2021 IEEE 22nd International Workshop on Signal Processing Advances in Wireless Communications (SPAWC) | 978-1-6654-2851-4/21/$31.00 ©2021 IEEE | DOI: 10.1109/SPAWC51858.2021.9593175

Nonlinear Energy Harvesting Evaluation through the Logit Pearson Distribution

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***Abstract*—In this paper, we introduce the logit Pearson type III distribution and we utilize it, for first time in the literature, to investigate the statistical behavior of wireless power transfer, assuming that the harvested energy follows a well-established nonlinear energy harvesting (EH) model based on the logistic function. Specifically, we present closed-form expressions for the statistical properties of the introduced logit Pearson type III distribution, e.g., the cumulative distribution function (CDF), the probability density function and the moments and we utilize this distribution to define the logit gamma distribution. Furthermore, taking into account that the logit Pearson type III distribution is closely related to the considered nonlinear EH model the statistical properties of the distribution of the harvested power are derived. These expressions are of high practical value, since useful insights for the EH system can be extracted through the evaluation of the CDF, as well as the average harvested power and the variance.**

***Index Terms*—Logit Pearson type III distribution, energy harvesting, wireless power transfer**

I. Introduction

The Pearson type III distribution [1], [2] attracted the interest of the research community, since it has been utilized in many scientific fields such as hydrology and communications. Specifically, in [3] it was applied for flood frequency analysis, while in [4] the Pearson type III distribution was fit in data of low flows. Regarding communication systems, Pearson type III distribution can be considered as a generalized form of the gamma distribution, which is frequently used in wireless communications when the channel fading is assumed to follow Nakagami-m distribution.

In this paper, we utilize the Pearson type III family distribution in wireless power transfer (WPT) and especially in investigating the statistical properties of the harvested energy, which is a prerequisite in order to analytically evaluate the capabilities and reliability of this technology. It should be highlighted that energy harvesting (EH) is a promising solution for prolonging the lifetime of Internet-of-Things (IoT) networks by offering self-sustainability to the devices, minimizing -if not eliminating- the use of battery power. This is of paramount importance especially when replacing or recharging the batteries is inconvenient, costly, or dangerous, such as in remote areas, harsh industrial environments, e.g., rotating and moving platforms, human bodies, or vacuum equipment [5]. However, the main disadvantage of basic EH methods is their reliability, since they depend solely on ambient natural energy sources, such as wind and solar, which are uncontrollable. To this end, WPT which utilizes radio frequency (RF) signals for EH is an interesting alternative and also benefits from high-density networks [6].

Scanning the open literature, a linear EH model was used to express the harvested energy, when WPT is performed [7], [8]. This model can be easily handled because of its simplicity, however, it can be considered impractical, since it is not accurate and cannot describe the saturation of the EH circuit. Nonlinear EH models were proposed in [9], [10]. Also, in [11] a practical parametric nonlinear EH model was proposed and its accuracy was verified through measurements. This model is based on the logistic function and due to its accuracy has been adopted in several research works [12], [13]. However, although this nonlinear EH model has received the researchers' attention, a comprehensive analytical framework which can assist in the design and evaluation of its performance has not been yet provided and its statistical properties, e.g., the CDF, the probability density function (PDF) and the moments, have not been derived. Considering that the basis of this model is the logistic function, the analytical investigation of WPT performance is facilitated by the use of logit Pearson type III distribution, which however has not been defined and studied in existing literature, in which solely the the logit normal distribution has been investigated [14].

In the present work, we introduce the logit Pearson type III distribution and we utilize the statistical properties of this distribution to investigate the performance of WPT systems where the nonlinear EH model proposed in [11] is considered. Specifically, we first introduce a new member of the Pearson type III family which is closely related to the considered nonlinear EH model, the logit Pearson type III distribution, and derive closed-form expressions for its statistical properties, e.g., the CDF, the PDF and the moments. Moreover, we utilize the above statistical results to provide a comprehensive analytical framework for the evaluation of the performance of the EH systems, when the considered nonlinear EH model is used, and to analytically evaluate the capabilities and reliability of WPT technology. Useful insights for the EH system can be extracted through the evaluation of the average harvested power and harvested power outage probability.

II. The Logit Pearson Type III Distribution

In this section, we introduce a new member of the Pearson type III family, the logit Pearson Type III distribution. To derive its statistical properties, firstly, we present the PDF and the CDF of the Pearson Type III distribution.

A. The Pearson Type III Distribution

If a RV X follows the Pearson type III distribution with parameters (a, b, m), where $a \in \mathbb{R}$ with $a > 0$ is the shape parameter, $b \in \mathbb{R}$ with $b \neq 0$ is the inverse scale parameter and $m \in \mathbb{R}$ is the shift parameter, then its PDF is given by [3]

$$f_X(x, a, b, m) = \frac{|b|}{\Gamma(a)}(b(x-m))^{a-1}e^{-b(x-m)}, \quad (1)$$

where $\Gamma(\cdot)$ is the gamma function and e is the base of the natural logarithm. If $b > 0$, $x \in (m, +\infty)$ and if $b < 0$, $x \in (-\infty, m)$.

In the following proposition, the CDF of the Pearson type III distribution is provided for $b < 0$. For $b > 0$, the CDF is provided in [2], but it is also included in the following proposition for completeness.

Proposition 1: The CDF of the Pearson type III distribution can be expressed as

$$F_X(x, a, b, m) = \begin{cases} \frac{1}{\Gamma(a)}\Gamma(a, b(x-m)), & b < 0 \\ \frac{1}{\Gamma(a)}\gamma(a, b(x-m)), & b > 0 \text{ [2]}, \end{cases} \quad (2)$$

where $\Gamma(\cdot,\cdot)$ and $\gamma(\cdot,\cdot)$ is the upper and lower incomplete gamma function, respectively.

Proof: If $b < 0$, $x \in (-\infty, m)$ the CDF is given by

$$F_X(x, a, b, m) = -\frac{b}{\Gamma(a)}\int_{-\infty}^{x}(b(y-m))^{a-1}e^{-b(y-m)}dy. \quad (3)$$

By using $z = b(y-m)$, (3) can be written as

$$F_X(x, a, b, m) = \frac{b}{\Gamma(a)}\int_{b(x-m)}^{\infty} z^{a-1}e^{-z}dy. \quad (4)$$

Considering the definition of the upper incomplete gamma function, the expression of the CDF when $b < 0$ is derived and the proof is completed. ■

Remark 1: The gamma distribution is a special case of the Pearson type III distribution with $b > 0$ and $m = 0$. If $b \neq 0$, a RV that follows the Pearson type III distribution with parameters (a, b, m) can be also multiplied with a constant c resulting in a RV that follows the Pearson type III distribution with parameters $(a, \frac{b}{c}, mc)$. Accordingly, if a RV follows the gamma distribution with parameters (a, b) with $b > 0$, multiplying this RV with a negative constant c results in a RV that follows the Pearson type III distribution with parameters $(a, \frac{b}{c}, 0)$, where the second parameter is negative.

B. Statistical Properties of The Logit Pearson Type III Distribution

In [14], the logit normal distribution is investigated where, considering that the RV A follows the normal distribution, the RV B follows the logit normal distribution, if $A = \text{logit}(B) = \ln\frac{B}{1-B}$, where $\ln(\cdot)$ is the natural logarithm, or $B = f(A)$, where $f(x) = \frac{1}{1+e^{-x}}$ is the logistic function. Accordingly, in this work, we introduce the logit Pearson type III distribution, which is defined below.

Definition 1: The RV $Z = \frac{1}{1+e^{-X}}$ follows the logit Pearson type III distribution with the parameters parameters (a, b, m), if the RV X follows the Pearson type III distribution with the same distribution or, equivalently, $X = \text{logit}(Z)$. For the domain of z, it holds that $z \in (\frac{1}{1+e^{-m}}, 1)$ if $b > 0$, while $z \in (0, \frac{1}{1+e^{-m}})$ if $b < 0$.

Proposition 2: The CDF of the logit Pearson type III distribution can be expressed as

$$F_Z(z, a, b, m) = \begin{cases} \frac{1}{\Gamma(a)}\Gamma\left(a, b\left(\ln\frac{z}{1-z} - m\right)\right), & b < 0 \\ \frac{1}{\Gamma(a)}\gamma\left(a, b\left(\ln\frac{z}{1-z} - m\right)\right), & b > 0. \end{cases} \quad (5)$$

Proof: The CDF of the logit Pearson type III distribution is derived by substituting $x = \ln\frac{z}{1-z}$ in (2). ■

In the following proposition, the PDF of Z is extracted.

Proposition 3: The PDF of the logit Pearson type III distribution is given by

$$f_Z(z, a, b, m) = \frac{|b|e^{bm}}{\Gamma(a)}\left(b\left(\ln\frac{z}{1-z} - m\right)\right)^{a-1} \times z^{-b-1}(1-z)^{b-1}. \quad (6)$$

Proof: The PDF of the logit Pearson type III distribution is derived as the first derivative of the CDF given by (5) and after some algebraic manipulations. ■

Proposition 4: The n-th moment of the logit Pearson type III distribution when $b > 0$ is given by (7) at the top of the next page, where $\binom{n}{k}$ denotes the binomial coefficient.

Proof: The proof is provided in Appendix A. ■

Corollary 1: The mean value of the logit Pearson type III distribution when $b > 0$ and $m \geq 0$ is given in closed-form by

$$\mu_Z^1(a, b, m) = b^a\Phi\left(-e^{-m}, a, b\right), \quad (8)$$

where $\Phi(\cdot,\cdot,\cdot)$ is the Lerch function [15].

Corollary 2: The second moment of the logit Pearson type III distribution when $b > 0$ and $m \geq 0$ is given in closed-form by

$$\mu_Z^2(a, b, m) = b^a\left(\Phi\left(-e^{-m}, a-1, b\right) - (b-1)\Phi\left(-e^{-m}, a, b\right)\right). \quad (9)$$

C. The Logit Gamma Distribution

Utilizing the above analysis, for the special case that $b > 0$ and $m = 0$, the logit gamma distribution can be derived. If the RV Z follows the logit gamma distribution, it holds that $z \in (0.5, 1)$. The CDF of the logit gamma distribution can be expressed as

$$F_Z(z, a, b) = \frac{1}{\Gamma(a)}\gamma\left(a, b\ln\frac{z}{1-z}\right). \quad (10)$$

The PDF of the logit gamma distribution is given by

$$f_Z(z, a, b) = \frac{b}{\Gamma(a)}\left(b\ln\frac{z}{1-z}\right)^{a-1}z^{-b-1}(1-z)^{b-1}. \quad (11)$$

$$\mu_Z^n(a,b,m)=\begin{cases}\sum_{l=0}^{\infty}\binom{n+l-1}{l}(-1)^l e^{-ml}\left(1+\frac{l}{b}\right)^{-a}, & m\geq 0\\ \frac{1}{\Gamma(a)}\sum_{l=0}^{\infty}\binom{n+l-1}{l}(-1)^l\left(e^{m(n+l)}\left(1-\frac{n+l}{b}\right)^{-a}\gamma\left(a,-mb\left(1-\frac{n+l}{b}\right)\right)\right. & \\ \qquad \left.+e^{-ml}\left(1+\frac{l}{b}\right)^{-a}\Gamma\left(a,-mb\left(1+\frac{l}{b}\right)\right)\right), & m<0\end{cases} \tag{7}$$

The n-th moment of the logit gamma distribution is given by

$$\mu_Z^n(a,b)=\sum_{l=0}^{\infty}\binom{n+l-1}{l}(-1)^l\left(1+\frac{l}{b}\right)^{-a}. \tag{12}$$

Corollaries 1 and 2 can be used to extract closed-form expressions for the first and the second moment of the logit gamma distribution.

III. Application of Pearson Type III Family of Distributions in WPT

A. System Model

In this section, a network is consisting of one power beacon (PB) that utilizes WPT to provide energy to the assigned EH sources. All nodes are equipped with one antenna. It is also assumed that the harvested power due to the processing noise is negligible and, thus, it can be ignored. The nonlinear EH model proposed in [11] is considered. In contrast with the linear EH model which is accurate only when the received power is constant, this nonlinear model captures the dynamics of the RF energy conversion efficiency for different input power levels and is based on the logistic function.

The power harvested by one of the sources can be expressed as [11]

$$Q^S=\frac{P_s\left(1+e^{AB}\right)}{e^{AB}\left(1+e^{-A(lp|h|^2-B)}\right)}-\frac{P_s}{e^{AB}}, \tag{13}$$

where P_s denotes the maximum harvested power when the EH circuit is saturated. Also, A and B are positive constants related to the circuit specification. Practically, A reflects the nonlinear charging rate with respect to the input power and B is related to the turn-on threshold. Given the EH circuit, the parameters P_s, A, and B can be determined by the curve fitting. Furthermore, l, p and h denote the path loss factor between the PB and the source, the transmitted power and the small scale fading coefficient between the PB and the source, respectively. We assume that the channel fading between the PB and the source is a stationary and ergodic random process, whose instantaneous channel realizations follow the Nakagami-m distribution with parameters $(a,\frac{a}{b})$, since the Nakagami channel model is general enough to describe the typical wireless fading environments. In this case, $|h|^2$ follows the gamma distribution with parameters (a,b) or the Pearson type III distribution with parameters $(a,b,0)$.

B. Statistical Properties

Some important statistical properties of the distribution of the harvested power are presented below.

Theorem 1: The CDF of the distribution of the harvested power is given by

$$F_{Q^S}(q)=\frac{1}{\Gamma(a)}\gamma\left(a,-\frac{b}{Alp}\times\left(\ln\left(\frac{P_s\left(1+e^{AB}\right)}{e^{AB}\left(q+\frac{P_s}{e^{AB}}\right)}-1\right)-AB\right)\right). \tag{14}$$

Proof: In (13), $|h|^2$ follows the gamma distribution with parameters (a,b), which is also the Pearson type III distribution with parameters $(a,b,0)$ and $lp|h|^2$ follows the Pearson type III distribution with parameters $(a,\frac{b}{lp},0)$. Therefore, $-A\left(lp|h|^2-B\right)$ follows the Pearson type III distribution with parameters $(a,-\frac{b}{Alp},AB)$ and $\frac{1}{1+e^{-A(lp|h|^2-B)}}$ follows the logit Pearson type III distribution with parameters $(a,\frac{b}{Alp},-AB)$.

The CDF of the distribution of the harvested power is obtained as

$$F_{Q^S}(q)=P\left(Q<q\right), \tag{15}$$

where $P(\cdot)$ denotes probability. After some algebraic manipulations and using (5), (15) can be rewritten as

$$F_{Q^S}(q)=F_Z\left(\frac{e^{AB}\left(q+\frac{P_s}{e^{AB}}\right)}{P_s\left(1+e^{AB}\right)}\right). \tag{16}$$

From (16), (14) is derived. ■

It should be highlighted that in EH systems, since the harvested power is a RV, it is possible that the available power may drop below the required power, causing a power outage. The CDF of the distribution of the harvested power indicated the probability that power outage occurs if we consider a threshold q.

In the following theorem, the PDF of Q^S is extracted.

Theorem 2: The PDF of the distribution of the harvested power can be expressed as

$$f_{Q^S}(q)=\frac{c\left(1+e^{AB}\right)\hat{b}^a}{\Gamma(a)e^{AB\hat{b}}}\left(ce^{AB}-q\right)^{-1+\hat{b}}(c+q)^{-1-\hat{b}}\times\left(AB-\ln\left(\frac{c\left(1+e^{AB}\right)}{c+q}-1\right)\right)^{a-1}, \tag{17}$$

where $\hat{b}=\frac{b}{Alp}$ and $c=\frac{P_s}{e^{AB}}$.

Proof: The PDF is obtained as the first derivative of the CDF given by (14) and after some algebraic manipulations. ■

$$\mu_{Q^S}^n = \frac{c^n}{\Gamma(a)} \sum_{l_1=0}^{n} \sum_{l_2=0}^{\infty} \binom{n}{l_1} \binom{l_1+l_2-1}{l_2} (-1)^{n-l_1+l_2} \left(e^{-AB}+1\right)^{l_1} \left(e^{-ABl_2} \left(1-\frac{l_1+l_2}{\hat{b}}\right)^{-a} \right.$$
$$\left. \times \gamma\left(a, AB\hat{b}\left(1-\frac{l_1+l_2}{\hat{b}}\right)\right) + e^{AB(l_1+l_2)} \left(1+\frac{l_2}{\hat{b}}\right)^{-a} \Gamma\left(a, AB\hat{b}\left(1+\frac{l_2}{\hat{b}}\right)\right)\right) \quad (18)$$

In the following theorem, the moments of Q^S are provided.

Theorem 3: The n-th moment of the distribution of the harvested power is given by (18) at the top of the next page, if $\hat{b} \notin \mathbb{Z}$.

Proof: The proof is provided in Appendix B. ■

Remark 2: The first moment, i.e., the mean value expresses the average power harvested by the EH source. From the second moment the variance of the harvested power can be derived which expresses how the values of the harvested power fluctuate around the mean value. It should be highlighted that the variance should be small, the majority of the harvested power is larger than the sensitivity threshold.

C. Simulation Results

In this subsection, simulations are provided to validate the theoretical results derived in the previous subsection. The path loss factor is given by $l = 1 - e^{-\frac{a_t a_r}{(c/f)^2 d^2}}$, where a_t is the aperture of the transmit antenna, a_r the aperture of the receive antenna, c the velocity of light, f_c the operating frequency and d the distance between the transmitter and the receiver. Assuming the receiver as a small sensor, we set $a_t = 0.5$m, $a_r = 0.01$m and $f_c = 2.4$GHz [12]. For the parameters of the nonlinear EH model, we set $A = 150$, $B = 0.014$ and $P_s = 24$mW [16]. We normalize the outage threshold q_t with respect to the maximum harvested power when the power harvesting circuit is saturated, termed as P_s. The outage performance improves as the distance and the threshold decrease as it can be observed from 1. Moreover, 2 illustrates that the outage performance also improves with the increase of the transmitted power.

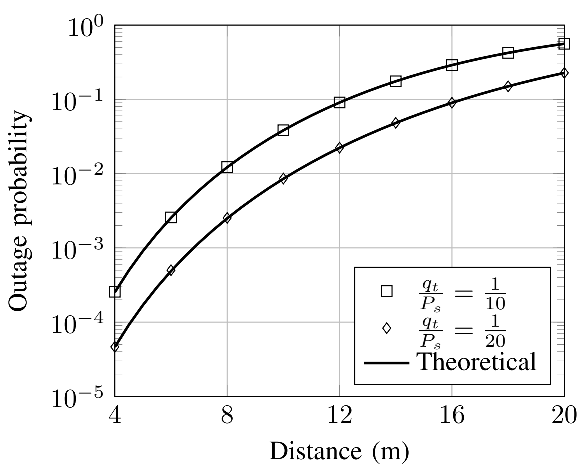

Fig. 1. Outage probability versus distance between the PB and the EH source.

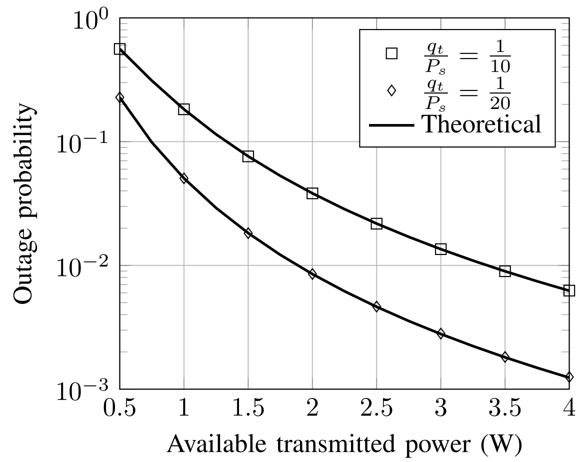

Fig. 2. Outage probability versus transmitted power.

IV. Conclusions

In this work, we have utilized the Pearson type III distribution in wireless communications and specifically in WPT, where a frequently-used nonlinear EH model is considered. We have introduced the logit Pearson type III distribution which is closely related to the specific model and derived closed form expressions for its CDF, PDF and moments. Moreover, we have utilized the derived results to extract closed-form expressions for the CDF, the PDF and the moments of the harvested power for an EH system with the considered nonlinear EH model. These statistical properties can provide useful insights such as the probability that outage occurs in the harvested power considering a specific threshold and the average harvested power by the source.

Acknowledgment

The research leading to these results has received funding from the EuropeanUnion's Horizon 2020 research and innovation programme under grant agreement No 957406. The research work of S. A. Tegos was supported by the Hellenic Foundation for Research and Innovation (HFRI) under the HFRI PhD Fellowship grant (Fellowship Number: 1394).

Appendix A
Proof of Proposition 4

The n-th moment of the logit Pearson type III distribution when $b > 0$ can be obtained by the integral $\int_{\frac{1}{1+e^{-m}}}^{1} z^n f_Z(z) dz$ which can be rewritten as

$$\mu_Z^n(a,b,m) = \frac{1}{\Gamma(a)} \int_0^{\infty} \left(\frac{1}{1+e^{-\frac{z}{b}-m}} \right)^n x^{a-1} e^{-x} dz. \quad (19)$$

When $m \geq 0$, it holds that $e^{-\frac{z}{b}-m} \leq 1$ and utilizing the binomial theorem for negative integer exponent, (19) can be rewritten as

$$\mu_Z^n(a,b,m) = \frac{1}{\Gamma(a)} \int_0^{\infty} \sum_{l=0}^{\infty} \binom{n+l-1}{l} (-1)^l e^{-ml} \times z^{a-1} e^{-\left(1+\frac{l}{b}\right)z} dz. \tag{20}$$

The infinite series in (20) converges from the utilization of the binomial theorem. When $m \geq 0$, (7) is derived by interchanging the order of summations and integrations and using the definition of gamma function and [15, eq.(3.381.4)].

When $m < 0$, utilizing the binomial theorem for negative integer exponent, the denominator can be written as

$$\left(1+e^{-\frac{z}{b}-m}\right)^{-n} = \begin{cases} \sum_{l=0}^{\infty} \binom{n+l-1}{l} (-1)^l e^{m(n+l)} e^{\frac{n+l}{b}z}, \\ \qquad x < -mb \\ \sum_{l=0}^{\infty} \binom{n+l-1}{l} (-1)^l e^{-ml} e^{-\frac{l}{b}z}, \\ \qquad x > -mb. \end{cases} \tag{21}$$

In this case, the n-th moment can be calculated as

$$\mu_Z^n(a,b,m) = \frac{1}{\Gamma(a)} \int_0^{-mb} \sum_{l=0}^{\infty} \binom{n+l-1}{l} (-1)^l e^{m(n+l)} \times z^{a-1} e^{-\left(1-\frac{n+l}{b}\right)z} dz + \frac{1}{\Gamma(a)} \int_{-mb}^{\infty} \sum_{l=0}^{\infty} \binom{n+l-1}{l} \times (-1)^l e^{-ml} z^{a-1} e^{-\left(1+\frac{l}{b}\right)z} dz. \tag{22}$$

Considering the definition of lower and upper incomplete gamma function and [15, eq.(3.381.1&3)], (7) is derived when $m < 0$ which completes the proof.

Appendix B
Proof of Theorem 3

Setting in $\int_0^{P_s} q^n f_{Q^S}(q) dq$

$$x = -\hat{b}\left(\ln\left(\frac{c\left(1+e^{AB}\right)}{q+c} - 1\right) - AB\right), \tag{23}$$

the n-th moment is calculated as

$$\mu_{Q^S}^n = \frac{c^n}{\Gamma(a)} \int_0^{\infty} e^{-x} x^{a-1} \left(\frac{e^{-AB}+1}{e^{-x/\hat{b}} + e^{-AB}} - 1\right)^n dx. \tag{24}$$

Using the binomial theorem, (24) can be rewritten as

$$\mu_{Q^S}^n = \frac{c^n}{\Gamma(a)} \sum_{l_1=0}^{n} \binom{n}{l_1} (-1)^{n-l_1} \left(e^{-AB}+1\right)^{l_1} \times \int_0^{\infty} \frac{x^{a-1} e^{-x}}{\left(e^{-x/\hat{b}} + e^{-AB}\right)^{l_1}} dx. \tag{25}$$

Utilizing the binomial theorem for negative integer exponent, the denominator can be written as

$$\left(e^{-x/\hat{b}} + e^{-AB}\right)^{-l_1} = \begin{cases} \sum_{l_2=0}^{\infty} \binom{l_1+l_2-1}{l_2} \left(e^{-x/\hat{b}}\right)^{-l_1-l_2} \\ \quad \times (-1)^{l_2} \left(e^{-AB}\right)^{l_2}, \ x < AB\hat{b} \\ \sum_{l_2=0}^{\infty} \binom{l_1+l_2-1}{l_2} \left(e^{-AB}\right)^{-l_1-l_2} \\ \quad \times (-1)^{l_2} \left(e^{-x/\hat{b}}\right)^{l_2}, \ x > AB\hat{b}. \end{cases} \tag{26}$$

In this case, the infinite series always converge. Using (26) and considering the definition of lower and upper incomplete gamma function and [15, eq.(3.381.1&3)], respectively, after some algebraic manipulations (18) is derived.

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