

# Student-T Prior Sparse Bayesian Learning for Improved Channel Estimation in OTFS Systems

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**Abstract**—Orthogonal Time-Frequency-Space (OTFS) modulation can effectively suppress the effects of Doppler shift in high-speed mobile scenarios. At the same time, the accuracy of OTFS channel estimation is an important factor that affects the performance of OTFS. In this paper, we propose a Sparse Bayesian Learning (SBL) algorithm to quickly and accurately estimate the OTFS channel by combining the pilot pattern and the sparsity of the OTFS channel. First, we propose a new pilot pattern to prevent the contamination of information symbols on pilot symbols. Since the pilot pattern uses only partial guard symbols, it can also improve the spectral efficiency. Then, we propose the Student-T prior SBL (STSBL) algorithm to improve the speed and accuracy of OTFS channel estimation by exploiting the sparsity of the OTFS channel. Simulation results show that the normalized mean squared error (NMSE), bit error rate (BER), and throughput of the proposed scheme outperform the benchmark schemes.

**Index Terms**—OTFS, channel estimation, pilot pattern, Student-T prior Sparse Bayesian Learning.

## I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) modulation is a key technology in Long Term Evolution (LTE) [1]. In fifth generation (5G) and next generation mobile communication systems, the effects of Doppler shift become more severe in high-speed mobile scenarios. To address the problems caused by Doppler shift in such scenarios [2], Orthogonal Time-Frequency-Space (OTFS) modulation technology has been proposed as an alternative to OFDM [3]. OTFS has its own advantages, such as the channel is sparse and can be treated as time-invariant, and the Doppler shift can be effectively handled in the delay Doppler (DD) domain [4]. However, OTFS also faces challenges: 1) as a multi-carrier multiplexing technique, OTFS also faces the problem of high peak-to-average power ratio (PAPR) similar to OFDM [5], [6]; 2) due to delay and Doppler shift, OTFS symbols overlap in the DD domain, necessitating the use of non-linear detectors, while reducing the complexity of data detection has become an important OTFS research direction [7]; 3) the accuracy of OTFS channel estimation is critical in affecting OTFS performance.

As an approach to solve the OTFS channel estimation problem, the embedded single pilot (EP) design has been proposed in [1]. By surrounding pilot symbols with guard symbols, EP can avoid contamination of information symbols by pilot symbols. However, the guard symbols of EP reduce the spectral efficiency. To solve this problem, [8] proposes to use the full superposition pilot, where information symbols and pilot symbols overlap. This improves efficiency, but results in information contamination of the pilot symbols. In [9], a guard symbol-free frame structure is designed and the Laplace Prior Sparse Bayesian Learning (SBL) algorithm is applied to estimate the OTFS channel. This scheme improves the spectral efficiency but also overlooks the pilot symbol contamination by information symbols. [10] optimizes the algorithm of the literature [9] by using the method of reducing algorithm complexity in compressed sensing [11]–[14], but it still cannot avoid the pilot symbol pollution by information symbols.

Inspired by both pilot design and algorithm optimization [10], [15], this paper first proposes an SBL algorithm combined with a pilot pattern using partial guard symbols to solve the integral order Doppler shift channel estimation problem for OTFS systems. Furthermore, we propose the Student-T prior SBL (STSBL) algorithm to achieve higher accuracy in OTFS channel estimation.

The contributions of the paper as follows:

- Unlike EP [1] and being guard symbol free [9], [10], our partial guard pilot symbol scheme prevents contamination of pilot symbols by information symbols along the delay axis.
- The relax method is used to obtain a lower bound on the likelihood function for the received signal, thereby reducing the complexity of the STSBL algorithm.
- Simulations show that the performance results of the proposed scheme are better than Laplace prior SBL [9], Generalized Approximate Message Passing SBL (GAMP-SBL) [14], EP [1] and conventional Minimum Mean Square Error (MMSE).

This paper is organized as follows. Section II describes the

OTFS system model with integer order Doppler shift. The pilot pattern and the proposed STSBL algorithm model are derived in Section III. Section IV presents the simulations results and Section V concludes the paper.

## II. OTFS SYSTEM MODEL

At the transmitter of the OTFS systems, user data are first modulated using binary phase shift keying (BPSK). Then the BPSK symbols are mapped to the DD field which is represented  $x_d(k, l)$ ,  $k \in [0, N - 1]$ ,  $l \in [0, M - 1]$ . In the two-dimensional DD domain,  $k$  is the Doppler axis index,  $l$  is the delay axis index,  $N$  is the number of Doppler axis grid, and  $M$  is the number of delay axis grid. After mapping the BPSK symbols into the DD domain,  $x_d(k, l)$  is first converted from the DD domain to the time-frequency domain symbol  $X(n, m)$ ,  $n \in [0, N - 1]$ ,  $m \in [0, M - 1]$  through the Inverse Symplectic Fast Fourier Transform (ISFFT). The details of ISFFT process is presented as follows:

$$X(n, m) = \frac{1}{\sqrt{NM}} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} x_d(k, l) \exp(j2\pi(\frac{nk}{N} - \frac{ml}{M})). \quad (1)$$

Then we need to utilize the Heisenberg transform to move  $X(n, m)$  from the time-frequency domain to the time domain. Based on Heisenberg transformation process, the transmitted symbol is

$$s(t) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} X(n, m) \exp(j2\pi m \Delta f (t - nT)) \times g_{tx}(t - nT), \quad (2)$$

where  $T$ ,  $g_{tx}$  are sampling interval and the transmit pulse respectively,  $\Delta f = \frac{1}{T}$ . In OTFS system, the channel impulse response model is as follows:

$$h(\tau, v) = \sum_{i=1}^P h_i \delta(\tau - \tau_i) \delta(v - v_i), \quad (3)$$

where,  $h_i$  represents gain coefficient of the  $i$ -th path,  $\delta(\cdot)$  is Dirac function,  $\tau_i = \frac{l\tau_i}{M\Delta f}$ ,  $v_i = \frac{k_{v_i} + \kappa_{v_i}}{NT}$  represents delay and Doppler shift in the multipath model, respectively. The model established in this paper is integer order Doppler shift, we set  $\kappa_{v_i} = 0$  [4]. According to the analysis in [4], OTFS channel is sparse and has only  $P$  non-zero values with a sparse factor of  $\frac{P}{NM}$ .

At the receiver of OTFS system, Wigner transform and Symplectic Fast Fourier Transform (SFFT) are successively applied and transform received symbol to the symbol  $y(k, l)$  in DD domain as:

$$y(k, l) \approx \sum_{i=1}^P h_i \exp(j2\pi(\frac{l - l_{\tau_i}}{M}) \frac{k_{v_i}}{N}) \chi_i(k, l) \times x([k - k_{v_i}]_N, [l - l_{\tau_i}]_M) + n(k, l), \quad (4)$$

$$\chi_i(k, l) = \begin{cases} 1 & l_{\tau_i} \leq l < M \\ \frac{N-1}{N} \exp(-j2\pi(\frac{[k - k_{v_i}]_N}{N})) & 0 \leq l < l_{\tau_i}, \end{cases} \quad (5)$$

where  $[\cdot]_N$  is mod  $N$  operation,  $n(k, l)$  represents the complex additive white Gaussian noise (AWGN). After the information symbol of DD domain is obtained,  $y[k, l]$  can be recovered by linear minimum mean square error data detector [7].

## III. PROPOSED STSBL FOR CHANNEL ESTIMATION

In this part, we first introduce our pilot pattern in this paper. Then the STSBL algorithm for OTFS channel estimation proposed in this paper is described.

### A. Pilot pattern

Different with the pilot patterns in the literature [9] and [1] which do not exist the guard symbols or use guard symbols surround the pilot symbol, our pilot pattern utilizes the partial guard symbols to reduce the pilot symbols pollution from information symbols. The pilot symbols distribution in this paper is as follows:

$$x[k, l] = \begin{cases} x_p, & k \in [k_p - k_{max}, k_p + k_{max}], \\ & l \in [l_p, l_p + M_p] \\ x_d, & k \notin [k_p - k_{max} - N_p, k_p + k_{max} + N_p], \\ & l \notin [l_p - l_{max}, l_p + M_p] \\ 0, & \text{otherwise,} \end{cases} \quad (6)$$

where  $k_p$  and  $l_p$  represent the coordinates of the reference pilot symbol in the DD domain.

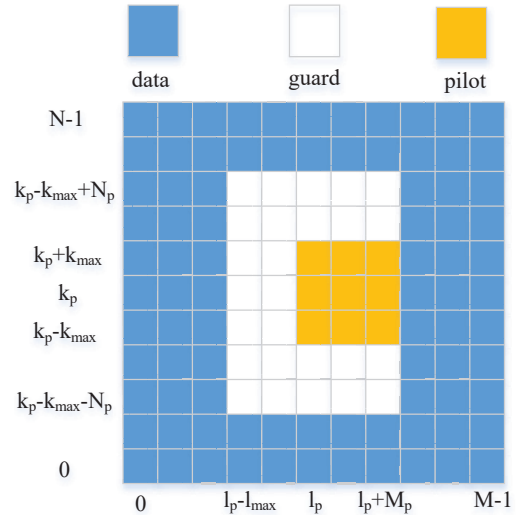


Fig. 1. The pilot pattern with partial guard symbols in DD domain.

After obtaining the position distribution of pilot symbols  $x_p$ , we can derive the symbol formula which will be used to channel estimation in DD domain:

$$y_p(k, l) = \sum_{l_1=0}^{l_{max}} \sum_{k_1=-k_{max}}^{k_{max}} w(k_1, l - l_1) h_{k_1, l_1} x_p(k - k_1, l - l_1) + n(k, l). \quad (7)$$

where  $w(k_1, l - l_1) = \exp(\frac{j2\pi(l - l_1)(k_1)}{MN})$ . Then we construct vector  $\mathbf{y}_p$  for channel estimation from the symbol  $y_p(k, l)$ . The

length of  $\mathbf{y}_p$  is  $(2k_{\max} + 1) \times (M_p + 1)$ . In SBL algorithm, the relationship between  $\mathbf{y}_p$ ,  $x_p$ , channel response vector  $\mathbf{h}$  is established using formula (8) and (9), as follows:

$$\mathbf{y}_p = \mathbf{A}_p \mathbf{h} + \mathbf{n}, \quad (8)$$

$$\mathbf{A}_p = (\mathbf{X}_p \odot \Phi), \quad (9)$$

where  $\odot$  is Hadamard product,  $\mathbf{y}_p$  is the observation vector;  $\mathbf{A}_p$  is sense matrix;  $\mathbf{X}_p$  is pilot symbol matrix which composed of pilot symbol  $x_p$ ;  $\mathbf{n}$  is complex AWGN vector,  $\Phi((l - l_p + (k + N_p)M_p), (l_1(2k_{\max} + 1) + k_1 + k_{\max})) = w(k_1, l - l_1)$ . Formula (8) is the mathematical model of STSBL algorithm proposed in this paper.

### B. Sparse Bayesian Learning

SBL is a machine learning algorithm. Due to the sparsity of signals or channels, SBL has been studied in the communication field [12]. This section introduces the STSBL algorithm proposed in this paper.

First, we assign the  $\mathbf{n}$  in the formula (8) to a normal distribution with zero mean and variance of  $\lambda^{-1}$ :

$$p(\mathbf{n}|\lambda) \sim \mathcal{CN}(\mathbf{n}|0, \lambda^{-1}\mathbf{I}), \quad (10)$$

where  $\mathbf{I}$  is unit matrix. According to the assumption of the hierarchical model in Bayesian inference [11], we make the hyperparameter  $\lambda$  obey the Gamma distribution and assign the parameters  $a, b$ .

$$p(\lambda) = \text{Gamma}(a, b) = \frac{b^a}{\Gamma(a)} \lambda^{a-1} \exp(-b\lambda), \quad (11)$$

where  $\Gamma(a) = \int_0^\infty t^{a-1} \exp(-t) dt$ .

After obtaining the distribution function of the noise, we can write the likelihood function of the symbols which are used to channel estimation as follows:

$$p(\mathbf{y}_p|\mathbf{A}_p, \lambda) = \mathcal{CN}(\mathbf{y}_p|\mathbf{A}_p \mathbf{h}, \lambda^{-1}\mathbf{I}). \quad (12)$$

Before the process of channel estimation using SBL, a prior model needs to be assigned to  $\mathbf{h}$ , which can reduce the variance. In this hierarchical model, the channel response  $\mathbf{h}$  is assigned a Student-T prior. The prior model is as follows:

$$p(\mathbf{h}) = \prod_{j=1}^n p(h_j) = \prod_{j=1}^n \int p(h_j|\alpha_j) p(\alpha_j) d\alpha_j, \quad (13)$$

$$p(h_j|\alpha_j) = \mathcal{CN}(h_j|0, \alpha_j^{-1}). \quad (14)$$

Because of assigning a prior distribution to  $\mathbf{h}$ , we assume that the hyperparameter  $\alpha_j$  follows the Gamma distribution of the parameter  $c, d$ :

$$p(\alpha_j) \sim \text{Gamma}(c, d). \quad (15)$$

After integrating  $\alpha_j$ , we can get Student-T prior of  $\mathbf{h}$ . In order to reduce the complexity of STSBL algorithm, we use Lemma 1 [11] to relax lower bound of equation (12). Let

$f(\mathbf{h}) : \mathbb{R}^n \rightarrow \mathbb{R}$  is a continuously differentiable function [13] with Lipschitz continuous gradient. Then we can get:

$$f(\mathbf{h}) \leq R(\mathbf{h}, \boldsymbol{\beta}) := \|\mathbf{y}_p - \mathbf{A}_p \boldsymbol{\beta}\| + 2(\mathbf{h} - \boldsymbol{\beta})^* (\mathbf{A}_p)^* (\mathbf{A}_p \boldsymbol{\beta} - \mathbf{y}_p) + s_0 \|\mathbf{h} - \boldsymbol{\beta}\|^2, \quad (16)$$

where  $(\cdot)^*$  represents the conjugate transpose operation,  $s_0 = \text{eig}(\Phi^T \Phi) + \tau$ ,  $\text{eig}(\cdot)$  is the maximum eigenvalue of a square matrix;  $\tau$  is a constant,  $\tau = 1e - 2$  in this paper;  $R(\cdot, \cdot)$  is correlation function; if and only if  $\mathbf{h} = \boldsymbol{\beta}$ ,  $f(\mathbf{h}) = R(\mathbf{h}, \boldsymbol{\beta})$ . Formula (12) can be rewritten:

$$p(\mathbf{y}_p|\mathbf{h}, \lambda) \geq \left(\frac{\lambda}{2\pi}\right)^{\frac{m_{\mathbf{A}_p}}{2}} \exp\left(-\frac{\lambda}{2} R(\mathbf{h}, \boldsymbol{\beta})\right), \quad (17)$$

where  $m_{\mathbf{A}_p}$  is row of matrix  $\mathbf{A}_p$ . Based on Bayesian inference [11], we can derive the posterior conditional probability distribution function of the channel response model  $\mathbf{h}$ :

$$p(\mathbf{h}|\mathbf{y}_p, \lambda, \boldsymbol{\alpha}^{-1}) \sim \mathcal{CN}(\boldsymbol{\mu}_h, \boldsymbol{\Sigma}_h), \quad (18)$$

$$\boldsymbol{\mu}_h = \lambda \boldsymbol{\Sigma}_h (s_0 \boldsymbol{\beta} - (\mathbf{A}_p)^* \mathbf{A}_p \boldsymbol{\beta} + (\mathbf{A}_p)^* \mathbf{y}_p), \quad (19)$$

$$\boldsymbol{\Sigma}_h = (\boldsymbol{\Gamma}^{-1} + s_0 \lambda \mathbf{I})^{-1}, \boldsymbol{\Gamma} = \text{diag}(\boldsymbol{\alpha}^{-1}), \quad (20)$$

where  $(\cdot)^{-1}$  represents inverse operation of matrix,  $\text{diag}(\cdot)$  denote change a vector to a diagonal matrix,  $\boldsymbol{\alpha} = [\alpha_1; \dots; \alpha_n]$ . The loss function of the model can be derived by combining formulas (13), (14):

$$\begin{aligned} \mathcal{L}(\mathbf{h}, \boldsymbol{\beta}, \sigma^2, \boldsymbol{\alpha}^{-1}) &= \sum_{j=1}^n (\ln(\sigma^2 + s_0 \alpha_j^{-1}) + c_1 \ln \alpha_j^{-1} \\ &\quad + \frac{2d + h_j^2}{\alpha_j^{-1}}) + \frac{R(\mathbf{h}, \boldsymbol{\beta}) + 2b_0}{\sigma^2} \\ &\quad - e \ln \sigma^2, \end{aligned} \quad (21)$$

where  $e = n_{\mathbf{A}_p} + 2 - m_{\mathbf{A}_p} - 2a$ ,  $c_1 = 2c - 2$ ,  $n_{\mathbf{A}_p}$  denotes column of matrix  $\mathbf{A}_p$ .

Finally, according to the steps of maximization and minimization (MM) [11], we can replace the minimization loss function with a iteratively minimize convex surrogate function:

$$(\hat{\mathbf{h}}^{(i+1)}, \hat{\boldsymbol{\beta}}^{(i+1)}, \boldsymbol{\alpha}^{-1(i+1)}) \in \arg \min_{\mathbf{h}, \boldsymbol{\beta}, \boldsymbol{\alpha}^{-1}} \hat{\mathcal{L}}(\mathbf{h}, \boldsymbol{\beta}, \boldsymbol{\alpha}^{-1}) \quad (22)$$

Due to the function  $\hat{\mathcal{L}}(\mathbf{h}, \boldsymbol{\beta}, \boldsymbol{\alpha}^{-1})$  is decomposable, we can use the block coordinate descent (BCD) method to iteratively update the parameters [11]. When the gradient is zero, we can update the parameter:

$$\boldsymbol{\Sigma}_h^{(i)} = (\boldsymbol{\Gamma}^{(i-1)} + \frac{s_0}{(\sigma^2)^{(i)}} \mathbf{I})^{-1}, \quad (23)$$

$$\hat{\mathbf{h}}^{(i+1)} = \frac{1}{(\sigma^2)^{(i)}} \boldsymbol{\Sigma}_h^{(i)} (s_0 \boldsymbol{\beta}^{(i)} - (\mathbf{A}_p)^* \mathbf{A}_p \boldsymbol{\beta}^{(i)} + (\mathbf{A}_p)^* \mathbf{y}_p), \quad (24)$$

$$\hat{\boldsymbol{\beta}}^{(i+1)} = \hat{\mathbf{h}}^{(i+1)}, \quad (25)$$

$$\rho^{(i+1)} = \sum_{j=1}^n \frac{1}{(\sigma^2)^{(i)} + s_0 \alpha_j^{-1(i)}}, \quad (26)$$

$$(\sigma^2)^{(i+1)} = \frac{e + \sqrt{e^2 + 4\rho^{(i+1)}(\|\mathbf{y}_p - \mathbf{A}_p \boldsymbol{\beta}^{(i+1)}\|^2 + 2b)}}{2\rho^{(i+1)}}, \quad (27)$$

$$\alpha_j^{-1(i+1)} = \sqrt{\frac{\alpha_j^{-1(i)}((\sigma^2)^{(i)} + s_0 \alpha_j^{-1(i)})(2d + h_j^{(i+1)^2})}{(c_1 + 1)s_0 \alpha_j^{-1(i)} + c_1(\sigma^2)^{(i)}}}, \quad (28)$$

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**Algorithm 1: The STSBL scheme for OTFS channel estimation**


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**Input:**  $\mathbf{y}_p, \mathbf{A}$ 
**Initialization:**

Set  $i = 1$ , and initialize  $a = b = d = 10^{-6}, c = 1 + 10^{-6}, \alpha_j^{-1(0)} = 1, \lambda(0) = (\sigma^2)^{(0)} = \text{var}(\mathbf{y}_p)/100, \beta(0) = (\mathbf{A}_p)^* \mathbf{y}_p$  and maximum iteration iter  $i_{\max} = 200$ ;

**Repeat:**

1. Calculate  $\hat{\mathbf{h}}^0$  and  $\Sigma_{\mathbf{h}}^0$  using  $s_0, (\sigma^2)^{(0)}$  and  $\alpha_i^{-1(0)}$ ;
2. Update  $\sigma^2$  and  $\alpha_j^{-1}$  use formula (25), (26), (27), (28);
3.  $i = i + 1$ ;

**Continue the iterations until:**

$$i = i_{\max} \text{ or } \frac{\|\hat{\mathbf{h}}^{(i+1)} - \hat{\mathbf{h}}^{(i)}\|}{\|\hat{\mathbf{h}}^{(i+1)}\|} \leq 10^{-5}.$$

**Output:**  $\hat{\mathbf{h}}$ 


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#### IV. SIMULATION RESULTS

In this section, we first compare the Normalised Mean Squared Error (NMSE) performance of the proposed STSBL scheme with four benchmark schemes, Laplace prior SBL [9], GAMP-SBL [14], EP [1] and MMSE.

$$\text{NMSE} = 10 \log_{10} \left( \frac{\mathbb{E}(\|\mathbf{h} - \hat{\mathbf{h}}\|_2^2)}{\mathbb{E}(\|\mathbf{h}\|_2^2)} \right). \quad (29)$$

We also verify the Bit Error Rate (BER) of the STSBL scheme and the benchmark schemes to further validate its channel estimation accuracy, comparing them with perfect channel state information (CSI). Additionally, we analyse the NMSE performance of the STSBL scheme under different pilot ratio by adjusting  $M_p$ .

##### A. Parameters

In our simulation, the OTFS channel follows the extended vehicle model [9], with using Jason's formula generate Doppler shift [9]. We set  $\Delta f = 20 \text{ kHz}$ , the carrier frequency is  $4 \text{ GHz}$ , and the average power of the pilot is  $p = 3$ . Finally,  $l_{\max}, k_{\max}$  also meet  $l_{\max} \leq 2N_p + 1, k_{\max} \leq M_p$ . The initial value of the proposed STSBL algorithm is set as follows:  $a = 10^{-6}, b = 10^{-6}, c = 1 + 10^{-6}, d = 10^{-6}$ , and the decision threshold is  $10^{-5}$ .

TABLE I  
SNR = 10dB, RUNNING TIME OF DIFFERENT SCHEMES.

Algorithm	runtime(s/frame)
Laplace prior SBL [9]	0.04
GAMP-SBL [14]	0.06
EP [1]	0.0001
MMSE	0.001
<b>STSBL</b>	<b>0.004</b>

##### B. NMSE

Fig. 2 illustrates the NMSE performance versus the average frame symbols, demonstrating that the proposed STSBL outperforms benchmark schemes. This can be attributed to the fact that the proposed pilot pattern alleviates contamination from information symbols. Table.I compares the running time of different schemes, showing that STSBL performs fairly close with MMSE and EP, and is obviously faster than the other two schemes.

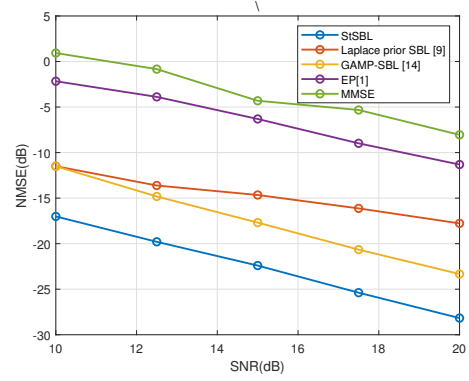


Fig. 2. NMSE (dB) of different schemes with  $N = 64, M = 128, P = 9, M_p = 10, N_p = 5$ .

##### C. BER

To further verify the effectiveness of STSBL scheme in improving channel estimation accuracy for the OTFS system, we compare the BER performance of the proposed STSBL scheme with benchmark schemes. Fig.3 illustrates the BER curves versus signal-to-noise ratio (SNR), showing that STSBL performs the best and exhibits BER performance similar to that of perfect CSI. The throughput results are depicted in Fig.4, where the proposed STSBL scheme also performs the best.

##### D. NMSE with different $M_p$

Finally, we compare the NMSE performance of STSBL with that of the benchmark schemes when the proportion of pilot symbols is different at  $M_p = 11, 12, 13, 14, 15$ . In the case of SNR = 18dB, it can be seen from Fig.5 that with the increase of  $M_p$ , the NMSE values of the five schemes are almost constant and the NMSE value of the STSBL scheme proposed is always lower than the other four benchmark schemes.

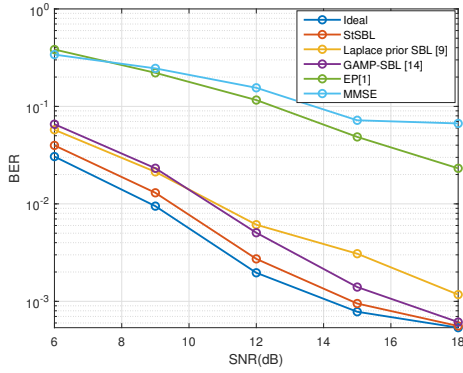


Fig. 3. Comparison of BER under the  $N = 64, M = 128, P = 9, M_p = 10, N_p = k_{max} = 5$ .

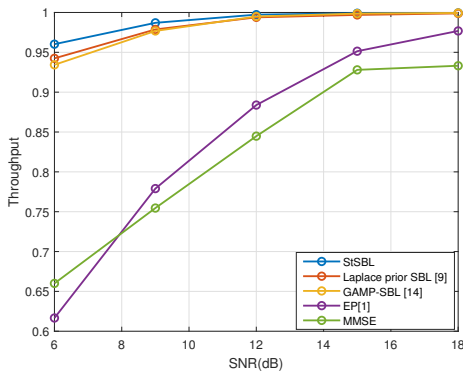


Fig. 4. Throughput vs SNR (dB) with  $N = 64, M = 128, P = 9, M_p = 10, N_p = k_{max} = 5$ .

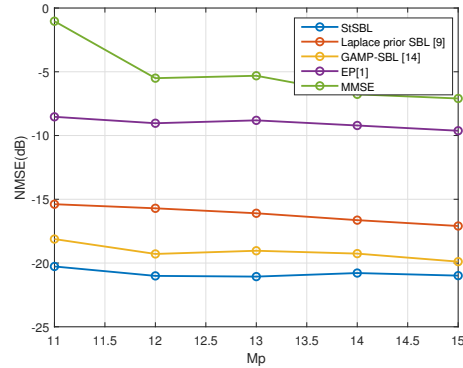


Fig. 5. Influence of different  $M_p$  for five schemes' NMSE.

## V. CONCLUSION

In this paper, a new pilot pattern and STSBL algorithm are proposed to estimate the OTFS channel more accurately. The new pilot pattern reduces the contamination of pilot symbols by information symbols along the delay axis by using partial guard symbols. The proposed STSBL algorithm achieves more accurate CSI in less time. Simulation results verify that the proposed scheme outperforms GAMP-SBL,

Laplace prior SBL, EP, and MMSE schemes in terms of NMSE, BER, and throughput performance.

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## REFERENCES

- [1] P. Raviteja, K. T. Phan and Y. Hong, "Embedded Pilot-Aided Channel Estimation for OTFS in Delay-Doppler Channels," in IEEE Transactions on Vehicular Technology, vol. 68, no. 5, pp. 4906-4917, May 2019, doi: 10.1109/TVT.2019.2906357.
- [2] P. Raviteja, K. T. Phan, Y. Hong and E. Viterbo, "Interference Cancellation and Iterative Detection for Orthogonal Time Frequency Space Modulation," in IEEE Transactions on Wireless Communications, vol. 17, no. 10, pp. 6501-6515, Oct. 2018, doi: 10.1109/TWC.2018.2860011.
- [3] R. Hadani et al., "Orthogonal Time Frequency Space Modulation," 2017 IEEE Wireless Communications and Networking Conference (WCNC), San Francisco, CA, USA, 2017, pp. 1-6, doi: 10.1109/WCNC.2017.7925924.
- [4] P. Raviteja, Y. Hong, E. Viterbo and E. Biglieri, "Practical Pulse-Shaping Waveforms for Reduced-Cyclic-Prefix OTFS," in IEEE Transactions on Vehicular Technology, vol. 68, no. 1, pp. 957-961, Jan. 2019, doi: 10.1109/TVT.2018.2878891.
- [5] X. Liu, L. Cao, Z. Zhang, J. Hu, Y. Zhou and J. Shi, "Low Complexity Design of Peak-to-average power ratio (PAPR) Reduction Scheme for OTFS Modulation," 2022 10th International Conference on Information Systems and Computing Technology (ISCTech), Guilin, China, 2022, pp. 151-155, doi: 10.1109/ISCTech58360.2022.00031.
- [6] W. Liu, L. Zou, B. Bai and T. Sun, "Low PAPR channel estimation for OTFS with scattered superimposed pilots," in China Communications, vol. 20, no. 1, pp. 79-87, Jan. 2023, doi: 10.23919/JCC.2023.01.007.
- [7] T. Thaj and E. Viterbo, "Low Complexity Iterative Rake Decision Feedback Equalizer for Zero-Padded OTFS Systems," in IEEE Transactions on Vehicular Technology, vol. 69, no. 12, pp. 15606-15622, Dec. 2020, doi: 10.1109/TVT.2020.3044276.
- [8] H. B. Mishra, P. Singh, A. K. Prasad and R. Budhiraja, "OTFS Channel Estimation and Data Detection Designs With Superimposed Pilots," in IEEE Transactions on Wireless Communications, vol. 21, no. 4, pp. 2258-2274, April 2022, doi: 10.1109/TWC.2021.3110659.
- [9] L. Zhao, W. -J. Gao and W. Guo, "Sparse Bayesian Learning of Delay-Doppler Channel for OTFS System," in IEEE Communications Letters, vol. 24, no. 12, pp. 2766-2769, Dec. 2020, doi: 10.1109/LCOMM.2020.3021120.
- [10] K. Shu, X. He, L. Shi and N. Chen, "An OTFS Channel Estimation Scheme Based on Efficient Sparse Bayesian Learning," 2022 IEEE/CIC International Conference on Communications in China (ICCC), Sanshui, Foshan, China, 2022, pp. 150-155, doi: 10.1109/ICCC55456.2022.9880804.
- [11] W. Zhou, H. -T. Zhang and J. Wang, "An Efficient Sparse Bayesian Learning Algorithm Based on Gaussian-Scale Mixtures," in IEEE Transactions on Neural Networks and Learning Systems, vol. 33, no. 7, pp. 3065-3078, July 2022, doi: 10.1109/TNNLS.2020.3049056.
- [12] X. Zhang, F. Labeau, L. Hao and J. Liu, "Joint Active User Detection and Channel Estimation Via Bayesian Learning Approaches in MTC Communications," in IEEE Transactions on Vehicular Technology, vol. 70, no. 6, pp. 6222-6226, June 2021, doi: 10.1109/TVT.2021.3077569.
- [13] H. Duan, L. Yang, J. Fang and H. Li, "Fast Inverse-Free Sparse Bayesian Learning via Relaxed Evidence Lower Bound Maximization," in IEEE Signal Processing Letters, vol. 24, no. 6, pp. 774-778, June 2017, doi: 10.1109/LSP.2017.2692217.
- [14] M. Al-Shoukairi, P. Schniter and B. D. Rao, "A GAMP-Based Low Complexity Sparse Bayesian Learning Algorithm," in IEEE Transactions on Signal Processing, vol. 66, no. 2, pp. 294-308, 15 Jan.15, 2018, doi: 10.1109/TSP.2017.2764855.
- [15] S. Wang, J. Guo, X. Wang, W. Yuan and Z. Fei, "Pilot Design and Optimization for OTFS Modulation," in IEEE Wireless Communications Letters, vol. 10, no. 8, pp. 1742-1746, Aug. 2021, doi: 10.1109/LWC.2021.3078527.