Error Analysis of Wireless Transmission over Generalized Multipath/Shadowing Channels

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***Abstract*—The η-μ / inverse gamma and κ-μ / inverse gamma distributions were recently introduced as particularly flexible and tractable composite fading models that provide accurate characterization of multipath and shadowing effects, which are encountered simultaneously during wireless transmission in emerging communication scenarios such as off-body, cellular and vehicular-to-vehicular communications. The present contribution analyzes the symbol error rate performance of digital communications over these fading channels. To this end, we derive novel analytic expressions for the symbol error rate of multiple amplitude based modulated systems under these fading conditions, which are subsequently used in the analysis of the corresponding system performance. In this context, numerous insights are developed on the effect of different fading conditions on the corresponding error rate, which are expected to be useful in the design of timely and demanding wireless technologies such as wearable, cellular and vehicular communication systems.**

I. Introduction

Accurate characterization and modeling of fading channels has been a core topic in wireless communications as fading phenomena affect significantly the performance of conventional and emerging communication systems. This has led to the proposition of numerous fading models that provide adequate modeling accuracy to specific types of fading conditions [1]–[5] and the references therein. In this context, it has been extensively shown that generalized fading models are capable of providing accurate characterization of multipath fading [6]–[10]. Nevertheless, it is also known that multipath fading and shadowing phenomena practically occur simultaneously and can be modeled with the aid of composite fading distributions [11]–[19]. However, the existing composite fading models typically provide partially accurate modeling of fading phenomena, while they often have a complicated mathematical form, which renders them analytically intractable. Motivated by this the authors in [1]–[3] proposed two novel distributions, namely the κ-μ / inverse gamma and the η-μ / inverse gamma that constitute effective composite fading models. The high modeling capability of these models has been validated by accurate fitting to results from extensive measurement campaigns in the context of wearable, cellular and vehicular communications, which constitute important and timely topics of interest. In addition, a distinct characteristic of the proposed models is their relatively convenient algebraic representation, which renders them tractable both analytically and numerically.

Fading distributions have been extensively used in the analysis and evaluation of wireless communications since they typically allow the derivation of explicit expressions for critical performance measures of interest. However, this task becomes considerably more challenging, if not impossible, in the case of generalized and/or composite fading conditions [6], [20]. Based on this, the authors in [21]–[23] analyzed the capacity over generalized fading channels under different adaptation policies. This topic was also addressed in [24] for the case of $K_{\rm G}$ fading channels, in [13] and [25] for the case of $\mathcal{G}$ fading channels and in [26] and [27] for the case of η-μ / gamma and κ-μ shadowed fading channels, respectively. In the same context, the outage probability (OP) over different generalized interference-limited scenarios was investigated in [28], whereas an analytical framework for device-to-device communications in cellular networks was proposed in [29]. Finally, the outage capacity (OC) of orthogonal space-time block codes over multi-cluster scattering multi-antenna systems along with the coverage capacity 5G millimeter wave cellular systems were addressed in [30] and [31], respectively.

Motivated by the above, the present work analyzes the error performance of digital communications over $\kappa-\mu$ / inverse gamma and $\eta-\mu$ / inverse gamma fading channels. To this end, we derive explicit expressions for the average symbol error probability (SEP) of multiple amplitude ($M-$AM) modulation under these composite fading conditions. These expressions are given in exact closed-form representation for the case of $\kappa-\mu$ / inverse gamma fading and in exact infinite series

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representation for the case of $\eta-\mu$ / inverse gamma. Based on this, a simple closed-form upper bound is also provided for the truncation error of the derived infinite series which can readily determine the number of terms required for achieving specific levels of accuracy. These expressions are subsequently employed in the analysis of the considered scenarios, which quantifies the effects of different fading conditions on the corresponding system performance. This leads to the development of useful insights that are expected to be useful in the design of emerging wireless technologies such as wearable, cellular and vehicular communications. For example, it is shown that the resulting performance of the considered scenarios varies significantly from the performance achieved by the conventional Rayleigh distributed multipath fading and that accurate modeling of the incurred fading conditions can enable effective wireless transmission using binary modulations even at moderate signal-to-noise ratio (SNR) values.

II. Inverse Gamma Based Composite Distributions

A. The κ-μ / Inverse Gamma Fading Model

The κ-μ / inverse gamma model assumes that the mean power of both the dominant and scattered signal components is subject to shadowing, which is weighted by an inverse gamma random variable (RV). This model was shown to provide remarkable accuracy in line of sight (LOS) communication scenarios and its envelope probability density function (PDF) is expressed as [1], [3]

$$f_R(r) = \frac{2\mu^\mu (1+\kappa)^\mu m_s^{m_s} \Omega^{m_s} e^{-\mu\kappa} r^{2\mu-1}}{B(m_s,\mu)[\mu(1+\kappa)r^2 + m_s\Omega]^{m_s+\mu}} \times {}_1F_1\left(m_s+\mu;\mu;\frac{\mu^2\kappa(1+\kappa)r^2}{\mu(1+\kappa)r^2+m_s\Omega}\right) \tag{1}$$

where r denotes the envelope of the signal, κ is the ratio of the total power of the dominant components to the total power of the scattered waves, μ is related to the number of multipath clusters, m_s is the shadowing parameter and Ω is the the mean signal power. Furthermore, $B(\cdot,\cdot)$ and ${}_1F_1(\cdot;\cdot;\cdot)$ denote the Beta function and the Kummer hypergeometric function, respectively [32].

Based on (1), the SNR PDF of the κ-μ / inverse gamma fading model is given by

$$f_\gamma(\gamma) = \frac{\mu^\mu (1+\kappa)^\mu m_s^{m_s} \overline{\gamma}^{m_s} e^{-\mu\kappa} \gamma^{\mu-1}}{B(m_s,\mu)[\mu(1+\kappa)\gamma + m_s\overline{\gamma}]^{m_s+\mu}} \times {}_1F_1\left(m_s+\mu;\mu;\frac{\mu^2\kappa(1+\kappa)\gamma}{\mu(1+\kappa)\gamma+m_s\overline{\gamma}}\right) \tag{2}$$

where γ and $\overline{\gamma} = E[\gamma]$ represent the instantaneous SNR and the corresponding average SNR, respectively, with $E[\cdot]$ denoting statistical expectation.

It is evident that the algebraic representation of the PDF of the $\kappa-\mu$ / inverse gamma fading model is relatively convenient both analytically and numerically.

B. The η-μ / Inverse Gamma Fading Model

The η-μ / inverse gamma model is based on the η-μ distribution and assumes that the mean power of the scattered component is subject to shadowing, which is weighted by an inverse gamma RV. This model was shown to provide remarkable accuracy in non-line of sight (NLOS) communications and its envelope and SNR PDFs are given by [2], [3]

$$f_R(r) = \frac{2^{2\mu+1}\mu^{2\mu}h^\mu (m_s\Omega)^{m_s} r^{4\mu-1}}{B(m_s,2\mu)(2\mu h r^2 + m_s\Omega)^{m_s+2\mu}} \times {}_2F_1\left(\frac{m_s}{2}+\mu, \frac{m_s+1}{2}+\mu; \mu+\frac{1}{2}; \frac{(2\mu H r^2)^2}{(2\mu h r^2+m_s\Omega)^2}\right) \tag{3}$$

and

$$f_\gamma(\gamma) = \frac{2^{2\mu}\mu^{2\mu}h^\mu m_s^{m_s}\overline{\gamma}^{m_s}\gamma^{2\mu-1}}{B(m_s,2\mu)(2\mu h\gamma + m_s\overline{\gamma})^{m_s+\mu}} \times {}_2F_1\left(\frac{m_s}{2}+\mu, \frac{m_s+1}{2}+\mu; \mu+\frac{1}{2}; \frac{(2\mu H\gamma)^2}{(2\mu h\gamma+m_s\overline{\gamma})^2}\right) \tag{4}$$

respectively, where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gaussian hypergeometric function [32]. Also, η is defined according to the two formats of the $\eta-\mu$ distribution. In *Format* 1, $h = (2+\eta+\eta^{-1})/4$ and $H = \eta^{-1} - \eta/4$, with η denoting the scattered wave power ratio between the in-phase and quadrature components of each cluster of multipath. In *Format* 2, $h = 1/(1-\eta^2)$ and $H = \eta/(1-\eta^2)$ with η denoting the correlation coefficient between the in-phase and quadrature components.

In what follows, we capitalize on the above statistical results to derive useful analytic expressions for the average SEP of $M-$AM modulation under these composite fading conditions.

III. SEP for Multiple Amplitude Modulation

A. Average SEP over κ-μ / Inverse Gamma Fading Channels

This subsection is devoted to the derivation of an exact closed-form representation for the average SEP of $M-$AM constellations under κ-μ / inverse gamma fading conditions.

Theorem 1. *For $\kappa,\mu,m_s,\overline{\gamma}\in\mathbb{R}^+$ and $M=2^n$ with $n\in\mathbb{N}$, the analytic expression in* (5)*, top of the next page, is valid for the symbol error rate in multilevel amplitude modulation over κ-μ / inverse gamma fading channels, with M denoting the corresponding modulation order.*

Proof. It is recalled that the SEP of $M-$AM under additive white Gaussian noise (AWGN) channels is given by [6, Eq. (8.3)]. To this effect, the average SEP in the case of κ-μ / inverse gamma fading channels is obtained by averaging [6, Eq. (8.3)] over the corresponding SNR PDF in (2), namely

$$P_s(E) = \frac{2(M-1)\mu^\mu(1+\kappa)^\mu\overline{\gamma}^{m_s}}{Me^{\mu\kappa}m_s^{-m_s}B(m_s,\mu)} \times \int_0^\infty \frac{\gamma^{\mu-1}Q\left(\sqrt{\frac{6\gamma\log_2(M)}{(M-1)(M+1)}}\right)}{[\mu(1+\kappa)\gamma+m_s\overline{\gamma}]^{m_s+\mu}} \times {}_1F_1\left(m_s+\mu;\mu;\frac{\mu^2\kappa(1+\kappa)\gamma}{\mu(1+\kappa)\gamma+m_s\overline{\gamma}}\right)\mathrm{d}\gamma \tag{6}$$

$$P_s(E) = \frac{M-1}{M}\left\{1 - \frac{m_s^{m_s-1}\overline{\gamma}^{m_s}\Gamma\left(\frac{1}{2}-m_s\right)3^{m_s}\log_2^{m_s}(M)}{\mu^{m_s}(M^2-1)^{m_s}\sqrt{\pi}(1+\kappa)^{m_s}B(m_s,\mu)e^{\mu\kappa}}\right.$$
$$\left.\times\, {}_2F_2\left(m_s+\mu:-,m_s;-:\,\mu,m_s+\frac{1}{2};-,1+m_s;\,\mu\kappa,\frac{3\overline{\gamma}\log_2(M)m_s}{\mu(1+\kappa)(M^2-1)}\right)\right\}$$
$$-\frac{\Gamma\left(m_s-\frac{1}{2}\right)\Gamma\left(\mu+\frac{1}{2}\right)\sqrt{3m_s(M-1)\overline{\gamma}\log_2(M)}}{2^{-1}Me^{\mu\kappa}\Gamma(\mu)\Gamma(m_s)\sqrt{\pi\mu(1+\kappa)(M+1)}}\,{}_2F_2\left(\mu+\frac{1}{2}:-,\frac{1}{2};-:\,\mu,\frac{3}{2};-,\frac{3}{2}-m_s;\,\mu\kappa,\frac{3\overline{\gamma}\log_2(M)m_s}{\mu(1+\kappa)(M^2-1)}\right) \quad (5)$$

$$P_s(E) = \frac{2(M-1)(1+\kappa)^{\mu}\overline{\gamma}^{m_s}}{M\mu^{-\mu}m_s^{-m_s}e^{\mu\kappa}B(m_s,\mu)}\sum_{l=0}^{\infty}\frac{(m_s+\mu)_l}{(\mu)_l}\frac{(1+\kappa)^l}{l!\mu^{-2l}\kappa^{-l}}\int_0^{\infty}\frac{\gamma^{\mu+l-1}}{[\mu(1+\kappa)\gamma+m_s\overline{\gamma}]^{m_s+\mu+l}}Q\left(\sqrt{\frac{6\gamma\log_2(M)}{(M-1)(M+1)}}\right)\mathrm{d}\gamma \quad (7)$$

$$\begin{aligned}P_s(E) =&\frac{M-1}{M} - \left\{\frac{2\Gamma\left(m_s-\frac{1}{2}\right)\sqrt{M-1}\sqrt{3m_s\overline{\gamma}\log_2(M)}}{M\sqrt{\pi}e^{\mu\kappa}\Gamma(m_s)\sqrt{\mu}\sqrt{1+\kappa}\sqrt{M+1}}\right.\\
&\left.\times\sum_{l=0}^{\infty}\frac{\kappa^l\mu^l}{l!\Gamma(\mu+l)}\Gamma\left(\mu+l+\frac{1}{2}\right){}_2F_2\left(\frac{1}{2},\mu+l+\frac{1}{2};\frac{3}{2},\frac{3}{2}-m_s;\frac{3m_s\overline{\gamma}\log_2(M)}{\mu(1+\kappa)(M^2-1)}\right)\right\}\\
&-\frac{m_s^{m_s-1}\overline{\gamma}^{m_s}(M-1)3^{m_s}\log_2^{m_s}(M)\Gamma\left(\frac{1}{2}-m_s\right)}{M\mu^{m_s}(M^2-1)^{m_s}\sqrt{\pi}(1+\kappa)^{m_s}B(m_s,\mu)e^{\mu\kappa}}\\
&\times\sum_{l=0}^{\infty}\frac{(m_s+\mu)_l\mu^l\kappa^l}{l!(\mu)_l}{}_2F_2\left(m_s,\mu+m_s+l;m_s+\frac{1}{2},1+m_s;\frac{3\overline{\gamma}m_s\log_2(M)}{\mu(1+\kappa)(M^2-1)}\right).\end{aligned} \quad (8)$$

$$P_s(E) = \frac{M-1}{M} - \frac{2\Gamma\left(m_s-\frac{1}{2}\right)\Gamma\left(\mu+\frac{1}{2}\right)\sqrt{3m_s\overline{\gamma}\log_2(M)}\sqrt{M-1}}{M\sqrt{\pi}e^{\mu\kappa}\Gamma(\mu)\Gamma(m_s)\sqrt{\mu}\sqrt{1+\kappa}\sqrt{M+1}}\sum_{l=0}^{\infty}\sum_{i=0}^{\infty}\frac{\left(\mu+\frac{1}{2}\right)_{l+i}\left(\frac{1}{2}\right)_i}{(\mu)_l\left(\frac{3}{2}\right)_i\left(\frac{3}{2}-m_s\right)_i}\frac{\mu^l\kappa^l}{l!}\frac{\left(\frac{2\overline{\gamma}\log_2(M)m_s}{\mu(1+\kappa)(M^2-1)}\right)^i}{i!}$$
$$-\frac{m_s^{m_s}\overline{\gamma}^{m_s}3^{m_s}\log_2^{m_s}(M)(M-1)\Gamma\left(\frac{1}{2}-m_s\right)e^{-\mu\kappa}}{M\mu^{m_s}(M^2-1)^{m_s}\sqrt{\pi}(1+\kappa)^{m_s}\sqrt{\pi}(1+\kappa)^{m_s}B(m_s,\mu)}\sum_{l=0}^{\infty}\sum_{i=0}^{\infty}\frac{(\mu+m_s)_{l+i}(m_s)_i}{(\mu)_l\left(m_s+\frac{1}{2}\right)_i(1+m_s)_i}\frac{\mu^l\kappa^l}{l!}\frac{\left(\frac{2\overline{\gamma}\log_2(M)m_s}{\mu(1+\kappa)(M^2-1)}\right)^i}{i!}. \quad (9)$$

where $Q(\cdot)$ denotes the one dimensional Gaussian $Q-$function [32], [33]. Based on this and by expanding the involved hypergeometric function in terms of its single series representation, one obtains (7), where $n! \triangleq \Gamma(n+1)$ and $(x)_n \triangleq \Gamma(x+n)/\Gamma(x)$ are the factorial and Pochhammer symbols, respectively, with $\Gamma(\cdot)$ denoting the Euler gamma function [32]–[34]. Importantly, the integral representation in (7) can be expressed explicitly in terms of [34, Eq. (2.8.1.3)]. To this effect, by performing the necessary variable transformation, recalling that the Gaussian $Q-$function is related to the complementary error function by the identity $Q(x) \triangleq \mathrm{erfc}(x/\sqrt{2})/2$ and after some algebraic manipulations, equation (8) is deduced, at the top of the page. To this effect, it is noted that the above infinite series representation converges quickly and only few terms are required to achieve sufficient accuracy. Furthermore, it can be used to derive a closed-form representation in terms of the extended hypergeometric function of two variables, which constitutes a special case of the generic Kampe de Feriet function. To this end, by expanding the hypergeometric functions in (8) and using the Pochhammer symbol identities $\Gamma(x+n) = (x)_n\Gamma(x)$ yields (9), at the top of the page. Notably, the above representation can be expressed in closed-form with the aid of the extended hypergeometric function of two variables. Based on this and after some algebraic manipulations (5) is deduced, which completes the proof. □

Remark 1. *For the special case of $M = 2$, equation* (5) *reduces to* (10)*, at the top of the next page, which also holds for the case of binary phase shift keying (BPSK).*

B. Average SEP over η-μ / Inverse Gamma Fading Channels

In this subsection, we derive a simple exact infinite series representation for the average SEP of $M-$AM modulated

$$P_b(E) = \frac{1}{2} - \frac{\Gamma\left(m_s - \frac{1}{2}\right)\Gamma\left(\mu + \frac{1}{2}\right)\sqrt{m_s\overline{\gamma}}}{e^{\mu\kappa}\Gamma(\mu)\Gamma(m_s)\sqrt{\pi\mu(1+\kappa)}}\, {}_2F_2\left(\mu + \frac{1}{2} : -, \frac{1}{2}; - : \mu, \frac{3}{2}; -, \frac{3}{2} - m_s;\ \mu\kappa, \frac{\overline{\gamma}m_s}{\mu(1+\kappa)}\right) - \frac{m_s^{m_s-1}\overline{\gamma}^{m_s}\Gamma\left(\frac{1}{2} - m_s\right)e^{-\mu\kappa}}{2\mu^{m_s}\sqrt{\pi}(1+\kappa)^{m_s}B(m_s,\mu)}\, {}_2F_2\left(\mu + m_s : -, m_s; - : \mu, m_s + \frac{1}{2}; -, 1 + m_s;\ \mu\kappa, \frac{\overline{\gamma}}{\mu(1+\kappa)}\right) \tag{10}$$

systems over η-μ / inverse gamma fading channels.

Theorem 2. *For $\mu, m_s, \overline{\gamma} \in \mathbb{R}^+$, $n \in \mathbb{N}$, $M = 2^n$, $\eta \in \mathbb{R}^+$ in Format 1 and $-1 < \eta < 1$ in Format 2, the following analytic expression is valid for the symbol error rate in the case of multilevel amplitude modulation over η-μ / inverse gamma fading channels*

$$\begin{aligned} P_s(E) &= \frac{\sqrt{\pi}2^{1-2\mu}(M-1)\Gamma(2\mu)}{M\Gamma(\mu)\Gamma\left(\mu+\frac{1}{2}\right)h^{\mu}} \\ &\times {}_3F_2\left(\frac{m_s}{2}+\mu, \frac{m_s+1}{2}+\mu, \mu; \mu+\frac{m_s}{2}, \mu+\frac{m_s+1}{2}; \frac{H^2}{h^2}\right) \\ &- \frac{m_s^{m_s-1}\overline{\gamma}^{m_s}\Gamma(m_s+2\mu)(M-1)\Gamma\left(\frac{1}{2}-m_s\right)}{M\Gamma(m_s)\Gamma(\mu)\Gamma\left(\mu+\frac{1}{2}\right)2^{2\mu+m_s-1}\mu^{m_s}h^{\mu+m_s}} \\ &\quad \times \frac{3^{m_s}\log_2^{m_s}(M)}{(M^2-1)^{m_s}}\sum_{l=0}^{\infty}\frac{\left(\frac{m_s}{2}+\mu\right)_l\left(\frac{m_s+1}{2}+\mu\right)_l}{l!\left(\mu+\frac{1}{2}\right)_l h^{2l}H^{-2l}} \\ &\times {}_2F_2\left(m_s, m_s+2\mu+2l; m_s+\frac{1}{2}, m_s+1; \frac{2\overline{\gamma}m_s\log_2(M)}{2\mu h(M^2-1)}\right) \\ &- \frac{\sqrt{\overline{\gamma}m_s}\Gamma(m_s+2\mu)\sqrt{M-1}\Gamma\left(m_s-\frac{1}{2}\right)\sqrt{3\log_2(M)}}{M\Gamma(m_s)\Gamma(\mu)\Gamma\left(\mu+\frac{1}{2}\right)\sqrt{M+1}2^{2\mu-\frac{3}{2}}\sqrt{\mu}h^{\mu+\frac{1}{2}}} \\ &\quad \times \sum_{l=0}^{\infty}\frac{\left(\frac{m_s}{2}+\mu\right)_l\left(\frac{m_s+1}{2}+\mu\right)_l\Gamma\left(2\mu+2l+\frac{1}{2}\right)H^{2l}}{l!\left(\mu+\frac{1}{2}\right)_l h^{2l}\Gamma(2\mu+2l+m_s)} \\ &\quad \times {}_2F_2\left(\frac{1}{2}, 2\mu+2l+\frac{1}{2}; \frac{3}{2}, \frac{3}{2}-m_s; \frac{2m_s\overline{\gamma}\log_2(M)}{2\mu h(M^2-1)}\right). \end{aligned} \tag{11}$$

Proof. By substituting (4) into [6, Eq. (8.3)], it follows that

$$\begin{aligned} P_s(E) &= \frac{2^{2\mu+1}\mu^{2\mu}h^{\mu}m_s^{m_s}\overline{\gamma}^{m_s}(M-1)}{MB(m_s,2\mu)} \\ &\times \int_0^{\infty}\frac{\gamma^{2\mu-1}}{(2\mu h\gamma+m_s\overline{\gamma})^{m_s+2\mu}}Q\left(\sqrt{\frac{6\gamma\log_2(M)}{(M-1)(M+1)}}\right) \\ &\times {}_2F_1\left(\frac{m_s}{2}+\mu, \frac{m_s+1}{2}+\mu; \mu+\frac{1}{2}; \frac{(2\mu H\gamma)^2}{(2\mu h\gamma+m_s\overline{\gamma})^2}\right)\mathrm{d}\gamma \end{aligned} \tag{12}$$

which upon expansion of the involved hypergeometric function in (12) yields

$$\begin{aligned} P_s(E) &= \frac{2^{2\mu+1}\mu^{2\mu}h^{\mu}m_s^{m_s}\overline{\gamma}^{m_s}(M-1)}{MB(m_s,2\mu)} \\ &\times \sum_{l=0}^{\infty}\frac{\left(\frac{m_s}{2}+\mu\right)_l\left(\frac{m_s+1}{2}+\mu\right)_l(2\mu H)^2}{l!\left(\mu+\frac{1}{2}\right)_l} \\ &\times \int_0^{\infty}\frac{\gamma^{2\mu+1}}{(2\mu h\gamma+m_s\overline{\gamma})^{m_s+2\mu+2}}Q\left(\sqrt{\frac{6\gamma\log_2(M)}{M^2-1}}\right)\mathrm{d}\gamma. \end{aligned} \tag{13}$$

Notably, the above integral can be expressed in closed-form in terms of the generalized hypergeometric function with the aid of [34, Eq. (2.8.2.4)]. To this end, by performing the necessary variable transformation and after some algebraic manipulations yields (11), which completes the proof. □

The derived infinite series representation in (11) has a rather simple algebraic representation. In addition, it is convergent and only few terms are required to achieve sufficient accuracy in the context of error rate analysis. However, the determination of the exact number of terms required for strict accuracy requirements is practically essential. Based on this, we derive a simple closed-form upper bound for the truncation error of (11), which can be computed straightforwardly with the aid of popular software packages such as MATLAB, MAPLE and MATHEMATICA.

Proposition 1. *For $\mu, m_s, \overline{\gamma} \in \mathbb{R}^+$, $n \in \mathbb{N}$, $M = 2^n$, $\eta \in \mathbb{R}^+$ in Format 1 and $-1 < \eta < 1$ in Format 2, the following closed-form upper bound is valid for the truncation error of the infinite series representation in (11)*

$$\begin{aligned} \mathcal{T} &\leq \frac{(1-M)^{1-m_s}\Gamma\left(\frac{1}{2}-m_s\right)}{M\Gamma(m_s)\Gamma(\mu)\Gamma\left(\mu+\frac{1}{2}\right)2^{2\mu+m_s}\mu^{m_s}h^{\mu+m_s}(M+1)^{m_s}} \\ &\times {}_2F_2\left(m_s, 2p+2\mu+m_s; m_s+\frac{1}{2}, 1+m_s; \frac{3\overline{\gamma}\log_2(M)m_s}{2\mu h(M^2-1)}\right) \\ &\quad \times {}_2F_1\left(\frac{m_s}{2}+\mu, \frac{m_s+1}{2}+\mu; \mu+\frac{1}{2}; \frac{H^2}{h^2}\right) \end{aligned} \tag{14}$$

where p denotes the corresponding number of terms that truncate the series.

Proof. The truncation error of (11) when truncating the two infinite series after p terms is given by

$$\begin{aligned} \mathcal{T} &= \frac{\sqrt{\pi}2^{1-2\mu}(M-1)\Gamma(2\mu)}{M\Gamma(\mu)\Gamma\left(\mu+\frac{1}{2}\right)h^{\mu}} \\ &\times {}_3F_2\left(\frac{m_s}{2}+\mu, \frac{m_s+1}{2}+\mu, \mu; \mu+\frac{m_s}{2}, \mu+\frac{m_s+1}{2}; \frac{H^2}{h^2}\right) \\ &- \frac{m_s^{m_s-1}\overline{\gamma}^{m_s}\Gamma(m_s+2\mu)(M-1)\Gamma\left(\frac{1}{2}-m_s\right)3^{m_s}\log_2^{m_s}(M)}{M\Gamma(m_s)\Gamma(\mu)\Gamma\left(\mu+\frac{1}{2}\right)2^{2\mu+m_s-1}\mu^{m_s}h^{\mu+m_s}(M^2-1)^{m_s}} \\ &\quad \times \sum_{l=p}^{\infty}\frac{\left(\frac{m_s}{2}+\mu\right)_l\left(\frac{m_s+1}{2}+\mu\right)_l}{l!\left(\mu+\frac{1}{2}\right)_l h^{2l}H^{-2l}} \\ &\times {}_2F_2\left(m_s, m_s+2\mu+2l; m_s+\frac{1}{2}, m_s+1; \frac{2\overline{\gamma}m_s\log_2(M)}{2\mu h(M^2-1)}\right) \\ &- \frac{\sqrt{\overline{\gamma}m_s}\Gamma(m_s+2\mu)\sqrt{M-1}\Gamma\left(m_s-\frac{1}{2}\right)\sqrt{3\log_2(M)}}{M\Gamma(m_s)\Gamma(\mu)\Gamma\left(\mu+\frac{1}{2}\right)\sqrt{M+1}2^{2\mu-\frac{3}{2}}\sqrt{\mu}h^{\mu+\frac{1}{2}}} \end{aligned}$$

$$\times \sum_{l=p}^{\infty} \frac{\left(\frac{m_s}{2}+\mu\right)_l \left(\frac{m_s+1}{2}+\mu\right)_l \Gamma\left(2\mu+2l+\frac{1}{2}\right) H^{2l}}{l! \left(\mu+\frac{1}{2}\right)_l h^{2l} \Gamma(2\mu+2l+m_s)}$$
$$\times {}_2F_2\left(\frac{1}{2}, 2\mu+2l+\frac{1}{2}; \frac{3}{2}, \frac{3}{2}-m_s; \frac{2m_s\overline{\gamma}\log_2(M)}{2\mu h(M^2-1)}\right). \quad (15)$$

Importantly, the above representation can be upper bounded by the following inequality

$$\mathcal{T} < \frac{\sqrt{\pi}2^{1-2\mu}(M-1)\Gamma(2\mu)}{M\Gamma(\mu)\Gamma\left(\mu+\frac{1}{2}\right)h^{\mu}}$$
$$\times {}_3F_2\left(\frac{m_s}{2}+\mu, \frac{m_s+1}{2}+\mu, \mu; \mu+\frac{m_s}{2}, \mu+\frac{m_s+1}{2}; \frac{H^2}{h^2}\right)$$
$$- \frac{m_s^{m_s-1}\overline{\gamma}^{m_s}\Gamma(m_s+2\mu)(M-1)\Gamma\left(\frac{1}{2}-m_s\right)3^{m_s}\log_2^{m_s}(M)}{M\Gamma(m_s)\Gamma(\mu)\Gamma\left(\mu+\frac{1}{2}\right)2^{2\mu+m_s-1}\mu^{m_s}h^{\mu+m_s}(M^2-1)^{m_s}}$$
$$\times {}_2F_2\left(m_s, m_s+2\mu+2p; m_s+\frac{1}{2}, m_s+1; \frac{2\overline{\gamma}m_s\log_2(M)}{2\mu h(M^2-1)}\right)$$
$$\times \sum_{l=0}^{\infty} \frac{\left(\frac{m_s}{2}+\mu\right)_l \left(\frac{m_s+1}{2}+\mu\right)_l}{l! \left(\mu+\frac{1}{2}\right)_l h^{2l} H^{-2l}}$$
$$- \frac{\sqrt{\overline{\gamma}m_s}\Gamma(m_s+2\mu)\sqrt{M-1}\Gamma\left(m_s-\frac{1}{2}\right)\sqrt{3\log_2(M)}}{M\Gamma(m_s)\Gamma(\mu)\Gamma\left(\mu+\frac{1}{2}\right)\sqrt{M+1}2^{2\mu-\frac{3}{2}}\sqrt{\mu}h^{\mu+\frac{1}{2}}}$$
$$\times \sum_{l=0}^{\infty} \frac{\left(\frac{m_s}{2}+\mu\right)_l \left(\frac{m_s+1}{2}+\mu\right)_l \Gamma\left(2\mu+2l+\frac{1}{2}\right)}{l! \left(\mu+\frac{1}{2}\right)_l h^{2l}\Gamma(2\mu+2l+m_s)H^{-2l}}. \quad (16)$$

It is evident that (16) can be expressed in closed-form in terms of the Gaussian hypergeometric function. Based on this and after some algebraic manipulations and simplifications, equation (14) is deduced, which completes the proof. □

IV. Numerical Results

This section utilizes the derived results in the previous sections in the quantification of the effects of composite multipath/shadowing conditions on $M-$AM modulated systems. It is noted that all scenarios account for realistic fading conditions and, without loss of generality, we consider only *Format* 1 in the cases of $\eta-\mu$ fading channels. However, since the derived analytic expressions are generic, numerical results for the corresponding *Format* 2 scenario can be deduced straightforwardly.

Fig. 1 demonstrates the average SEP versus the average SNR for the case of $M-$AM under $\kappa-\mu$ / inverse gamma fading conditions, for different values of fading parameters and modulation order. It is evident that the modulation order is, as expected, the primary factor that determines the corresponding error performance. Nevertheless, it is also observed that the value of both μ and m_s have detrimental effects on the system performance as the corresponding differences exceed an order of magnitude, particularly in the high SNR regime. For example, a variation of about 5dB is noticed in both $M=2$ and $M=16$ even at slight variations of the severity of multipath fading and shadowing conditions, which turns out to be critical in demanding emerging applications of increased quality of service requirements.

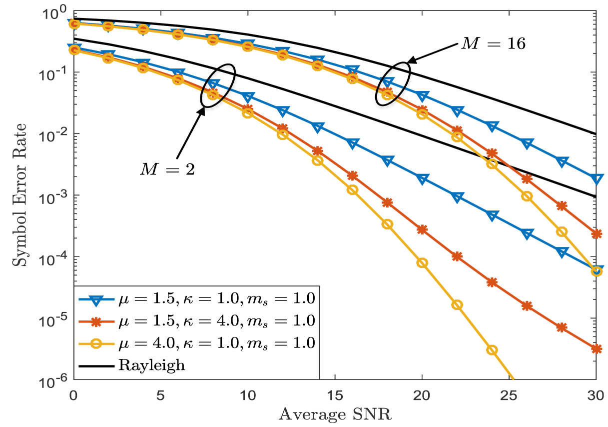

Fig. 1: Average SEP vs $\overline{\gamma}$ for $M-$AM modulation under κ-μ / inverse gamma fading channels with different M, μ, κ, η and m_s values.

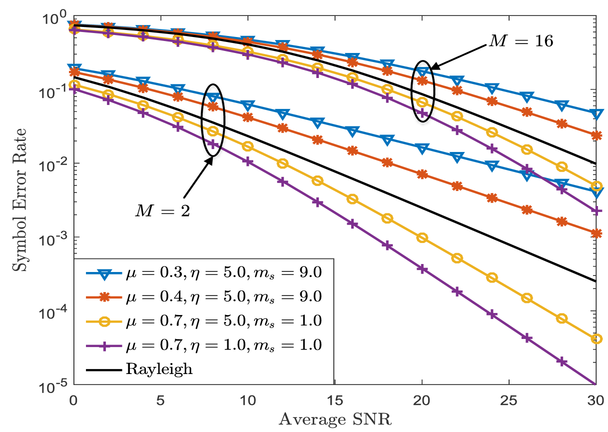

Fig. 2: Average SEP vs $\overline{\gamma}$ for $M-$AM modulation under η-μ / inverse gamma fading channels with different M, μ, κ, η and m_s values.

A similar trend is also observed in the case of $\eta-\mu$ / inverse gamma fading as illustrated in Fig. 2. Is is observed that the corresponding pre-Rayleigh and post-Rayleigh effects are considerable compared to the conventional case of Rayleigh fading conditions. For example, it is evident that the differences between the effect of the fading conditions characterized by the proposed composite models and Rayleigh fading is even three orders of magnitude is certain cases. This verifies the need for highly accurate composite fading models in order to avoid unrealistic evaluation of conventional and emerging communication scenarios. Regarding the considered scenario, it is finally noted that the value of the modulation order is, as expected, the most critical parameter in the achieved performance as it is evident that the 16$-$AM option is practically problematic in the case of severe fading conditions.

Yet, it is also shown that accurate channel modeling can allow adequate transmission at moderate and small SNR values, when using binary modulation at non-severe fading conditions.

V. Conclusion

This work was devoted to the average symbol error probability analysis of multiple amplitude modulation systems under κ-μ / inverse gamma and η-μ / inverse gamma fading conditions. Novel exact analytic expressions were derived which were subsequently employed in quantifying the effects of severity of multipath fading and shadowing fading conditions on the overall system performance. It was shown that the effect of different types of composite fading have a considerable effect across all SNR regimes and that acceptable performance can be achieved at moderate and low SNR values in non-severe fading conditions using binary modulation. This verifies that accurate channel characterization is highly essential in the realistic design and deployment of emerging wireless systems such as wearable, cellular and vehicular communications.

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