Performance of MRC Diversity Receivers over Correlated Nakagami-m Fading Channels

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***Abstract*— We present exact closed-form expressions for the statistics of the sum of non-identical squared Nakagami-m random variables and it is shown that it can be written as a weighted sum of Erlang distributions. The analysis includes both independent and correlated cases with distinct average powers and integer-order fading parameters. The proposed formulation significantly improves previously published results which are in the form of infinite sums or higher order derivatives. The obtained formulae can be applied on the performance analysis of maximal-ratio combining diversity receivers operating over Nakagami-m fading channels.**

I. Introduction

Performance analysis of digital wireless communications systems usually deals with complicated and cumbersome statistical tasks. One of them arises in the study of diversity combining receivers operating over Nakagami-m fading channels [1], where the statistics of the sum of squared Nakagami-m random variables (RVs) (or equivalently the sum of Gamma RVs) are required. Well-known applications in the field of mobile radio systems where such sums could be useful are maximal-ratio combining (MRC) and post-detection equal-gain combining (EGC), or in the evaluation of the outage probability in cellular systems with co-channel interference (see [2]–[8] and references therein). Ω The most general approach related to the distribution of the sum of Gamma RVs has been presented by Moschopoulos in [9], where an infinite series representation for the probability density function (PDF) of the sum of independent Gamma RVs, with non-identical parameters, has been proposed. Alouini *et al.* in [8], have extended the result of [9], for the case of arbitrarily correlated Gamma RVs and studied the performance of MRC and post-detection EGC receivers, as well as the cochannel interference in cellular mobile radio systems. However, to the best of the authors' knowledge, there are not available in the open technical literature any simple closed-form expressions for both PDF and cumulative distribution function (CDF) of the sum of squared non-identically distributed Nakagami-m RVs. Consequently, there have not been presented any closed-form expressions for the performance metrics of the above mentioned diversity receivers.

In this paper, novel closed-form expressions for the PDF and the CDF of the sum of non-identical squared Nakagami-m RVs, with integer-order fading parameters, are derived. Our results include both the statistical independent and correlated cases. Furthermore, in order to reveal the importance of the proposed statistical formulation, we study the performance of L-branch MRC receivers, in the presence of Nakagami-m multipath fading. Exact formulae for the outage probability, the channel average spectral efficiency (SE) and the average symbol error probability (ASEP) for several coherent, non-coherent, binary and multilevel modulation signallings are obtained.

After this short introduction, in Section II, novel closed-form expressions for the PDF and the CDF of the sum of squared Nakagami-m RVs are obtained. In Section III, the theoretical results of Section II are applied to derive useful expressions for performance metrics of MRC diversity receivers, operating over Nakagami-m fading channels. Finally, in Section V, useful concluding remarks are provided.

II. Closed-Form Statistics for the Sum of Squared Nakagami-m RVs

Let $\{X_\ell\}_{\ell=1}^{L}$ be L Nakagami-m distributed RVs, with PDF given by[1] [1]

$$f_X(x; m_\ell, \eta_\ell) = \frac{2\, x^{2\, m_\ell - 1}}{\eta_\ell^{m_\ell}\; (m_\ell - 1)!}\; \exp\left(-\frac{x^2}{\eta_\ell}\right) \mathrm{U}(x) \quad (1)$$

where $\mathrm{U}(x)$ is the well-known unit step function defined as $\mathrm{U}\,(x \geq 0) = 1$ and zero otherwise, m_ℓ denotes the Nakagami-m fading parameter, here considered as a positive integer parameter and $\eta_\ell = E\left\langle X_\ell^2 \right\rangle / m_\ell$, with $E\left\langle \cdot \right\rangle$ denoting expectation. Moreover, the squared value of X_ℓ, $Y_\ell = X_\ell^2$, follows the Erlang distribution[2] with PDF given by

$$f_Y(y; m_\ell, \eta_\ell) = \frac{y^{m_\ell - 1}}{\eta_\ell^{m_\ell}\; (m_\ell - 1)!}\; \exp\left(-\frac{y}{\eta_\ell}\right) \mathrm{U}(y) \quad (2)$$

and the CDF can be expressed as [10]

$$F_Y(y; m_\ell, \eta_\ell) = 1 - \frac{\Gamma\left(m_\ell, y/\eta_\ell\right)}{(m_\ell - 1)!} \quad (3)$$

or, using [11, eq. (8.3502.2)] the CDF can be re-written as

$$F_Y(y; m_\ell, \eta_\ell) = 1 - \exp\left(-\frac{y}{\eta_\ell}\right) \sum_{\mu=1}^{m_\ell - 1} \frac{1}{\mu} \left(\frac{y}{\eta_\ell}\right)^{\mu}. \quad (4)$$

A. Independent RVs

Theorem 1 (PDF of sum of squared Nakagami-m RVs): Let $\{Y_\ell\}_{\ell=1}^{L}$ be a set of RVs which follow the PDF defined in (2). Then, the PDF of the sum

$$Z_L = \sum_{i=1}^{L} Y_i \quad (5)$$

[1]Note that, (1) is an alternative form of the classical Nakagami-m PDF [1], in the case where the fading parameter m_ℓ is a positive integer.

[2]The Erlang distribution is a special case of the well-known Gamma distribution for integer values of m_ℓ.

$$\Xi_L\left(i,k,\{m_q\}_{q=1}^{L},\{\eta_q\}_{q=1}^{L},\{l_q\}_{q=1}^{L-2}\right)=$$
$$=\sum_{l_1=k}^{m_i}\sum_{l_2=k}^{l_1}\cdots\sum_{l_{L-2}=k}^{l_{L-3}}\left[\frac{(-1)^{R_L-m_i}\,\eta_i^{k}}{\prod_{h=1}^{L}\eta_h^{m_h}}\,\frac{\left(m_i+m_{1+\mathrm{U}(1-i)}-l_1-1\right)!}{\left(m_{1+\mathrm{U}(1-i)}-1\right)!\,(m_i-l_1)!}\left(\frac{1}{\eta_i}-\frac{1}{\eta_{1+\mathrm{U}(1-i)}}\right)^{l_1-m_i-m_{1+\mathrm{U}(1-i)}}\right.$$
$$\times\frac{\left(l_{L-2}+m_{L-1+\mathrm{U}(L-1-i)}-k-1\right)!}{\left(m_{L-1+\mathrm{U}(L-1-i)}-1\right)!\,(l_{L-2}-k)!}\left(\frac{1}{\eta_i}-\frac{1}{\eta_{L-1+\mathrm{U}(L-1-i)}}\right)^{k-l_{L-2}-m_{L-1+\mathrm{U}(L-1-i)}} \tag{7}$$
$$\left.\times\prod_{s=1}^{L-3}\frac{\left(l_s+m_{s+1+\mathrm{U}(s+1-i)}-l_{s+1}-1\right)!}{\left(m_{s+1+\mathrm{U}(s+1-i)}-1\right)!\,(l_s-l_{s+1})!}\left(\frac{1}{\eta_i}-\frac{1}{\eta_{s+1+\mathrm{U}(s+1-i)}}\right)^{l_{s+1}-l_s-m_{s+1+\mathrm{U}(s+1-i)}}\right]$$

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is a nested finite weighted sum of Erlang PDFs, given by

$$f_{Z_L}(z)=\sum_{i=1}^{L}\sum_{k=1}^{m_i}\Xi_L\left(i,k,\{m_q\}_{q=1}^{L},\{\eta_q\}_{q=1}^{L},\{l_q\}_{q=1}^{L-2}\right)\times f_Y(z;k,\eta_i) \tag{6}$$

where the parameter $\Xi_L\left(i,k,\{m_q\}_{q=1}^{L},\{\eta_q\}_{q=1}^{L},\{l_q\}_{q=1}^{L-2}\right)$ is given in (7) (see at top of this page) with R_L defined as $R_L\triangleq\sum_{i=1}^{L}m_i$.

Proof: See [12, Appendix] ∎

Note, that for $m_\ell=1$ (i.e., Rayleigh fading), it can be easily verified that (6) is reduced to a well-known result in the literature [13, eq. (10)].

Corollary 1 (CDF of sum of squared Nakagami-m RVs): The CDF of Z_L is given by

$$F_{Z_L}(z)=\sum_{i=1}^{L}\sum_{k=1}^{m_i}\Xi_L\left(i,k,\{m_q\}_{q=1}^{L},\{\eta_q\}_{q=1}^{L},\{l_q\}_{q=1}^{L-2}\right)\times F_Y(z;k,\eta_i). \tag{8}$$

Proof: The CDF of Z_L can be easily obtained by integrating (6) from 0 to z, interchanging the order of summations and integration. ∎

To the best of the authors' knowledge, (6) and (8) are novel. Both expressions can be easily evaluated due to the fact that are sums of simple elementary functions (i.e., powers and exponentials). Moreover, (6) is simpler compared to the PDF expression presented in [2, eq. (10)], which is apparently not in closed-form since it includes higher order derivatives as functions of the parameter m. Note also, that with the use of [2, eq. (10)] is difficult, if not impossible, to study other statistical metrics such as CDF.

B. Correlated RVs

In order to obtain the sum of correlated squared Nakagami-m RVs, the following assumptions, made also in [14]–[16], are taken into account and repeated here for the reader's convenience:

i) Without loss of generality, it can be assumed that statistical parameters m_ℓ are is increasing order, i.e., $m_1\le m_2\le\cdots\le m_L$,

ii) Let $\{X_\ell\}$ be arbitrarily correlated Nakagami-m RVs with marginal PDFs given by (1),

iii) Let $\mathbf{W}_\ell$ be $2\,m_\ell\times 1$ dimensional vectors defined as $\mathbf{W}_\ell=[W_{\ell,1}\,W_{\ell,2}\cdots W_{\ell,2m_\ell}]^\dagger$, where $(\cdot)^\dagger$ denotes transpose and the elements $\{W_{\ell,k}\}_{k=1}^{2m_\ell}$ are independent and identically distributed zero mean Gaussian RVs with variance $E\left\langle W_{\ell,k}^2\right\rangle=\eta_\ell/2$,

iv) Let $\mathbf{W}$ is a vector $D_T\times 1$ order, defined as $\mathbf{W}=\left[\mathbf{W}_1^\dagger\,\mathbf{W}_2^\dagger\cdots\mathbf{W}_L^\dagger\right]^\dagger$, where $D_T=\sum_{i=1}^{N}2\,m_i$, with covariance matrix given by $\mathcal{K}_W=E\left\langle\mathbf{W}\,\mathbf{W}^\dagger\right\rangle$,

v) The correlation among the elements of $\mathbf{W}$ is constructed such that

$$E\left\langle W_{i,k}\,W_{j,l}\right\rangle=\begin{cases}\eta_i/2, & \text{if } i=j\ \&\ k=l\\ \frac{\rho_{i,j}}{2}\sqrt{\eta_i\,\eta_j}, & \text{if } i\ne j\ \&\ k=l\\ & =1,2,\ldots,\\ & 2\,\min\{m_i,m_j\}\\ 0, & \text{otherwise.}\end{cases}$$

It can be shown that the relationship between the covariance of Y_i and Y_j and the correlation of the elements of $\mathbf{W}$ is given by

$$\begin{aligned}\rho_{Y_i,Y_j}=&\frac{E\left\langle(Y_i-m_i\,\eta_i)\,(Y_j-m_j\,\eta_j)\right\rangle}{\sqrt{\mathrm{var}(Y_i)\,\mathrm{var}(Y_j)}}\\=&\frac{\min\{m_i,m_j\}}{\sqrt{m_i\,m_j}}\,\rho_{i,j}^2,\end{aligned} \tag{9}$$

vi) Let $\{\lambda_\ell\}$ be the set of L distinct eigenvalues of $\mathcal{K}_W$, where each λ_ℓ has algebraic multiplicity μ_ℓ, such that $\sum_{i=1}^{L}\mu_i=D_T$.

Theorem 2: (PDF of the sum of squared correlated Nakagami-m RVs): If

$$Z_L=\sum_{i=1}^{L}X_i^2 \tag{10}$$

then it holds that

$$Z_L\overset{d}{=}\sum_{i=1}^{L}V_i \tag{11}$$

where V_ℓ is the ℓth Erlang distributed RV with parameters $m_\ell=\mu_\ell/2,\eta_\ell=4\,\lambda_\ell/\mu_\ell$ and the notation "$\overset{d}{=}$" means "equality in distribution".

Proof: See [14], where the Karhunen-Loeve expansion is used to de-correlate arbitrarily correlated, non-identical, Gamma distributed RVs, with integer-orders for m_ℓ's. ∎

Lemma 1: (CDF of the sum of squared correlated Nakagami-m RVs): The CDF of the sum of arbitrarily correlated squared Nakagami-m RVs, can be found in closed-form using *Corollary 1* together with *Theorem 2*.

$$\overline{S}_e = \frac{1}{\ln 2} \sum_{i=1}^{L} \sum_{k=1}^{m_i} \Xi_L \left(i, k, \{m_q\}_{q=1}^{L}, \left\{ \frac{\overline{\gamma}_q}{m_q} \right\}_{q=1}^{L}, \{l_q\}_{q=1}^{L-2} \right) \frac{1}{(k-1)!} \left(\frac{m_i}{\overline{\gamma}_i} \right)^k \sum_{w=1}^{k} \binom{k-1}{w-1} (-1)^{k-w}$$
$$\times \exp\left(\frac{m_i}{\overline{\gamma}_i}\right) \left\{ \frac{1}{w^2}\, {}_2F_2\left(w, w; w+1, w+1; -\frac{m_i}{\overline{\gamma}_i}\right) + \left(\frac{\overline{\gamma}_i}{m_i}\right)^w (w-1)! \left[\ln\left(\frac{\overline{\gamma}_i}{m_i}\right) + \sum_{h=1}^{w-1} \frac{1}{h} - \mathcal{C} \right] \right\} \quad (15)$$

III. Performance of MRC Receivers

We consider an L-branch diversity receiver operating in a flat fading environment. The baseband received signal at the ℓth, $\ell = 1, 2, \ldots, L$, diversity branch is

$$\zeta_\ell = s\, X_\ell \exp(\jmath\, \theta_\ell) + n_\ell \quad (12)$$

where s is the transmitted symbol, with energy $E_s = E\left\langle |s|^2 \right\rangle$, X_ℓ is the Nakagami-m distributed fading envelope, $\jmath = \sqrt{-1}$, n_ℓ is the additive white Gaussian noise, with a single-sided power spectral density N_0 and θ_ℓ is the random phase due to Doppler shift and oscillators frequency mismatch. The phase θ_ℓ is uniformly distributed over the range $[0, 2\pi)$ and the noise components are assumed to be statistically independent of the signal and uncorrelated with each other. The channel is considered slowly time varying and thus, the phase can be easily estimated.

The instantaneous SNR per symbol $\gamma_\ell = X_\ell^2\, E_s/N_0$ in the ℓth input branch follows a two parameter Erlang distribution $f_{Y_\ell}(\gamma_\ell; m_\ell, \overline{\gamma}_\ell/m_\ell)$, with m_ℓ and $\overline{\gamma}_\ell = E\left\langle X_\ell^2 \right\rangle E_s/N_0$ being the corresponding Nakagami-m fading parameter and the average input SNR per symbol, respectively. The performance analysis of the MRC receivers, in which the instantaneous SNR per symbol at the output is given by the well-known expression $\gamma = \sum_{i=1}^{L} \gamma_i$, can be tackled using the analysis presented in Section II, both for independent and correlative fading.

A. Outage Probability

The outage probability in noise-limited systems, P_{out}, is defined as the probability that the instantaneous MRC output SNR falls below a given outage threshold, γ_{th}. This probability can be easily obtained by replacing z with γ_{th} in (8) as

$$P_{out}(\gamma_{th}) = F_{Z_L}(\gamma_{th}) \quad (13)$$

with $\eta_q = \overline{\gamma}_q/m_q$, for the independent case, or using *Lemma 1* for the correlative case. Furthermore, our approach can be efficiently applied to evaluate the outage probability in cellular systems, when co-channel interference is considered.

B. Average SE

The Shannon channel capacity provides an upper bound of maximum transmission rate in a given Gaussian environment [17]. The average SE, in Shannon's sense, defined as the normalized, by the transmitted signal's bandwidth, average channel capacity is given by

$$\overline{S}_e = \int_0^\infty \log_2(1+\gamma)\, f_{Z_L}(\gamma)\, d\gamma. \quad (14)$$

By substituting (6) in the above integral and using [11, eq. (4.358/1)], in case of independent fading, the average channel SE can be written in closed-form as in (15) (see at top of this page), where $\mathcal{C}$ is the Euler's constant [11, Sec. (9.73)], ${}_2F_2(\cdot,\cdot;\cdot,\cdot;\cdot)$ is a generalized hypergeometric series [11, eq. (9.14/1)] and $\binom{k-1}{w-1}$ is the binomial coefficient defined as $\binom{k-1}{w-1} = (k-1)!/[(w-1)!\,(k-w)!]$. In case of correlative fading, the average SE can be obtained using (15) and substituting m_i with $\mu_i/2$ and $\overline{\gamma}_i$ with $\lambda_i/2$.

C. ASEP

The most straightforward approach to obtain the ASEP, $\overline{P}_{se}$, is to average the conditional symbol error probability, $P_{se}(\gamma)$, over the PDF of the combiner output SNR [10], i.e.,

$$\overline{P}_{se} = \int_0^\infty P_{se}(\gamma)\, f_{Z_L}(\gamma)\, d\gamma. \quad (16)$$

It is well known, that for several signaling constellations, $P_{se}(\gamma)$ can be written as follows:

i) For binary phase shift keying (BPSK), binary frequency shift keying (BFSK) and for high values of average input SNR for Gaussian minimum shift keying (GMSK)[3], M-ary-differentially encoded phase shift keying (M-DEPSK), quadrature phase shift keying (QPSK), M-ary-phase shift keying (M-PSK), M-ary-frequency shift keying (M-FSK), square M-ary-quadrature amplitude modulation (M-QAM), and M-ary-differential PSK (M-DPSK) in the form of $P_{se}(\gamma) = A\,\mathrm{erfc}\left(\sqrt{B\,\gamma}\right)$, where erfc$(\cdot)$ is the complementary error function [11, eq. (8.250.4)],

ii) For differential binary PSK (DBPSK) and M-ary-non-coherent frequency shift keying (M-NFSK), in the form of $P_{se}(\gamma) = A\,\exp(-B\,\gamma)$.

The particular values of A and B depend on the considered modulation scheme and summarized in [12, Table 1]. Into the following, $\overline{P}_{se}$ is obtained in closed-form expressions for each one of the above two cases.

By substituting (6) in (16), it can be easily recognized that for coherent binary and M-ary modulation schemes, such as i) BPSK and BFSK and ii) for high values of average input SNR for GMSK, M-DEPSK, QPSK, M-PSK, M-FSK, M-QAM and M-DPSK, the evaluation of integrals of the form $\Upsilon = \int_0^\infty x^{k-1}\,\mathrm{erfc}\left(\sqrt{B\,x}\right) \exp(-x/\eta_i)\, dx$ with $\eta_i = \overline{\gamma}_i/m_i$, is required. The above integral can be evaluated via [11, eq. (6.455-1)], by noting that $\mathrm{erfc}(\cdot)$ can be expressed as an incomplete Gamma function with the using of [18, eq.

[3] $\mathcal{B}$ is determined by the bandwidth of the premodulation Gaussian filter.

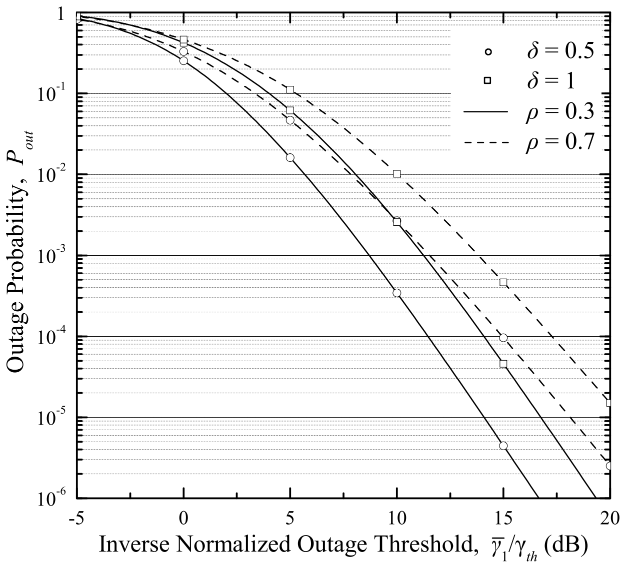

Fig. 1. Outage probability versus inverse, normalized to the average SNR of the first input branch, outage threshold, for $L = 3$, with an exponentially decaying PDP and $m_1 = m_2 = 1$ and $m_3 = 2$.

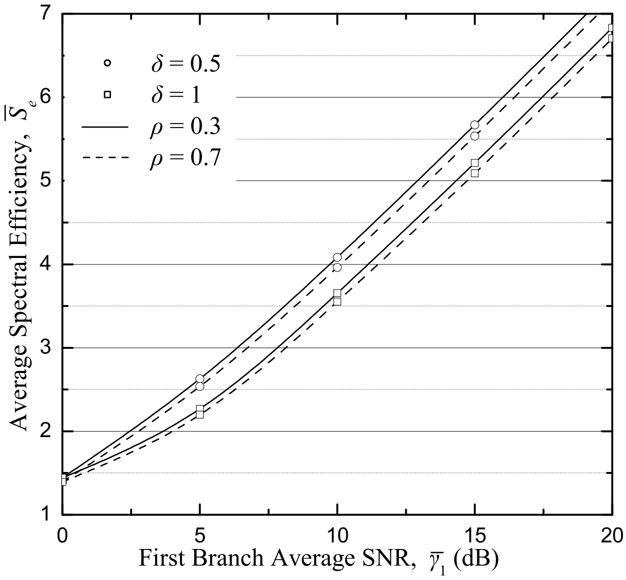

Fig. 2. Average SE versus first branch average SNR, for $L = 3$, with an exponentially decaying PDP and $m_1 = m_2 = 1$ and $m_3 = 2$.

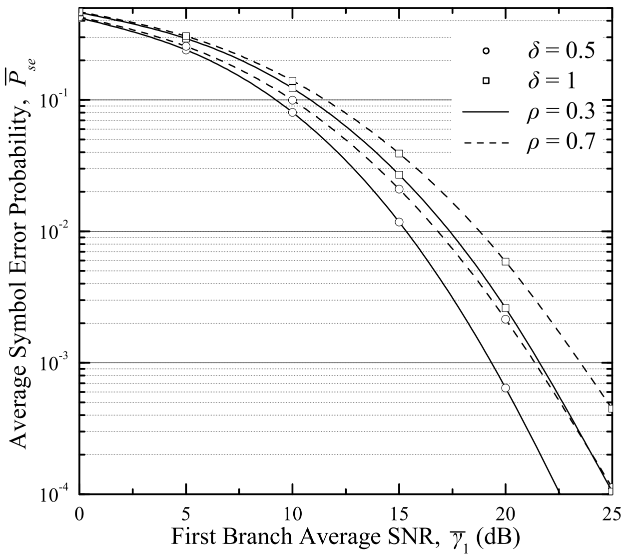

Fig. 3. ASEP of 16-QAM versus first branch average SNR, for $L = 3$, with an exponentially decaying PDP and $m_1 = m_2 = 1$ and $m_3 = 2$.

(06.06.03.0004.01)]. Therefore, the ASEP can be derived in closed-form as

$$\overline{P}_{se} = A \sum_{i=1}^{L} \sum_{k=1}^{m_i} \Xi_L \left(i, k, \{m_q\}_{q=1}^{L}, \left\{ \frac{\overline{\gamma}_q}{m_q} \right\}_{q=1}^{L}, \{l_q\}_{q=1}^{L-2} \right) \times \frac{(2\,k-1)!!}{k!\,(2\,B)^k} \left(\frac{m_i}{\overline{\gamma}_i} \right)^k {}_2F_1 \left(k, k+\frac{1}{2}; k+1; -\frac{m_i}{B\,\overline{\gamma}_i} \right) \quad (17)$$

where ${}_2F_1\left(\cdot,\cdot;\cdot;\cdot\right)$ is the Gauss hypergeometric function [11, eq. (9.100)].

The ASEP for non-coherent modulation schemes, such as NBFSK and DBPSK, can be extracted by substituting (6) in (16) and using [11, eq. (3.381/4)], yielding

$$\overline{P}_{se} = A \sum_{i=1}^{L} \sum_{k=1}^{m_i} \Xi_L \left(i, k, \{m_q\}_{q=1}^{L}, \left\{ \frac{\overline{\gamma}_q}{m_q} \right\}_{q=1}^{L}, \{l_q\}_{q=1}^{L-2} \right) \times \left(\frac{m_i}{\overline{\gamma}_i} \right)^k \left(B + \frac{m_i}{\overline{\gamma}_i} \right)^{-k}. \quad (18)$$

For correlative fading, the ASEP can be obtained using (17) and (18) after substituting m_i with $\mu_i/2$ and $\overline{\gamma}_i$ with $\lambda_i/2$.

IV. Numerical Results

Based on the analysis presented in the previous section, some representative numerical examples for the outage probability, the average SE and the ASEP, are presented in Figs. 1, 2 and 3, respectively. In these numerical results, it is considered an MRC receiver with $L = 3$ antennae, operating in a Nakagami-m multipath fading environment, with fading parameters $m_1 = m_2 = 1$ and $m_3 = 2$, having an exponentially decaying power delay profile (PDP) $\overline{\gamma}_\ell = \overline{\gamma}_1 \exp[-\delta(\ell-1)]$ with power decaying factors $\delta = 0.5$ and 1, and exponential correlation among the input channels [4] $\rho_{\gamma_i,\gamma_j} = \rho^{|i-j|}$, with $\rho = 0.3$ and 0.7. Note, that the correlation matrix of this model corresponds to the scenario of multichannel reception from equispaced diversity antennas, since the correlation between the pairs of combined signals decays as the spacing between the antennas increases [3].

In Fig. 1, the outage probability, P_{out}, is plotted as a function of the inverse, normalized, to $\overline{\gamma}_1$, outage threshold, i.e., $\overline{\gamma}_1/\gamma_{th}$. The obtained results clearly show that the outage performance degrades with an increase of the fading correlation and/or the decaying factor. In Fig. 2, the average SE, $\overline{S}_e$, is plotted as a function of the average SNR of the first input branch, $\overline{\gamma}_1$. It can be

[4] When $\rho \rightarrow 1$, it is easily verified that the covariance matrix $\mathcal{K}_W$ is not a positive definite matrix (i.e., some eigenvalues are not greater than zero or complex). Therefore, it is impossible to study cases for values of ρ near to 1. This is true due to the fact that two Nakagami RVs with different distributions can not be completely correlated [19].

observed that, the higher the values of ρ and δ, the less the maximum errorless transmission rate which can be achieved. In Fig. 3, the ASEP of 16-QAM, $\overline{P}_{se}$, is plotted as a function of the average SNR of the first input branch. The obtained performance evaluation results show that the error performance improves with the decrease of ρ, while as expected the diversity gain decreases with increasing values of δ.

V. CONCLUSIONS

We derived novel closed-form expressions for the PDF and the CDF of the sum of squared, non-identical, independent or correlated Nakagami-m RVs in case of integer-order Nakagami-m fading parameters, which improve previously published results. It was shown that these expressions can be written as a weighted sum of Erlang distributions. Based on the statistical formulae obtained, MRC receivers were studied and important performance metrics, such as outage probability, average SE and ASEP, were expressed in closed form.

REFERENCES

[1] M. Nakagami, "The m-distribution–A general formula of intensity distribution of rapid fading," in *Statistical Methods in Radio Wave Propagation*. Oxford, U.K.: Pergamon Press, 1960, pp. 3–36.

[2] E. K. Al-hussaini and A. A. M. Al-bassiouni, "Performance of MRC diversity systems for the detection of signals with Nakagami fading," *IEEE Trans. Commun.*, vol. COM-33, no. 12, pp. 1315–1319, Dec. 1985.

[3] V. A. Aalo, "Performance of maximal-ratio diversity systems in a correlated Nakagami-fading environment," *IEEE Trans. Commun.*, vol. 43, no. 8, pp. 2360–2369, Aug. 1995.

[4] P. Lombardo, G. Fedele, and M. M. Rao, "MRC performance for binary signals in Nakagami fading with general branch correlation," *IEEE Trans. Commun.*, vol. 47, no. 1, pp. 44–52, Jan. 1999.

[5] Q. T. Zhang, "Exact analysis of postdetection combining for DPSK and NFSK systems over arbitrarily correlated Nakagami channels," *IEEE Trans. Commun.*, vol. 46, no. 11, pp. 1459–1467, Nov. 1998.

[6] ——, "Maximal-ratio combining over Nakagami fading channels with an arbitrary branch covariance matrix," *IEEE Trans. Veh. Technol.*, vol. 48, no. 4, pp. 1141–1150, July 1999.

[7] ——, "Error performance of noncoherent MFSK with L-diversity on correlated fading channels," *IEEE Trans. Wireless Commun.*, vol. 1, no. 3, pp. 531–539, July 2002.

[8] M.-S. Alouini, A. Abdi, and M. Kaveh, "Sum of gamma variates and performance of wireless communication systems over Nakagami-fading channels," *IEEE Trans. Veh. Technol.*, vol. 50, no. 6, pp. 1471–1480, Nov. 2001.

[9] P. G. Moschopoulos, "The distribution of the sum of independent gamma random variables," *Ann. Inst. Statist. Math. (Part A)*, vol. 37, pp. 541–544, 1985.

[10] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New York: Wiley, 2005.

[11] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. New York: Academic Press, 2000.

[12] G. K. Karagiannidis, N. C. Sagias, and T. A. Tsiftsis, "Closed-form statistics for the sum of squared Nakagami-m variates and its applications," *IEEE Trans. Commun.*, submitted.

[13] P. Gupta, N. Bansal, and R. K. Mallik, "Analysis of minimum selection H-S/MRC in Rayleigh fading," *IEEE Trans. Commun.*, vol. 53, no. 5, pp. 780–784, May 2005.

[14] M. Z. Win, G. Chrisikos, and J. H. Winters, "MRC performance for M-ary modulation in arbitrarily correlated Nakagami fading channels," *IEEE Commun. Lett.*, vol. 4, no. 10, pp. 301–303, Oct. 2000.

[15] Q. T. Zhang, "A generic correlated Nakagami fading model for wireless communications," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1745–1748, Nov. 2003.

[16] I. Ghareeb and S. Abu-Surra, "Differential detection of GMSK signal with postdetection MRC over correlated and unbalanced Nakagami-m fading channels," *IEE Proc.-Commun*, vol. 152, no. 2, pp. 221–228, Apr. 2005.

[17] W. C. Y. Lee, "Estimate of channel capacity in Rayleigh fading environment," *IEEE Trans. Veh. Technol.*, vol. 39, no. 3, pp. 187–189, Aug. 1990.

[18] Wolfram. (2005) The Wolfram functions site. Internet. [Online]. Available: http://functions.wolfram.com

[19] M. Z. Win and J. H. Winters, "Exact error probability expressions for MRC in correlated Nakagami channels with unequal fading parameters and branch powers," in *Proc. IEEE Global Commun. Conf. (GLOBECOM'99)*, Rio de Janeiro, Brazil, Dec. 1999, pp. 2331–2335.