Capacity of Wireless Powered Communication Systems over Rician Fading Channels

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***Abstract*—This paper investigates the ergodic capacity of a multi-input multi-output (MIMO) wireless powered communication system with partial channel state information at the power beacon (PB). Employing time splitting protocol, the PB first transmit energy-bearing signals to the energy constrained source S through beamforming, and then S uses this energy to transmit information to the destination. Unlike several prior works, we assume that the energy harvesting link is subjected to Rician fading. Base on this, we present a comprehensive analysis of the system achievable ergodic capacity. Specifically, closed-form expressions for the ergodic capacity bounds are derived. Besides, we investigated the capacity performance in both high and low signal-to-noise ratio regimes and the optimal time split that maximizes the capacity performance. Numerical results and simulations are provided to validate the theoretical analysis. The results show that the Rician factor K has a significant impact on the ergodic capacity performance.**

I. Introduction

Radio-frequency (RF) signal based energy harvesting (EH) has become a promising solution to address the issue of energy supply for energy constrained devices such as mobile phones and various type of sensors, and received considerable research attentions. Leveraging on the fact that RF signals can carry both information and energy, a new paradigm referred to as simultaneous wireless information and power transfer (SWIPT) has recently emerged [1–4].

The fundamental tradeoff between the information transmission (IT) and EH for SWIPT systems has been studied in the pioneering work of Varshney [5]. Later in [6], practical architectures for SWIPT systems were proposed, where the optimal transmit covariance achieving the rate-energy region were characterized. In addition, the application of SWIPT in relaying system has been studied in [7, 8]. However, all the aforementioned works adopt the hybrid base station (BS) model, where the BS acts both as energy and information source. Nevertheless, due to the fact that the operational sensitivity level of the information decoder differs substantially from that of the energy harvester, the hybrid BS model can only power mobile users within a very short range, which makes the full network coverage for SWIPT impractical.

Responding to this, the authors in [9] proposed a new network architecture, where a dedicated power beacon (PB) is introduced to power the wireless devices. Since then, significant research efforts has been devoted to understand the performance of PB-assisted systems. More specifically, the optimal resource allocation problem of PB-assisted wireless powered communication (WPC) network was investigated in [10]. The maximum achievable throughput of a wireless powered cooperative relaying network was investigated in [11, 12], where both user and relay were powered by a dedicated PB. Later in [13], the authors investigated the average throughput of PB assisted WPC networks, where it was shown that the employment of multiple antennas at the PB have a positive impact on the average throughput.

Most of the prior works adopt the Rayleigh fading distribution to model the wireless power transfer channels. However, due to the relatively short power transfer distance, the line-of-sight (LOS) path is very likely to exist between the PB and the EH node. Thus, the Rician fading distribution is the most appropriate model for the WPT channel. Motivated by this, we consider a multi-input multi-output (MIMO) system powered by a dedicated PB, where the Rician fading channels is considered for the WPT link. Also, we consider the realistic scenario with only partial channel state information (CSI) at PB.

Assuming the time splitting protocol, we present a detailed investigation on the ergodic capacity performance of the system. Specifically, we present closed-form expressions for the upper and lower bounds of the ergodic capacity. In addition, we look into the high and low SNR regimes, and derive simple and concise expressions for key performance measures such as high SNR slope, power offset and minimum required energy per information bit, which reveals the impact of key system parameters on the ergodic capacity. The findings of the paper

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suggests that the Rician factor K has a significant impact on the system performance, and a large K is desirable.

Notation: We use bold upper case and lower case letters to represent matrices and vectors, respectively. $||\cdot||$ stands for the Frobenius norm, $\mathrm{E}(\cdot)$ denotes the expectation of a random variable (RV), the symbol $\dagger$ is the conjugate transpose operator, $\det(\cdot)$ represents the determinant of a matrix, $\mathcal{CN}(\cdot)$ means complex Gaussian distribution and $\mathrm{tr}(\cdot)$ is the trace operator for matrix.

II. System Model

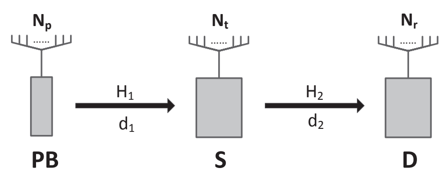

Fig. 1. System model.

We consider a point-to-point WPC system where an energy constrained source S communicates with a destination D, as depicted in Fig.1. It is assumed that S does not have external energy supply, and is powered by a dedicated PB [13, 14]. The PB is equipped with N_p antennas, while S and D are equipped with N_t and N_r antennas, respectively. Due to significant path-loss, current WPT technique is suitable for relatively short range energy transfer, as such the LOS path is very likely to exist between PB and S. Hence, the Rician distribution can be used to model the channel $\mathbf{H}_1$ between PB and S, namely

$$\mathbf{H}_1 = \sqrt{\frac{K}{K+1}}\mathbf{H_0} + \sqrt{\frac{1}{K+1}}\mathbf{H}_w, \tag{1}$$

where K is the Rician factor, and $\mathbf{H}_w \in \mathbb{C}^{N_t \times N_p}$ is the channel matrix with entries being independent and identically distributed (i.i.d.) zero-mean circularly symmetric Gaussian RVs with unit variance, $\mathbf{H}_0 \in \mathbb{C}^{N_t \times N_p}$ is a full rank deterministic matrix, normalized such that $||\mathbf{H}_0||^2 = N_p N_t$. On the other hand, Rayleigh distribution is adopted to model the IT channel $\mathbf{H}_2$ due to the relatively large distance between S and D, i.e., the entries of $\mathbf{H}_2$ are i.i.d. $\mathcal{CN}(0,1)$ RVs.

As in [13], we consider the time splitting protocol. Hence, an entire transmission block of length T is split into two separate phases. In the first phase of duration τT $(0 < \tau < 1)$, S harvests energy from PB. In the second phase, S transmits information to D using the harvested energy.

A. Energy Harvesting

Due to the fact that the instantaneous CSI is difficult to acquire in practice, we assume that the PB only has partial CSI of the channel $\mathbf{H}_1$. Specifically, we assume the statistical channel mean, $\mathbf{H}_0$, is known a prior at PB. Therefore, energy beamforming can be performed to achieve higher energy efficiency during the EH phase. And the received signal at S can be written as

$$\mathbf{y}_s = \sqrt{\frac{P_b}{d_1^l}}\alpha \mathbf{H}_0 \mathbf{w}_p x + \sqrt{\frac{P_b}{d_1^l}}\beta \mathbf{H}_w \mathbf{w}_p x + \mathbf{n}_s. \tag{2}$$

where $\alpha = \sqrt{K/K+1}$, $\beta = \sqrt{1/K+1}$, $\mathbf{w}_p$ is the beamforming vector with $||\mathbf{w}_p||_2^2 = 1$, d_1 is the distance between PB and S, l is the path-loss exponent, x is the normalized signal satisfying $\mathrm{E}\{|x|^2\} = 1$, and $\mathbf{n}_s$ is the additive white gaussian noise (AWGN) at S with covariance matrix of $N_0 \mathbf{I}_{N_t}$.

Note that PB is aware of $\mathbf{H}_0$, we choose the beamforming vector $\mathbf{w}_p$ to be the eigenvector associated with the largest eigenvalue of matrix ${\mathbf{H}_0}^\dagger \mathbf{H}_0$. Thus, the harvested energy at S can be obtained as

$$E_h = \frac{\eta \tau T P_b}{d_1^l}\left(\alpha^2 \lambda_1 + \beta^2 ||\mathbf{H}_w \mathbf{w}_p||_2^2\right), \tag{3}$$

where $0 < \eta < 1$ denotes the energy conversion efficiency, λ_1 represents the largest eigenvalue of $\mathbf{H}_0^\dagger \mathbf{H}_0$.

B. Information Transfer

In the remaining time of $(1-\tau)T$, S transmits information to D using all of the harvested energy. As the major objective is to characterize the impact of the LOS effect on the system ergodic capacity, we assume no CSI is available at S but perfect CSI at D during the IT phase in order to simplify the analysis. Hence the equal power allocation is performed and the received signal at D is given by

$$\mathbf{y}_d = \sqrt{\frac{P_s}{d_2^l N_t}}\mathbf{H}_2 \mathbf{x} + \mathbf{n}_d, \tag{4}$$

where d_2 is the distance between S and D. $P_s = \frac{E_h}{(1-\tau)T}$ is the transmit power of S, $\mathbf{x} \in \mathbb{C}^{N_t \times 1}$ indicates the normalized signal vector with $E\{\mathbf{x}\mathbf{x}^\dagger\} = \mathbf{I}_{N_t}$ and $\mathbf{n}_d \in \mathbb{C}^{N_r \times 1}$ is the AWGN at D with covariance matrix $N_0 \mathbf{I}_{N_r}$. The total transmit SNR at S is given by $\gamma_t = \frac{P_s}{N_0}$.

C. Ergodic Capacity

The system ergodic capacity can be expressed as [15]

$$C = (1-\tau)\mathrm{E}\left\{\log_2 \det\left(\mathbf{I}_{N_r} + \frac{\gamma_t}{d_2^l N_t}\mathbf{H}_2 \mathbf{H}_2^\dagger\right)\right\}. \tag{5}$$

Considering that the transmit power of S comes from the EH phase, we define the system SNR as $\rho = \frac{P_b}{N_0}$. Next, substituting γ_t into (5) we have

$$C(\rho) = (1-\tau)\mathrm{E}\left\{\log_2 \det\left(\mathbf{I}_n + \frac{\rho}{N_t} abA\mathbf{W}\right)\right\}, \tag{6}$$

where $a = \frac{\tau}{1-\tau}$, $b = \frac{\eta}{d_1^l d_2^l}$ and $A = \alpha^2 \lambda_1 + \beta^2 ||\mathbf{H}_w \mathbf{w}_p||_2^2$. $\mathbf{W} \in \mathbb{C}^{n \times n}$ is the Wishart matrix which equals $\mathbf{H}_2 \mathbf{H}_2^\dagger$ when $N_r \leq N_t$, or $\mathbf{H}_2^\dagger \mathbf{H}_2$ when $N_r > N_t$. Note that $n = \min\{N_t, N_r\}$ and $m = \max\{N_t, N_r\}$.

III. Preliminaries

In this section, we present some statistical results for certain RVs, which will be frequently invoked in the ensuing analysis.

Proposition 1: The probability density function (PDF) of the RV, $A = \alpha^2\lambda_1 + \beta^2||\mathbf{H}_w\mathbf{w}_p||_2^2$, is given by

$$f_A(x) = \frac{(x-\alpha^2\lambda_1)^{N_t-1}}{\Gamma(N_t)\beta^{2N_t}}\exp\left(-\frac{x-\alpha^2\lambda_1}{\beta^2}\right). \quad (7)$$

Where $\Gamma(\cdot)$ is the gamma function [16].

Proof: According to [17], when $\mathbf{h}_i$ is an arbitrary row vector of $\mathbf{H}_w$, then $|\mathbf{h}_i{}^\dagger\mathbf{w}_p|^2$ follows the exponential distribution with mean being 1. Therefore $||\mathbf{H}_w\mathbf{w}_p||_2^2$ is a gamma RV. After some algebraic manipulations, (7) can be obtained. ■

Lemma 1: The moments of the RV, A, are given by

$$\mathrm{E}\{A\} = \beta^2 N_t + \alpha^2\lambda_1, \quad (8)$$

$$\mathrm{E}\{A^2\} = (\beta^2 N_t + \alpha^2\lambda_1)^2 + \beta^4 N_t, \quad (9)$$

$$\mathrm{E}\{A^{i-n}\} = \frac{\exp\left(\frac{\alpha^2\lambda_1}{\beta^2}\right)}{\Gamma(N_t)}\sum_{r=0}^{N_t-1}\binom{N_t-1}{r}(-\alpha^2\lambda_1)^r\times \beta^{2(i-r-n)}\Gamma\left(N_t-r-n+i,\frac{\alpha^2\lambda_1}{\beta^2}\right), \quad (10)$$

where $\Gamma(a,b)$ is the incomplete gamma function [16], and $i\in\{0,1,2...n\}$.

Proof: Exploiting the PDF in (7) and using the integration formula [16, (3.351), (3.381)], (8)–(10) can be easily derived. ■

Lemma 2: The expected logarithm of A is given by

$$\mathrm{E}\{\ln A\} = \ln(\alpha^2\lambda_1) + \sum_{u=0}^{N_t-1}\frac{1}{(N_t-1-u)!}\times \left(\sum_{k=1}^{N_t-1-u}(k-1)!\left(\frac{-\alpha^2\lambda_1}{\beta^2}\right)^{N_t-1-u-k} - \left(\frac{-\alpha^2\lambda_1}{\beta^2}\right)^{N_t-1-u}\exp\left(\frac{\alpha^2\lambda_1}{\beta^2}\mathrm{E_i}\left(\frac{-\alpha^2\lambda_1}{\beta^2}\right)\right)\right). \quad (11)$$

Where $\mathrm{E_i}(\cdot)$ denotes the exponential integral function [16].

Proof: The result can be obtained with the help of the integration formula [16, (4.337), (3.381)]. ■

IV. Capacity Analysis

This section presents a detailed study of the system ergodic capacity. Due to the involvement of zonal polynomial, an exact analysis of the ergodic capacity appears extremely challenging. Therefore we mainly focus on the capacity bounds and its behavior in both high and low SNR regimes.

A. Capacity Upper Bound

The following Theorem provides an upper bound to the ergodic capacity.

Theorem 1: The ergodic capacity upper bound is expressed in closed-form as (12) on the top of the next page.

Proof: Omitted due to space limitation, please see the journal version [18] for details. ■

B. Capacity Lower Bound

Theorem 2: The ergodic capacity is lower bounded by

$$C_{lb}(\rho) = n(1-\tau)\log_2\left(1+\frac{\rho ab}{N_t}\exp\left(\sum_{i=1}^{m-n}\frac{1}{i} + \frac{m}{n}\sum_{i=m-n+1}^{m}\frac{1}{i} - 1 - \gamma + \sum_{u=0}^{N_t-1}\frac{1}{(N_t-1-u)!}\times \left(\sum_{k=1}^{N_t-1-u}(k-1)!\left(\frac{-\alpha\lambda_1}{\beta^2}\right)^{N_t-1-u-k} - \exp\left(\frac{\alpha^2\lambda_1}{\beta^2}\right)\times \mathrm{E_i}\left(\frac{-\alpha^2\lambda_1}{\beta^2}\right)\left(\frac{-\alpha^2\lambda_1}{\beta^2}\right)^{N_t-1-u}\right) + \ln(\alpha^2\lambda_1)\right)\right) \quad (13)$$

where γ is the Euler's constant.

Proof: Following similar steps as in [19], the ergodic capacity can be lower bounded by

$$C(\rho) \geq t\log_2\left(1+\exp\left(\mathrm{E}\left\{\ln\left(\frac{\rho abA}{N_t}(\det\mathbf{W})^{\frac{1}{n}}\right)\right\}\right)\right), \quad (14)$$

where $t = n(1-\tau)$ and the expectation in (14) can be further expressed as

$$\mathrm{E}\{\cdot\} = \ln\left(\frac{\rho ab}{N_t}\right) + \mathrm{E}\{\ln A\} + \frac{1}{n}\mathrm{E}\{\ln\det\mathbf{W}\}. \quad (15)$$

The last term in (15) can be computed according to [15, Appendix B]. Then, invoking *Lemma 2*, the desired result can be obtained after some algebraic manipulations. ■

C. High SNR regime

1) High SNR approximation: At high SNRs, it was proved that the ergodic capacity of a wireless MIMO communication system behave as [15]

$$\mathcal{C}(\rho) = S_\infty(\log_2\rho - L_\infty) + o(1), \quad (16)$$

where S_∞ and L_∞ are the key performance measures that dictate the capacity behavior at high SNRs. The capacity slope S_∞ in bit/s/Hz/3 dB, is given by

$$S_\infty = \lim_{\rho\to\infty}\frac{C(\rho)}{\log_2\rho}, \quad (17)$$

while the quantity L_∞ in (16), which denotes the power offset that anchors the capacity expansion, is given by

$$L_\infty = \lim_{\rho\to\infty}\left(\log_2\rho - \frac{C(\rho)}{S_\infty}\right) \quad (18)$$

in 3 dB units. Note that this performance measure was first introduced in [20].

Capitalizing on the capacity expression in (6), we obtain the following theorem.

Theorem 3: The key parameters that dominate the capacity

$$C_{ub}(\rho)=(1-\tau)\left(\log_2\left(\frac{\exp\left(\frac{\alpha^2\lambda_1}{\beta^2}\right)}{\beta^{2N_t}\Gamma(N_t)}\sum_{i=0}^{n}\sum_{r=0}^{N_t-1}\left(\frac{\rho ab}{N_t}\right)^{i-n}\binom{N_t-1}{r}(-\alpha^2\lambda_1)^r\beta^{2(N_t-r-n+i)}\times\right.\right.$$
$$\left.\Gamma\left(N_t-r-n+i,\frac{\alpha^2\lambda_1}{\beta^2}\right)\binom{n}{i}\frac{m!}{(m-i)!}\right)+n\log_2\left(\frac{\rho ab\alpha^2\lambda_1}{N_t}\right)+\frac{n}{\ln 2}\left(\sum_{u=0}^{N_t-1}\frac{1}{(N_t-1-u)!}\times\right.$$
$$\left.\left.\left(\sum_{k=1}^{N_t-1-u}(k-1)!\left(\frac{-\alpha^2\lambda_1}{\beta^2}\right)^{N_t-1-u-k}-\exp\left(\frac{\alpha^2\lambda_1}{\beta^2}\right)\mathrm{E_i}\left(\frac{-\alpha^2\lambda_1}{\beta^2}\right)\left(\frac{-\alpha^2\lambda_1}{\beta^2}\right)^{N_t-1-u}\right)\right)\right),\quad(12)$$

behavior in the high SNR regime is give by

$$L_\infty=\log_2\frac{N_t}{ab}-\frac{1}{\ln 2}\left(\sum_{i=1}^{m-n}\frac{1}{i}+\frac{m}{n}\sum_{i=m-n+1}^{m}\frac{1}{i}-\gamma-1\right)-\frac{1}{\ln 2}\left(\ln(\alpha^2\lambda_1)+\sum_{u=0}^{N_t-1}\frac{1}{(N_t-1-u)!}\times\left(\sum_{k=1}^{N_t-1-u}(k-1)!\left(\frac{-\alpha^2\lambda_1}{\beta^2}\right)^{N_t-1-u-k}-\exp\left(\frac{\alpha^2\lambda_1}{\beta^2}\right)\times\mathrm{E_i}\left(\frac{-\alpha^2\lambda_1}{\beta^2}\right)\left(\frac{-\alpha^2\lambda_1}{\beta^2}\right)^{N_t-1-u}\right)\right),\quad(19)$$

and

$$S_\infty=n(1-\tau).\quad(20)$$

Proof: The high SNR slope in (20) can be easily obtained by substituting (17) into (6). Next, substituting (20) and (6) into (18), and utilizing the independence between A and $\mathbf{W}$, we can write the power offset as

$$L_\infty=\log_2\frac{N_t}{ab}-\frac{1}{\ln 2}\mathrm{E}\left\{\ln A\right\}-\frac{1}{n\ln 2}\mathrm{E}\left\{\ln\det\mathbf{W}\right\}.\quad(21)$$

Invoking *Lemma 2*, we have the second term in (21), while the last term in (21) can be derived according to [15, Appendix B]. This completes the proof. ■

2) Optimal time split: Based on *Theorem 3* and (16), we can write the high SNR capacity approximation as a function with respect to the time split τ:

$$C_h(\tau)=n(1-\tau)\left(\log_2\frac{\tau}{1-\tau}+c_2\right),\quad(22)$$

where c_2 denotes a constant value, which is irrelevant to the time split τ, and is given by

$$c_2=\frac{1}{\ln 2}\left(\ln\frac{\rho b\alpha^2\lambda_1}{N_t}+\frac{1}{\ln 2}\left(\sum_{i=1}^{m-n}\frac{1}{i}+\frac{m}{n}\sum_{i=m-n+1}^{m}\frac{1}{i}-\gamma-1\right)+\sum_{u=0}^{N_t-1}\frac{1}{(N_t-1-u)!}\times\left(\sum_{k=1}^{N_t-1-u}(k-1)!\times\left(\frac{-\alpha^2\lambda_1}{\beta^2}\right)^{N_t-1-u-k}-\exp\left(\frac{\alpha^2\lambda_1}{\beta^2}\right)\mathrm{E_i}\left(\frac{-\alpha^2\lambda_1}{\beta^2}\right)\times\left(\frac{-\alpha^2\lambda_1}{\beta^2}\right)^{N_t-1-u}\right)\right).\quad(23)$$

Based on (22), we have the following optimization problem

$$\tau_h^*=\arg\max_{\tau}C_h(\tau)\quad \text{s.t.}\quad 0<\tau<1.\quad(24)$$

Note that since the above optimization maximizes the capacity approximation in the high SNR regime, the solution τ_h^* can be regarded as an approximation of the optimal time split that maximizes the exact system ergodic capacity at high SNRs. Therefore, such optimization problem is of great significance.

The optimization problem in (24) could be solved analytically. Thus we have the following result:

Corollary 1: The optimal time split that maximizes the system capacity in high SNR regime is given by

$$\tau_h^*=\frac{\exp\left(\mathrm{W}(\exp\left(c_2\ln 2-1\right))-c_2\ln 2+1\right)}{1+\exp\left(\mathrm{W}(\exp\left(c_2\ln 2-1\right))-c_2\ln 2+1\right)},\quad(25)$$

where $\mathrm{W}(\cdot)$ is the Lambert W function [21].

Proof: The derivation follows the same steps in [13, Lemma 4]. ■

D. Low SNR regime

In this section, we study the capacity performance of the considered WPC system in the low SNR regime. Similarly, we first characterize the low SNR approximation of the ergodic capacity in (6), and then investigate the optimal time split.

1) Low SNR approximation: In the low SNR regime, it is useful to investigate the capacity, in terms of the normalized transmit energy per information bit, E_b/N_0, rather than the system SNR, ρ. The key performance measures at low SNRs are thus $E_b/N_{0_{min}}$, i.e., the minimum energy per information bit required to convey any positive rate reliabily, and S_0, the capacity slope therein in bit/s/Hz/(3-dB)). Using these two parameters, the capacity of wireless powered multi-antenna communication system can be well approximated by [22]

$$C\left(\frac{E_b}{N_0}\right) \approx S_0 \log_2\left(\frac{\frac{E_b}{N_0}}{\frac{E_b}{N_0}_{min}}\right). \quad (26)$$

Considering the fact that the transmit power at S is originated from the RF signals emitted by PB, in this paper, we define $\frac{E_b}{N_0}$ to be the normalized energy per-information bit required from the PB. Therefore, we have

$$\frac{E_b}{N_0} = \frac{P_s}{N_0}\frac{1}{C(\rho)} = \frac{\rho}{C(\rho)}. \quad (27)$$

With the definition above, the key performance measures that dictate the capacity behavior in the low SNR regime can be obtained from $C(\rho)$ via [23]:

$$\frac{E_b}{N_0}_{min} = \frac{1}{\dot{C}(0)} \quad \text{and} \quad S_0 = -\frac{2\left[\dot{C}(0)\right]^2}{\ddot{C}(0)} \ln 2, \quad (28)$$

where $\dot{C}(\cdot)$ and $\ddot{C}(\cdot)$ denote the first and second order derivatives taken with respect to ρ, respectively.

Capitalizing on the ergodic capacity expression in (6), we have the following theorem.

Theorem 4: At low SNRs, the key performance measures for the system capacity are given by

$$S_0 = \frac{2N_tN_r(1-\tau)}{(N_t+N_r)\left(1+\frac{N_t}{(N_t+K\lambda_1)^2}\right)}, \quad (29)$$

and

$$\frac{E_b}{N_0}_{min} = \frac{d_1^l d_2^l \ln 2}{\tau\eta N_r(\beta^2 N_t + \alpha^2\lambda_1)}. \quad (30)$$

Proof: Omitted due to space limitation, please see the journal version [18] for details. ■

Remark 1: From (29) and (30), it is observed that S_0 is an increasing function with respect to N_r, and $E_b/N_{0_{min}}$ is a decreasing function with respect to N_r, indicating double benefits of increasing N_r.

Remark 2: Increasing the Rician factor K is also beneficial for both S_0 and $E_b/N_{0_{min}}$, i.e., it increase the slope and decrease the minimum energy required at PB.

From (29), it is rather intuitive that S_0 is an increasing function of K. As for the $E_b/N_{0_{min}}$, we observe from (30) that K affects $E_b/N_{0_{min}}$ only through the term $\beta^2 N_t + \alpha^2\lambda_1$. To establish the monotonicity of $E_b/N_{0_{min}}$ with respect to K, it is sufficient to determine the sign of the first derivative of $1/(\beta^2 N_t + \alpha^2\lambda_1)$ with respect to K. Since $\lambda_1 \geq N_t$, it can be shown that $E_b/N_{0_{min}}$ is a decreasing function of K.

2) Optimal time split: With (26), (29) and (30), the low SNR capacity approximation can be expressed as a function of τ:

$$C_l(\tau) = c_3(1-\tau)\log_2(c_4\tau). \quad (31)$$

Note that c_3 and c_4 are both irrelevant to τ, given the specific $\frac{E_b}{N_0}$, and are given by

$$c_3 = \frac{2N_tN_r}{(N_t+N_r)\left(1+\frac{N_t}{(N_t+K\lambda_1)^2}\right)}, \quad (32)$$

$$c_4 = \frac{E_b}{N_0}\cdot\frac{\eta N_r(\beta^2 N_t + \alpha^2\lambda_1)}{d_1^l d_2^l \ln 2}. \quad (33)$$

Thus, to characterize the optimal time split that maximizes the capacity approximation in low SNR regime, we have the following optimization problem

$$\begin{aligned} &\tau_l^* = \arg\max_{\tau} C_l(\tau) \\ &\text{s.t.} \quad 0 < \tau < 1. \end{aligned} \quad (34)$$

Note that solving the above optimization problem improves the capacity performance for WPC systems that operating in the low SNR regime.

Corollary 2: The optimal time split that maximizes the system capacity in low SNR regime is given by

$$\tau_l^* = \exp\left(\mathrm{W}(\exp(1+\ln c_4)) - \ln c_4 - 1\right). \quad (35)$$

Proof: Taking the first derivative of $C_l(\tau)$ with respect to τ and set $\frac{dC_l(\tau)}{d\tau} = 0$, we have

$$\ln(c_4\tau) = \frac{1}{\tau} - 1. \quad (36)$$

Inspired by [13], and after some algebraic manipulations, we can rewrite the last equation as

$$\begin{aligned} \ln(\tau\exp(1+\ln c_4))\exp(\ln(\tau\exp(1+\ln c_4))) \\ = \exp(1+\ln c_4), \end{aligned} \quad (37)$$

which is in the form of the standard definition of lambert W function. This completes the proof. ■

V. Numerical results and simulations

In this section, simulation results are provided to validate the analytical expressions presented in the previous sections, and illustrate the impact of key system parameters on the system performance. Unless otherwise specified, we adopt the following set of parameters: the distances d_1 and d_2 are set to be 7 and 15 meters, respectively. The path-loss exponent l is set to be 2.5, while the energy conversion efficiency η is set to be 0.4. For the channel $\mathbf{H}_1$, we choose the Rician factor $K = 3 + \sqrt{12}$ [13]. All the simulation results are obtained by averaging over 10^6 independent trials.

Fig. 2 depicts the the upper bound, lower bound, as well as the high SNR approximation of the system ergodic capacity under different antenna numbers at the destination D. As can be readily observed, the proposed upper and lower bounds are sufficiently tight. Moreover, as the number of N_r increases, the system achieves higher capacity, which indicates the benefits of deploying multiple antennas at D.

Fig. 3 investigates the impact of Rician factor K on the low SNR capacity. It can be readily observed that the rician factor K affects both the low SNR slope S_0 and $E_b/N_{0_{min}}$.

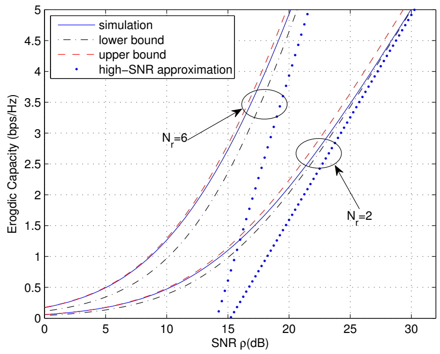

Fig. 2. Performance comparison with different antenna number at D. $N_t = 4, N_p = 2$.

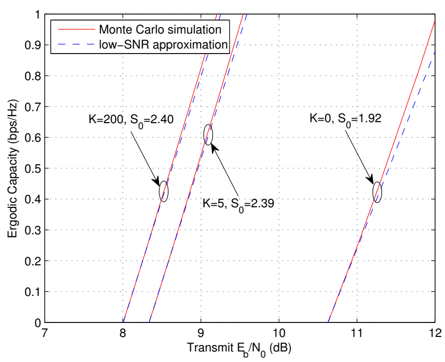

Fig. 3. Impact of Rician factor K on the ergodic capacity with partial CSI at PB.

Specifically, with larger K, the minimum energy required at PB become less while the capacity slope becomes larger, which indicates the double benefits of increasing K. However, the benefits become less substantial when K is large enough, i.e., $K > 5$, especially the gain on the low SNR slope.

VI. Conclusion

We have comprehensively investigated the capacity behavior of a MIMO WPC system. Specifically, the closed-form expressions for the lower bound and the upper bound of the ergodic capacity are derived. Besides, we investigated the capacity behavior under two SNR scenarios, namely, the high SNR regime and the low SNR regime, where we derived the closed-form approximations of the ergodic capacity for both cases. Furthermore, the optimal time split that maximizes the capacity performance under these two SNR levels are examined, and exact solutions were obtained. Numerical results and simulations have verified our theoretical analysis. The findings suggest that the capacity slope in high SNR regime only depends on the minimum antenna number of the IT link. However, adding more antennas at PB will improve the system capacity. It is also observed that the larger the Rician factor K, the better quality the approximation of the ergodic capacity.

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