Optimal Relay Control in Power-Constrained Dual-Hop Transmissions over Arbitrary Fading Channels

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***Abstract*— This paper examines the performance of a dual-hop, noise-limited, non-regenerative system where the relay gain is allowed to vary as an unknown deterministic function of the channel states, subject to instantaneous (i.e. peak) and average power constraints, the latter directly affecting battery lifetime in wireless systems. A convex optimization problem, in terms of a generic performance metric, is formulated and solved algorithmically for the Signal-to-Noise Ratio (SNR), Shannon rate and BPSK Bit-Error-Rate (BER) utilities. The presented model allows for arbitrary statistics of the fading channels, including correlation between them. Special emphasis is placed on the relation between the optimal solutions obtained when observing the channels of either the first or both hops. The regions in channel state space where zero and full power is allocated are determined in closed form, with numerical results being presented for the Shannon rate and BER utilities. It is observed that, for sufficiently high first hop SNR, monitoring both channels instead of just the first hop can lead to a significant performance increase.**

I. Introduction

Relayed transmission is a promising technique for improving the quality of wireless communications. Originally proposed for bent-pipe satellite communications, it has thereafter been extensively studied for wireless cellular, ad-hoc WLANs and hybrid networks. Its advantages relative to direct link communication include: ease of implementation and good scalability, increased connectivity, robustness to changing channel conditions and reduced operating power levels. The latter implies lower interference levels and, hence, increased capacity. In further support of the above advantages comes the concept of co-operative user diversity, which can potentially offer even higher capacity with a reduced outage probability [1]–[4].

In accordance with standard relay nomenclature, the relays are classified into regenerative and non-regenerative, according to whether they fully decode the received signal and re-transmit it or they simply forward an amplified version of the received signal. Furthermore, the relays can either transmit with fixed gains (blind relays) or exploit channel state information (CSI) to adjust their gain. In [5], Boyer *et al.* examine, for an isolated single-user link, various combinations of the above cases in a "serial" multi-hop scenario, where each relay can potentially receive all signals transmitted by the previous relays of the link, while [6] proposes distributed codes for a "parallel" multi-hop case where the original transmitter broadcasts to all relays and a suitably selected subset of them re-transmits to the desired destination. In a similar context, [7] examines the effect of relay co-operation on the overall capacity region. Rayleigh fading is assumed in both cases. Since diversity is outside the scope of this paper, it will not be further discussed although it will become apparent that the ultimate goal is to incorporate it into our proposed model.

In [8], the authors examine a semi-blind relay and compare a regenerative to a non-regenerative system, while [9] presents an instantaneous-power-constrained optimization problem for a dual-hop scenario and offers a generalization to a multi-hop regenerative system. In both cases, Rayleigh fading is assumed and the outage probability is used as the performance metric. Recently, Karagiannidis has studied in [10] the performance bounds for multi-hop systems with blind relays over various Nakagami channels using a moments-based approach.

In this work we consider a dual-hop, noise-limited, non-regenerative system with arbitrary fading statistics and correlation. The relay node has perfect knowledge of the CSI of either the first only or both channels — transmitter to relay or relay to receiver. Our objective is to investigate the effect of the relay's knowledge of each channel's fading state on the overall performance, which is measured by a generic utility metric containing SNR, BER and Shannon rate as special cases. We depart from the traditional trend of optimizing relay performance by choosing an (optimal) fixed relay transmission power (or gain) or a given function of the fading state (such as the inverse gain formula of [1]) and we seek instead to determine the optimal relay gain as a deterministic function of the channel state. The price to pay for this generalization is that closed form solutions are no longer available, except for trivial cases. We do not consider this a hindrance, since most closed form expressions already involve transcendental functions which can only be numerically evaluated. Finally, in addition to instantaneous power constraints (such as those appearing in [9]), we also take into account a constraint on the (long term) average power output by the relay. The latter is

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especially useful in cases where the relay is mobile and relies on batteries for its operation.

II. System and channel model

Consider a transmitter, relay and receiver that communicate over wireless channels. Denote the channels between transmitter-relay and relay-receiver as channels 1–2, both exhibiting random fading (for example due to mobility or multipath propagation) with coefficients a, b, respectively, defined as the appropriate **power** ratios between the transmitted and received power on each link. We denote $\mathcal{A}, \mathcal{B}$ the finite sets over which a, b range, respectively. In each channel, a positive power gain (g_1, g_2 respectively) is used to amplify the base signal at the corresponding transmitter. Hence, assuming the first transmitter transmits at unity power, the total power exiting the transmitter is g_1 and the power arriving at the relay is $g_1 a + N_0$, where the first term is due to fading and the second due to the presence of AWGN. Similarly, the power exiting the relay is $g_2(g_1 a + N_0)$ and the total power arriving at the final receiver is $g_2(g_1 a + N_0)b + N_0$, where it is assumed for simplicity that the same amount of noise exists in both channels.

Under the previous assumptions and notation, the instantaneous SNR at the receiver (end-to-end), conditioned upon the fading coefficients a, b, is given by

$$\gamma(a,b) = \frac{g_1 g_2 ab}{(1+bg_2)N_0} = \frac{g_1 a}{N_0}\left(1 - \frac{1}{1+bg_2}\right). \quad (1)$$

The term "instantaneous" is used here in a loose context to denote a time average of the randomly varying SNR over a time interval equal to the channel's coherence time. Since, by definition, a and b remain constant in such an interval, $\gamma(a,b)$ is actually a short-term time average, similar in concept to [11], and it is in this context that we can regard it as instantaneous. The long-term SNR time average is by ergodicity equal to the statistical average over all fading values.

The transmitter gain g_1 is arbitrarily fixed, whereas g_2 is allowed to vary as a (yet unknown) deterministic function of either a or both a, b. The various dependencies physically correspond to different fading states being known at the relay, i.e. the relay bases its decisions on knowledge of both a, b, or only a. The case where the relay observes b only is of little practical interest and has received little, if any, attention since the relay can always estimate a from its incoming signal $g_1 a + N_0$ (assuming that g_1 and N_0 are known). Therefore, the b only dependence will not be examined in this paper.

III. Problem statement

Since $\gamma(a,b)$ is a random variable, we take the expectation of a utility function of it as a figure of merit and seek to maximize this subject to average and instantaneous power constraints on the relay. Casting the problem in its equivalent "minimize" form results in the following formulation

$$\underset{g_2}{\text{minimize}} \sum_{a\in\mathcal{A}}\sum_{b\in\mathcal{B}} -\Phi\left(\frac{g_1 a}{N_0}\frac{bg_2}{bg_2+1}\right)\pi_{ab} \quad (2a)$$

$$\text{(D)} \qquad \text{s.t.} \sum_{a\in\mathcal{A}}\sum_{b\in\mathcal{B}} g_2\left(g_1 a + N_0\right)\pi_{ab} \le \bar{P}_2 \quad (2b)$$

$$0 \le g_2\left(g_1 a + N_0\right) \le p_2, \quad (2c)$$

where π_{ab} is the a, b joint pdf, p_2 and $\bar{P}_2$ are design parameters independent of the optimization problem and g_2 is a function of either a or both a, b according to the rationale of Section II. It is implicitly assumed that $\pi_{ab} > 0 \ \forall (a,b)$ otherwise the corresponding channel state would never be observed and it would not contribute to the objective functional. The function Φ is required to be positive, increasing, concave and continuously differentiable. Functions that satisfy the above properties include $\Phi(x) = x$ (i.e. maximize SNR itself), $\Phi(x) = \log(1+x)$ (maximize Shannon rate) and most complementary bit error rates (i.e. $1 - P_e$) of practical modulation schemes. Hence, the above properties cover a large range of suitable performance metrics.

Existence of a solution to problem (D) is guaranteed from compactness of the constraint set while uniqueness follows from strict convexity of the functional, viewed as the composition of a convex and a strictly concave function, and the constraint set. It is also noteworthy that when there is no average power constraint (which is the standard case in the existing literature on relays), the optimization problem always has the trivial solution $g_2 = p_2/(g_1 a + N_0)$, since the function $kx/(1+kx)$ is increasing with respect to x, Φ is increasing with respect to its argument and the objective is a positively weighted sum of increasing functions. Hence, the standard inverse fading gain formula proposed in [1] is a special case of our general formulation. However, it is clear that imposing the average power constraint makes the inverse gain function suboptimal by definition.

A final note is in place regarding the discrete nature of the fading state sets. We avoid the usual assumption of continuous fading states in order to reduce the mathematical complexity of the analysis. Though a treatment of continuous fading sets based on calculus of variations is given in [12], the main concepts are essentially the same as those presented here and involve the limiting case of $|\mathcal{A}|, |\mathcal{B}| \to \infty$. Since all constraints are either point-wise or of integral type, it can be shown that choosing a sufficiently large number of states allows us to approach arbitrarily close to the continuous effect. Hence, nothing important is essentially lost.

IV. Dual formulation and algorithm

Since the function $kx/(1+kx)$ is increasing with respect to x for all $k > 0$, the minimizing functional in (2a) is decreasing with respect to g_2. Hence, the solution is either the trivial one of setting all g_2 to the maximum value allowed by (2c) and checking if (2b) is satisfied or, if that fails, the solution to the same problem as above with the inequality in the mean power constraint replaced by equality. The latter solution is

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obtained by exploiting the separable structure of problem (D). Specifically, we can write (2a)–(2c) in the more general form

$$\text{minimize} \sum_j f_j(x_j) \tag{3a}$$

$$\text{s.t. } 0 \le x_j \le p_j \quad \forall j \tag{3b}$$

$$\sum_j c_j x_j = \bar{P}, \quad c_j \ge 0, \tag{3c}$$

where j ranges from 1 to N and each f_j is a continuously differentiable decreasing and strictly convex function with respect to x_j. Existence and uniqueness of solution is again guaranteed from compactness/convexity and [13] provides a necessary and sufficient condition, based on duality theory, for a vector x^* to be the solution to the problem above. Since the analysis is quite standard, we only present the final results.

Specifically, if $\sum_j c_j p_j \le \bar{P}$, the optimal solution is $x_j^* = p_j \; \forall j$. Otherwise it is given parametrically by

$$x_j^*(\mu^*) = \begin{cases} G_j^{-1}(\mu^*) & \text{if} \quad G_j(p_j) \le \mu^* \le G_j(0) \\ 0 & \text{if} \quad \mu^* \ge G_j(0) \\ p_j & \text{if} \quad \mu^* \le G_j(p_j) \end{cases}, \tag{4}$$

where $G_j(x) = -f_j'(x)/c_j$ and the parameter μ^* is defined through

$$\sum_j c_j x_j^*(\mu^*) - \bar{P} = 0. \tag{5}$$

Equivalently, we need to construct a partition of the index set $\mathcal{J}$ as $\mathcal{J} = \mathcal{J}_0 \cup \mathcal{J}_p \cup \mathcal{J}_i$ where x_j^* attains the value 0, p_j or $G_j^{-1}(\mu^*)$ if j belongs to set $\mathcal{J}_0$, $\mathcal{J}_p$, $\mathcal{J}_i$, respectively. The structure of this decomposition will be examined later. In computational terms, viewing (4) as a function of μ^*, this function is obviously continuous and decreasing $\forall j$, so that the weighted sum of all such functions in (5) retains these properties, since $c_j \ge 0$. Also, viewing the LHS of (5) as a function of μ^*, this function changes signs in the interval $[\min_j G_j(p_j), \quad \max_j G_j(0)]$. Hence, a simple bisection algorithm can be used for the numerical computation of μ^*.

Clearly, an effective algorithm depends crucially on the ability to efficiently compute G^{-1}. Barring a closed form solution, a standard binary search algorithm that exploits the strict monotonicity of G can be used to numerically compute $x = G^{-1}(\mu) \Rightarrow \mu - G(x) = 0$. The algorithm stops when the length of the search interval becomes less than $\delta < 1$ times the initial search interval, which corresponds to $O(-\log\delta) \cdot O\,(\text{time to compute } G)$ time for the computation of G^{-1}. Since the average power constraint must be checked for each μ and a second binary search must be performed for μ^* in the interval $[\min_j G_j(p_j), \quad \max_j G_j(0)]$ to find the μ^* that satisfies the average constraint, it follows that the optimal μ^* can be calculated in $O\left(N(-\log\delta)^2\right) \cdot O\,(\text{time to compute } G)$.

In principle, the analysis above solves problem (D) in its most general case and further insight may be gained by examining different dependence cases. This distinction of cases is one of the most important aspects of the paper and is performed next.

A. g_2 depends on both a, b

A straightforward application of the previous theory, with the obvious substitutions for c_j, x_j, results in

$$G_{a,b}(g_2) = \frac{1}{g_1 a + N_0} \Phi' \left(\frac{g_1 a}{N_0} \frac{b g_2}{1 + b g_2} \right) \frac{g_1 a}{N_0} \frac{b}{(1 + b g_2)^2}. \tag{6}$$

In general, $G_{a,b}^{-1}$ cannot be computed from (6) in closed form. However for relatively simple utilities, such as the SNR and Shannon rate, an analytical solution is found as

$$G_{a,b}^{-1}(y) = \begin{cases} -\frac{1}{b} + \sqrt{\frac{1}{bN_0 y} \frac{g_1 a}{g_1 a + N_0}} & \text{for SNR} \\ \frac{-g_1 a y - 2yN_0 + \sqrt{g_1^2 a^2 y^2 + 4yabg_1}}{2by(g_1 a + N_0)} & \text{for Shannon} \end{cases} \tag{7}$$

In all other cases, a binary search can be used to compute $x = G_{a,b}^{-1}(y)$ since $G_{a,b}$ is strictly decreasing. Assuming that $G_{a,b}$ can be computed in $O(1)$ time, it turns out from the discussion above that a binary search for μ^* can be performed in $O\left((-\log\delta)^2 \left|\mathcal{A} \times \mathcal{B}\right|\right)$, where δ is determined from the iteration stopping criterion.

Power allocation regions: From (6), the optimal g_2 solution is identically $0 \; \forall a, b$ s.t.

$$G_{a,b}(0) \le \mu^* \Rightarrow b \le \mu^* \frac{N_0}{\Phi'(0)} \left(1 + \frac{N_0}{g_1 a}\right), \tag{8}$$

the last relation being a displaced hyperbola of the form $b \le b^* + c_1/a$ in the a, b space. Hence, the optimal g_2 practically acquires non-zero values only when b exceeds a threshold $b^* = \mu^* N_0/\Phi'(0)$. A similar threshold does not exist for a, which can receive non-zero power in the optimal allocation policy regardless of its value.

Characterizing the region in a, b space where the optimal g_2 attains its max inst. power requires solving for b as a function of a the implicit equation $G_{a,b}\left(p(a)\right) \ge \mu^*$ where $p(a) = p_2/(g_1 a + N_0)$. Since Φ is concave and increasing, it holds $0 < \Phi'(x) \le \Phi'(0)$. Additionally, since $b/(1 + bp(a))^2 \le b/(1 + bp(a)) \le 1/p(a)$, it is easy to show that $G_{a,b}\left(p(a)\right) \ge \mu^*$ implies

$$\frac{g_1 a}{p_2 N_0} \Phi'(0) \ge \mu^* \Rightarrow a \ge \frac{\mu^* N_0 p_2}{g_1 \Phi'(0)} = \hat{a}. \tag{9}$$

Hence, in general, maximum power is allocated only to sufficiently high a, if at all (i.e. if the maximum power region is not the empty set, it will always be a subset of the region $a \ge \hat{a}$).

For the SNR and Shannon rate utilities, the maximum power regions can be computed analytically. Only the salient points of this procedure are described here with [14] providing all the details. Specifically, the maximum power regions satisfy the following trinomial inequalities

$$\begin{gathered} p_2^2 b^2 + b\,(g_1 a + N_0) \left(2p_2 - \frac{g_1 a}{\mu^* N_0}\right) + (g_1 a + N_0)^2 \le 0 \\ \frac{p_2^2}{N_0} b^2 + b \left[2p_2 + \frac{g_1 a}{N_0}\left(p_2 - \frac{1}{\mu^*}\right)\right] + (g_1 a + N_0) \le 0 \end{gathered} \tag{10}$$

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TABLE I

CHARACTERISTIC POINTS FOR OPTIMAL SNR AND RATE UTILITIES: a, b DEPENDENCE.

	a^*	b_p^*
SNR	$\frac{4p_2\mu^* N_0}{g_1}$	$\frac{N_0}{p_2}(1+4p_2\mu^*)$
Rate	$\frac{4p_2 N_0}{g_1\mu^*\left(p_2-\frac{1}{\mu^*}\right)^2}$	$\frac{N_0}{p_2}\frac{1+\mu^* p_2}{1-\mu^* p_2}$

for the SNR and Shannon rate, respectively. Denote the trinomial discriminants as Δ_s, Δ_r, respectively. For each utility, let the trinomial roots be b_+, b_-, where the subscript indicates whether the positive or negative root is taken, and define a^* as the solution of the equation $\Delta = 0$. Finally, let $b_p^* = b_+(a^*) = b_-(a^*)$ for each utility and denote $a_{max} = \sup \mathcal{A}$, $b_{max} = \sup \mathcal{B}$. After some algebraic manipulations, the following conclusion is drawn [14]

Corollary 1 *For the SNR and Shannon rate utilities, a^* and b_p^* are given in Table I. For both utilities, the maximum power region is the intersection of $\{(a,b) : a^* \leq a \leq a_{max}\}$ and $\{(a,b) : \max(0, b_-(a)) \leq b \leq \min(b_{max}, b_+(a))\}$. Furthermore, b_+ and b_- are strictly increasing/decreasing, respectively. For the SNR utility, it holds $b_+, b_- \geq 0 \quad \forall\, a \geq a^*$, while for the rate, the extra condition $1/\mu^* - p_2 > 0$ must be satisfied, otherwise it either holds $b_+, b_- \leq 0 \quad \forall\, a \geq a^*$, or b_+, b_- are complex.*

The above corollary holds for general dependent channels. Its pictorial representation is shown in Fig. 1 where, for comparison purposes, the zero power region is also shown (b^* refers to the zero power region threshold defined in (8)). The maximum power region, when non-empty, is a singly connected set which, in general, is neither convex nor concave.

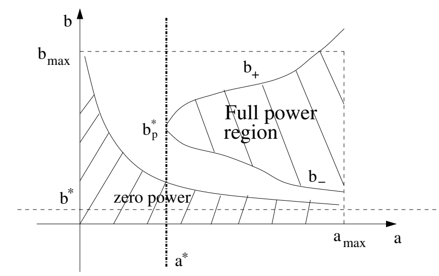

Fig. 1. Characterization of the maximum power region for SNR and rate utilities.

B. g_2 depends on a only

In this case, (2a)–(2b) is reduced to

$$\underset{g_2}{\text{minimize}} \sum_{a \in \mathcal{A}} -\mathbb{E}\left\{ \Phi\left(\frac{g_1 a}{N_0}\frac{b g_2}{1+b g_2}\right) \middle| a \right\} \pi_a \tag{11a}$$

$$\text{s.t.} \sum_{a \in \mathcal{A}} g_2(a)(g_1 a + N_0)\pi_a = \bar{P}_2 \tag{11b}$$

and (2c) remains unchanged so that

$$G_a(g_2) = \frac{g_1 a p(a)}{N_0}\mathbb{E}\left\{ \frac{b}{(1+b g_2(a))^2}\Phi'\left(\frac{g_1 a}{N_0}\frac{b g_2}{1+b g_2}\right) \middle| a \right\}. \tag{12}$$

Contrary to the a, b dependence of Section IV-A, in the current case G_a^{-1} cannot be computed in closed form even for the SNR utility, the reason being the presence of the expectation operator. Therefore, $G_a^{-1}(y)$ must be numerically computed through a standard binary search. Assuming that $\Phi(x)$ is computable in $O(1)$ time, $G_a(x)$ can be computed in $O(|\mathcal{B}|)$ time so that G_a^{-1} is computed in $O((-\log\delta)|\mathcal{B}|)$ and μ^* is found in $O\left((-\log\delta)^2|\mathcal{A}|\cdot|\mathcal{B}|\right)$ time.

Power allocation regions: The region where the optimal $g_2(a)$ solution is identically zero is given by

$$\frac{g_1 a}{g_1 a + N_0}\mathbb{E}\{b|a\} \leq \frac{\mu^* N_0}{\Phi'(0)}. \tag{13}$$

Denoting the LHS of (13) as $q(a)$, it is clear that if the two channels are independent, or if $\mathbb{E}\{b|a\}$ is an increasing function of a, then $q(a)$ is also increasing, so that (13) is reduced to the form $a \leq a^*$, where $a^* = \sup\{a : q(a) \leq \mu^* N_0/\Phi'(0)\}$. Hence, the zero power region for independent channels or increasing $\mathbb{E}\{b|a\}$ is non-empty iff $a^* < a_{max}$ and it is always a singly connected set. However, if $\mathbb{E}\{b|a\}$ is decreasing or of variable monotonicity, no definite conclusions can be drawn for the general case.

For the maximum power regions, we examine the SNR and rate utilities separately. For $\Phi(x) = x$, the maximum power region is described through

$$G_a(p) = \frac{g_1 a p(a)}{p_2 N_0}\mathbb{E}\left\{ \frac{b}{(1+b p(a))^2} \middle| a \right\} \geq \mu^*, \tag{14}$$

with $p(a) = p_2/(g_1 a + N_0)$. Since $ap(a)$ is strictly increasing with respect to a, and $(1 + bp(a))$ appears in the denominator of the expectation in (14), it follows that $G_a(p)$ is a strictly increasing function of a so that (14) is reduced to

$$a \geq G_a^{-1}(\mu^*) = \tilde{a}. \tag{15}$$

The actual computation of G_a^{-1} requires knowledge of the channel fading pdfs and is omitted. However, it follows from (15) that the maximum power region is non-empty iff $\tilde{a} < a_{max}$ and is always singly connected.

For $\Phi(x) = \log(1 + x)$, the maximum power region is described through

$$G_a(p) = \frac{1}{N_0}\frac{g_1 a}{g_1 a + N_0}\mathbb{E}\left\{ \frac{b}{1+b p(a)}\frac{N_0}{N_0 + b p_2} \middle| a \right\} \geq \mu^*, \tag{16}$$

and, following the same argument as for the SNR utility above, it is easy to show that $G_a(p)$ is strictly increasing so that (16) is reduced to the same form as (15), with the obvious difference in $\tilde{a}$. The conditions for the maximum power region to be non-empty are formally identical to the SNR utility, and the region is again singly connected.

As a final remark, we see from (6) that when power allocation depends on both a and b, $G_{a,b}^{-1}$ is independent of

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the fading pdf, i.e. fading affects only the power allocation region, but **not** the functional form of the optimal solution. This is obviously not true for the a only dependence, since (12) contains an expectation operator which depends crucially on the fading pdf.

V. Numerical results

An extensive set of simulations was performed to test the relation between the various dependencies for the most important utilities: SNR, Shannon rate and BPSK BER. To this end, it was initially assumed that the channels are independent and a, b are exponentially distributed (i.e. the fading amplitudes $\sqrt{a}, \sqrt{b}$ are Rayleigh distributed) with the same expected value. The value of g_1 was set to 1 for all cases and the ratio $\bar{\gamma}_1 = \mathbb{E}\{a\}/N_0$, which corresponds to the first hop average SNR, was set to vary in the interval 0 to 30 dB with a 2 dB increment. The exponential distribution allows for unbounded fading values so, in order to keep the fading set bounded for simulation purposes, a larger than 99% confidence interval was used (i.e. the fading set was defined as $[\alpha_1\ \alpha_2]$ where $\Pr(a < \alpha_1) = \Pr(a > \alpha_2) = 10^{-3}$). This set, which contained all fading values of practical importance, was then uniformly discretized into 500 states. Additionally, since the initial power used by the transmitter is unity, the average and inst. power constraints have to be normalized with respect to that.

Following this approach, a set of 11 simulations was performed, one for each $(\bar{P}_2, p_2)$ pair. The used pairs were: $(0.6, 1.5), (0.6, 3), (0.6, 6), (0.2, 1.2), (0.2, 2), (0.1, 1.2), (0.1, 1.5), (0.1, 0.5), (0.05, 0.8), (0.2, 0.5)$ and $(0.01, 0.1)$. For each pair, the entire range 0–30 dB for $\bar{\gamma}_1$ was swept and the optimal solutions for both dependencies were obtained. Due to space restrictions, only the most interesting or representative cases will be given. Specifically, results for the case $\Phi(x) = x$ (i.e. maximize average SNR) will be omitted, since it was generally observed that the optimal SNR was almost a linear function of $\bar{\gamma}_1$. The important case of statistically correlated channels is also omitted for lack of space (see [12] for details). In all figures, the solutions for a dependence are shown with lines (solid,dotted,dashed) while the a, b dependency is shown with markers. As a measure of comparison, we also plot the utilities for the optimally chosen **fixed** relay gain (i.e. the solution to (D) when g_2 is restricted to be a constant). For this case, it is easy to show that the optimal constant gain is $\hat{g}_2 = \min\left\{p_2/\left(g_1 A_{max} + N_0\right), \bar{P}_2/\left(g_1 \mathbb{E}\{a\} + N_0\right)\right\}$. Finally, we denote $r = p_2/\bar{P}_2$.

An examination of the results suggests the following trends. As expected, for a fixed $p_2/\bar{P}_2$ ratio, all optimal utilities increase when $\bar{P}_2, p_2$ increase. This is verified by the scales of Figs. 2, 3 for the pairs $(0.01, 0.1)$ and $(0.6, 6)$ in both utilities. For a fixed $\bar{P}_2$, the optimal utility obtained from a dependence is insensitive to changes in r, as shown in Fig. 2(a,b) for the rate and BER utilities (notice that there exists only one continuous line instead of three, which would be the case if the solutions varied with r). The same fact holds for the SNR utility for all possible dependencies, although the graph is omitted. This is due to the fact that the inst. power constraint is inactive for the chosen $(\bar{P}_2, p_2)$ pairs (i.e. the optimal solution is always strictly less than the peak constraint). From the same figures, it is also evident that the a, b optimal solution exhibits a similar insensitivity to r for the rate utility, although for the BER there is a slight improvement as r increases. This improvement is more pronounced in higher $\bar{\gamma}_1$. Finally, the fixed gain solution gradually improves when increasing r for a fixed $\bar{P}_2$, so that at sufficiently high r it can be considered an acceptable suboptimal solution (i.e. there is no practical need for CSI monitoring).

The comparison of the a, b solution to the a solution, assisted by Figs. 2, 3, suggests the following: For the rate utility, a, b and a solutions, presented in Figs. 2a, 3a, yield so close values that the a solution can be practically considered the true "optimal" solution. For the BER utility (results shown in Figs. 2b, 3b), the superiority margin of a, b with respect to a starts from minimal in low $\bar{\gamma}_1$ values and grows to considerable proportions as $\bar{\gamma}_1$ increases (for example, at 30 dB the a, b solution is less than a quarter of the BER of the a solution, as shown in Fig. 3b). This indicates that a substantial performance gain can be achieved if the relay monitors both channels. The fixed gain solution is relatively close to the optimal adaptive solution in relatively low SNR (below 8 dB), but as γ_1 increases it gradually becomes much worse. Hence, at high first hop SNRs, it is not advisable to use such a solution. This is not totally unexpected since the fixed gain solution captures in a sense the worst-case channel behavior and tries to do the best it can under the circumstances. If a channel is "bad" most of the time, meaning low $\bar{\gamma}_1$, then the worst case scenario is the dominant one. For the SNR utility (graph omitted) and not very low $\bar{P}_2$, the a, b superiority margin is of the order of 1 dB at most, so that the a solution can be considered a very good suboptimal solution. As $\bar{P}_2$ increases, however, the superiority margin of $g_2(a, b)$ also increases, so that for large $\bar{P}_2$ it is advantageous to monitor both channels.

VI. Conclusions

This paper presented a general methodology for computing the utility-optimal relay gain (as a CSI-based function of both channels) of a dual-hop system, subject to both average and instantaneous power constraints, for arbitrary fading pdfs. Emphasis was placed on examining the different dependencies of the gain on the channel states, and their effect on system performance. We showed that the problem needs to be solved only once in an off-line manner and the obtained solution can be stored in the relay. Simulations were performed for the rate and BER utilities and the numerical results indicate that a considerable performance gain is possible by exploiting both channel states in sufficiently high first hop SNRs.

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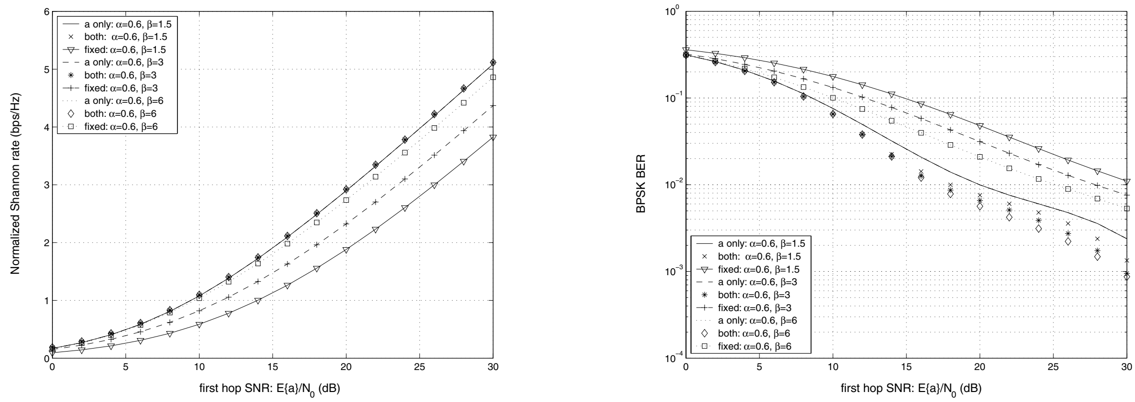
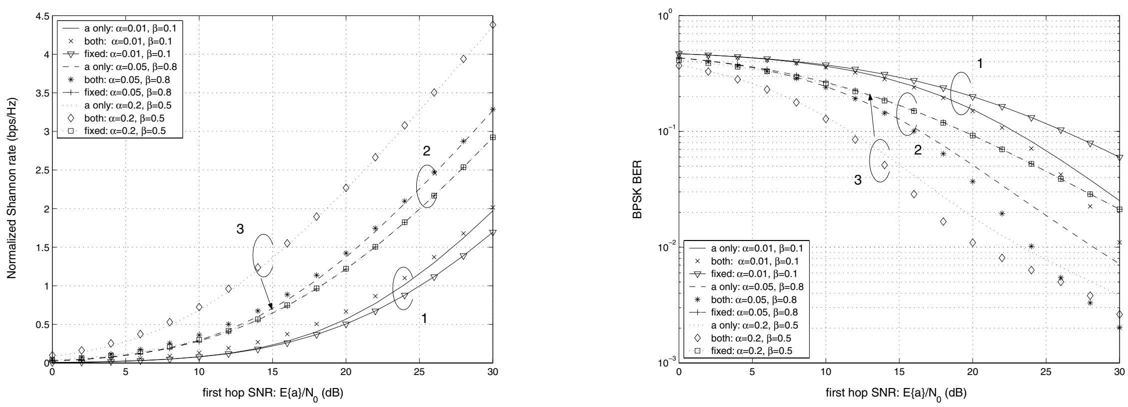


(a) Optimal Shannon rate for fixed $\bar{P}_2$ and increasing r.

(b) Optimal BER for fixed $\bar{P}_2$ and increasing r.

Fig. 2. Optimal rate and BER, as well as fixed gain solution, for $\bar{P}_2 = 0.6$ and $r = 2.5, 5, 10$.



(a) Optimal Shannon rate.

(b) Optimal BER.

Fig. 3. Optimal rate and BER for some $(\bar{P}_2, p_2)$ pairs and all possible dependencies. Used pairs are: 1. (0.01,0.1), 2. (0.05,0.8) and 3. (0.2,0.5).

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