Selection Diversity for Wireless Communications with Non-Identical Weibull Statistics

Nikos C. Sagias*, George K. Karagiannidis†, Dimitris A. Zogas*, and P. Takis Mathiopoulos*

*Institute for Space Applications and Remote Sensing, National Observatory of Athens,
Metaxa & Vas. Pavlou Street, Palea Penteli, GR-15236 Athens, Greece, Email: {nsagias;zogas;mathio}@space.noa.gr
†Electrical & Computer Engineering Department, Aristotle University of Thessaloniki, GR-54124 Thessaloniki, Greece, Email: geokarag@auth.gr

***Abstract*— Motivated by the suitability of the Weibull distribution to model multipath fading channels, the performance of L-branch selection combining (SC) receivers, with non-identical statistic, is presented. Deriving a useful expression for the probability density function of the SC output signal-to-noise ratio (SNR), important performance metrics are studied. More specifically, a closed-form expression is derived for the moments of the combiner output SNR, which is used to study the average output SNR and the amount of fading. The average symbol error probability for several coherent and non-coherent, binary and multilevel modulations schemes is also obtained in closed-form.**

I. Introduction

Selection combining (SC) receivers are utilized to mitigate the detrimental effects of channel fading and co-channel interference in wireless digital communications systems. Their major advantage is the reduced complexity compared to other well-known diversity techniques, such as equal-gain combining (EGC), maximal-ratio combining (MRC) and generalized-selection combing (GSC). In L-branch selection diversity receivers, the instantaneous signal-to-noise ratio (SNR) of the L branches are estimated and the one with the highest value is selected [1].

Experimental fading channel measurements have shown, that the Weibull model also exhibits an excellent fit for both indoors [2], as well as outdoors [3] environments. Furthermore, due to the well-known fact that the land-mobile satellite channel has some similarities with the terrestrial radio propagation environment [4], the Weibull distribution could be also considered as an alternative channel model for land-mobile satellite systems. For example, the Weibull distribution is more general than the Rice, which is the most commonly used for land mobile satellite systems. Moreover, for rainfall signal attenuation, extensive experimental measurements for satellite communications systems operating at frequencies above 10 GHz have shown the existence of multipath received signals. The survivorship function of this signal attenuation, i.e., the ratio between the number of crossings lasting longer than a given duration and the total number of crossings of that attenuation level, was found to be well represented by the Weibull distribution [5].

The performance of selection diversity receivers has been extensively studied in the open technical literature for several well-known fading statistical models, such as Rayleigh, Rice and Nakagami-m, for both independent and correlative fading [1], [6]–[10]. Surprisingly, published works, related to the diversity receivers performance in Weibull fading, are scarce. For example, an analysis for the evaluation of the GSC receivers performance has been presented in [11], while in [12], the performance analysis of dual-branch SC receivers over correlative Weibull fading, has been studied. Recently in [13], the outage probability and the average output SNR of L-branch SC receivers operating over independent and identically distributed (i.i.d.) Weibull fading channels have been obtained in closed-form. However, a performance study concerning SC receivers with non-identical Weibull fading statistics, has not been previously published.

In this paper, by deriving a useful formula for the probability density function (PDF) of the output SNR of an L-branch SC receiver with SNR branch unbalance, a novel closed-form expression for the moments is derived. This expression is used to study important performance criteria, such as average output SNR, amount of fading (AoF), and average symbol error probability (ASEP) for several coherent modulations schemes, such as BFSK, DEBPSK, M-QAM, MSK, and M-PSK, as well as for non-coherent modulations schemes, such as DBPSK, NBFSK, $\pi/4$-DQPSK with Gray encoding and M-DPSK. Selected numerical examples are presented, outlining the mathematical analysis and showing the effects of various channels and systems parameters, such as the fading severity, the power delay profile (PDP) and the number of diversity branches on the combiners performance.

II. System and Channel Models

We consider an L-branch SC receiver operating in a non-identical Weibull fading environment. The PDF and the cumulative distribution function (CDF) of the instantaneous SNR in the ℓth, $\ell = 1, 2, \ldots, L$, input branch is

$$p_{\gamma_\ell}(\gamma) = \frac{\beta_\ell}{2a_\ell\,\overline{\gamma}_\ell}\left(\frac{\gamma}{a_\ell\,\overline{\gamma}_\ell}\right)^{\frac{\beta_\ell}{2}-1}\exp\left[-\left(\frac{\gamma}{a_\ell\,\overline{\gamma}_\ell}\right)^{\beta_\ell/2}\right] \quad (1)$$

and

$$F_{\gamma_\ell}(\gamma) = 1 - \exp\left[-\left(\frac{\gamma}{a_\ell\,\overline{\gamma}_\ell}\right)^{\beta_\ell/2}\right] \quad (2)$$

respectively. In the above equations, $\overline{\gamma}_\ell$ is the corresponding average input SNR, $a_\ell = 1/\Gamma\left(d_{2,\ell}\right)$, where $\Gamma(\cdot)$ is the Gamma function [14, eq. (8.310/1)], $d_{n,\ell} = 1 + n/\beta_\ell$ (n is a positive integer) and β_ℓ is the Weibull fading parameter ($\beta_\ell \geq 0$) and describes the severity of fading. As β_ℓ increases, the severity of the fading decreases, while for the special case of $\beta_\ell = 2$, it becomes the well-known Rayleigh model.

The CDF of the instantaneous SC output SNR γ_{sc} is the probability that the signal levels of all branches fall below a certain level, i.e., $F_{\gamma_{sc}}(\gamma) = \prod_{k=1}^{L} F_{\gamma_k}(\gamma)$, which using (2), is expressed as [13, eq. (3)]

$$F_{\gamma_{sc}}(\gamma) = \prod_{k=1}^{L} \left\{ 1 - \exp\left[-\left(\frac{\gamma}{a_k \overline{\gamma}_k} \right)^{\beta_\ell/2} \right] \right\}. \quad (3)$$

The PDF of γ_{sc} can be obtained by differentiating (3) with respect to γ_{sc}, yielding

$$p_{\gamma_{sc}}(\gamma) = \frac{1}{2} \sum_{k=1}^{L} \frac{\beta_k \, \gamma^{\beta_k/2-1}}{(a_k \overline{\gamma}_k)^{\beta_k/2}} \exp\left[-\left(\frac{\gamma}{a_k \, \overline{\gamma}_k} \right)^{\beta_k/2} \right] \times \prod_{\substack{i=1 \\ i \neq k}}^{L} \left\{ 1 - \exp\left[-\left(\frac{\gamma}{a_i \, \overline{\gamma}_i} \right)^{\beta_i/2} \right] \right\}. \quad (4)$$

The above expression for the PDF of γ_{sc} can not be easily mathematically manipulated in the current form. Therefore, we rearrange (4), performing all the multiplications within the product and thus, for $\beta_\ell = \beta$, $\forall \ell$ ($a_\ell = a$ and $d_{n,\ell} = d_n$), valid for practical applications, after manipulations (4), can be written as

$$p_{\gamma_{sc}}(\gamma) = \frac{1}{2} \gamma^{\frac{\beta}{2}-1} \sum_{k=1}^{L} (-1)^{k+1} \sum_{\lambda_1=1}^{L-k+1} \sum_{\lambda_2=\lambda_1+1}^{L-k+2} \cdots \sum_{\lambda_k=\lambda_{k-1}+1}^{L} \prod_{j=1}^{k} t_{\lambda_j} \sum_{i=1}^{k} u_{\lambda_i} \quad (5)$$

where $u_\ell = \beta / (a \, \overline{\gamma}_\ell)^{\beta/2}$ and $t_\ell = \exp\left(u_\ell \, \gamma^{\beta/2} / \beta \right)$. Equation (5) includes only sums of simple products of power and exponential functions, which are mathematically trackable. Note, that for i.i.d. input paths (i.e., $\overline{\gamma}_\ell = \overline{\gamma}$ and $\Omega_\ell = \Omega, \forall \ell$), using the binomial identity [14, eq. (1.111)], (4) reduces to [13, eq. (5)].

III. Moments of the SC Output SNR

The first and the second order moments of the SC output SNR are statistical parameters used to evaluate important performance measures of the combiner, such as average output SNR, variance and AoF. The higher order moments can be also useful in signal processing algorithms for signal detection, classification, and estimation and they play a fundamental role in understanding the performance of wideband communications systems in the presence of fading [15].

The nth moment of the output SNR is given by

$$E \langle \gamma_{sc}^n \rangle = \int_0^\infty \gamma^n \, p_{\gamma_{sc}}(\gamma) \, d\gamma \quad (6)$$

where $E \langle \cdot \rangle$ denotes expectation and n is a positive integer. Substituting (5) in (6), interchanging the order of summation and integration and using [14, eq. (3.326/2)], after some manipulations, the nth moment of the output SNR can be obtained in closed-form as

$$E \langle \gamma_{sc}^n \rangle = \mathcal{G}_{2n/\beta} \left(\mathbf{y} \right) \Gamma \left(d_{2n} \right) / \Gamma^n \left(d_2 \right) \quad (7)$$

where $\mathcal{G}.(\cdot)$ is a *symmetric function*, defined in Appendix I and $\mathbf{y} = \left[\overline{\gamma}_1^{-\beta/2} \, \overline{\gamma}_2^{-\beta/2} \ldots \overline{\gamma}_L^{-\beta/2} \right]$. For i.i.d. input paths, (7) simplifies to

$$E \langle \gamma_{sc}^n \rangle = L \left(a \overline{\gamma} \right)^n \, \Gamma \left(d_{2n} \right) \sum_{k=0}^{L-1} \binom{L-1}{k} \frac{(-1)^k}{(k+1)^{\frac{2n}{\beta}+1}}. \quad (8)$$

A. Average Output SNR

Setting $n = 1$ in (7), the SC average output SNR with non-identical input branches can be obtained in closed-form as

$$\overline{\gamma}_{sc} = \mathcal{G}_{2/\beta} \left(\mathbf{y} \right) \quad (9)$$

where for i.i.d. input paths reduces to [13, eq. (8)].

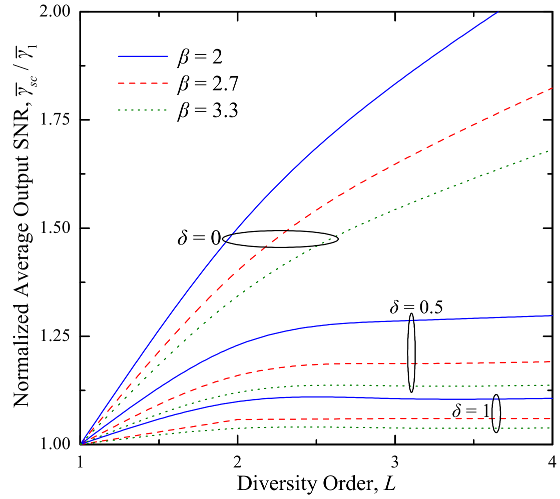

Fig. 1. First branch normalized average output SNR vs. diversity order with an exponentially decaying PDP.

In Fig. 1, using (9), the first branch normalized average output SNR, $\overline{\gamma}_{sc}/\overline{\gamma}_1$ is plotted as a function of L, for non-identically distributed input branches, with an exponentially decaying PDP, i.e., $\overline{\gamma}_\ell = \overline{\gamma}_1 \exp[-\delta \left(\ell - 1 \right)]$, for several values of β and the power decaying factor $\delta \geq 0$. As expected, the diversity gain increases as L increases, while for a fixed β and L, $\overline{\gamma}_{sc}/\overline{\gamma}_1$ degrades rapidly as δ increases. Additionally, for a fixed δ, $\overline{\gamma}_{sc}/\overline{\gamma}_1$ increases as the severity of fading increases (i.e., as β decreases).

B. Amount of Fading (AoF)

The first two moments of γ_{sc} can be used to evaluate the AoF at the output of the combiner, which is considered as a unified measure for the severity of fading [1]. The AoF is defined as the ratio of the variance to the square mean of γ_{sc} and using (7), it can be expressed as

$$\text{AoF} = \frac{\mathcal{G}_{4/\beta} \left(\mathbf{y} \right) \Gamma \left(d_4 \right)}{\Gamma^2 \left(d_2 \right) \, \mathcal{G}_{2/\beta}^2 \left(\mathbf{y} \right)}. \quad (10)$$

TABLE I

A, B AND Λ FOR SEVERAL SIGNALING CONSTELLATIONS

Signaling	A	B	Λ
BPSK	1/2	1	–
BFSK	1/2	1/2	–
DEBPSK	1	1	–
QPSK and MSK	1	1/2	–
Square M-QAM	$2-2/\sqrt{M}$	$1.5/(M-1)$	–
NBFSK	1/2	1/2	–
DBPSK	1/2	1	–
$\pi/4$-DQPSK	$1/(2\pi)$	$\frac{2}{2-\sqrt{2}\cos(\theta)}$	π
M-PSK	$1/\pi$	$\frac{\sin^2(\pi/M)}{\sin^2(\theta)}$	$\pi-\pi/M$
M-DPSK	$1/\pi$	$\frac{\sin^2(\pi/M)}{1+\cos(\pi/M)\cos(\theta)}$	$\pi-\pi/M$

IV. ERROR RATE PERFORMANCE

The most straightforward approach to obtain the ASEP $\overline{P}_{se}$, is to average the conditional symbol error probability $P_{se}(\gamma)$ over the PDF of the combiner's output SNR [1], i.e.,

$$\overline{P}_{se} = \int_0^\infty P_{se}(\gamma)\, p_{\gamma_{sc}}(\gamma)\, d\gamma. \tag{11}$$

It is well-known, that $P_{se}(\gamma)$ can be written [1]:

i) For coherent binary and M-ary modulation schemes, such as BPSK and BFSK and for higher values of average input SNR for DEBPSK, QPSK, MSK and square M-QAM in the form of $P_{se}(\gamma) = A\,\mathrm{erfc}\left(\sqrt{B\,\gamma}\right)$, where $\mathrm{erfc}(\cdot)$ is the complementary error function [14, eq. (8.250/4)],

ii) For non-coherent modulation schemes, such as NBFSK and DBPSK in the form of $P_{se}(\gamma) = A\,\exp(-B\,\gamma)$,

iii) Furthermore, for multilevel modulation schemes, such as $\pi/4$-DQPSK with Gray encoding, M-PSK and M-DPSK in the form of $P_{se}(\gamma) = A\int_0^\Lambda \exp[-B(\theta)\,\gamma]\,d\theta$.

The particular values of A, B and Λ depend on the considered modulation scheme and are summarized in Table I.

A. ASEP for Coherent BPSK, BFSK, DEBPSK, QPSK, MSK and Square M-QAM

Using (5), it is easily recognized that (11) requires evaluation of integrals of the form

$$\Upsilon_1 = \int_0^\infty x^{\beta/2-1}\,\mathrm{erf}\left(\sqrt{B\,x}\right)\exp\left(-\xi\, x^{\beta/2}\right)dx \tag{12}$$

where ξ is a positive real value. Since the above integral is not a tabulated one, a solution is given in Appendix II in terms of the Meijer's G-function, helping us to express the ASEP in a closed-form as in (13) (top of this page), where $G[\cdot]$ is the Meijer's G-function [14, eq. (9.301)], with $\mathrm{I}(h,\mu) = h/\mu, (h+1)/\mu, \ldots, (h+\mu-1)/\mu$, μ being positive integer and h real constant. The variables κ and μ are positive integers so that $\mu/\kappa = \beta/2$ holds. Depending upon the value of β, a set with minimum values for κ and μ must be properly chosen (e.g. for $\beta = 2.6$ we have to choose $\kappa = 10$ and $\mu = 13$), while for the special case where β is an integer we must be chose $\kappa = 2$ and $\mu = \beta$.

For i.d.d. input paths, (13) reduces to

$$\overline{P}_{se} = A\left\{1 - \frac{\beta L}{2\,(a\overline{\gamma})^{\frac{\beta}{2}}}\frac{\sqrt{\kappa}\,\mu^{\frac{\beta}{2}-1}B^{-\frac{\beta}{2}}}{\sqrt{\pi}\,(2\pi)^{\frac{\kappa+\mu-2}{2}}}\sum_{n=0}^{L-1}\binom{L-1}{n}(-1)^n \times G^{\kappa+\mu,\,\mu}_{2\mu,\,\kappa+\mu}\left[\frac{(n+1)^\kappa}{\kappa^\kappa\,(a\,\overline{\gamma})^{\frac{\kappa\beta}{2}}}\left(\frac{\mu}{B}\right)^\mu \middle| \begin{matrix}\mathrm{I}\left(\mu,\frac{1-\beta}{2}\right), \mathrm{I}\left(\mu,1-\frac{\beta}{2}\right)\\ \mathrm{I}(\kappa,0)\ ,\ \mathrm{I}\left(\mu,-\frac{\beta}{2}\right)\end{matrix}\right]\right\} \tag{14}$$

where setting $L = 1$, the ASEP for the single channel receivers can be obtained as

$$\overline{P}_{se} = A\left\{1 - \frac{\beta}{2\,(a\overline{\gamma})^{\frac{\beta}{2}}}\frac{\sqrt{\kappa}\,\mu^{\frac{\beta}{2}-1}B^{-\frac{\beta}{2}}}{\sqrt{\pi}\,(2\pi)^{\frac{\kappa+\mu-2}{2}}} \times G^{\kappa+\mu,\,\mu}_{2\mu,\,\kappa+\mu}\left[\frac{\kappa^{-\kappa}}{(a\,\overline{\gamma})^{\frac{\beta\,\kappa}{2}}}\left(\frac{\mu}{B}\right)^\mu \middle| \begin{matrix}\mathrm{I}\left(\mu,\frac{1-\beta}{2}\right), \mathrm{I}\left(\mu,1-\frac{\beta}{2}\right)\\ \mathrm{I}(\kappa,0)\ ,\ \mathrm{I}\left(\mu,-\frac{\beta}{2}\right)\end{matrix}\right]\right\}. \tag{15}$$

B. ASEP for Non-coherent BFSK and DBPSK

For non-coherent modulation schemes, such as NBFSK and DBPSK, (11) requires evaluation of integrals of the form

$$\Upsilon_2 = \int_0^\infty x^{\beta/2-1}\exp(-B\,x)\exp\left(-\xi\,x^{\beta/2}\right)dx. \tag{16}$$

This type of integral, is analytically solved in terms of the Meijer's G-function (see Appendix III) and the ASEP can be obtained in closed-form expression as in (17) (top of this page), where the integers κ and μ must be chosen so that $\mu/\kappa = \beta/2$ holds.

For i.d.d. input paths (17) reduces to

$$\overline{P}_{se} = \frac{\beta L A}{2\,(a\,\overline{\gamma})^{\frac{\beta}{2}}}\frac{\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\mu^{\frac{\beta}{2}}\,B^{-\frac{\beta}{2}}}{(2\pi)^{\frac{\kappa+\mu-2}{2}}}\sum_{n=0}^{L-1}\binom{L-1}{n}(-1)^n \times G^{\kappa,\mu}_{\mu,\kappa}\left[\frac{(n+1)^\kappa}{\kappa^\kappa\,(a\,\overline{\gamma})^{\frac{\kappa\beta}{2}}}\left(\frac{\mu}{B}\right)^\mu \middle| \begin{matrix}\mathrm{I}\left(\mu,1-\frac{\beta}{2}\right)\\ \mathrm{I}(\kappa,0)\end{matrix}\right] \tag{18}$$

and setting $L = 1$ in (18), the ASEP for the single channel receivers can be obtained as

$$\overline{P}_{se} = \frac{\beta A}{2\,(a\,\overline{\gamma})^{\frac{\beta}{2}}}\frac{\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\mu^{\frac{\beta}{2}}\,B^{-\frac{\beta}{2}}}{(2\pi)^{\frac{\kappa+\mu-2}{2}}} \times G^{\kappa,\mu}_{\mu,\kappa}\left[\frac{\kappa^{-\kappa}}{(a\,\overline{\gamma})^{\frac{\beta\,\kappa}{2}}}\left(\frac{\mu}{B}\right)^\mu \middle| \begin{matrix}\mathrm{I}\left(\mu,1-\frac{\beta}{2}\right)\\ \mathrm{I}(\kappa,0)\end{matrix}\right]. \tag{19}$$

C. ASEP of Gray Encoded $\pi/4$-DQPSK, M-PSK and M-DPSK

For multilevel modulation schemes such as $\pi/4$-DQPSK with Gray encoding, M-PSK and M-DPSK, (11) requires evaluation of integrals with finite limits of the form

$$\overline{P}_{se} = \int_0^\Lambda \overline{P}_{se}[B(\theta)]\,d\theta \tag{20}$$

$$\overline{P}_{se} = A\left\{1 - \frac{\beta\sqrt{\kappa}\,\mu^{\frac{\beta}{2}-1}B^{-\frac{\beta}{2}}}{2\sqrt{\pi}\,(2\pi)^{\frac{\kappa+\mu}{2}-1}} \sum_{n=1}^{L}(-1)^{n+1}\sum_{\lambda_1=1}^{L-n+1}\sum_{\lambda_2=\lambda_1+1}^{L-n+2}\cdots\sum_{\lambda_n=\lambda_{n-1}+1}^{L}\sum_{i=1}^{n}\left(a\,\overline{\gamma}_{\lambda_i}\right)^{-\frac{\beta}{2}} \times G_{2\mu\,,\,\kappa+\mu}^{\kappa+\mu\,,\,\mu}\left[\left[\frac{1}{\kappa}\sum_{j=1}^{n}\left(a\,\overline{\gamma}_{\lambda_j}\right)^{-\frac{\beta}{2}}\right]^{\kappa}\left(\frac{\mu}{B}\right)^{\mu}\middle|\begin{matrix}\mathrm{I}\left(\mu,\frac{1-\beta}{2}\right),\mathrm{I}\left(\mu,1-\frac{\beta}{2}\right)\\ \mathrm{I}(\kappa,0)\;,\;\mathrm{I}\left(\mu,-\frac{\beta}{2}\right)\end{matrix}\right]\right\} \tag{13}$$

$$\overline{P}_{se} = \sqrt{\frac{\kappa}{\mu}}\frac{A\,\beta\,\mu^{\frac{\beta}{2}}B^{-\frac{\beta}{2}}}{2\,(2\pi)^{\frac{\kappa+\mu}{2}-1}}\sum_{n=1}^{L}(-1)^{n+1}\sum_{\lambda_1=1}^{L-n+1}\sum_{\lambda_2=\lambda_1+1}^{L-n+2}\cdots\sum_{\lambda_n=\lambda_{n-1}+1}^{L}\sum_{i=1}^{n}\left(a\,\overline{\gamma}_{\lambda_i}\right)^{-\frac{\beta}{2}} \times G_{\mu\,,\,\kappa}^{\kappa\,,\,\mu}\left[\left[\frac{1}{\kappa}\sum_{j=1}^{n}\left(a\,\overline{\gamma}_{\lambda_j}\right)^{-\frac{\beta}{2}}\right]^{\kappa}\left(\frac{\mu}{B}\right)^{\mu}\middle|\begin{matrix}\mathrm{I}\left(\mu,1-\frac{\beta}{2}\right)\\ \mathrm{I}(\kappa,0)\end{matrix}\right] \tag{17}$$

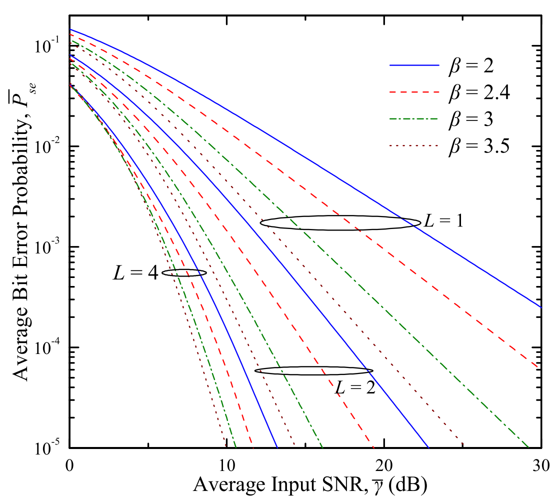

Fig. 2. ASEP of SC as a function of input SNR for BPSK signaling.

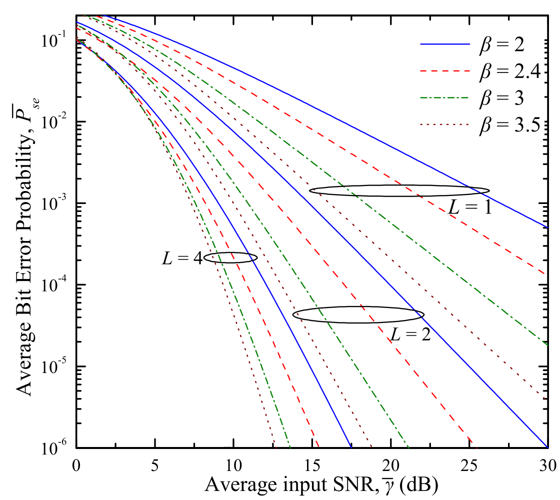

Fig. 3. ASEP of SC as a function of input SNR for DBPSK signaling.

where $\overline{P}_{se}[B(\theta)]$ is similar to (17), but now B is a function of θ, i.e., $B(\theta)$, where its corresponding expressions are listed in Table I. The integral in (20) can be numerically evaluated, using any of the well-known mathematical software packages, such as Maple and Mathematica.

As indicative examples of the above three cases, using (14), (18), and (20), ASEPs of BPSK, DBPSK, and 8-PSK, are plotted in Figs. 2, 3, and 4, respectively, for i.i.d. input branches and for several values of β and L. The applicability of the satellite channel model is expressed in these figures, comparing β with the corresponding Rician factor and using proper values for β. The obtained performance evaluation results show that $\overline{P}_{se}$ improves with an increase of $\overline{\gamma}$. Furthermore, for a fixed value of $\overline{\gamma}$, as β and L increases better performance is provided.

Appendix I
Definition of $\mathcal{G}_q(\cdot)$

Let $\mathbf{u}$ be an L dimensional vector, $\mathbf{u} = [u_1\, u_2\, \cdots\, u_L]$, with elements $\{u_l\}$ real constant values ($l = 1, 2, \ldots, L$). We define the *symmetric function* $\mathcal{G}_q(\mathbf{u})$ as

$$\mathcal{G}_q(\mathbf{u}) = \sum_{n=1}^{L}(-1)^{n+1} \sum_{\lambda_1=1}^{L-n+1}\sum_{\lambda_2=\lambda_1+1}^{L-n+2}\cdots\sum_{\lambda_n=\lambda_{n-1}+1}^{L}\left(\sum_{i=1}^{n}u_{\lambda_i}\right)^{-q} \tag{I.1}$$

where q is a real value and n is a positive integer ($n \leq L$).

Appendix II
Evaluation of Integral in (12)

The complicated integrals (12) and (16) can be efficiently solved using (II.1) (top of the next page) [16, eq. (21)], where $G[\cdot]$ is the Meijer's G-function [14, eq. (9.301)], $c^* = m + n - (p+q)/2$, $\mu = \sum_{j=1}^{q} b_j - \sum_{j=1}^{p} a_j + (p+q)/2 + 1$, $b^* = s + t - (u+v)/2$, $\varrho = \sum_{j=1}^{\nu} d_j - \sum_{j=1}^{u} c_j + (u - v)/2 + 1$ and $\mathrm{I}(k,h) = h/k, (h+1)/k, \ldots, (h+k-1)/k$, with $k, l, s, t, u, v, m, n, p$ and q being positive integers and $\eta, a_p, b_q, c_u, d_v, h, \sigma$ and ω being real constant values.

$$\int_0^\infty x^{\eta-1}\, G_{u,v}^{s,t}\left[\sigma x \,\middle|\, \begin{smallmatrix}\{c_u\}\\ \{d_v\}\end{smallmatrix}\right] G_{p,q}^{m,n}\left[\omega x^{\frac{l}{k}} \,\middle|\, \begin{smallmatrix}\{a_p\}\\ \{b_q\}\end{smallmatrix}\right] dx = \frac{k^{\mu}\, l^{\varrho+a(v-u)-1}\, \sigma^{-a}}{(2\pi)^{b^*(l-1)+c^*(k-1)}}$$
$$\times\, G_{kp+lv,kq+lu}^{km+lt,kn+ls}\left[\frac{\omega^k\, k^{k(p-q)}}{\sigma^l\, l^{l(u-v)}} \,\middle|\, \begin{smallmatrix}\mathrm{I}(k,a_1),\ldots,\mathrm{I}(k,a_n),\mathrm{I}(l,1-\eta-d_1),\ldots,\mathrm{I}(l,1-\eta-d_\nu),\mathrm{I}(k,a_{n+1}),\ldots,\mathrm{I}(k,a_p)\\ \mathrm{I}(k,b_1),\ldots,\mathrm{I}(k,b_m),\mathrm{I}(l,1-\eta-c_1),\ldots,\mathrm{I}(l,1-\eta-c_u),\mathrm{I}(k,b_{m+1}),\ldots,\mathrm{I}(k,b_q)\end{smallmatrix}\right] \quad \text{(II.1)}$$

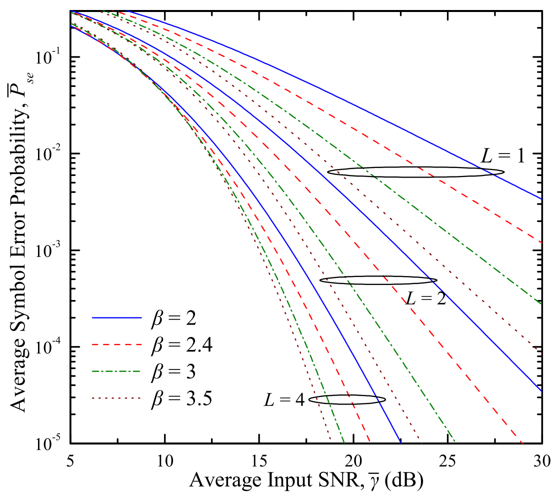

Fig. 4. ASEP of SC as a function of input SNR for 8-PSK signaling.

The integral in (12) can be solved by expressing the error and the exponential functions as Meijer's G-functions, i.e., $\mathrm{erf}\left(\sqrt{Bx}\right) = G_{1,2}^{1,1}\left[B\, x\middle|\,{}_{0.5\,,\,0}^{\;1}\right]/\sqrt{\pi}$ [16, eq. (12)] and $\exp\left(-\xi x^{\beta/2}\right) = G_{0,1}^{1,0}\left[\xi x^{\beta/2}\middle|\,{}_{0}^{-}\right]$ [16, eq. (11)]. Thus, the integral in Section IV-A can be written as $\pi^{-1/2}\int_0^\infty x^{\beta/2-1}\, G_{1,2}^{1,1}\left[B\, x\middle|\,{}_{0.5\,,\,0}^{\;1}\right] G_{0,1}^{1,0}\left[\xi x^{\beta/2}\middle|\,{}_{0}^{-}\right] dx$ and using (II.1), yields

$$\Upsilon_1 = \frac{\sqrt{k}\, l^{\beta/2-1} B^{-\beta/2}}{\sqrt{\pi}\,(2\pi)^{(k+l-2)/2}} \times G_{2l\,,\,k+l}^{\,k+l\,,\,l}\left[\frac{\xi^k}{k^k}\frac{l^l}{B^l}\,\middle|\, \begin{smallmatrix}\mathrm{I}(l,(1-\beta)/2)\,,\,\mathrm{I}(l,1-\beta/2)\\ \mathrm{I}(k,0)\;\;,\;\;\mathrm{I}(l,-\beta/2)\end{smallmatrix}\right] \quad \text{(II.2)}$$

where k and l are positive integers so that $l/k = \beta/2$ holds. Depending upon the value of β, a set with minimum values of k and l must be properly chosen.

Appendix III
Evaluation of Integral in (16)

Similarly, and as previously discussed in Appendix II, by expressing the integral in (16) using [16, eq. (11)] as $\int_0^\infty x^{\beta/2-1}\, G_{0,1}^{1,0}\left[B\, x\middle|\,{}_{0}^{-}\right] G_{0,1}^{1,0}\left[\xi x^{\beta/2}\middle|\,{}_{0}^{-}\right] dx$ and employing (II.1), (16) can be expressed in closed-form as

$$\Upsilon_2 = \frac{\sqrt{k/l}\, l^{\beta/2}\, B^{-\beta/2}}{(2\pi)^{(k+l-2)/2}}\, G_{l,k}^{k,l}\left[\left(\frac{\xi}{k}\right)^k \left(\frac{l}{B}\right)^l \middle|\, \begin{smallmatrix}\mathrm{I}(l,1-\frac{\beta}{2})\\ \mathrm{I}(k,0)\end{smallmatrix}\right] \quad \text{(III.1)}$$

where k and l are chosen using so that $l/k = \beta/2$ holds.

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