi_{22}|^2}{|\xi_{11}|^2-x|\xi_{12}|^2}}\right)}{\left(\left|\xi_{11}\right|^2-x\left|\xi_{12}\right|^2\right)\left(\left|\xi_{11}\right|^2-\left|\xi_{22}\right|^2+x\left(\left|\xi_{21}\right|^2-\left|\xi_{12}\right|^2\right)\right)},\qquad 0\leq\gamma\leq\frac{|\xi_{11}|^2}{|\xi_{12}|^2} \tag{33}$$

$$P_b\approx D\left[\frac{\sqrt{2}Q\left(w,z\right)}{\sqrt{\left(t+2\right)^2-2}}-e^{-t\frac{|\xi_{11}|^2}{|\xi_{12}|^2}}Q\left(a\frac{|\xi_{11}|}{|\xi_{12}|},b\frac{|\xi_{11}|}{|\xi_{12}|}\right)+\left(\frac{1}{2}-\frac{\sqrt{2}}{2\sqrt{\left(t+2\right)^2-2}}\right)\left(1+e^{-\frac{|\xi_{11}|^2}{|\xi_{12}|^2}(t+2)}I_0\left(\sqrt{2}\frac{|\xi_{11}|^2}{|\xi_{12}|^2}\right)\right)\right] \tag{34}$$

B. Multi-carrier systems impaired by IQI

In this subsection, we derive an analytic expression for the BER in multi-carrier transmission for the case of DQPSK constellation and Rayleigh fading conditions in the presence of IQI.

1) TX impaired by IQI: The corresponding BER is evaluated using (14), where $F_{\gamma}\left(x\right)$ is derived as

$$F_{\gamma}\left(x\right)=F_{ideal}\left(\frac{\left|\mu_T\right|^2}{\left|\nu_T\right|^2+\frac{1}{x}}\right). \tag{27}$$

Hence, for multi-carrier transmission, the corresponding BER is given by (22) and is lower bounded by (23).

2) RX impaired by IQI: Assuming that the RX is impaired by IQI while the TX is ideal, the corresponding BER is evaluated using (14), where $F_{\gamma}\left(x\right)$ is derived as

$$F_{\gamma}\left(x\right)=\int_0^{\infty}F_{\gamma}\left(x|\gamma_{ideal}\left(-k\right)\right)f_{\gamma_{ideal}}\left(y\right)dy \tag{28}$$

Substituting (11), (15) and (16) and solving the above integral yields

$$f_{\gamma}\left(\gamma\right)=\frac{e^{-\frac{\gamma}{\overline{\gamma}}\left(\frac{|\nu_R|^2}{|\mu_R|^2}+1\right)}\left(1+\frac{|\nu_R|^2}{|\mu_R|^2}+\frac{\overline{\gamma}|\nu_R|^2}{|\mu_R|^2\left(\frac{\gamma|\nu_R|^2}{|\mu_R|^2}+1\right)}\right)}{\overline{\gamma}\left(\frac{\gamma|\nu_R|^2}{|\mu_R|^2}+1\right)} \tag{29}$$

which is valid for $0\leq x\leq\infty$. By also substituting (29) in (14) and integrating by parts yields

$$P_b=\frac{1}{2}\left(\sqrt{2}|\mu_R|^2\int_0^{\infty}\frac{e^{-\gamma\left(2+\frac{|\nu_R|^2}{\overline{\gamma}|\mu_R|^2}+\frac{1}{\overline{\gamma}}\right)}I_0\left(\sqrt{2}\gamma\right)}{|\mu_R|^2+\gamma|\nu_R|^2}\right)d\gamma \tag{30}$$

Following a binomial expansion of $(|\mu_R|^2+\gamma|\nu_R|^2)^{-1}$, the integral in (30) can be solved by [20, Eq. 2.15.3]. Moreover, since $\forall\gamma\in[0,\infty]$, it follows that $\frac{1}{\left(\frac{\gamma|\nu_R|^2}{|\mu_R|^2}+1\right)}<1$, and since, $I_0\left(\sqrt{2}\gamma\right)e^{-\gamma\left(2+\frac{|\nu_R|^2}{\overline{\gamma}|\mu_R|^2}+\frac{1}{\overline{\gamma}}\right)}\geq 0$, the BER for multi-carrier systems can be upper bounded by

$$P_b<\frac{1}{2}\left(1-\frac{\sqrt{2}|\mu_R|^2}{\sqrt{\left(2+\frac{|\nu_R|^2}{\overline{\gamma}|\mu_R|^2}+\frac{1}{\overline{\gamma}}\right)^2-2}}\right) \tag{31}$$

3) Joint TX/RX impaired by IQI: In this subsection, it is assumed that both TX and RX are impaired by IQI. Thus, substituting (12), (15) and (16) in (28) and solving the integral yields (33) at the top of the page. Substituting (33) in (14) yields the expression of the BER for multi-carrier systems with joint TX/RX IQI. To the best of the authors' knowledge, this integral cannot be expressed in closed-form. Yet, assuming $|\xi_{21}|^2\approx|\xi_{12}|^2\approx 0$, the corresponding BER can be approximated by (34), at the top of the page, where

$$t=\frac{\left(|\mu_R|^2+|\nu_R|^2\right)}{\overline{\gamma}|\xi_{11}|^2}, \tag{35}$$

$$D=\frac{|\mu_R|^2+|\nu_R|^2}{t\overline{\gamma}\left(|\xi_{11}|^2-|\xi_{22}|^2\right)}+\frac{|\xi_{21}|^2|\xi_{11}|^2-|\xi_{12}|^2|\xi_{22}|^2}{t\left(|\xi_{11}|^2-|\xi_{22}|^2\right)^2}, \tag{36}$$

$$w=\sqrt{\frac{|\xi_{11}|^2}{|\xi_{12}|^2}\left(t+2-\sqrt{\left(t+2\right)^2-2}\right)} \tag{37}$$

and

$$z=\sqrt{\frac{|\xi_{11}|^2}{|\xi_{12}|^2}\left(t+2+\sqrt{\left(t+2\right)^2-2}\right)}. \tag{38}$$

It is shown in the following section that the offered results are quite accurate as they are quite in tight agreement with respective results from computer simulations.

IV. Numerical Results

In this section, we quantify the effects of IQI on the performance of DQPSK over Rayleigh fading channel in terms of the corresponding BER.

To this end, Fig. 1 and Fig. 2 illustrate the BER for the case of single-carrier and multi-carrier systems, respectively. Furthermore, the cases when both TX/RX are ideal, when the TX is impaired while RX is ideal, when the RX is impaired and the TX is ideal and when both the TX/RX are impaired are presented for two different IRRs. Also, the realistic case of $\mathrm{IRR}_T=\mathrm{IRR}_R=27$dB and $\phi_{T/R}=2°$ is compared to the more severe case of $\mathrm{IRR}_T=\mathrm{IRR}_R=10$dB with $\phi=25°$. The dashed lines correspond to simulation results while the markers display the results from the derived analytic expressions. One can clearly see the detrimental effects of IQI on both systems; however for a fixed IRR, IQI causes slightly more performance degradation on multi-carrier systems than on single-carrier systems. For instance, when considering the realistic assumption of $\text{IRR}_T = \text{IRR}_R = 27\text{dB}$, there is a 9.5% increase in the BER for SNR $= 10\text{dB}$ and a 12% increase for SNR $= 20\text{dB}$ for single-carrier systems versus an 11% increase when SNR $= 10\text{dB}$ and a 32% increase for SNR $= 20\text{dB}$ for multi-carrier systems. Moreover, in Fig. 2, it is noticed that high IQI ($|\nu_{T/R}| >> 0$) can generate an error floor, which is explained by the fact that the SNR reaches a limit since the signal is actually interfered by its conjugate or the image carrier signal. As a consequence, increasing the signal also increases the noise. Finally, it is observed that for both systems, IQI at the RX is more detrimental on the system performance than IQI at the TX. This is explained by the fact that RX IQI affects both the noise and the signal, whereas TX IQI affects only the signal. In this case the noise power is multiplied by $|\mu_R|^2 + |\nu_R|^2 > 1$. Moreover, for a fixed IRR, IQI causes more performance degradation on multi-carrier systems than on single-carrier systems.

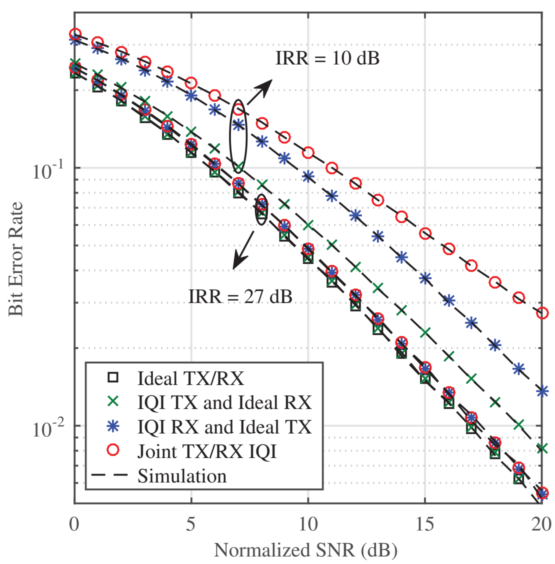

Fig. 1: Single-carrier system BER as a function of the normalized SNR for different IRRs.

Fig. 3 demonstrates the derived lower bound for single-carrier systems with joint TX/RX IQI . The lower bound is displayed in solid line while the markers display the exact BER. We show the results for $\text{IRR}_T = \text{IRR}_R = 27\text{dB}$ and $\phi = 2°$ as well as for $\text{IRR}_T = \text{IRR}_R = 15\text{dB}$ and $\phi = 7°$. It is observed that the derived lower bound is rather tight for high IRR values, which is explained by the fact that in these cases $|\xi_{12}|^2 \approx 0$ and $|\xi_{21}|^2 \approx 0$, while for lower IRRs this assumption is no longer valid. However for practical analog RF front-ends, the value of IRR is in the range of $20\text{dB}-40\text{dB}$ [13]-and the references therein, which implies that the derived lower bound represents an accurate approximation for the BER of DQPSK modulated systems over Rayleigh channel in the presence of IQI.

Fig. 4 compares the derived approximations to the exact BER for $\text{IRR}_T = \text{IRR}_R = 15dB$ and $\phi = 7°$. The lower bound derived for the case of ideal RX with IQI TX is the less accurate approximation, where the error reaches 8% at

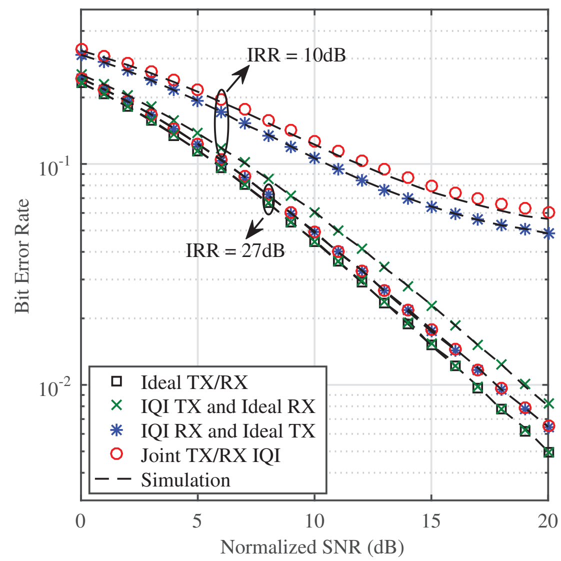

Fig. 2: Multi-carrier system BER as a function of the normalized SNR for different IRRs.

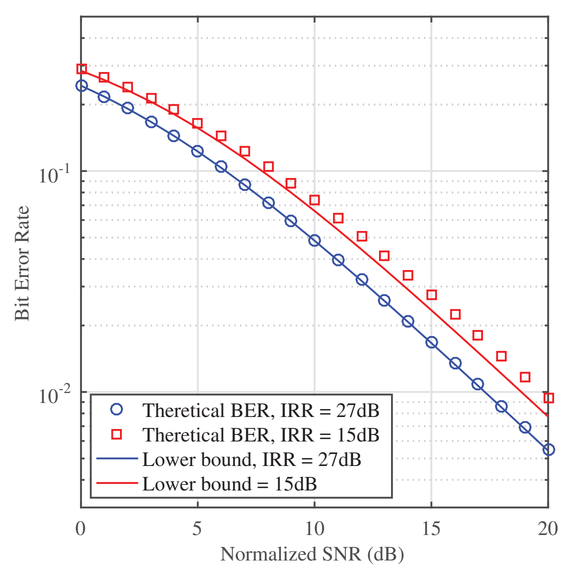

Fig. 3: Single-carrier system, derived lower bound (solid line) and exact BER (markers) for joint TX/RX IQI.

SNR $= 20\text{dB}$. Meanwhile the upper bound provided for the case of ideal TX with IQI RX in only 6% away from the exact value for the same SNR. Finally, the approximation provided for the joint TX/RX IQI case is the most accurate with only 1% error.

Fig. 5 compares the effects of IQI on the performance of QPSK and DQPSK for single-carrier systems and for the extreme case of $\text{IRR}_T = \text{IRR}_R = 10\text{dB}$. It is observed that the effects of IQI are more severe on the BER performance of DQPSK than QPSK. For instance, when SNR $= 15\text{dB}$, IQI at the transmitter generates around 1dB loss for QPSK versus around 3dB for DQPSK.

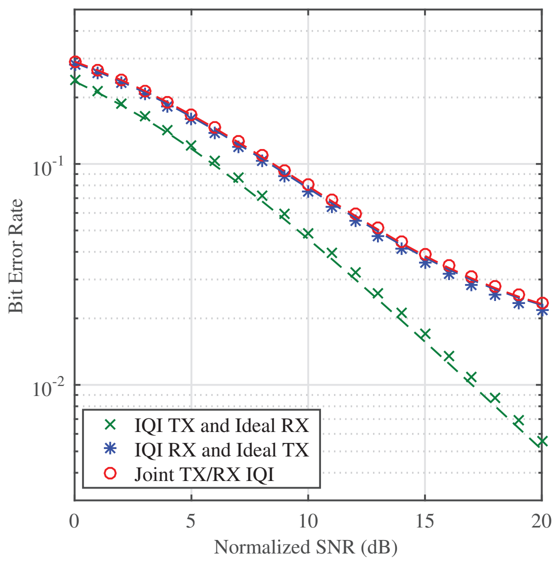

Fig. 4: Multi-carrier system comparison between exact BER (markers) and approximations (dashed lines) for $\mathrm{IRR}_T = \mathrm{IRR}_R = 15\mathrm{dB}$ and $\phi = 7°$.

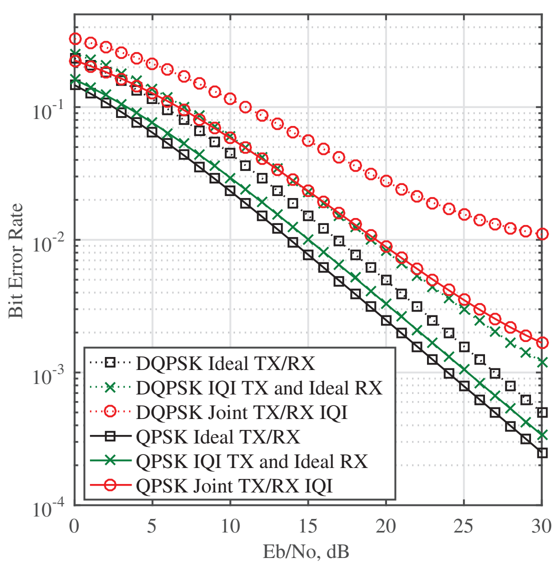

Fig. 5: Single-carrier system BER comparison between QPSK and DQPSK for $\mathrm{IRR}_T = \mathrm{IRR}_R = 10\mathrm{dB}$ and $\phi = 25°$.

V. Conclusion

In this paper, the performance of DQPSK modulated systems over Rayleigh multipath fading channels in the presence of IQI has been studied for both single-carrier and multi-carrier system. The cases of ideal TX with RX IQI, ideal RX with TX IQI and joint TX/RX IQI have been considered. Novel analytic expressions for the BER as well as simple lower bounds have been derived for the single -carrier case. For multi-carrier systems, analytic expressions are derived for the BER for ideal RX with TX IQI along with as a lower bound, while a lower bound and an approximation have been proposed for the cases of ideal TX with RX IQI and joint TX/RX IQI, respectively. Simulation results were provided to verify the validity of the derived expressions. It was observed that for DM, IQI can lead to a significant decrease of the BER of both single-carrier and multi-carrier systems. In fact, IQI induces more performance loss in the case of DQPSK than in QPSK which highlights the importance of considering this impairment in the performance analysis of DM.

References

[1] J. Abouei, K. N. Plataniotis, and S. Pasupathy, "Green modulations in energy-constrained wireless sensor networks," *IET commun.*, vol. 5, no. 2, pp. 240–251, 2011.

[2] L. Mohjazi, S. Muhaidat, and M. Dianati, "Performance analysis of differential modulation in swipt cooperative networks," *IEEE Signal Process. Lett.*, vol. 23, no. 5, pp. 620–624, May 2016.

[3] V. K. Nguyen, "Performance analysis of differential space-time modulation," *IEEE Trans. on Inf. Theory*, vol. 58, no. 5, pp. 2620–2632, May 2012.

[4] S. AlMaeeni, P. C. Sofotasios, S. Muhaidat, G. K. Karagiannidis, and M. Valkama, "Distributed differential modulation over asymmetric fading channels," *IEEE Signal Process. Lett.*, vol. 23, no. 12, pp. 1712–1716, Dec 2016.

[5] M. R. Bhatnagar, "Average ber analysis of differential modulation in df cooperative communication system over gamma-gamma fading fso links," *IEEE Commun. Lett*, vol. 16, no. 8, pp. 1228–1231, August 2012.

[6] M. Hofer, M. Łimko, S. Schwarz, and M. Rupp, "Performance evaluation of differential modulation in lte - downlink," in *20th International Conference on Systems, Signals and Image Processing*, July 2013, pp. 179–182.

[7] M. Valkama, M. Renfors, and V. Koivunen, "Advanced methods for I/Q imbalance compensation in communication receivers," *IEEE Trans. Signal Process.*, vol. 49, no. 10, pp. 2335–2344, Oct 2001.

[8] B. Selim, P. Sofotasios, S. Muhaidat, and G. Karagiannidis, "The effects of i/q imbalance on wireless communications: A survey," in *IEEE 59th International Midwest Symposium on Circuits and Systems (MWSCAS)*, to be published.

[9] . zdemir, R. Hamila, and N. Al-Dhahir, "Exact average ofdm subcarrier sinr analysis under joint transmit receive I/Q imbalance," *IEEE Trans. Veh. Technol.*, vol. 63, no. 8, pp. 4125–4130, Oct 2014.

[10] J. Qi, S. Aissa, and M. S. Alouini, "Dual-hop amplify-and-forward cooperative relaying in the presence of tx and rx in-phase and quadrature-phase imbalance," *IET Commun.*, vol. 8, no. 3, pp. 287–298, Feb 2014.

[11] M. K. Simon and M.-S. Alouini, *Digital communication over fading channels*. John Wiley & Sons, 2005, vol. 95.

[12] T. Schenk, *RF Imperfections in High-Rate Wireless Systems*. The Netherlands: Springer, 2008.

[13] A.-A. Boulogeorgos, P. Sofotasios, B. Selim, S. Muhaidat, G. Karagiannidis, and M. Valkama, "Effects of rf impairments in communications over cascaded fading channels," 2015.

[14] J. G. Proakis and M. Salehi, *Digital communications*. McGraw Hill, 2014.

[15] A. H. Nuttall, "Some integrals involving the q-function," DTIC Document, Tech. Rep., 1972.

[16] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Inegrals and Series, Vol. 2: Special Functions*. Gordon and Breach Science Publishers, 1992.

[17] M. K. Simon, *Probability distributions involving Gaussian random variables: A handbook for engineers and scientists*. Springer Science & Business Media, 2007.

[18] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. New York: Academic, 2000.

[19] F. Yilmaz and M.-S. Alouini, "A novel unified expression for the capacity and bit error probability of wireless communication systems over generalized fading channels," *IEEE Trans. Commun.*, vol. 60, no. 7, pp. 1862–1876, 2012.

[20] A. Prudnikov, Y. Brychkov, and O. Marichev, "Integrals and series, volume 2: Special functions, second printing with corrections," *Gordon and Breach Science Publishers*, 1988.