BER Analysis of Dual Equal Gain Diversity over Generalized Fading Channels

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Abstract

In this paper, a novel characteristic function (CHF)-based approach to the evaluation of the BER performance in predetection dual Equal Gain Combining (EGC) systems over correlated Nakagami-m fading channels is presented. Expressions for the average symbol error probability (ASEP) in both coherent and non-coherent modulation schemes are derived. Numerical results and simulations are also presented to illustrate the proposed mathematical analysis and to point out the effect of the input SNRs unbalancing, the fading severity as well as the fading correlation on the average error probability.

1. Introduction

Signals from multiple antennas, or "spatial diversity," can be used to reduce the effects of fast fading in wireless communications systems and to improve received signal strength. The most popular diversity techniques are Selection Combining (SC), Equal Gain Combining (EGC) and Maximal Ratio Combining (MRC). Among these types of diversity combining, EGC provides an intermediate solution as far as the performance and the implementation complexity are concerned. The performance analysis of predetection EGC, assuming independent channel fading, has been studied extensively in the literature. A comprehensive summary with most of the related work is included in [1]. However, independent fading assumes antenna elements be placed sufficiently apart, which is not always realized in practice due to insufficient antenna spacing, when diversity is applied in compact terminals. In this kind of terminals, the fading among the channels is correlated resulting in a degradation of the diversity gain obtained. Therefore, it is important to understand how the correlation between received signals affects the offered diversity gain. From reviewing the literature there are few approaches for the performance evaluation of predetection EGC over correlated fading channels. In [2] a formula for the error probability of EGC with orthogonal Binary Frequency Shieft Keying (BFSK) operating in correlative Rician time-selective fading, is proposed. Recently, Malik et. al. in [3] presented a useful approach to the performance analysis for coherent detection of binary signals with dual-diversity predetection EGC over correlated Rayleigh channels.

In this paper, deriving an infinite series representation for the CHF of two correlated Nakagami-*m* variables and using the Annamalai et. al. [4] unified framework for the performance analysis of EGC systems in independent fading, an efficient approach to the BER performance of dual predetection EGC in correlative Nakagami-*m* fading is proposed. Coherent, non-coherent and multilevel signaling schemes are assumed. Simulations are performed to check the accuracy of the proposed mathematical analysis. The effect of the input SNR unbalance as well as the fading correlation on the average error probability is pointed out.

The remainder of this paper is organized as follows: In Section 2, useful expressions are derived for several modulation schemes, while in Section 3 simulations and numerical results are presented to illustrate the proposed mathematical analysis. Finally, some concluding remarks are offered in Section 4.

2. BER Analysis

In a dual predetection EGC system, the instantaneous output SNR is $\gamma = a^2$, where a is given by [1, eq. (9.46)]

$$a = \sqrt{\frac{E_s}{2N_0}}\left(a_1 + a_2\right) \qquad (1)$$

with a_1, a_2 be the correlated Nakagami-*m* envelopes of the input signals [5], E_s is the energy per symbol and N_0 is the AWGN power spectral density, assumed equal for both two branches. The joint pdf of a_1, a_2 is [5]

$$f_{a_1,a_2}\left(a_1,a_2\right) = \frac{4\left(a_1 a_2\right)^m e^{-\frac{\Omega_2 a_1^2 + \Omega_1 a_2^2}{\Omega_1 \Omega_2 (1-\rho)}}}{\Gamma(m)\Omega_1 \Omega_2 (1-\rho)\left(\sqrt{\rho \Omega_1 \Omega_2}\right)^{m-1}} \times I_{m-1}\left(\frac{2\sqrt{\rho}\, a_1 a_2}{\sqrt{\Omega_1 \Omega_2}\,(1-\rho)}\right) \qquad (2)$$

with $\Gamma(\bullet)$ being the Gamma function, $I_\nu(\bullet)$ being the first kind and ν^{th} order modified Bessel function,

$\Omega_i = \overline{a_i^2}/m$, with $\overline{a_i^2}$ being the average signal power at the i^{th} branch, ρ is the correlation coefficient and m is a parameter describing the fading severity. The CHF of $f_{a_1,a_2}(a_1,a_2)$ is by definition [6]

$$\Phi_{a_1,a_2}(s_1,s_2) = \int_0^\infty\int_0^\infty f_{a_1,a_2}(a_1,a_2) e^{-js_1a_1 - js_2a_2} da_1 da_2 \quad (3)$$

and the CHF of $f_{a_1+a_2}(a_1,a_2)$ is given by [6]

$$\Phi_{a_1+a_2}(s) = \int_0^\infty\int_0^\infty f_{a_1,a_2}(a_1,a_2) e^{-js(a_1+a_2)} da_1 da_2 \quad (4)$$

Substituting the Bessel function in (2) with its infinite series representation [7, eq. (9.6.10)] and as the summand is Riemann integrable and converges uniformly in the range $[0,\infty)$, we may interchange the order of summation and integration resulting in

$$\Phi_{a_1+a_2}(s) = \frac{4(1-\rho)^{-m}}{\Gamma(m)[\Omega_1\Omega_2]^m} \sum_{k=0}^{\infty} \frac{(1-\rho)^{-2k}}{k!\Gamma(m+k)} \left(\frac{\rho}{\Omega_1\Omega_2}\right)^k$$
$$\times \int_0^\infty a_1^{2m+2k-1} e^{-jsa_1 - \frac{a_1^2}{\Omega_1(1-\rho)}} da_1 \int_0^\infty a_2^{2m+2k-1} e^{-jsa_2 - \frac{a_2^2}{\Omega_2(1-\rho)}} da_2 \quad (5)$$

The integrals in (5) can be solved using [8, eq.(3.462/1)] which gives

$$\Phi_{a_1+a_2}(s) = \frac{(1-\rho)^m 2^{2-2m}}{\Gamma(m)} \sum_{k=0}^{\infty} \frac{[\Gamma(2m+2k)]^2 e^{\frac{s^2(1-\rho)(\Omega_1+\Omega_2)}{8}}}{2^{2k}\rho^{-k} k!\Gamma(m+k)}$$
$$\times D_{-(2m+2k)}\left(\frac{s\sqrt{\Omega_1(1-\rho)}}{\sqrt{2}}\right) D_{-(2m+2k)}\left(\frac{s\sqrt{\Omega_2(1-\rho)}}{\sqrt{2}}\right) \quad (6)$$

where $D_{-\nu}(z)$ is the parabolic cylinder function of order ν and argument z [8, eq. (9.240)]. Finally using [8 eq. (9.240)], [7, eq. (6.118)] and after some algebraic manipulations the CHF of a_1+a_2 can be restated in terms of the more familiar confluent hypergeometric function of the first kind ${}_1F_1$ [8, eq. (9.210)] as

$$\Phi_{a_1+a_2}(s) = \frac{(1-\rho)^m}{\Gamma(m)} \sum_{k=0}^{\infty} \frac{\rho^k A(\Omega_1) A(\Omega_2)}{k!\Gamma(m+k)} \quad (7)$$

with

$$A(x) = \Gamma(m+k)\, {}_1F_1\left(m+k; \frac{1}{2}; -\frac{x(1-\rho)s^2}{4}\right)$$
$$+ js\sqrt{x(1-\rho)}\,\Gamma\left(m+k+\frac{1}{2}\right) {}_1F_1\left(m+k+\frac{1}{2}; \frac{3}{2}; -\frac{x(1-\rho)s^2}{4}\right) \quad (8)$$

Taking into account that if $y = kx$ then $\Phi_y(s) = \Phi_x(ks)$ [6] and that the input average SNRs are $\overline{\gamma_i} = \overline{a_i^2}\frac{E_s}{N_0} = m\Omega_i\frac{E_s}{N_0}$, the CHF of a can be finally written as

$$\Phi_a(s) = \frac{(1-\rho)^m}{\Gamma(m)} \sum_{k=0}^{\infty} \frac{\rho^k}{k!\Gamma(m+k)} A\left(\frac{\overline{\gamma_1}}{2m}\right) A\left(\frac{\overline{\gamma_2}}{2m}\right) \quad (9)$$

2.1. BPSK, non-coherent BFSK, Multilevel Signaling (MPSK, MQAM)

Using the Parseval's Theorem approach, presented in [4], the ASEP can be evaluated for several coherent and non-coherent modulation schemes as

$$P_s = \frac{1}{\pi}\int_0^\infty \text{Real}\{G^*(s)\Phi(s)\} ds \quad (10)$$

where $G^*(s)$ denotes the complex conjugate of the Fourier transform of the conditional error probability. For Binary Differential Phase Shift Keying (BDPSK) and non-coherent Binary Frequency Shift Keying (BFSK), $G(s)$ is given in [4, eq.(10)] as

$$G(s) = \frac{b_1}{\sqrt{b_2}}\left[\frac{\sqrt{\pi}}{2} e^{-\frac{s^2}{4b_2}} + jF\left(\frac{s}{2\sqrt{b_2}}\right)\right] \quad (11)$$

where $b_1 = 0.5, b_2 = 1$ for BDPSK, $b_1 = 0.5, b_2 = 0.5$ for BFSK and $F(\bullet)$ denotes the Dawson's integral

$$F(x) = e^{-x^2}\int_0^x e^{t^2} dt = x\, {}_1F_1\left(1, \frac{3}{2}, -x^2\right) \quad (12)$$

since for M-ary Phase Shift Keying (MPSK) or M-ary Quadrature Amplitude Modulation (MQAM), $G(s)$ is given by [4, eq. (13)] as

$$G(s) = \frac{2b_1}{s\sqrt{\pi}} F\left(\frac{s}{2\sqrt{b_2}}\right)$$
$$- \frac{4b_3}{s\sqrt{\pi}}\left[F\left(\frac{s}{2\sqrt{b_2}}\right) - F\left(\frac{s}{2\sqrt{2b_2}}\right) e^{-\frac{s^2}{8b_2}}\right]$$
$$+ j\left[\frac{b_1}{s}\left[1 - e^{-\frac{s^2}{4b_2}}\right] - \frac{b_3}{s}\left[1 - e^{-\frac{s^2}{4b_2}} - \frac{4}{\pi}F^2\left(\frac{s}{2\sqrt{2b_2}}\right)\right]\right] \quad (13)$$

where $b_1 = 1$, $b_2 = 0.5$, $b_3 = 0.25$ for QPSK and $b_1 = 2(1 - 1/\sqrt{M})$, $b_2 = 1.5/(M-1)$, $b_3 = (1 - 1/\sqrt{M})^2$ for MQAM.

Some points about the evaluation of the integral in (10) are necessary to be discussed here. Let

$$g(s) = \text{Real}\{G^*(s)\Phi(s)\} \quad (14)$$

As $s \to 0$, the limit of $g(s)$ is

$$\frac{b_l(1-\rho)}{2\Gamma(m)\sqrt{b_2}\,\pi}\sum_{k=0}^{\infty}\frac{\rho^k\Gamma(m+k)}{k!}$$ for non-coherent NDPSK and BFSK and zero for MPSK and MQAM. Moreover, while $s\to\infty$, it is obvious through observation that $g(s)=0$ for both cases. Hence, if numerical integration is performed, it is only needed to integrate $g(s)$ over a finite interval $[0,u]$. However, although $g(s)$ is continuous in $[0,u]$, it oscillates between peaks and valleys near to zero, making difficult its approximation with a single polynomial, as e.g. Tsebyshev. Hence in this case is suitable instead of using a single polynomial for the whole range $[0,u]$, to use piecewise Gaussian Quadrature numerical integration. Such a technique was used in [9] for the evaluation of the outage probability in cellular systems.

2.2. BPSK and coherent BFSK

In the case of coherent binary signaling schemes with EGC, an expression for the average BER performance is given in [4, eq. (19)]

$$P_s=\frac{1}{2}-\frac{1}{2\pi}\int_0^{\infty}t^{-1}e^{-t}\,\mathrm{Imag}\left\{\Phi_\alpha\left(2\sqrt{bt}\right)\right\}dt \tag{15}$$

where $b=1$ for Binary Phase Shift Keying (BPSK) and $b=0.5$ for coherent BFSK. After some algebraic manipulations, it can be shown that

$$\mathrm{Imag}\left\{\Phi_\alpha\left(2\sqrt{bt}\right)\right\}=\frac{(1-\rho)^{m+1/2}\sqrt{2b}}{\sqrt{m}\,\Gamma(m)}\times\sum_{k=0}^{\infty}\frac{\rho^k\Gamma(m+k+1/2)}{k!}\left[G\left(t,\overline{\gamma_1},\overline{\gamma_2}\right)+G\left(t,\overline{\gamma_2},\overline{\gamma_1}\right)\right] \tag{16}$$

where

$$G(t,x,y)=\sqrt{ty}\,{}_1F_1\left(m+k;\frac{1}{2};-ctx\right)\times{}_1F_1\left(m+k+\frac{1}{2};\frac{3}{2};-cty\right) \tag{17}$$

and $c=b(1-\rho)/2m$. Applying the Kummer's transformation formula [7, eq. (13.1.27)] for the confluent hypergeometric functions in (16), the integrals produced in (15) have the following form

$$I=\int_0^{\infty}t^{v-1}e^{-wt}{}_1F_1\left(u_1,v_1,z_1t\right){}_1F_1\left(u_2,v_2,z_2t\right)dt \tag{18}$$

and can be solved using [4, Appendix C]. Finally, combining (15), (16) and (17) we obtain the final expression for the average BER performance of BPSK and coherent BFSK as shown at the bottom of the page, where F_2 is the hypergeometric function of two variables [8, eq. (9.180/20].

For $m=1$ (19) gives the same results as in [3, eq. (15)]. The differences in the final formulae are due to the use of the Hermite representation for ${}_1F_1$ in [3]. For $\rho=0$ in (19), only the $k=0$ term of the summation over k is nonzero which leads to the same formulation as in [4, eq. (20)].

3. Simulations and Numerical Results

Simulations were performed and the results were compared to the corresponding of the proposed mathematical analysis in order to check the accuracy of the derived formulae. The computer simulation was written in C^{++} programming language using the algorithm presented in the Appendix of [10]. For the generation of the correlated Nakagami-*m* fading envelops, over a million samples were used. Figs. 1 and 2 plot the performance of BPSK employing EGC versus the average SNR per branch, $\left(\overline{\gamma_1}+\overline{\gamma_2}\right)/2$, for equal average SNRs $\left(\overline{\gamma_1}=\overline{\gamma_2}\right)$ as well as for unequal branch SNRs $\left(\overline{\gamma_1}=\overline{\gamma_2}/5\right)$ correspondingly and for several values of the parameters *m* and ρ. Furthermore in Fig. 3 the dual EGC receiver performance of BPSK is plotted versus the correlation coefficient ρ, for several values of the input branch SNRs and the parameter *m*. It is evident that the proposed mathematical analysis gives exact results compared to simulations. As it was expected, for a given SNR per branch the BER performance deteriorates with an increase of ρ, while it improves with an increase of *m*. Moreover, it is observed that a system with equal branch SNRs performs better than that with unequal branch SNRs. Finally, it is evident from Fig. 3, that with an increase of the input branch SNR, the BER performance is more sensitive to the fading correlation.

$$P_s=\frac{1}{2}-\frac{\sqrt{b}(1-\rho)^{m+1/2}\Gamma(1/2)}{\sqrt{2m}\,\pi\Gamma(m)\left[c\left(\overline{\gamma_1}+\overline{\gamma_2}\right)+1\right]^{1/2}}\sum_{k=0}^{\infty}\frac{\rho^k\Gamma(m+k+1/2)}{k!}\left[\sqrt{\overline{\gamma_2}}F_2\left(\frac{1}{2};\frac{1}{2}-m-k,1-m-k;\frac{1}{2},\frac{3}{2};\frac{c\overline{\gamma_1}}{c\left(\overline{\gamma_1}+\overline{\gamma_2}\right)+1},\frac{c\overline{\gamma_2}}{c\left(\overline{\gamma_1}+\overline{\gamma_2}\right)+1}\right)+\sqrt{\overline{\gamma_1}}F_2\left(\frac{1}{2};1-m-k,\frac{1}{2}-m-k;\frac{3}{2},\frac{1}{2};\frac{c\overline{\gamma_2}}{c\left(\overline{\gamma_1}+\overline{\gamma_2}\right)+1},\frac{c\overline{\gamma_1}}{c\left(\overline{\gamma_1}+\overline{\gamma_2}\right)+1}\right)\right] \tag{19}$$

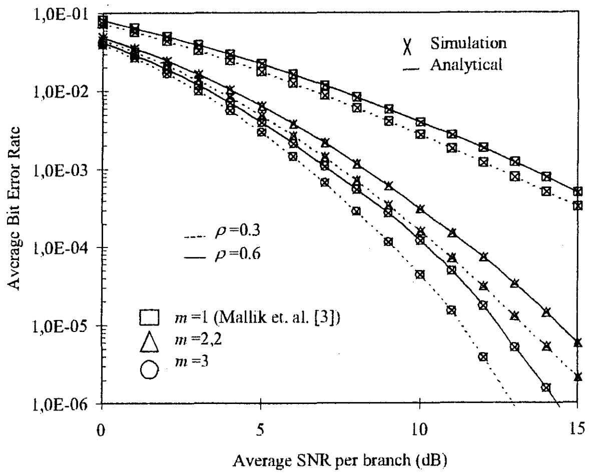

Figure 1. Average BER, P_s, versus average SNR per branch for $\overline{\gamma_1} = \overline{\gamma_2}$, in BPSK for several values of ρ and *m*.

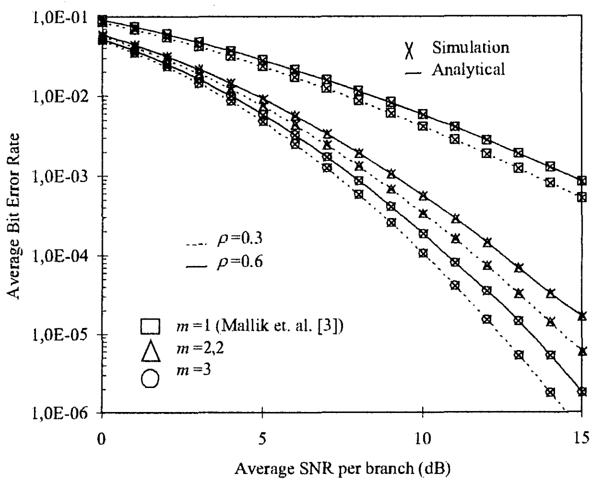

Figure 2. Average Bit Error Rate, P_s, versus average SNR per branch for $\overline{\gamma_1} = \overline{\gamma_2}/5$, in BPSK for several values of ρ and *m*.

4. Conclusions

In this paper, using a useful expression for the CHF of two correlated Nakagami-*m* envelopes, an approach to the BER performance of dual predetection EGC system in correlative Nakagami-*m* fading is presented. Simulations showed the accuracy of the proposed mathematical analysis and numerical results depict clearly the effect of fading correlation, fading severity and input SNRs unbalancing on the EGC BER performance.

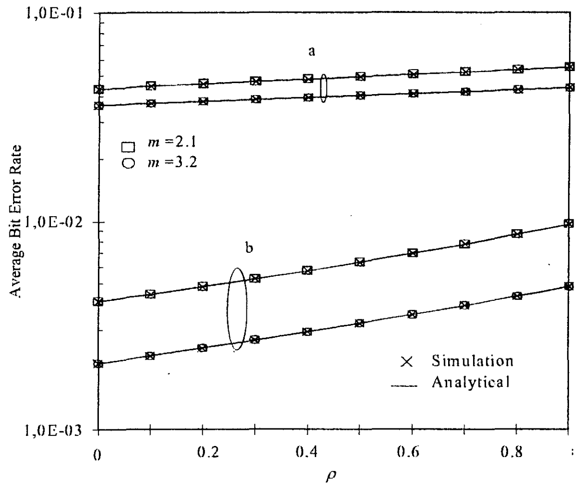

Figure 3. Average BER, P_s, versus the correlation coefficient ρ, in BPSK for a) $\overline{\gamma_1} = \overline{\gamma_2} = 0\text{dB}$, b) $\overline{\gamma_1} = \overline{\gamma_2} = 5\text{dB}$ and for several values of the parameter *m*.

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