2023 30th International Conference on Systems, Signals and Image Processing (IWSSIP) | 979-8-3503-3729-7/23/$31.00 ©2023 IEEE | DOI: 10.1109/IWSSIP58668.2023.10180234

Indoor 3D Visible Light Positioning Analysis with Channel Estimation Errors

Dimitrios Bozanis[1], Nikos G. Evgenidis[1], Vasilis K. Papanikolaou[2], Pavlos S. Bouzinis[1],
Sotiris A. Tegos[3], Alexis A. Dowhuszko[4], Panagiotis D. Diamantoulakis[3], and George K. Karagiannidis[1,5]

[1] Wireless Communications and Information Processing Group (WCIP),
Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, Thessaloniki 54124, Greece
[2] Institute for Digital Communications (IDC), Friedrich-Alexander Universitat Erlangen-Nurnberg (FAU), Germany
[3] Department of Applied Informatics, University of Macedonia, 54636 Thessaloniki, Greece
[4] Department of Communications and Networking, Aalto University, 02150 Espoo, Finland
[5] Cyber Security Systems and Applied AI Research Center, Lebanese American University (LAU), Lebanon

dimimpoz@ece.auth.gr, nevgenid@ece.auth.gr, vpapanikk@ieee.org, mpouzinis@auth.gr,
sotiris.a.tegos@gmail.com, alexis.dowhuszko@aalto.fi, padiaman@ieee.org, geokarag@auth.gr

***Abstract*—Since radio-frequency signals are affected significantly by multipath fading, the localization accuracy they offer cannot meet the sixth generations (6G) demanding specifications. On the other hand, visible light positioning (VLP) has emerged as a low cost alternative that takes into account the highly deterministic channel characteristics of visible light communications to provide cm-level accuracy provided that the investigated spaces are adequately covered by LEDs. However, imperfect channel state information and modeling can lead to positioning errors. In this paper, localization accuracy is quantified in terms of channel estimation errors via low complexity lateration algorithm. The channel estimation errors can be caused by a variety of issues and the provided analysis is general so that it can be easily applied to a plethora of applications. Finally, Monte Carlo simulations validate the proposed method and offer valuable insights for the design and performance bounds of VLP systems.**

***Index Terms*—visible light positioning (VLP), localization, channel estimation errors**

I. Introduction

As we pave the way for the sixth generation (6G) of wireless access, one of the key performance indicators (KPIs) has been identified as the increase in localization accuracy [1]. In the indoor case, where the service of global navigation satellite systems (GNSSs) is poor, radio-frequency (RF) systems, such as cellular and WiFi, have long attracted attention as possible alternatives. However, in most cases, the accuracy is unsatisfactory, primarily due to the fading effects that RF signals experience. On the other hand, visible light communications (VLC) have increased in popularity as an energy efficient way to complement current indoor wireless access. Additionally, visible light positioning (VLP), the solution of VLC for indoor localization, offers many benefits over the RF options, primarily, the fact that the VLC channel gain experiences much fewer non line-of-sight effects, such as fading. The ubiquitous coverage of buildings with energy efficient light emitting diodes (LEDs) as part of the illumination infrastructure means that in large indoor spaces, such as offices, airports, malls, etc., there are many possible access points (APs) to exploit for localization purposes.

In this context, various works on VLP have emerged in the last decade. Most works focus on received signal strength (RSS) techniques, while different methods of obtaining a location estimate using the time of arrival, angle of arrival and time difference of arrival are also presented. These aspects of VLP can be found in recent surveys such as [2]–[4]. Besides these, in [5], a trilateration method based on RSS was utilized to locate users and experimental results showed an estimated position error of 2.4 cm for fixed height. Furthermore, the authors of [6] achieved a three-dimensional (3D) accuracy of 9 cm with the use of Gaussian mixture sigma-point particle filter technique. The authors of [7] used a gyroscope and tilted receivers to increase the localization accuracy by utilizing the readings of the gyroscope and adjusting the estimated location accordingly. Moreover, in [8], the authors utilized the RSS indication technique to estimate that unless moderate to high LED power and low reflectivity walls are used, the error could exceed 1 m with such methods. In [9], the authors also used an accelerometer on the device with tilted receivers to adjust for the orientation. The performed experiment validated their method and the resulting error was about 6 cm even when the mobile device was moving in the 3D space with up to 1.3 m/s velocity. Additionally, in [10], orthogonal frequency division multiplexing (OFDM) in VLC was utilized to counter the multipath effects both for communications and localization. Simultaneously with the location, the orientation was also estimated in [11] by utilizing the RSS with a particle-assisted stochastic search algorithm. The resulting scheme was able to estimate the location and orientation with high accuracy without information on the height of receiver in the space. More recently, in [12], a highly accurate VLP system was presented with tilted transmitters considering the non line-of-sight (nLoS) paths and achieved cm-level error.

In the above works, specific assumptions about the channel model were made, while also assuming the availability of the channel state information (CSI). This can often be difficult to be obtained, especially when designing a system with low complexity such as RSS-based VLP systems. In this work, in order to quantify how the channel estimation imperfections, either because of imprecise channel modeling or due to the unavailability of perfect CSI, we propose a way to connect the channel estimation errors with the localization accuracy of an RSS VLP system. Monte Carlo simulations validate the proposed analysis and valuable insights are derived that can assist in defining performance bounds of such systems.

II. System Model

We consider a VLP system that consists of a group of visible light communication APs and a user that requires localization services, as shown in Fig. 1. In VLC systems it is known that line-of-sight (LoS) is the most dominant path [13]. The nLoS paths' influence is negligible, thus they are usually omitted [14]–[16]. Moreover, it is assumed that the user's device is fitted with a photodetector (PD) vertically facing upwards. From this assumption, it is extracted that the angle of incidence, which is denoted by ψ is equal to the angle of irradiance denoted by ϕ. Furthermore, d denotes the corresponding Euclidean distance and we assume that the VLC APs are indexed from the set $\mathcal{I} = \{1, \ldots, I\}$. Thus, the VLC channel from the i-th AP with $i \in \mathcal{I}$ to a user is given by [13], [17]

$$h_i = \frac{(m+1)A_{\mathrm{PD}}}{2\pi d_i^2} \cos^m(\phi_i) T_f g_c(\psi_i) \cos(\psi_i), \quad (1)$$

where $m = -\frac{\ln 2}{\ln(\cos \Phi_{1/2})}$ is the Lambertian emission order and $\Phi_{1/2}$ is the radiation angle at which the intensity is half of the intensity at the main-beam direction. Also, T_f is the gain of the optical filter, A_{PD} denotes the physical area of the PD, and $g_c(\psi_i)$ stands for the optical concentrator gain, which is given as [13], [18]

$$g_c(\psi_i) = \begin{cases} \frac{n_c^2}{\sin^2(\Psi_{\max})}, & 0 \leq \psi_i \leq \Psi_{\max} \\ 0, & \psi_i > \Psi_{\max}, \end{cases} \quad (2)$$

where n_c denotes the refractive index, and $\Psi_{\max}$ is the semi-angle of the field of view of the PD. Without loss of generality, the use of an orthogonal multiple access (OMA) scheme, e.g., optical orthogonal frequency division multiple access (OFDMA), single-carrier FDMA (SC-FDMA), time division multiple access (TDMA), and frequency reuse are applied in the network, thus there is neither intra- nor inter-cell interference. The SNR of the optical wireless link from the i-th AP to the user can be written as

$$\gamma_i = \frac{(h_i \eta P_i)^2}{\sigma_n^2}, \quad (3)$$

where η denotes the responsivity of the PD in the VLC receiver, P_i denotes the optical power that AP i allocates for communication to transmit the data-carrying signal, σ_n^2 is the variance of the photocurrent noise, accounting for thermal, background, dark, and shot noises, and modeled as additive white Gaussian noise with

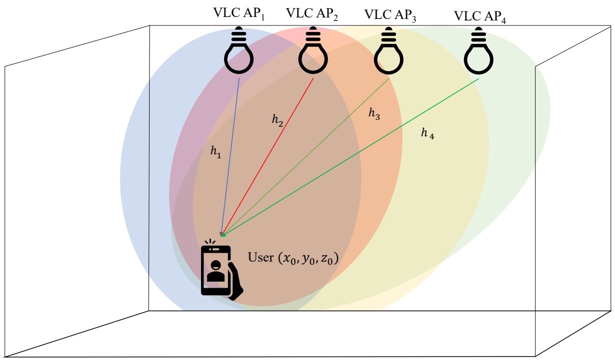

Fig. 1. System model.

$$\sigma_n^2 = \sigma_t^2 + \sigma_b^2 + \sigma_d^2 + \sigma_s^2, \quad (4)$$

where σ_t^2, σ_b^2, σ_d^2, and σ_s^2 denote the variances of thermal, background, dark, and shot noise, respectively, which depend on the load resistor of the trans-impedance amplifier (TIA) and the average photo-current I_b due to background noise.

III. Positioning with Channel Estimation Errors

In this section, the 3D trilateration algorithm is proposed that takes into account channel estimation errors. We assume that a user in the room can estimate their true channel, h_i, with an estimate $\hat{h}_i$. The estimation of the channel is given as [19]

$$h_i = \hat{h}_i + \epsilon_i, \quad (5)$$

where the error ϵ_i is a random variable following the Gaussian distribution with zero mean and for the i-th AP the variance is described by $\sigma_{\epsilon,i}^2$. The noise during the CSI estimation is AWGN [13], which means that it has zero mean and therefore the estimation error will also follow a Gaussian distribution with zero mean.

The user estimates the channel value from the received messages transmitted by the APs, with a certain SNR from each AP. Let S_i be the surface that characterizes the channel of the i-th AP, described by (1), which encloses a certain amount of volume. In order to uniquely determine the position of the user, the surfaces $S_i, \forall i \in \mathcal{I}$ must have a single intersection point. For this to happen we need more than 3 APs, since the intersection of two surfaces, without loss of generality say S_1, S_2, is a curve $f_{1,2}$ and the intersection of $f_{1,2}$ with S_3 would be at least 2 points, except for the special case that S_3 has only one touching point with $f_{1,2}$. This makes it impossible to identify the correct position of the user by deploying up to 3 APs.

Hence, for the proposed positioning system we assume that $I = 4$ VLC APs are deployed. Let their positions in Cartesian coordinates be described as $(x_i, y_i, z_i), \forall i \in \mathcal{I}$, and let z_i be the same for all APs. As stated in Section II, for the sake of simplicity, we have $\phi_i = \psi_i$. Also, in general, $\cos(\phi_i) = z_0/d_i$, where z_0 denotes the z-coordinate of the user. Solving (1) with respect to d_i yields the following relation

$$d_i = \left(c_i z_0^{m+1}\right)^{1/(m+3)}, \tag{6}$$

where

$$c_i = \frac{T_f g_c(\psi_i)(m+1)A_{\text{PD}}}{2\pi h_i}. \tag{7}$$

For the position of the user, we know that the Euclidean distance is given as

$$d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2}, \tag{8}$$

where (x_0, y_0, z_0) is the position of the user in space expressed in Cartesian coordinates. Combining (6) and (8), we get that for every AP the following equation holds

$$\left(c_i z_0^{m+1}\right)^{\frac{1}{m+3}} = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2}. \tag{9}$$

Using (9) for $i \in \mathcal{I}$, we obtain a system of non-linear equations in terms of x_0, y_0, z_0. To solve this system, we can implement numerical methods such as the Levenberg-Marquardt algorithm.

IV. Numerical Results

In this section, we present simulation results of the proposed method. For the simulations, we assumed that the user is inside a room of dimensions $7\text{m} \times 7\text{m} \times 3\text{m}$. The VLC APs were deployed in the positions shown in Table I, described in Cartesian coordinates. A Monte Carlo of over 10^4 user position realizations was utilized in order to evaluate the performance of the proposed system in the studied room. Also, we assumed that the channel estimation error is due to the noise variance σ_n^2, which is assumed to be constant and equal to a value estimated from a reference point, the center of the room. Therefore, for each AP i, $\sigma_{\epsilon,i}^2$ is expressed as

$$\sigma_{\epsilon,i}^2 = \sigma_n^2 = \frac{(\eta P_i)^2}{\gamma} h_{\text{av}}^2, \tag{10}$$

where $h_{\text{av}} = \left(\sum_{i\in\mathcal{I}} h_i\right)/I$ is the average channel. To better evaluate the performance of the proposed system, it is important to investigate the influence of the error in the channel estimation and how this affects the accuracy in the localization problem. For this reason, inspired by the amount of fading (AoF) metric, we introduce the amount of estimation error metric, defined as $\mathcal{E} = \left(\mathbb{E}[\hat{h}^2] - (\mathbb{E}[\hat{h}])^2\right)/(\mathbb{E}[\hat{h}])^2$ and in the considered system can be expressed as

$$\mathcal{E} = \frac{\sigma_\epsilon^2}{(\mathbb{E}[\hat{h}])^2} \tag{11}$$

with $\mathbb{E}[\cdot]$ denoting expectation. The main advantage of this metric is that it can be used to compare the main characteristic of the error, i.e., its variance, with the main characteristic of the localization accuracy, i.e., the real channel. Hence, smaller values of $\mathcal{E}$ correspond in better channel estimation, since the error is small compared to the mean channel of the user at his current location.

TABLE I
VLC APs Coordinates

VLC AP	Coordinates
1	$(-1.5, 1.5, 3)$
2	$(2, 2, 3)$
3	$(1.5, -1.5, 3)$
4	$(-2, -2, 3)$

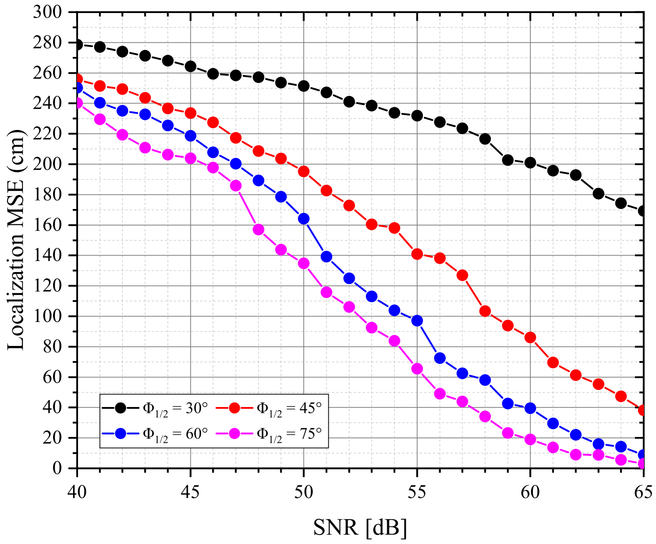

Fig. 2. Localization MSE error for varying values of transmit SNR, γ, and radiation angles, $\Phi_{1/2}$.

In Fig. 2, we investigate the performance of the proposed indoor VLP system in terms of mean squared error (MSE). As can be seen, the error performance of the system has two characteristic regions based on the transmit SNR that is used. For the low SNR region, it can be observed that the localization error in positioning attains relatively large values regardless of the radiation angle that is utilized. On the other hand, for the high SNR region, it is evident that the error in positioning grows smaller. This behavior is to be expected, because, as showcased by (10), increasing values of γ result in smaller values of σ_ϵ, which means that the estimation error grows smaller compared to the real value of the channel h as well. Hence, as γ increases, we can expect an improvement in the accuracy of the proposed positioning system. Furthermore, we can see that the greater the radiation angle $\Phi_{1/2}$ is the better the localization gets and with a steeper curve. This improvement is a result of the better coverage of the studied room as $\Phi_{1/2}$ increases because the equi-channel surfaces that describe the greater radiation angles are not as directional as those of the smaller radiation angles. Thus, in the case of small radiation angles the performance of the system is limited for two basic reasons, the fact that for most locations in the room, the user's channel is poor, but also because even if the user is located close to a VLC AP resulting in good channel conditions, their distance from the rest of the VLC APs remains large which deteriorates the accuracy of the system.

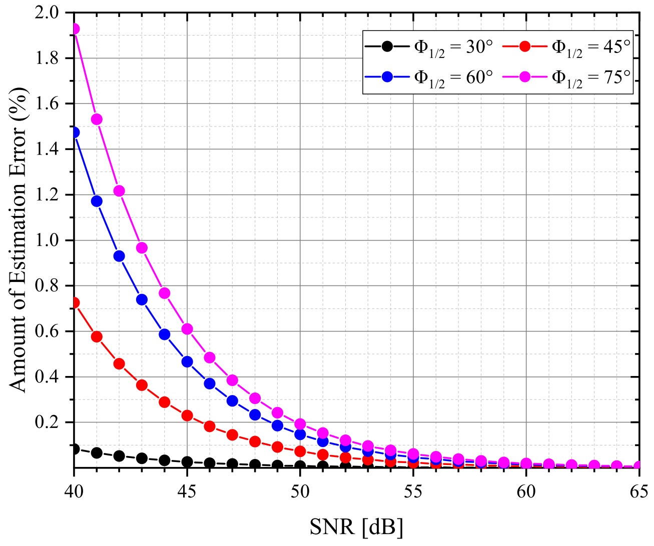

Fig. 3. Percentage amount of fading for varying values of transmit SNR, γ, and radiation angles, $\Phi_{1/2}$.

In Fig. 3, the percentage $\mathcal{E}$ of the proposed system is presented with regard to the SNR. As explained in Section III, the performance of the system improves, as $\mathcal{E}$ gets smaller. As expected, the large decrease of $\mathcal{E}$ in the larger radiation angles results in greater improvement in the accuracy, because the channel estimation is more robust. This decrease is of paramount importance since it is the main reason that allows the proposed VLP system to attain small MSE localization error regardless of the user's position in the room. This also highlights the fact that even though VLC conditions can be far from ideal in the vast majority of the room, i.e., the coverage is such that the channels can be relatively small compared to typical values, VLC can still be used to assist the user in retrieving accurate localization information.

V. Conclusions

In this paper, a VLP system based on the RSS method was evaluated when channel estimation errors plagued the reception. The channel estimation errors were modeled as an additive noise to the conventional LoS estimation of the VLC channel, so that they are general enough to capture both modeling errors and channel state imperfections. The performance of the VLP system was quantified under these terms and valuable design insights were obtained. The derived results agree with existing literature on the performance of VLP systems, while the use of the estimation errors can lead to performance bounds for similar RSS-based visible light localization systems.

Acknowledgment

This work was based upon work from European Union's Horizon 2020 COST Action CA19111 (European Network on Future Generation Optical Wireless Communication Technologies, NEWFOCUS). The work of P. S. Bouzinis and G. K. Karagiannidis was supported by the European Union's Horizon 2020 Research and Innovation Program under Agreement 957406. The work of P. D. Diamantoulakis and S. A. Tegos was supported by the Hellenic Foundation for Research and Innovation (H.F.R.I.) under the "3rd Call for H.F.R.I. Research Projects to support Post-Doctoral Researchers" (Project Number: 7280).

References

[1] Z. Zhang, Y. Xiao, Z. Ma, M. Xiao, Z. Ding, X. Lei, G. K. Karagiannidis, and P. Fan, "6g wireless networks: Vision, requirements, architecture, and key technologies," *IEEE Vehicular Technology Magazine*, vol. 14, no. 3, pp. 28–41, 2019.

[2] T.-H. Do and M. Yoo, "An in-Depth Survey of Visible Light Communication Based Positioning Systems," *Sensors*, vol. 16, no. 5, p. 678, May 2016.

[3] A. Yassin, Y. Nasser, M. Awad, A. Al-Dubai, R. Liu, C. Yuen, R. Raulefs, and E. Aboutanios, "Recent Advances in Indoor Localization: A Survey on Theoretical Approaches and Applications," *IEEE Commun. Surv. Tutorials*, vol. 19, no. 2, pp. 1327–1346, 2017.

[4] N. Chaudhary, L. N. Alves, and Z. Ghassemblooy, "Current Trends on Visible Light Positioning Techniques," in *2019 2nd West Asian Colloquium on Optical Wireless Communications (WACOWC)*. Tehran, Iran: IEEE, Apr. 2019, pp. 100–105.

[5] H.-S. Kim, D.-R. Kim, S.-H. Yang, Y.-H. Son, and S.-K. Han, "An Indoor Visible Light Communication Positioning System Using a RF Carrier Allocation Technique," *J. Lightwave Technol.*, vol. 31, no. 1, pp. 134–144, Jan. 2013.

[6] W. Gu, W. Zhang, J. Wang, M. R. A. Kashani, and M. Kavehrad, "Three dimensional indoor positioning based on visible light with Gaussian mixture sigma-point particle filter technique," in *Broadband Access Communication Technologies IX*, B. B. Dingel and K. Tsukamoto, Eds., vol. 9387. International Society for Optics and Photonics, 2015.

[7] E. Jeong, S. Yang, H. Kim, and S. Han, "Tilted receiver angle error compensated indoor positioning system based on visible light communication," *Electron. lett.*, vol. 49, no. 14, pp. 890–892, Jul. 2013.

[8] N. A. Mohammed and M. A. Elkarim, "Exploring the effect of diffuse reflection on indoor localization systems based on RSSI-VLC," *Opt. Express*, vol. 23, no. 16, p. 20297, Aug. 2015.

[9] M. Yasir, S.-W. Ho, and B. N. Vellambi, "Indoor Position Tracking Using Multiple Optical Receivers," *J. Lightwave Technol.*, vol. 34, no. 4, pp. 1166–1176, Feb. 2016.

[10] M. Aminikashani, W. Gu, and M. Kavehrad, "Indoor location estimation with optical-based orthogonal frequency division multiplexing communications," *Opt. Eng*, vol. 55, no. 5, p. 056116, May 2016.

[11] B. Zhou, V. Lau, Q. Chen, and Y. Cao, "Simultaneous Positioning and Orientating for Visible Light Communications: Algorithm Design and Performance Analysis," *IEEE Trans. Veh. Technol.*, vol. 67, no. 12, pp. 11 790–11 804, Dec. 2018.

[12] N. Chaudhary, O. I. Younus, L. N. Alves, Z. Ghassemlooy, S. Zvanovec, and H. Le-Minh, "An Indoor Visible Light Positioning System Using Tilted LEDs with High Accuracy," *Sensors*, vol. 21, no. 3, p. 920, Jan. 2021.

[13] T. Komine and M. Nakagawa, "Fundamental analysis for visible-light communication system using LED lights," *IEEE Transactions on Consumer Electronics*, vol. 50, no. 1, pp. 100–107, 2004.

[14] D. A. Basnayaka and H. Haas, "Design and Analysis of a Hybrid Radio Frequency and Visible Light Communication System," *IEEE Transactions on Communications*, vol. 65, no. 10, pp. 4334–4347, 2017.

[15] M. R. Zenaidi, Z. Rezki, M. Abdallah, K. A. Qaraqe, and M.-S. Alouini, "Achievable Rate-Region of VLC/RF Communications with an Energy Harvesting Relay," in *GLOBECOM 2017 - 2017 IEEE Global Communications Conference*. IEEE Press, 2017, p. 1–7.

[16] H. Kazemi, M. Safari, and H. Haas, "A Wireless Optical Backhaul Solution for Optical Attocell Networks," *IEEE Transactions on Wireless Communications*, vol. PP, pp. 1–1, 12 2018.

[17] H. Ma, L. Lampe, and S. Hranilovic, "Coordinated Broadcasting for Multiuser Indoor Visible Light Communication Systems," *IEEE Trans. Commun.*, vol. 63, pp. 3313–3324, 09 2015.

[18] J. Kahn and J. Barry, "Wireless infrared communications," *Proceedings of the IEEE*, vol. 85, no. 2, pp. 265–298, 1997.

[19] A. Papoulis and S. Pillai, *Probability, Random Variables, and Stochastic Processes*, ser. McGraw-Hill series in electrical engineering: Communications and signal processing. Tata McGraw-Hill, 2002.