Regenerative versus Non-Regenerative Relaying in Cooperative Diversity Systems

Lydia-Chaido A. Siafara*, Diomidis S. Michalopoulos*, Theodoros A. Tsiftsis†
and George K. Karagiannidis*
*Telecommunications Division, Department of Electrical & Computer Engineering, Aristotle University of Thessaloniki, GR-54124, Thessaloniki, Greece
Email:{lsiafara, dmixalo, geokarag}@auth.gr
†Wireless Telecommunications Laboratory, Department of Electrical & Computer Engineering, University of Patras, Rion, GR-26500, Patras, Greece
Email: tsiftsis@ee.upatras.gr

***Abstract*— Cooperative diversity is a technique which improves the performance of the wireless communication systems which suffer from channels impairement due to fading. This paper evaluates the performance of practical cooperative diversity systems when two types of relays are used: regenerative and non-regenerative. Specifically, the average bit-error probability is studied over Rayleigh fading channels for both types of relays via numerical integration and Monte Carlo simulations. A number of numerical examples verify that non-regenerative cooperative diversity systems outperform those using regenerative relays due to better exploitation of the spatial diversity on the destination node.**

I. Introduction

Multihop relaying technology is a promising solution for future cellular and ad-hoc wireless communications systems in order to achieve broader coverage and to mitigate wireless channels impairment without the need to use large power at the transmitter [1]–[5]. More recently, the concept of *cooperative relaying* has emerged. By allowing cooperation among the users-relays and by combining all relays' transmissions at the destination terminal, the realization of spatial diversity is achieved. Cooperative diversity exploits two fundamental features of the wireless medium: its broadcast nature and its ability to achieve diversity through independent channels. In the up to date study, cooperation protocols have been introduced, involving the sharing of information on an internode channel, such that each user communicates with the base station via a number of cooperating users. For the case of a single available relay, an efficient cooperative protocol was proposed in [1], concerning activation of the relay only when the fading coefficient of the channel at the input or at the output of the relay is above a given threshold.

Realying transmisson can be classified into two main categories, namely regenerative (or amplify-and-forward) and non-regenerative (or decode-and-forward), depending on the relay type they use: 1) *Non-regenerative relays* act as analog repeaters by retransmitting an amplified version of their received signals and also a scalled version of the Gaussian noise. 2) *Regenerative relays* decode, regenerate and retransmit an exact copy of the original signal, potentially propagating decoding errors. Additionally, the relays of non-regenerative systems are classified in two main subcategories, as a) channel state information (CSI)-assisted relays, where they use the CSI from the previous hop to produce their gain, leading to a power control of the retransmitted signal and b) fixed-gain relays with lower complexity compared to CSI-assisted ones, which introduce a fixed gain and thus a variable signal power at the output.

In this paper, we investigate which type of relay performs better in dual-hop cooperative transmissions in terms of the average bit-error rate (BER). Since user cooperation is most useful when channels are varying very slowly and it is assumed as independent and not necessarily identically distributed (i.i.d.) Rayleigh fading channels. We investigate cases with 2 or 4 users-relays between the source and destination terminals and a thorough comparison between the two types of relays is executed. Closed-form expressions are given for the non-regenerative relays and the study of regenerative one is conducted only via simulations. Also, the destination terminal combines all the signals via a maximal ratio combiner (MRC) receiver.

The remainder of this paper is organized as follows. The dual-hop cooperative relayed system under consideration is described in the next section and formulae for the end-to-end SNR is presented. In Section III the overall performance analysis of the system is studied and closed-form expressions for the average BER are presented. In Section IV, several numerical and simulation results are presented, while some concluding remarks are offered in Section V.

II. System and Channel Model

A. Non-Regenerative Cooperative Diversity Systems

We consider a cooperative diversity system where the source terminal S, communicates with the destination terminal D via L other parallel cooperating terminals which act as relays, denoted by $R_i,\ \ i=1,...,L$. This system is depicted in Fig. 1. Each transmission period T is divided into two slots: in the first time slot, terminal S communicates with the relays and the destination terminal, while in the second slot, only the relays communicate with terminal D. The above transmission protocol was originally proposed in [6] which realizes maximum degrees of bradcasting and exhibits no receive collision. The destination terminal combines the received signals from L relaying channels and the direct one (i.e., $S \rightarrow R$) using an MRC receiver at terminal D. The system is operating

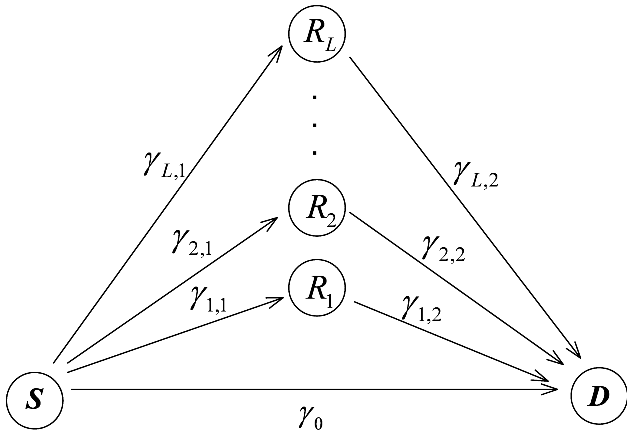

Fig. 1. A typical cooperative diversity system

over independent but not necessarily identically distributed Rayleigh fading channels [7].

Assume that S transmits a signal $x(t)$ with an average power normalized to unity. The received signal $y_i(t)$ at relay R_i can be written as:

$$y_i(t) = \alpha_{i,1}x(t) + n_{i,1},\ i = 1, ..., L \tag{1}$$

where $n_{i,1}$ is an additive white Gaussian signal (AWGN) with one sided power spectral density (PSD) N_0, and $\alpha_{i,1}$ is the fading amplitude of the first hop (i.e., $S \rightarrow R_i$).

The signal $y_i(t)$ is then multiplied by the gain G_i of the i_{th} relay and retransmitted to the destination terminal, where the received signal $y_i^{'}(t)$ can be expressed by

$$y_i^{'}(t) = G_i\alpha_{i,2}\left[\alpha_{i,1}s(t) + n_{i,1}\right] + n_{i,2}, \tag{2}$$

where $n_{i,2}$, $\alpha_{i,2}$ are the the AWGN with one sided PSD N_0 and the Rayleigh distributed fading amplitude of the second hop (i.e., $R_i \rightarrow D$), respectively. Note, that the subscripts numbers 1, 2 in α denote the first and the second hop, respectively. Each of the received signals $y_i^{'}(t)$ corresponds to an individual relay branch, denoted by $b_i,\ i = 1, ..., L$. Hence, the equivalent instantaneous end-to-end SNR γ_i of the branch b_i can be written as [8]

$$\gamma_i = \frac{\left(G_i\alpha_{i,2}\alpha_{i,1}\right)^2}{\left[\left(G_i\alpha_{i,2}\right)^2 + 1\right]N_0} \tag{3}$$

1) CSI-assisted Relays: If the relay R_i has full knowledge of the channel state information (CSI) in its input, its gain can act as a counterbalance of the signal degradation in the hop $S \rightarrow R_i$, [1], in order to limit its output power if $(\alpha_{i,1})^2$ is low, e.g.,

$$G_i^2 = \frac{1}{(\alpha_{i,1})^2 + N_0}. \tag{4}$$

Substituting (4) into (3) yields

$$\gamma_i = \frac{\gamma_{i,1}\gamma_{i,2}}{\gamma_{i,1} + \gamma_{i,2} + 1} \tag{5}$$

where $\gamma_{i,1} = (\alpha_{i,1})^2/N_0$, $\gamma_{i,2} = (\alpha_{i,2})^2/N_0$ are the instantaneous SNR of the hop $S \rightarrow R_j$ and $R_j \rightarrow D$ respectively. Since $\alpha_{i,1}$, $\alpha_{i,2}$ are Rayleigh distributed, $\gamma_{i,1}$, $\gamma_{i,2}$ are exponentially distributed with probability density function (PDF)

$$f_{\gamma_{i,k}} = \frac{1}{\overline{\gamma}_{i,k}}\exp\left(-\frac{\gamma_{i,k}}{\overline{\gamma}_{i,k}}\right),\quad k \in \{1, 2\},\ i = 1, ..., L, \tag{6}$$

Assuming an MRC receiver at the output of the destination terminal, the overall end-to-end SNR can be written as

$$\gamma_{end} = \gamma_0 + \sum_{i=1}^{L}\frac{\gamma_{i,1}\gamma_{i,2}}{\gamma_{i,1} + \gamma_{i,2} + 1} = \sum_{i=0}^{L}\gamma_i, \tag{7}$$

where γ_0 is the instantaneous SNR of the direct $S \rightarrow D$ channel. The system branch corresponding to the $S \rightarrow D$ channel is denoted by b_0.

2) Fixed-Gain Relays: When the relay R_i introduce fixed-gain to the received signal given by [5], [9]

$$G_i^2 = \frac{1}{C_i\ N_0}. \tag{8}$$

where C_i is a constant positive real number. By substituting (8) to(3) , the equivalent end-to-end SNR of the γ_i can be expresed by

$$\gamma_i = \frac{\gamma_{i,1}\gamma_{i,2}}{C + \gamma_{i,2}} \tag{9}$$

and the end-to-end SNT at the output of the MRC receiver as

$$\gamma_{end} = \gamma_0 + \sum_{i=1}^{L}\frac{\gamma_{i,1}\gamma_{i,2}}{C + \gamma_{i,2}} = \sum_{i=0}^{L}\gamma_i. \tag{10}$$

It is emphasized here, that fixed-gain relays provide reduced implemetation complexity in the CSI part, at the expense of the requirements for high-transmission-power amplifiers. Also, as it is shown in [9], CSI-assisted relays are slightly outperform those with fixed-gain relays and only in to medium to high average SNR region.

B. Regenerative Cooperative Diversity Systems

When the relays regenerate the original incoming signal, the signal transmission in the first time slot is the same as the one presented above for the non-regeneretive relays (eq. (1)). Upon the reception of the incoming signal the relay demodulates, decodes and finally re-encodes (using or not a different codebook) the signal and retransmit it to the destination terminal. Thus, the received signal at the destination can be given by

$$y_i^{''}(t) = \alpha_{i,2}x'(t) + n_{i,2} \tag{11}$$

where $x'(t)$ is the equivalent signal retransmitted by the relay. When the regenerative cooperative system is in the outage case (i.e., fails to decode tha data correctly), the spatial diversity effect of cooperative diversity approach does not exist anymore. Then, the relay has two options: One is to remain silent and the other to transmit its own data. As presented in [**?**], the performance improvement brought by the latter is so minor that it does not worth the additional energy consumed.

III. Performance Analysis

In this section we investigate only the average probability of error which has a practical interest.

A. Non-Regenerative Cooperative Diversity Systems

The error performance, for several digital modulation schemes, can be efficiently studied using the well-known moment generating function (MGF)-based approach [7]. The MGF, defined here as

$$\mathcal{M}_{\gamma_{end}}(s) := E\left\langle \exp\left(s\gamma_{end}\right)\right\rangle, \tag{12}$$

where $E\langle\cdot\rangle$ stands fo the expectation value of its argument.

Since the intermediate channels are independent Rayleigh fading channels the $\mathcal{M}_{\gamma_{end}}(s)$ can be expressed by

$$\mathcal{M}_{\gamma_{end}}(s) = \prod_{j=0}^{L} \mathcal{M}_j(s). \tag{13}$$

where $\mathcal{M}_{\gamma_0}(s) = (1 - s\gamma_0)^{-1}$ and $\mathcal{M}_{\gamma_j}(s), \;\; j = 1, 2..., L$ can be evaluated numerically by the following double integral

$$\mathcal{M}_{\gamma_j}(s) = \int_0^\infty \int_0^\infty \exp\left(s\gamma_j\right) f_{\gamma_{i,1}}(\gamma_{i,1}) f_{\gamma_{i,2}}(\gamma_{i,2}) d\gamma_{i,1} d\gamma_{i,2}. \tag{14}$$

Using the pdfs $f_{\gamma_{i,1}}(\gamma_{i,1})$, $f_{\gamma_{i,2}}(\gamma_{i,2})$ given in (6), a closed-form expression is presented in [10, eq. (6)].

With the aid of $\mathcal{M}_{\gamma_{end}}(s)$, and using the MGF-based approach for the performance evaluation of digital modulations over fading channels presented in [7] , the error rates can be calculated directly for non-coherent binary signalling, such as BFSK and DPSK, while for other cases including M-QAM and M-PSK, single integrals with finite limits and integrands composed of elementary functions have to be readily evaluated via numerical integration.

For example, the ABEP for BPSK modulation scheme, which is used as an example in Section IV, can be evaluated via numerical integration as

$$\text{ABEP} = \frac{1}{\pi}\int_0^{\pi/2} \mathcal{M}_{\gamma_{end}}\left(-\frac{1}{\sin^2\phi}\right) d\phi, \tag{15}$$

using any of the well-known mathematical software packages, such as Mathematica or Mapple.

B. Regenerative Cooperative Diversity Systems

In regenerative dual-hop cooperative diversity systems, the transmitted signal corresponding to each branch b_j undergoes two states of decoding in cascade, and the end-to-end conditionally probability of error using the M-QAM signal modulation is given in [10, eq. (9)] as

$$\begin{aligned} P_i(E) &= P_i(E|\gamma_{i,1}) + P_i(E|\gamma_{i,2}) \\ &\quad - \frac{M}{M-1} P_i(E|\gamma_{i,1}) P_i(E|\gamma_{i,2}), \end{aligned} \tag{16}$$

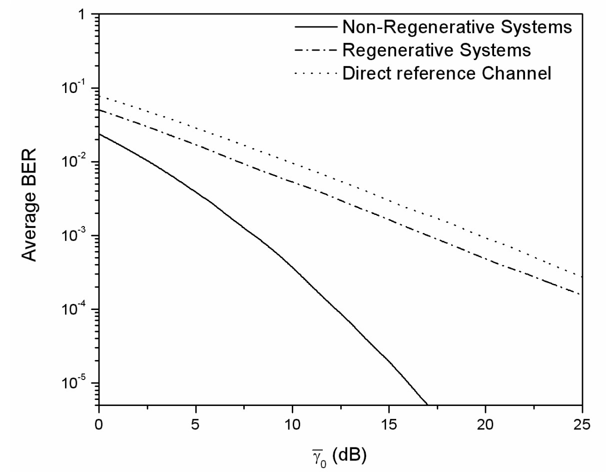

Fig. 2. ASEP vs $\overline{\gamma}_0$ using BPSK modulation for $L = 2$, $\mu_1 = 2$, $\mu_2 = 4$.

where $P_i\left(E|\gamma_{i,1}\right)$ and $P_i(E|\gamma_{i,1})$ are the individual ASEP corresponding to the first and second hop of the b_i branch respectively. Then, assuming that each branch b_i can be substituted by an equivalent Rayleigh fading channel, and using the approximation in [7, p. 275] we finally obtain

$$\text{ASEP} \simeq \left(\frac{M}{M-1}\right)^2 \prod_{i=0}^{L} P_i\left(E\right). \tag{17}$$

IV. Numerical-Simulation Examples and Discussion

In this section the simulation results concerning the performance of the regenerative and non-regenerative relaying are discussed. For simplicity of exposition, BPSK signal modulation is assumed for both scenarios, and the SNR on each S-R_j and R_j-D channel is considered independent and exponentially distributed, with normalized expected value with respect to $\overline{\gamma}_{SD}$ equal to μ_1 and μ_2 respectively. For the non-regenerative case, the relay gains are given in (4), resulting in identical transmiting power by all system nodes, and in the regenerative case the relays' transmitting power is also identical, same as previous. Both regenerative and non-regenerative relaying systems are compared in terms of average BER with the direct reference channel, which is a hypothetical S-D channel with average transmission power equal to the sum of the average transmission powers of all system nodes. Hence, a fair comparison is provided between these two systems and also between them and the conventional telecommunication ones, consisting of a single transmitting and receiving node, in terms of energy consumption.

Figs. 2-3 depict the average BER versus $\overline{\gamma}_{SD}$ for the 2-user regenerative and non-regenerative scenario assuming MRC at the destination terminal, accompanied with the average BER corresponding to the direct reference channel. In Fig. 2, μ_1 and μ_2 are equal to 2 and 4, while in Fig. 3, μ_1 and μ_2 are equal to 4 and 2 respectively. As it was expected, cooperative diversity systems outperform the conventional telecommunication ones;

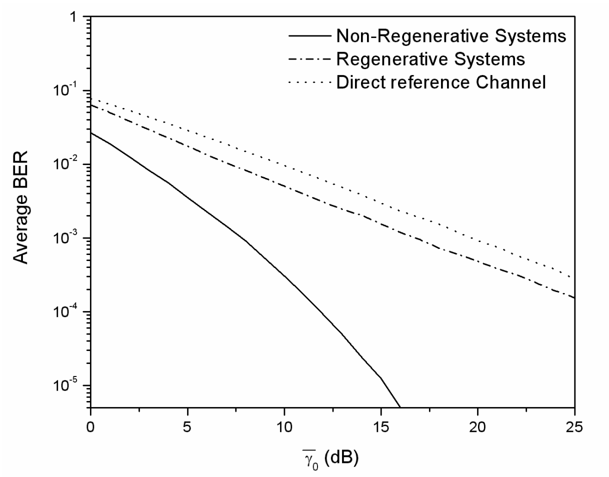

Fig. 3. ASEP vs $\overline{\gamma}_0$ using BPSK modulation for $L = 2$, $\mu_1 = 4$, $\mu_2 = 2$.

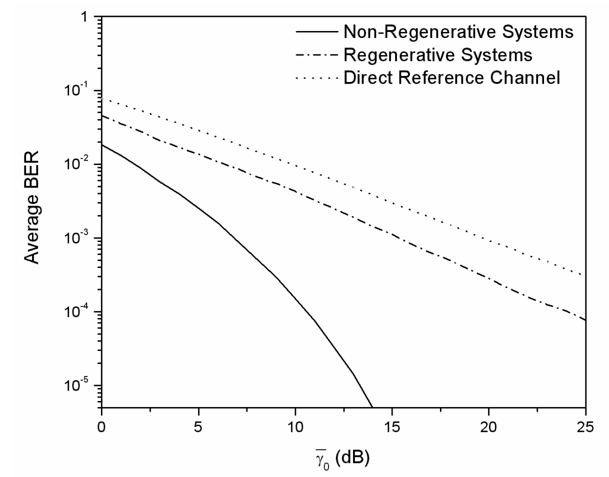

Fig. 5. ASEP vs $\overline{\gamma}_0$ using BPSK modulation for $L = 4$, $\mu_1 = 2$, $\mu_2 = 4$.

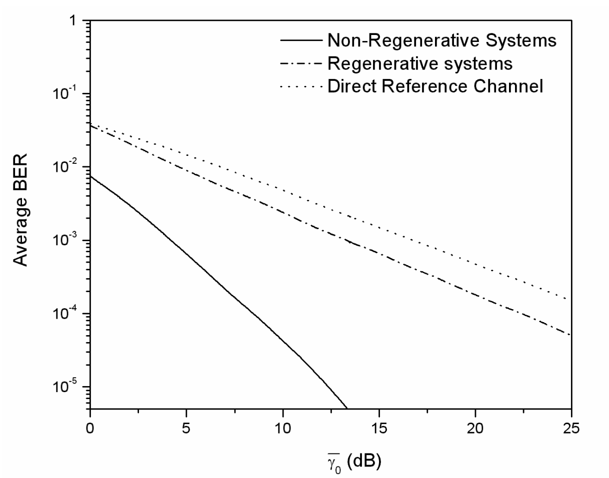

Fig. 4. ASEP vs $\overline{\gamma}_0$ using BPSK modulation for $L = 4$, $\mu_1 = 4$, $\mu_2 = 2$.

however, non-regenerative diversity leads to an average BER which is generally lower than the equivalent regenerative one. In particular, the difference between regenerative and non-regenerative performance increases as the $\overline{\gamma}_{SD}$ grows large. Moreover, regenerative systems with small number of cooperating users appear to perform in a way similar to the direct reference channel, as it is shown in Figs. 2 and 3.

In Figs. 5 and 4, the same comparison assuming 4 co-operating users is presented. The results derived from these two Figs. are generally the same, and again non-regenerative systems seem to perform better in terms of average BER. Furthermore, observing Figs. 2-4, it is clear that this difference on the error performance widens as the number of cooperating users increases.

Considering all the above we conclude that, *in cases when each relay gain is implied by (4), or (8) non-regenerative co-operative diversity systems outperform the regenerative ones, in terms of avera.* It is evident that the former systems have the disadvantage of propagating the noise received from the first-hop channel, whereas the main drawback of the latter ones is the propagation of the erroneously detected symbols. However, other relaying protocols have been proposed that improve the performance of regenerative relays, [11] which involves coding techniques with out-of-band feedback, indicating success of reception and adaptive behavior based on receiver channel information. Then, the question "is the amplify-and-forward technique better than the decode-and-forward one?" can be rephrased as "in which case is propagating the noise better than propagating the erroneous symbols?". The answer lies in the existence of diversity at the receiver. In amplify-and-forward systems, terminal D combines concurrently the signals incident from node S and all the cooperating relays, the SNR of the latter ones having the form of (5). Then, it is evident that any possible severe attenuation in a single first-hop channel can be identified by the MRC, and thus has a slight impact on its output signal quality. On the contrary, a possible error on symbol detection in decode-and-forward systems cannot be distinguished by the destination node, which in fact combines the erroneous symbol resulting finally in a noteworthy degradation on error performance. In other words, non-regenerative systems take full advantage of diversity whereas regenerative do not. Consequently, it is easy to realize that the higher the number of cooperating users the better the non-regenerative error performance compared to the equivalent regenerative one, since diversity operates better as the number of input branches increases.

V. Conclusion

In this paper, a comparison between the performance of regenerative and non-regenerative relaying in cooperative diversity systems was presented. Numerical and simulation examples revealed that non-regenerative cooperative diversity systems lead to significantly lower ASEP, since regenerative relaying limits the diversity's benefical effects on error performance. In fact, amplify-and-forward seems to represent an efficient relaying method, whereas decode-and-forward results in an error performance which does not considerably differ from the equivalent conventional one, especially when a small number of cooperating users is available.

REFERENCES

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behaviour," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[2] P. A. Anghel and M. Kaveh, "Exact symbol error probability of a cooperative network in a Rayleigh-fading environment," *IEEE Trans. Wir. Commun.*, vol. 3, pp. 1416–1421, Sept. 2004.

[3] A. Ribeiro, X. Cai, and G. Giannakis, "Symbol error probabilities for general cooperative links," *IEEE Trans. Wir. Commun.*, vol. 4, pp. 1264–1273, May 2005.

[4] G. K. Karagiannidis, T. A. Tsiftsis, and R. K. Mallik, "Bounds for multihop relayed communications in Nakagami-m fading," *IEEE Trans. Commun.*, vol. 54, pp. 18–22, Jan. 2006.

[5] G. K. Karagiannidis, "Performance bounds of multihop wireless communications with blind relays over generalized fading channels," *IEEE Trans. Wir. Commun.*, to appear.

[6] J. N. Laneman, "Cooperative Diversity in Wireless Networks: Algorithms and Architectures," Ph.D. dissertation, Massachusetts Institute of Technology, Cambridge, MA, Aug. 2002.

[7] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New York: Wiley, 2005.

[8] M. O. Hasna and M. S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh fading channels," *IEEE Trans. Wir. Commun.*, vol. 2, pp. 1126–1131, Nov. 2003.

[9] ——, "A performance study of dual-hop transmissions with fixed gain relays," *IEEE Trans. Wir. Commun.*, vol. 3, pp. 1963–1968, Nov. 2004.

[10] P. A. Anghle, G. Leus, and M. Kaveh, "Multi-user space-time coding in cooperative networks," in *Proc. of International Conference on Acoustics, Speech, and Signal Processing (ICASSP 2003)*. Hong Kong: IEEE, Apr. 2003, pp. 73–76.

[11] T. E. Hunter, S. Sanayei, and A. Nosratinia, "Outage analysis of coded cooperation," *IEEE Trans. Inform. Theory*, vol. 52, pp. 375–391, Feb. 2006.