

# Energy Efficient Power and Subcarrier Allocation for Downlink Non-Orthogonal Multiple Access Systems

Alemu Jorgi Muhammed\*, Zheng Ma\*, Li Li\*, Panagiotis D. Diamantoulakis<sup>†</sup>, and George K. Karagiannidis<sup>†</sup>

\*School of Information Science and Technology, Southwest Jiaotong University, Chengdu, China

<sup>†</sup>Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, Thessaloniki, Greece  
e-mails: alemu\_jorgi@my.swjtu.edu.cn, {zma, ll5e08}@swjtu.edu.cn padiaman@ieee.org, geokarag@auth.gr

**Abstract**—Non-orthogonal multiple access (NOMA) has attracted both academic and industrial interest since it has been considered as one of the promising 5G technologies in order to increase connectivity and spectral efficiency. In this paper, we focus on a downlink NOMA network, where a single base station serves a set of users through multiple subchannels. The goal is to jointly optimize energy efficiency (EE) and fairness among users with respect to the subcarrier and power allocation parameters. To achieve this with acceptable complexity, we propose a novel greedy subcarrier assignment scheme. Due to the fractional form of the EE expression and the existence of interference, the power allocation problem is non-convex. To this end, we first transform this into an equivalent subtractive form, which is then solved by using fractional programming with sequential optimization of the power allocation vectors. Simulation results reveal the effectiveness of the proposed scheme in terms of EE and fairness among users compared to baseline schemes. Finally, the proposed algorithms are of fast convergence, low complexity, and insensitive to the initial values.

## I. INTRODUCTION

With the explosive growth of mobile users, the internet-of-things, and cloud-based applications, wireless technology will require a paradigm shift to support large-scale connectivity and different data and latency requirements. To this direction, many potential techniques have been introduced in recent years. Among these, non-orthogonal multiple access (NOMA) has aroused great interest in both academia and industry [2], due to its superiority in gaining spectral efficiency, mass connectivity, and low latency compared to orthogonal multiple access (OMA). Even though intra-cell interference is increased, NOMA can simultaneously serve multiple users over the power domain, by using the same spectrum [1]. NOMA scheme uses superposition coding (SC) to broadcast multiple users' message signals in power domain by considering differences in channel conditions. At the receiving end, each user device applies a successive interference cancellation (SIC) to extract its own signal from the aggregate received signal.

Driven by the rapid growth of data traffic and wireless terminals, energy efficiency (EE) has drawn significant attention since the greenhouse-gas emissions produced by information and communication technology (ICT) accounts more than 2% and that percentage is expected to double by 2020, which is becoming one of the major social and economic concerns worldwide [13]. Thus, resource allocation scheme which aim to improve the EE and decreases the carbon emission caused

by energy consumption have become an important motivations for the research on energy efficient communications in NOMA networks. Currently, very few works has been investigated for NOMA to improve the EE of the wireless system. In [4], an energy-efficient power allocation strategy in millimeter wave massive MIMO with NOMA has been investigated. In [5], an energy efficient transmission has been studied for SISO-NOMA systems. Moreover, the joint power allocation and channel assignment for maximizing the EE in NOMA systems was considered in [6]. The same authors in [7] further extends the work on a joint subchannel and power optimization for the downlink NOMA heterogeneous network to improve the EE.

Different from the aforementioned works [4]–[7], which mainly focus on maximizing the network EE, in this paper, we study a fairness based EE problem to maximize the minimum user EE which is defined as the ratio of the user rate to its power consumption [3], [8]. To the best of our knowledge, the resource allocation problem that simultaneously optimizes energy efficiency and fairness among users has not been considered before in the open literature for NOMA systems, which is of vital importance for the fifth generation (5G) of wireless networks. Motivated by this observations, we formulate the resource allocation problem for energy efficient communication in downlink NOMA systems as an optimization problem. In this setting, we are interested in maximizing the minimum individual EE under the total power and minimum rate constraints by optimally allocating the subchannels and transmit power. However, the formulated optimization problem is a non-convex and thus difficult to solve directly. To tackle this, the original problem is addressed by a two-stage algorithm that involves approximation and relaxations. Thus, we first propose a greedy subcarrier allocation algorithm by assuming equal power allocation under each subchannel in NOMA transmission. Given the subchannel allocation scheme, we solve the power allocation to further improve the energy efficiency of NOMA system. However, the power allocation subproblem is in fractional structure, and thus it is nonconvex. By exploiting the property of fractional programming, we first transformed into an equivalent subtractive form and then a low complexity iterative power allocation scheme is proposed. As a result, a suboptimal power and subcarrier allocation policies are obtained for maximizing the EE in each iteration. Numerical simulations show the performance compared to

existing NOMA and OFDMA schemes.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

### A. System Model

A single-cell based downlink NOMA system scenario is considered, where a BS sends information to  $K$  users. All transceivers are equipped with a single-antenna. The BS sends data to all users simultaneously, subject to the transmit power constraint, denoted by  $P_t$ . The total bandwidth  $B$  is equally divided into  $N$  subcarriers, each with a bandwidth of  $W = \frac{B}{N}$ . In this paper, the terms subchannel and subcarrier are used interchangeably. The channel between user  $k$  and the BS on subcarrier  $n$  is denoted by  $h_{k,n}$ , and we assume that the BS has perfect knowledge of CSI. Based on the CSI of each channel, the BS assigns a subset of subchannels to the users and allocates different levels of power to them. Let  $K_n \in \{K_1, K_2, \dots, K_N\}$  be the number of users using subchannel  $n \in \{1, 2, 3, \dots, N\}$  and  $UE_{k,n}$  denotes user  $k$  on each subchannel  $n$  for  $k \in \{1, 2, 3, \dots, K_n\}$ . Then, the corresponding transmitted signal on each subchannel  $n$  is represented by

$$x_n = \sum_{k=1}^{K_n} \sqrt{p_{k,n}} s_k, \quad (1)$$

where  $s_k$  is the symbol of  $UE_{k,n}$  and  $p_{k,n}$  is the power allocated to  $UE_{k,n}$ . The received signal at  $UE_{k,n}$  is

$$y_{k,n} = \sqrt{p_{k,n}} h_{k,n} s_k + \sum_{i=1, i \neq k}^{K_n} \sqrt{p_{i,n}} h_{k,n} s_i + z_{k,n}, \quad (2)$$

where  $h_{k,n} = g_{k,n} d_n^{-\gamma}$  is the channel coefficient from the BS to  $UE_{k,n}$  and  $g_{k,n}$  is the small scale fading parameter that follows a complex Gaussian distribution, i.e.,  $g_{k,n} \sim CN(0, 1)$ ,  $d_n$  is the distance between the BS and  $UE_{k,n}$ ,  $\gamma$  is the path loss exponent, and  $z_{k,n} \sim CN(0, \alpha_n^2)$  is the additive white Gaussian noise (AWGN).

Using the main principle of power-domain NOMA, multi-user signal separation is conducted at the receiver side using the SIC approach [1]. By exploiting SIC and assuming perfect CSI, the users with better channel conditions can successfully decode the messages of the weaker users. Let  $\Upsilon_{k,n} = \frac{|h_{k,n}|^2}{\alpha_n^2}$  denotes the channel response normalized by noise (CRNN) and consider that  $K_n$  users are allocated on the  $n$ -th subchannel. Without loss of generality, the users at the  $n$ -th subchannel are sorted in a descending order as  $\Upsilon_{1,n} \geq \dots \geq \Upsilon_{k,n} \dots \geq \Upsilon_{K_n,n}$ . Thus,  $UE_{1,n}$  is the user which has the best channel conditions on subcarrier  $n$ , while  $UE_{K_n,n}$  is the user which has the worst channel condition on the same subcarrier on channel  $n$ . According to the NOMA protocol [2], the BS will allocate more power to the weaker users to provide fairness and facilitate the SIC process, which results in  $p_{1,n} \leq \dots \leq p_{k,n} \leq \dots \leq p_{K_n,n}$ . Note that the first user (the user with the best channel conditions) will cancel interference from all other users. So, the last user ( $K_n$ ) will see interference from all other users when decoding its own message. In general,  $UE_{k,n}$  is able to decode signals of  $UE_{i,n}$  for  $i > k$  and remove them from its own signals, but treats the signals from  $UE_{i,n}$  for  $i < k$  as interference. Thus, the interference ( $I_{k,n}$ ) experienced by each user on each subcarrier with this decoding order will be [6]

$$I_{k,n} = \sum_{i=1, i \neq k}^{k-1} p_{i,n} \Upsilon_{k,n}. \quad (3)$$

Hence, the received signal to the interference plus noise ratio (SINR) of the  $k$ -th user on the  $h_{k,n}$  is written as

$$SINR_{k,n} = \frac{P_{k,n} |h_{k,n}|^2}{\alpha_n^2 + I_{k,n}} = \frac{P_{k,n} \Upsilon_{k,n}}{1 + \sum_{i=1, i \neq k}^{k-1} p_{i,n} \Upsilon_{k,n}}, \quad (4)$$

where  $\alpha_n^2 = E[|z_{k,n}|^2]$  is the noise power on  $h_{k,n}$ . Thus, the data rate of  $k$ -th user is [6]

$$R_{k,n} = W \log_2(1 + SINR_{k,n}), \quad (5)$$

where  $P_{k,n}$  is the power allocated to the  $k$ -th user over the  $n$ -th subchannel satisfying

$$\sum_{k \in K} P_{k,n} = P_n, \quad \text{and} \quad \sum_{n=1}^N P_n \leq P_t. \quad (6)$$

Accordingly, as there are  $K_n$  users on subchannel  $n$  and  $N$  subchannels in the system, the data rate on subchannel  $n$  and the total sum rate is given by

$$R_n(P_n) = \sum_{n=1}^{K_n} R_{k,n}(P_{k,n}) \quad \text{and} \quad (7)$$

$$R = \sum_{n=1}^N R_n(P_n), \quad (8)$$

respectively. Moreover, the power consumed by user  $k$  can be expressed as

$$P_k = \zeta P_{k,n} + P_{k,n}^C, \quad (9)$$

where  $\zeta$  represents the inverse of the power amplifier efficiency, while  $P_{k,n}^C$  is the circuit power consumption. The EE for individual user provides the ratio between the data rate and consumed power for each user, while it becomes particularly important when a balance between these two metrics is desired for all users. Thus, given (5) and (9), the EE for each user  $k$  is defined as [10]

$$E_\eta(P_{k,n}) = \frac{R_{k,n}(P_{k,n})}{P_k(P_{k,n})}. \quad (10)$$

### B. Problem Formulation

In this section, we introduce an optimization problem for downlink NOMA systems. Thus, given the expression for the individual EE for each user, the optimization problem can be formulated as

$$\begin{aligned} \max_{Q, P} \min_{k=1, \dots, K} E_\eta(Q, P) &= \frac{R_{k,n}(Q, P)}{P_k(Q, P)} \\ \text{s.t.} \quad C_1: R_k &\geq R_{\min}, \quad \forall k \in K, \\ C_2: \sum_{n=1}^N P_n &\leq P_t, \\ C_3: \sum_{k=1}^{K_n} q_{k,n} P_{k,n} &\leq P_n, \quad \forall k \in K, \\ C_4: \sum_{k=1}^K q_{k,n} &\leq K_n, \quad \forall n \in N, \\ C_5: P_{k,n} &\geq 0, \quad \forall k, n, \\ C_6: q_{k,n} &\in \{0, 1\}, \quad \forall k, n, \end{aligned} \quad (11)$$

where the set  $Q$  with elements  $q_{k,n}$  and  $P$  with elements  $P_{k,n}$  are the subcarrier allocation policy and the power allocation strategy, respectively. Constraint  $C_1$  guarantees that all users meet their minimum QoS requirements,  $R_{\min}$  for each user  $k$ .  $C_2$  and  $C_3$  are constraints for the transmission power of the BS and power budget for each subchannel  $n$ , respectively.  $C_4$  ensures that one subcarrier can be with at most  $K_n$  users.  $C_5$  retains the power allocation variables to non-negative values.  $C_6$  is a subcarrier allocation variable indicator, which becomes 1 if the user  $k$  is multiplexed on subcarrier  $n$ , and zero otherwise. Thus, problem (11) is a non-convex optimization problem due to the binary constraint in  $C_6$  and the existence of the interference term and fractional expression in the objective function [10]. To make the problem tractable, we can

relax  $q_{k,n}$  from discrete value of 0 or 1 to continuous real numbers that range in  $0 \leq q_{k,n} \leq 1, \forall k, n$  [12]. This considered as a time sharing factor for subchannel  $n$  that user  $k$  is assigned during one block of transmission. Now, the optimization problem in (11) can be reformulated as.

$$\begin{aligned} \max_{Q, P} \min_{k=1, \dots, K} \quad & E_\eta(Q, P) = \frac{R_{k,n}(Q, P)}{P_k(Q, P)} \\ \text{s.t.} \quad & C_1, C_2, C_3, C_4, C_5 \\ & C_6: q_{k,n} \in [0, 1], \quad \forall k, n. \end{aligned} \quad (12)$$

Due to the interference term and fractional structure, (12) is still nonconvex, and thus it is challenging to find an optimal solution. Thus, it is beneficial to transform the problem into a sequence of linear programs (LPs) and develop a customized low-complexity algorithm for its solution. To this end, we next propose a two-stage algorithm, according to which the subchannel and power allocation processes are sequentially performed.

### III. ENERGY EFFICIENT SUBCARRIER ASSIGNMENT SCHEME

In this section, we propose a low complexity greedy based subchannel assignment algorithm by assuming equal power allocation across the subchannels and fractional transmitted power allocation (FTPA) among multiplexed users on each subcarrier. In the FTPA scheme, the transmit power of  $UE_k$  on subchannel  $n$  is assigned based on the channel gains of all the multiplexed users on subchannel  $n$ , as described in [6], is given by

$$P_{k,n} = P_n \frac{(H_{k,n})^{-\sigma}}{\sum_{i=1}^{K_n} (H_{i,n})^{-\sigma}}, \quad (13)$$

where  $\sigma$  ( $0 \leq \sigma \leq 1$ ) is a decay factor. From (13), it can be seen that as  $\sigma$  increases more power is allocated to users with lower channel gain. Inspired by [11], in this paper, we introduce the worst case user first subcarrier allocation (WCUFSA) algorithm. The WCUFSA algorithm is a greedy-based algorithm that allows the users with the worst channel quality to select their desired subcarrier first. To this end, users are arranged in ascending order with respect to the worst channel qualities of all users. Then, the algorithm first finds the worst channel qualities of the unassigned users and then assigns the best subcarrier to the user with the poorest channel value. Thus, the algorithm carried out iteratively till all subcarriers are assigned to all users (i.e., two users per subcarrier bases.) As a summary, the WCUFSA subcarrier allocation scheme is presented in Algorithm 1.

### IV. ENERGY EFFICIENT POWER ALLOCATION FOR NOMA SYSTEM

We assume that the users are assigned to different subchannels by using the subcarrier assignment algorithm, proposed in the Algorithm 1. In this section, the main objective is to maximize the EE by optimizing the transmit power of each user. Thus, the resulting optimization problem can be expressed as

$$\begin{aligned} \max_P \min_{k=1, \dots, K} \quad & E_\eta(Q, P) = \frac{R_{k,n}(Q, P)}{P_k(Q, P)} \\ \text{s.t.} \quad & C_1: R_k \geq R_{\min}, \quad \forall k \in K, \\ & C_2: \sum_{n=1}^N P_n \leq P_t, \\ & C_3: \sum_{k=1}^{K_n} q_{k,n} P_{k,n} \leq P_n, \quad \forall k \in K, \\ & C_5: P_{k,n} \geq 0, \quad \forall k, n. \end{aligned} \quad (14)$$

The optimization problem in (14) is still non-convex due to the fact that the objective function is the ratio of two functions [3],

### Algorithm 1 Subcarrier Allocation Algorithm

- 1: Initialize  $U^u = K, A = N, S_i = \emptyset, P_n = \frac{P_t}{N}$
- 2: Construct channel gain  $H \equiv |h_{k,n}|_{N \times K}$
- 3: Obtain the minimum channel gain of each user:  $H_k^{\min} = \min_{k \in K} \{H_{k,n}\}, i \in A, k \in U$ . Then the number of worst channel quality arranged in ascending order (i.e. from the worst to best) as  $H_{i_0}^{\min} \leq H_{i_1}^{\min} \leq \dots \leq H_{i_{N-1}}^{\min}$ , where  $i_0, i_1, \dots, i_{N-1}$  indicates subcarrier index in  $A$ .
- 4: **while**  $U^u \neq \emptyset$  **do**
- 5:   **for**  $k = 1$  to  $K$  **do**
  - (a) Find the user with the minimum channel quality:  $k = \arg \min_{k \in U} \{H_{k,i}^{\min}\}, \forall k \in K$
  - (b) Assign user  $k$  with the subcarrier with the best channel quality:  $n = \arg \max_{n \in A} \{H_{k,n}\}$
  - (c) Update  $S_k = S_k \cup \{k\}$ , then  $U^u = U^u - \{k\}$
- 6:   **if**  $(|S_k|)=2$  **then**,  $A = A - \{n\}$
- 7: A set of two users  $S_k$  are assigned on every Subcarrier  $n$  satisfying the maximum EE
- 8:   **end if**
- 9: Obtain power allocation for every two users using FTPA in (13):  $P_{k,n} = |S_k| P_n$
- 10: Update user data rate  $R_{k,n}$
- 11: **set**  $EE_{k,n} = \frac{R_{k,n}}{\zeta P_{k,n} + P_{k,n}^C}$
- 12:   **end for**
- 13: **Until**  $U^u = \emptyset$
- 14: **end while**

[10]. In order to efficiently solve (14), we transform the optimization problem into the subtractive form, which is more tractable. Thus, we need to introduce the following problem transformation.

#### A. Problem Transformation and Iterative Algorithm Design

Since the numerator of the objective in (14) is not concave, then fractional programming can not be directly used [9]. Thus, the standard convex optimization algorithm is not guaranteed to solve (14), and thus specific algorithms are required. As a result, we first transform (14) into its equivalent more tractable subtractive form. Without loss of generality, we assume that  $R_{k,n}(Q, P) > 0$  and  $P_k(Q, P) > 0$ . For the sake of simplicity, we define  $D$  as a set of feasible solutions of the optimization in (12) and  $\{P, Q\} \in D$ . Let  $e_k^*$  and  $P^*$  denote the maximum EE and optimal solution of power allocation, respectively. Thus, we define the maximum EE  $e_k^*$  of (14) as

$$\begin{aligned} e_k^* &= \max_P \min_{k=1, \dots, K} \frac{R_{k,n}(Q, P)}{P_k(Q, P)} \\ &= \min_k \frac{R_{k,n}(Q^*, P^*)}{P_k(Q^*, P^*)} \end{aligned} \quad (15)$$

where  $(\cdot)^*$  denotes optimality. Based on (15), we present the following essential theorem.

*Theorem 1:* A vector  $P^* \in D$  solves (15) if and only if [9]

$$\begin{aligned} & \max_{P \in D} \min_{k=1, \dots, K} \{R_{k,n}(Q, P) - e_k^* P_k(Q, P)\} \\ &= \min_{k=1, \dots, K} \{R_{k,n}(Q, P^*) - e_k^* P_k(Q, P^*)\} = 0. \end{aligned} \quad (16)$$

*Proof:* See the proof in [9]. ■

*Theorem 1* implies we can solve (14) via (16) equivalently. Thus, the optimal solution of the auxiliary problem (16) is also the optimal solution of (14) [9]. Let  $F(e_k)$  is the optimum objective value of (14). Thus, solving (14) is essentially equivalent to finding  $e_k = e_k^*$

with  $F(e_k) = 0$ . Moreover, the function  $F(e_k)$  is strictly decreasing in  $e_k$  [9]. Thus, with a given reasonable range, there is an optimal minimum EE  $e_k^*$ , satisfying  $F(e_k^*) = 0$ . In addition,  $F(e_k)$  is negative for  $e_k \rightarrow +\infty$  and positive for  $e_k \rightarrow -\infty$ . Thus, the bisection iterative algorithm can be employed to determine  $e_k$  since the monotonicity of  $F(e_k)$  and the opposite signs at the two sides of  $e_k^*$ . To this end, the  $e_k$  will reach its solution when  $F(e_k^*) = 0$  and the solution for  $P^*$  is achieved by addressing the auxiliary problem of (16) at the given minimum EE. The iterative algorithm based on the bisection method is summarized as Algorithm 2. Hence, the  $e_k$

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**Algorithm 2** Main Procedure for  $e_k^*$

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- 1: Initialize
  - 2: set iteration index  $j=0$ , the maximum iteration  $I_{max}$  and termination precision  $\epsilon > 0$
  - 3: set  $\eta_k^{min}$  and  $\eta_k^{max}$ , such that  $\eta_k^{min} \leq e_k^* \leq \eta_k^{max}$
  - 4: **repeat**
  - 5:  $e_k^j = (\eta_k^{min} + \eta_k^{max})/2$
  - 6: solve (17) for a given  $\eta_k^j$  and obtain power allocation  $P^j$
  - 7: **if**  $|F(e_k^j)| = |\min[R_k(P) - e_k^j P_k(P)]| \leq \epsilon$  **then**
  - 8:  $P^* = P^j$  and  $e_k^* = \min_k[\frac{R_k(P^j)}{P_k(P^j)}]$
  - 9: **break**
  - 10: **else**
  - 11: **if**  $|F(e_k^j)| < 0$  **then**
  - 12:  $\eta_k^{max} = e_k^j$
  - 13: **else**
  - 14:  $\eta_k^{min} = e_k^j$
  - 15: **end if**
  - 16: **end if**
  - 17: set  $j = j+1$
  - 18: **until**  $j > I_{max}$
- 

will reach its solution when  $F(e_k^*) = 0$ , and  $e_k^*$  can be obtained by bisection method [10], while the transmit power  $P^*$  can be achieved by addressing the optimization problem of (17), which needs to be solved at line 6 of Algorithm 2 for a given  $\eta_k^j$ . Thus, hereinafter, we focus on the following objective function:

$$\begin{aligned} \max_P \min_k \quad & \{R_{k,n}(Q, P) - e_k P_k(Q, P)\} \\ \text{s.t.} \quad & C_1, C_2, C_3, C_5. \end{aligned} \quad (17)$$

As it can be observed, the optimization problem in (17) involves non-convex with respect to transmit power,  $P_{k,n}$ , due to the terms of multi-user interference. Hence, sequential convex programming method (SCP) is adopted to make the problem more tractable. For simplicity, the proposed iterative power allocation scheme for this paper is named as non-orthogonal multiple access-sequential convex programming (NOMA-SCP). In each iteration, all non-convex constraints are replaced by their inner convex approximations [9]. To this end, the objective function in (17) can be rearranged into a difference of two concave function with respect to  $P_k$  as

$$R_{k,n}(P) - e_k P_k(P) = f_k(P) - g_k(P) \quad (18)$$

where,

$$f_k(P) = \log_2 \sum_{i=1}^N W(1 + P_{k,n} \Upsilon_{k,n}) - \eta_k P_k(P), \quad (19)$$

$$g_k(P) = \log_2 \sum_{i=1, i \neq k}^N (P_{i,n} \Upsilon_{k,n} + \alpha_{k,n}^2). \quad (20)$$

Now, we can equivalently rewrite problem (17) as

$$\begin{aligned} \max_P \min_k \quad & \{f_k(P) - g_k(P)\} \\ \text{s.t.} \quad & C_1, C_2, C_3, C_5. \end{aligned} \quad (21)$$

It is noted that the objective function in (21) is not smooth at each iteration of different minimum of  $f_k(P) - g_k(P)$ . Thus, we introduce a new variable  $\mathcal{R}$  to the optimization problem (22) to transform into a smooth optimization problem. Thus, problem (21) can be equivalently formulated as

$$\begin{aligned} \max_{P_n, \mathcal{R}} \quad & \mathcal{R} \\ \text{s.t.} \quad & C_1, C_2, C_3, C_5 \\ & C_7: \{f_k(P) - g_k(P)\} \geq \mathcal{R}, \forall k. \end{aligned} \quad (22)$$

It is noted that constraint  $C_6$  in problem (22) is the difference of two concave functions which can be effectively solved by sequential convex programming [9]. At step  $t$  we can get an iterative power allocation  $P^t$ . Thus, we approximate  $g_k(P)$  by first-order Taylor expansion at  $P^t$ , i.e.,

$$g_k(P^t) + \nabla g_k^T(P^t)(P - P^t), \quad (23)$$

where  $\nabla g_k(P)$  is the gradient of  $g_k(P)$  at  $P$  and is given by

$$\nabla g_k(P) = \frac{m_k}{\sum_{i=1, i \neq k} P_{i,n} \Upsilon_{k,n} + \alpha_{k,n}^2}. \quad (24)$$

In the given equation  $m_k$  is a  $K$  dimensional column vector with  $m_k(k) = 0$  and  $m_k(i) = \frac{g_{k,i}}{\ln 2}$ ,  $k \neq i$ . Furthermore, the minimum data rate constraint requirement  $R_{k,n} \geq R_{min}$  can be equivalently written as

$$C'1: P_{k,n} \Upsilon_{k,n} + (1 - 2^{R_{min}/W}) \left( \sum_{i=1, i \neq k}^{n-1} P_{i,n} \Upsilon_{k,n} + \alpha_{k,n}^2 \right) \geq 0 \quad (25)$$

Combining (23) and (22), we can rewrite (22) as

$$\begin{aligned} \max_{P_n, \mathcal{R}} \quad & \mathcal{R} \\ \text{s.t.} \quad & C'1, C_2, C_3, C_5 \\ & C_7: f_k(P) - [g_k(P^t) + \nabla g_k^T(P^t)(P - P^t)] \geq \mathcal{R}. \end{aligned} \quad (26)$$

Now, (26) is a smooth and standard convex approximation of (17). The local optimal transmit power can be efficiently calculated by solving (26). The algorithm iteratively solves the convex optimization problem in (26). Using successive iteration of the algorithm, the value of  $E^{(t)} = \min(f_k(P^t) - g_k(P^t))$  decreases. For every  $E^{(t)} = \min(f_k(P^t) - g_k(P^t))$ , the power vector that maximize the objective function in (R) is found. The algorithm stops when  $E^{(t)} = \min(f_k(P^t) - g_k(P^t))$  is less than or equal to the given tolerance value. We show the detailed power control algorithm in Algorithm 3.

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**Algorithm 3** Iterative Algorithm Procedure for  $P_n^*$

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- 1: Initialize  $t = 0$  and maximum tolerance  $\epsilon > 0$
  - 2: Set  $P^{(0)}$  calculate  $E^0 = \min_k [f_k(P^{(0)}) - g_k(P^{(0)})]$
  - 3: **while**  $\|E^{(t+1)} - E^{(t)}\| > \epsilon$  **do**
  - 4: Solve (26) to obtain the solution  $P^*$ .
  - 5: Set  $t = t + 1$ ,  $P^t = P^*$
  - 6:  $E^{(t)} = \min(f_k(P^t) - g_k(P^t))$
  - 7: **end while**
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## V. SIMULATION RESULTS

We consider a single BS located in the cell center and users are uniformly distributed inside a circular ring with radius of 300  $m$ . We set the value of path loss exponent  $\gamma$  as 2. The minimum distance from users to BS is limited 50  $m$ . The bandwidth of the system is set as 5 MHz. As it has already been mentioned, two users are assigned per subcarrier to reduce the complexity of SIC. For power coefficient to optimize the power allocation among

the multiplexed users of each subchannel  $n$ , we adopt the method fractional transmit power allocation (FTPA) used in (13), and scale a parameter  $0 \leq \delta \leq 1$ , where  $\gamma = 0$  enable equal power allocation, while increasing  $\delta$  results in more power allocated to the UE with poor channel condition. In the simulation, we set BS peak power  $P=12\text{ W}$  and circuit power consumption  $P_c=0.5\text{ W}$  [6], and  $\alpha_n^2 = \frac{BW \cdot N_0}{N}$ , where  $N_0=-174\text{ dBm/Hz}$  is the AWGN power spectral density. For simplicity, we consider each user has the same weighted bandwidth  $\frac{BW}{N}$ .

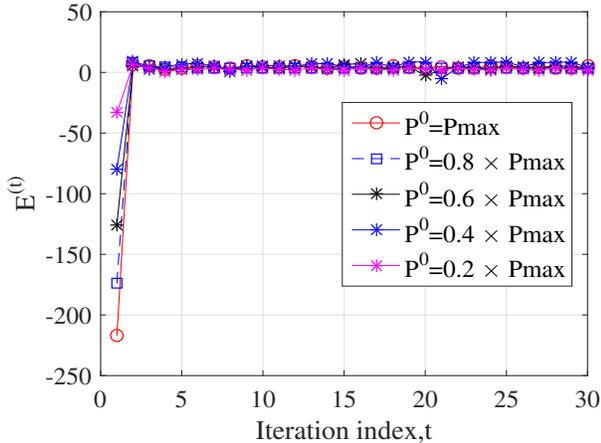


Fig. 1. The convergence of the iterative power allocation algorithm.

Fig. 1 demonstrates the convergence behavior of the proposed iterative algorithm for maximizing the EE. The algorithm stops iterating when the difference between two successive values of the EE returned by the algorithm is less than or equal to its tolerance value,  $\epsilon = 0.0001$ . It can be seen that the proposed algorithm converges to the solutions very fast and only requires about 8 iterations to reach the convergence point and insensitive to the selection of the different initial values of  $P^0$ . Thus, the fast convergence confirms the practicality of the proposed algorithm

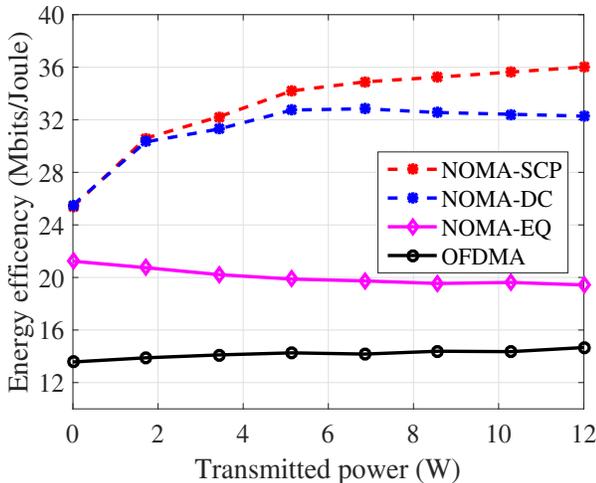


Fig. 2. Efficiency of the system versus transmitted power.

Fig. 2 shows the EE versus the maximum transmit power for different number of users,  $K$ . The proposed scheme, NOMA-SCP, is compared with differential convex programming (NOMA-DC) in [6] and OFDMA system in [3] as well as NOMA with equal power allocation (NOMA-EQ) used in our proposed subcarrier assignment scheme. It can be observed that the NOMA-SCP can achieve higher EE than all benchmark schemes. It can also be seen that the EE

initially increases with respect to transmitted power. However, as the transmit power increases further, the EE starts to converge with slow growth. This indicates that any additional transmit power as well as circuit power causes decline in EE.

## VI. CONCLUSION

In this work, we have investigated the downlink of NOMA system where a single base station transmits a block of messages to multiple users. Since the optimization problem was non-convex, we formulated the subcarrier assignment and power allocation as a two stage-problem to reduce computational complexity. For the subcarrier allocation, we proposed a greedy-based low complexity worst-case user first subcarrier allocation algorithm (WCUFSA). Besides, sequential convex programming was used to allocate power across each subchannel. Thus, the suboptimal power and subcarrier allocation policies have been obtained for maximizing the EE in each iteration. The provided simulation results have shown that the proposed resource optimization method achieves fast convergence and guarantees fairness. Consequently, the proposed resource allocation method is particularly promising, since remarkable gains are achieved compared to existing techniques, while it remains appropriate for the practical case.

## REFERENCES

- [1] Z. Wei, J. Yuan, D. W. K. Ng, M. Elkashlan, and Z. Ding, "A survey of downlink non-orthogonal multiple access for 5G wireless communication networks," *ZTE Communications*, vol. 14, pp. 17-25, Oct. 2016.
- [2] L. Dai, B. Wang, Y. Yuan, S. Han, C. I. I, and Z. Wang, "Non-orthogonal multiple access for 5G: Solutions, challenges, opportunities, and future research trends," *IEEE Commun. Mag.*, vol. 53, no. 9, pp. 74-81, Sept. 2015
- [3] Y. Li, M. Sheng, C. W. Tan, Y. Zhang, Y. Sun, X. Wang, Y. Shi and J. Li, "Energy-efficient subcarrier assignment and power allocation in OFDMA systems with Max-Min fairness guarantees," *IEEE Trans. Commun.*, vol. 63, no. 9, Sept 2015.
- [4] W. Hao, M. Zeng, Z. Chu, and S. Yang, "Energy-efficient power allocation in millimeter wave massive mimo with non-orthogonal multiple access," *IEEE Wireless Commun. Lett.*, vol. PP, no. 99, pp. 782-785, 2017.
- [5] Y. Zhang, H. M. Wang, T. X. Zheng, and Q. Yang, "Energy-efficient transmission design in non-orthogonal multiple access," *IEEE Trans. Veh. Technol.*, vol. 66, no. 99, pp. 2852-2857, Sept. 2016.
- [6] F. Fang, H. Zhang, J. Cheng, and V. C. M. Leung, "Energy-efficient resource allocation for downlink non-orthogonal multiple access network," *IEEE Trans. Commun.*, vol. 64, no. 9, pp. 3722-3732, May. 2016.
- [7] F. Fang, J. Cheng and Z. Ding, "Joint Energy Efficient Subchannel and Power Optimization for a Downlink NOMA Heterogeneous Network," *IEEE Trans. on vehicular technology*, vol. 68, no. 2, Feb. 2019.
- [8] S. He, Y. Huang, S. Jin, F. Yu, and L. Yang, "Max-min energy efficient beamforming for multicell multiuser joint transmission systems," *IEEE Commun. Lett.*, vol. 17, no. 10, pp. 1956-1959, Oct. 2013.
- [9] A. Zappone and E. Jorswieck, "Energy efficiency in wireless networks via fractional programming theory," *Foundations and Trends in Communications and Information Theory*, vol. 11, no. 3-4, pp. 185-396, 2015.
- [10] Y. Li, M. Sheng, X. Wang, Y. Zhang, and J. Wen, "Max-min energy efficient power allocation in interference-limited wireless networks," *IEEE Trans. Veh. Technol.*, vol. 64, no. 9, pp. 4321-4326, Sep. 2015.
- [11] J. Shi and L.-L. Yang, "Novel subcarrier-allocation schemes for downlink MC DS-CDMA systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 10, pp. 5716-5728, Oct. 2014.
- [12] F. Fang ; J. Cheng ; Z. Ding ; H. V. Poor, "Energy Efficient Resource Optimization for a Downlink NOMA Heterogeneous Small-cell Network," *IEEE conf.* , pp. 51-55, 2018.
- [13] A. Fehske, G. Fettweis, J. Malmodin, and G. Biczok, "The global footprint of mobile communications: the ecological and economic perspective," *IEEE Commun. Mag.*, vol. 49, no. 8, pp. 55-62, Aug. 2011.