Performance of Broadband Multihop Networks with Cooperative Diversity over Fading Channels

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Abstract

We present closed-form bounds for the performance of broadband multihop networks with cooperative diversity over Nakagami-m fading channels. The end-to-end signal-to-noise ratio is formulated and upper bounded by using the inequality between harmonic and geometric means of positive random variables (RVs). Novel closed-form expression is derived for the moment generating function of the product of arbitrary powers of statistically independent Gamma RVs. Using this theoretical result, closed-form lower bounds are obtained for the average error probability. Numerical results are compared to computer simulations, to show the tightness of the proposed bounds.

Keywords

Average error probability, cooperative diversity, Gamma RVs, moment generating function, multihop networks, Nakagami-m fading.

1. Introduction

Multihop networks realize a number of advantages over traditional communications networks in the areas of deployment, connectivity and capacity while minimize the need for fixed infrastructure. Relaying techniques enable network connectivity where traditional architectures are impractical due to location constraints and can be applied to cellular, wireless local area networks (WLAN), and hybrid networks. In multihop networks the source-terminal communicates with the destination-terminal through a number of relays-terminals. Therefore, multihop networks have the advantage of broadening the coverage without using large transmitting power (Hasna and Alouini, 2003a, and Boyer *et al.*, 2004). Recently, the concept of cooperative diversity, where the mobile users cooperate/colaborate each other in order to exploit the benefits of spatial diversity without the need of using physical antenna arrays, has gained great interest. In general, cooperative networks are multihop communication networks where the destination-terminal combines the signals received from both source-terminal and relays (Sendonaris *et al.*, 2003a,b, Anghel and Kaveh, 2004, and Laneman *et al.*, 2004).

The performance analysis of multihop wireless communication networks operating in fading channels has been an important field of research in the past few years. Hasna and Alouini have presented a useful and semi-analytical framework for the evaluation of the end-to-end outage probability of multihop wireless networks with non-regenerative channel state information (CSI)-assisted relays over Nakagami-m fading channels (Hasna and Alouini, 2003a). Moreover, Boyer *et al.* (Boyer *et al.*, 2004), have proposed and characterized four channel models for multihop wireless communication and also have introduced the concept of multihop diversity. Recently, Laneman *et al.* (Laneman *et al.*, 2004), have developed and analyzed low-complexity cooperative diversity protocols that combat fading induced by multipath propagation in wireless networks. Also, Anghel and Kaveh (Anghel and Kaveh, 2004), have developed tight bounds on the probability of error in a cooperative network over Rayleigh fading environment. Finally, Karagiannidis *et al.* have studied the performance bounds for multihop wireless communication networks with blind (fixed gain) relays over Nakagami-n (Rice), Nakagami-q (Hoyt) and Nakagami-m fading channels (Karagiannidis *et al.*, 2004a), using the moments-based approach (Karagiannidis *et al.*, 2004b). However, to the best of the authors knowledge, the performance of multihop relayed networks with cooperative diversity has never been addressed in terms of tabulated functions in Nakagami-m fading.

In this paper, using the well-known inequality between harmonic and geometric means of positive random variables (RVs), we present an efficient performance bound for the end-to-end signal-to-noise ratio (SNR) of broadband multihop networks with cooperative diversity operating in non-identical Nakagami-m fading channels. Motivated by the fact that the proposed bound, in its general form, is a product of arbitrary powers of statistically independent squared Nakagami-m (Gamma) RVs, we derive a novel closed-form expression for its moment generating function (MGF). Using this expression, closed-form lower bounds are presented for the average error probability for different signalling constellations. Numerical and computer simulation examples verify the accuracy of the presented mathematical analysis and show the tightness of the proposed bounds.

2. Statistical Background

Theorem 1: (MGF of the product of arbitrary powers of Gamma RVs): Let $\{X_i\}_{i=1}^{N}$ be N independent, but not necessarily identically distributed (i.n.i.d.), Gamma RVs, with PDF given by

$$f_{X_i}(x) = \frac{x^{\alpha_i - 1}}{\beta_i^{\alpha_i}\Gamma\left(\alpha_i\right)} \exp\left(-\frac{x}{\beta_i}\right) \tag{1}$$

where $\Gamma\left(\cdot\right)$ is the Gamma function (Gradshteyn and Ryzhik, 2000, eq. (8.310/1)) and α_i, β_i be positive real numbers. Then, the MGF of the new RV Y_1, defined as the product of arbitrary powers of N RVs X_i, i.e.,

$$Y_1 \triangleq \prod_{i=1}^{N} X_i^{\ell_i/k} \tag{2}$$

with $\ell_1, \ell_2, \ldots, \ell_N$ and k, being positive integers, can be expressed in closed-form as

$$\begin{aligned}\mathcal{M}_{Y_1}(s) = &\frac{\sqrt{k}\prod\limits_{i=1}^{N}\ell_i^{\alpha_i - 1/2}}{\left(\sqrt{2\pi}\right)^{r-N+k-1}\prod\limits_{i=1}^{N}\Gamma\left(\alpha_i\right)} \\ &\times G_{r,k}^{k,r}\left[\frac{\left(-1\right)^k\,\left(s/k\right)^k}{\prod\limits_{i=1}^{N}\left(\beta_i\ell_i\right)^{-\ell_i}}\;\middle|\;\begin{matrix}\Delta\left(\ell_1, 1-\alpha_1\right), \Delta\left(\ell_2, 1-\alpha_2\right), \ldots, \Delta\left(\ell_N, 1-\alpha_N\right)\\ \Delta\left(k, 0\right)\end{matrix}\right]\end{aligned} \tag{3}$$

where $r = \sum_{i=1}^{N}\ell_i$, $\Delta(k,u) \triangleq u/k,\ (u+1)/k, \ldots, (u+k-1)/k$, with u real, and $G\left[\cdot\right]$ is the Meijer's G-function (Gradshteyn and Ryzhik, 2000, eq. (9.301)).

Note, that Meijer's G-function is a standard built-in function in most of the well-known mathematical software packages such as MAPLE, MATHEMATICA and MATLAB. In addition, using (Adamchik and Marichev, 1990, eq. (18)), can be written in terms of the more familiar generalized hypergeometric functions (Gradshteyn and Ryzhik, 2000, eq. (9.14/1)).

Proof: See (Karagiannidis *et al.*, 2004c).

3. An Upper Bound for the End-to-End SNR

We consider an N-hop wireless communication network which operates over i.n.i.d. Nakagami-m fading channels, shown in Fig.1. The source terminal S communicates with the destination terminal D through a direct link and via $N-1$ nodes-terminals, $R_1, R_2, \ldots, R_{N-1}$.

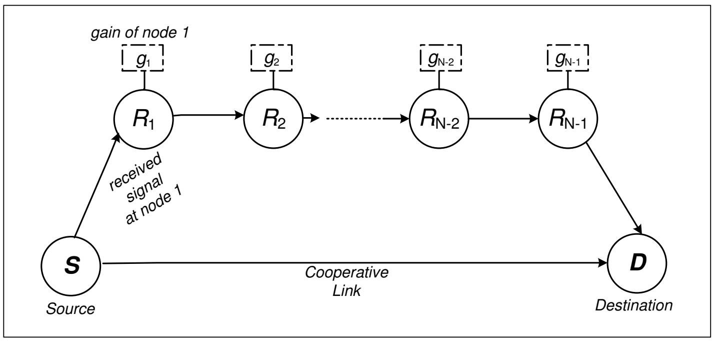

Figure 1 - Multihop wireless communication network with cooperative diversity.

These terminals act as intermediate non-regenerative CSI-assisted relays from one hop to the next. It is also assumed that all nodes-relays can simultaneously receive and transmit (in the same frequency band), and no delay is incurred in the whole chain of transmissions. Assuming that terminal S is transmitting a signal with an average power normalized to unity and maximal ratio combining (MRC) at the destination terminal, the end-to-end SNR, i.e., the SNR at D, can be written as (Hasna and Alouini, 2003a)

$$\gamma_{end} = \gamma_0 + \frac{\prod_{i=1}^{N} \upsilon_i^2 \, g_{i-1}^2}{\sum_{i=1}^{N} N_{0,i} \left(\prod_{j=i+1}^{N} g_{j-1}^2 \upsilon_j^2 \right)} \tag{4}$$

where γ_0 is the instantaneous SNR between S and D, υ_i is the fading amplitude of the ith hop, $N_{0,i}$ is the one sided power spectral density at the input of the ith relay, and g_i is the gain of the ith relay with $g_0 = 1$.
Due to the fact that, υ_i is Nakagami-m distributed, the corresponding instantaneous SNR, γ_i, defined as $\gamma_i = \upsilon_i^2 / N_{0,i}$, is Gamma distributed, with PDF given by (Simon and Alouini, 2005)

$$f_{\gamma_i}(\gamma) = \frac{m_i^{m_i}}{\overline{\gamma}_i^{m_i}\, \Gamma(m_i)} \gamma^{m_i - 1} \exp\left(-m_i \frac{\gamma}{\overline{\gamma}_i} \right) \tag{5}$$

where $m_i \geq 1/2$ is a parameter describing the fading severity of the ith hop and $\overline{\gamma}_i$ is the average SNR, i.e., $\overline{\gamma}_i = E\left\langle \upsilon_i^2 \right\rangle / N_{0,i}$, with $E\left\langle \cdot \right\rangle$ denoting expectation. It is obvious, that by setting $\alpha_i = m_i$ and $\beta_i = \overline{\gamma}_i / m_i$ in (1), yields (5).
One choice of gain is proposed in (Hasna and Alouini 2003a,b and 2004) as

$$g_i^2 = \frac{1}{\upsilon_i^2} \tag{6}$$

where the relay just amplifies the incoming signal with the inverse of the channel of the previous hop regardless the fading state (i.e., the noise) of that hop. As mentioned in (Hasna and Alouini 2003a,b and 2004), such a kind of relay serves as benchmark for all practical multihop networks using non-regenerative relays. Furthermore, its performance in the high SNR region, is equal to the performance of the CSI-assisted relays which satisfy the average power constraint, with an amplifying gain given by (Laneman *et al.*, 2004, eq. (9))

$$g_i^2 = \frac{1}{\upsilon_i^2 + N_{0,i}}. \tag{7}$$

By applying (6) to (4), the end-to-end SNR becomes

$$\gamma_{end} = \gamma_0 + \left(\sum_{i=1}^{N} \frac{1}{\gamma_i} \right)^{-1}. \tag{8}$$

In order to study important performance metrics of the end-to-end SNR, (8) should be expressed in a more mathematically tractable form. To achieve it, we propose an upper bound for (8) using the well-known inequality between geometric and harmonic means of N positive RVs $x_1, x_2, \ldots, x_N$ given by

$$\mathcal{H}_N \leq \mathcal{G}_N \tag{9}$$

where $\mathcal{H}_N \triangleq N \left(\sum_{i=1}^{N} 1/x_i \right)^{-1}$ and $\mathcal{G}_N \triangleq \prod_{i=1}^{N} x_i^{1/N}$ are the harmonic and geometric means, respectively. In (9), the equality holds only when $x_1 = x_2 = \cdots = x_N$. Using (8) and (9), an upper bound for the end-to-end SNR, γ_b, for multihop networks with CSI-assisted relays can be obtained as

$$\gamma_{end} \leq \gamma_b = \gamma_0 + \frac{1}{N} \prod_{i=1}^{N} {\gamma_i}^{1/N}. \tag{10}$$

4. Average Error Probability

Due to the independency of γ_i and γ_0, the MGF of the bound of the end-to-end SNR given in (10), equals to the product

$$\mathcal{M}_{\gamma_b}(s) = \mathcal{M}_{\gamma_0}(s)\, \mathcal{M}_{\frac{1}{N}\prod\limits_{i=1}^{N} {\gamma_i}^{1/N}}(s) \tag{11}$$

where $\mathcal{M}_{\gamma_0}(s)$ and $\mathcal{M}_{\frac{1}{N}\prod_{i=1}^{N} {\gamma_i}^{1/N}}(s)$ are the MGFs of γ_0 and $\frac{1}{N}\prod_{i=1}^{N} {\gamma_i}^{1/N}$ respectively. Using (3) and the MGF expression of γ_0 presented in (Simon and Alouini, 2005, Table 2.2), the MGF of γ_b can be written as

$$\mathcal{M}_{\gamma_b}(s) = \left(1 - \frac{s\overline{\gamma}_0}{m}\right)^{-m} \frac{\sqrt{N}}{\left(\sqrt{2\pi}\right)^{N-1} \prod\limits_{i=1}^{N} \Gamma(m_i)} G_{N,N}^{N,N}\left[\frac{(-1)^N\ (s/N^2)^N}{\prod\limits_{i=1}^{N} (\overline{\gamma}_i/m_i)^{-1}} \middle| \begin{matrix} m_1, m_2, \ldots, m_N \\ \Delta(N, 0) \end{matrix} \right]. \tag{12}$$

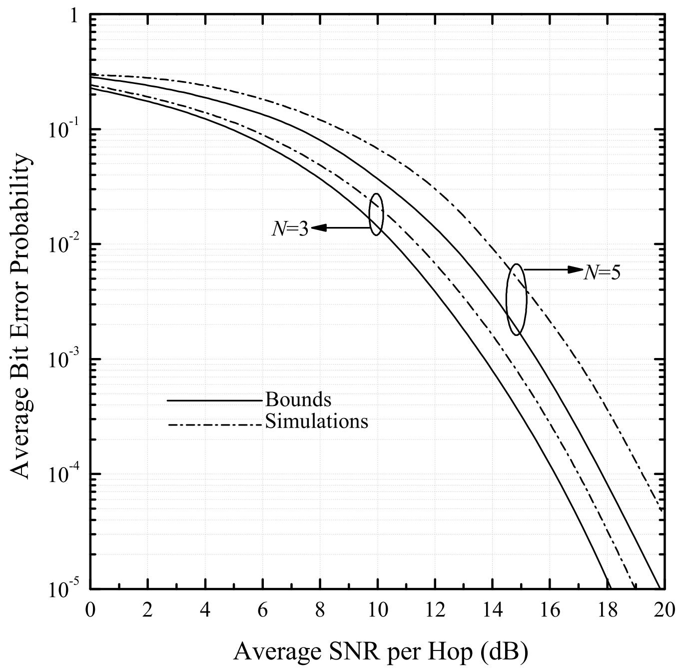

Figure 2 - BPSK error bounds for a multihop network with cooperative diversity in Nakagami-m fading ($\overline{\gamma}_i = \overline{\gamma}_0 = \overline{\gamma}$ and $m_i = m = 2$).

Having the MGF of γ_b in closed-form, as given in (12), and using the MGF-based approach for the performance evaluation of digital modulations over fading channels (Simon and Alouini, 2005), the average bit and symbol error probability can be evaluated for a wide variety of M-ary modulations such as M-ary phase-shift keying (M-PSK) and M-ary quadrature amplitude modulation (M-QAM).

In Fig. 2, assuming equal average SNR per hop (for all hops, $\overline{\gamma}_i = \overline{\gamma}_0 = \overline{\gamma}$), lower bounds for the average bit error probability (ABEP) for binary phase shift-keying (BPSK) of a multihop network with cooperative diversity are plotted only for $N = 3$ and $N = 5$ in order to avoid entanglement with the curve $N = 4$. It is evident that the proposed bounds are accurate and tight and as expected, the ABEP deteriorates with an increase in the number of hops.

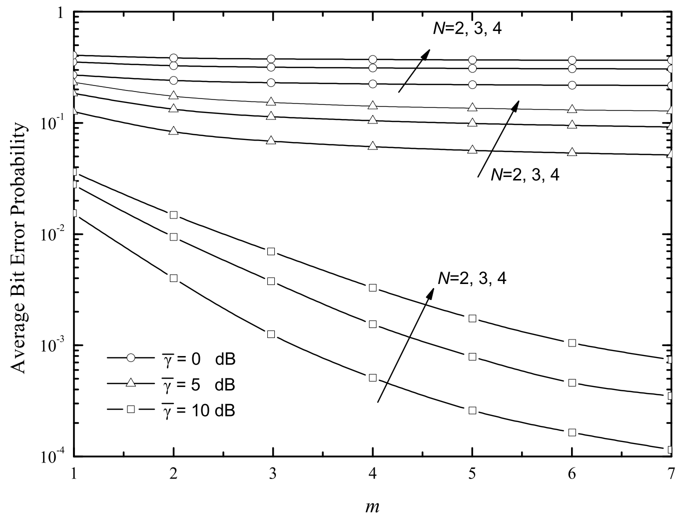

Figure 3 - BPSK error bounds for a multihop network with cooperative diversity in Nakagami-m fading versus m ($\overline{\gamma}_i = \overline{\gamma}_0 = \overline{\gamma}$ and $m_i = m$).

In Fig. 3, ABEP versus m is plotted for BPSK showing that the performance is improved with an increase of m. This improvement is sensitive both for small number of hops N and for higher values of $\overline{\gamma}$.

5. Conclusion

Performance bounds for multihop networks with cooperative diversity operating over i.n.i.d. Nakagami-m fading channels, were presented. The end-to-end SNR was formulated and upper-bounded by using the harmonic - geometric means inequality of positive RVs. Since the proposed bound are in the form of product of arbitrary powers of statistically independent Gamma RVs, novel closed-form expression for the MGF of this product was derived. Therefore, average error probability lower bounds were presented for different number of hops and values of fading severity parameter m.

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