New Results for Wireless Multihop Diversity Systems

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***Abstract*: We study the performance of multihop diversity systems with non-regenerative relays over independent and non-identical Rayleigh fading channels. The analysis is based on the evaluation of the instantaneous end-to-end signal-to-noise ratio (SNR), depending on the type of the relay and the diversity scheme used. A closed-form expression is derived for the average end-to-end SNR, when fixed-gain relays and a maximal ratio combiner (MRC) are used, whereas an analytical expression formula for the average symbol-error rate, is presented. The results show that multihop diversity systems outperform conventional telecommunication systems in terms of ASER when the same amount of energy is assumed to be consumed in both cases.**

1. Introduction

Wireless multihop communications systems have recently proposed as a viable option versus traditional communication networks due to the benefits in the area of network deployment, connectivity and capacity. Relaying techniques can solve problems of network connectivity where traditional architectures are impractical due to location constraints and can be applied to cellular, wireless local area networks (WLAN), and hybrid networks. In multihop communications systems, the source communicates with the receiver via a number of relays. Therefore, these systems have the advantage of broadening the coverage without using large transmitting power in the transmitter [1]-[5]. Recently, the concept of cooperative/collaborative diversity has gained great interest. Mobile users cooperate/collaborate with each other in order to exploit the benefits of spatial diversity without the need of using physical antenna arrays [6]-[9].

The performance analysis of multihop wireless communication systems operating in fading channels has been an important field of research in the past few years. Hasna and Alouini have evaluated the end-to-end outage probability of multihop wireless systems both for regenerative and non-regenerative channel state information (CSI)-assisted relays over Nakagami-m fading channels [3]. Moreover, the same authors have studied the outage and the error performance of dual-hop systems with regenerative and non-regenerative relays over Rayleigh [1], [4] and Nakagami-m [2] fading channels. Furthermore, Tsiftsis *et al.* presented a new upper bound for the end-to-end SNR and have efficiently evaluated the average error probability in dual-hop collaborative diversity systems, especially at low SNRs [10]. Additionally, in [11], Karagiannidis *et al.* have presented closed-form lower bounds for the performance of multihop transmissions with non-regenerative relays over independent and non-identical distributed Nakagami-m fading channels. The concept of multihop diversity, where the nodes of a multihop system using spatial diversity combine concurrent reception of signals that have been transmitted by all the preceding nodes, has been introduced recently by Boyer *et al.* in [5]. Tight upper bounds for the end-to-end probability of outage and error for decoded and amplified relaying, have been derived.

In this paper, a multihop diversity relaying system is considered employed CSI-assisted or fixed gain relays. Closed-form expressions for the instantaneous end-to-end SNR is studied for both types of relays and for the case where selection combining (SC) or maximal-ratio combining (MRC) receivers are considered in each node. These general formulae for the instantaneous SNR are applied to evaluate the performance of mulithop diversity systems over Rayleigh fading channels. Average end-to-end SNR and average symbol error probability (ASER) are studied. A variety of numerical examples are presented for a two-class multihop diversity system (i.e., two relays, six links) and computer simulation examples verify the accuracy of the presented mathematical analysis.

The remainder of this paper is organized as follows. In Section II, the multihop diversity system under consideration is presented and a brief analysis of gain relays is executed. Next, in Section III the instantaneous end-to-end SNR is derived for both types of gain relays and diversity schemes. In Section IV, a performance evaluation of the system is studied giving results on average SNR and ASER. Finally, numerical examples are discussed in Section V and concluding remarks are given in Section VI.

2. System and Channel Model

We consider a N-class non-regenerative multihop diversity system, where N non regenerative relays cooperate in order to transmit the information signal from a source node S to a destination node D [5], as shown in Fig. 1. Each node receives the signals transmitted by all the preceding terminals and combines them using one of the well known diversity techniques. After that, it amplifies and transmits the combined signal to all the following nodes, including the destination terminal. Denote by R_k, $1 \leq k \leq N$ the system relays. Let γ_{ij} be the instantaneous SNR at the input of the j_{th} terminal concerning the hop between the i_{th} and the j_{th} terminal, e.g., $\gamma_{ij} = (P_i/N_0)\, a_{ij}^2$, where a_{ij} is the fading

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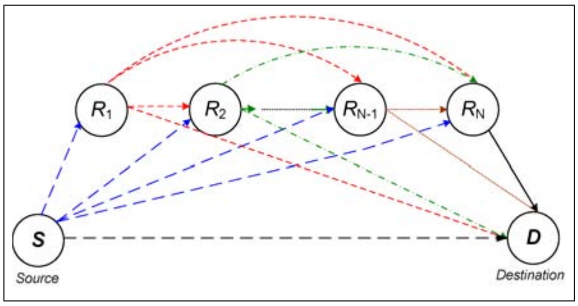

Figure 1: The N-class multihop diversity system

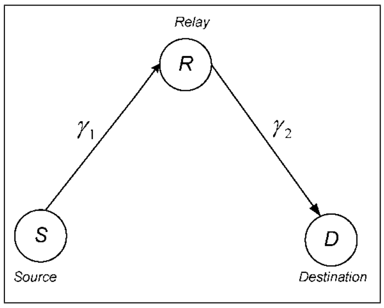

Figure 2: A simple dual-hop system

amplitude of that hop, P_i is the instantaneous transmitting power of the i_{th} terminal and N_0 is the Additive White Gaussian Noise (AWGN) power, which is considered identical in each intermediate channel. We assume identical flat fading in each channel. Let G_k be the gain of the relay R_k,which can be either fixed or non-fixed. In the former case, the relay gain is set to be

$$G_k^2 = \frac{1}{C_k N_0} \tag{1}$$

where C_k is a constant describing G_k, while in the latter case, the relay gain is variable, (i.e., aiming to limit the output power of the relay [12])

$$G_k^2 = \frac{1}{a_{eqk}^2 + N_0} \tag{2}$$

or simply

$$G_k^2 = \frac{1}{a_{eqk}^2} \tag{3}$$

called respectively as CSI-assisted and modified CSI-assisted relays. These gains require knowledge of the a_{eqk} of the equivalent channel in the input of the relay R_k. We denote as equivalent the single channel between the transmitter and the R_k, which can be regarded as the substitute of the complicated multihop relaying channel in terms of system performance.

3. Instantaneous end-to-end SNR

3.1. Fixed Gain Relays

Consider the non-regenerative dual-hop system with fixed gain relay shown in Fig. 2. The end-to-end SNR is given in [2] as

$$\gamma_{dualhop,fG} = \frac{\gamma_1 \gamma_2}{C + \gamma_2} \tag{4}$$

where C is a constant describing the relay gain and γ_1, γ_2 are the instantaneous SNR at the hop between the source and the relay terminal and at the hop between the relay and the destination terminal, respectively. Also, it is assumed that R_2 is the destination terminal, and that is equipped with one of the two most frequently used combiners, i.e., an MRC or an SC. The R_2 terminal receives signals incident from the two preceding terminals, S and R_1, concurrently. Thus, the end-to-end SNR at the output of the R_2 combiner is

$$\gamma_{eq,1,fG} = \mathcal{D}\left(\gamma_{SR_2}, \frac{\gamma_{SR_1}\gamma_{R_1R_2}}{C_1 + \gamma_{R_1R_2}}\right) \tag{5}$$

where

$$\mathcal{D}\left(x_1, x_2, ..., x_n\right) = \begin{cases} \max\left(x_1, ..., x_n\right) & \text{when using SC} \\ x_1 + ... + x_n & \text{when using MRC} \end{cases} \tag{6}$$

is a function describing the diversity type used each time. It is obvious that (5) gives the end-to-end SNR in a multihop diversity system when a single non-regenerative relay is considered. Thus*, the original complicated system has been simplified to an equivalent one, which has the classical structure of a single transmitter and receiver, the SNR of which is given in (5).*

3.1.1. Two-class Multihop Diversity System

Next, a special case of a multihop diversity system which is employing two non-regenerative relays, is considered. Each relay combines the receiving signals using one of the above diversity techniques. In such case, the destination terminal combines the received signals transmitted by the terminals S, R_1,and R_2. Let P_N represents the noise power at the input of the destination terminal and $P_{D,R_i}, P_{D,S}$ represent the instantaneous power associated with the signal incident from the relay R_i and the source terminal respectively. The ratio $P_{D,R_2}/P_N$ can be evaluated by applying (5) into (4) as

$$\frac{P_{D,R_2}}{P_N} = \frac{\left[\mathcal{D}\left(\gamma_{SR_2}, \frac{\gamma_{SR_1}\gamma_{R_1R_2}}{C_1+\gamma_{R_1R_2}}\right)\right]\gamma_{R_2D}}{C_2 + \gamma_{R_2D}}. \tag{7}$$

Equivalently, the ratio $P_{D,R1}/P_N$ describes the overall SNR in a non-regenerative dual-hop system and is given using (4) by

$$\frac{P_{D,R_1}}{P_N} = \frac{\gamma_{SR_1}\gamma_{R_1D}}{C_1 + \gamma_{R_1D}}. \tag{8}$$

The ratio $P_{D,S}/P_N$ is by definition

$$\frac{P_{D,S}}{P_N} = \gamma_{SD}. \tag{9}$$

Therefore, the overall SNR can be written as

$$\gamma_{eq,2,fG} = \mathcal{D}\left(\frac{P_{D,R_2}}{P_N}, \frac{P_{D,R1}}{P_N}, \frac{P_{D,S}}{P_N}\right)$$
$$= \mathcal{D}\left(\gamma_{SD}, \frac{\gamma_{SR_1}\gamma_{R_1D}}{C_1 + \gamma_{R_1D}}, \frac{\left[\mathcal{D}\left(\gamma_{SR_2}, \frac{\gamma_{SR_1}\gamma_{R_1R_2}}{C_1+\gamma_{R_1R_2}}\right)\right]\gamma_{R_2D}}{C_2 + \gamma_{R_2D}}\right). \tag{10}$$

3.1.2. N-class Multihop Diversity System

Now, let us assume that a new relay, R_3, is added to the existing set of two relays, while all the other system nodes remain constant. The destination terminal combines the signal incident from the preceding terminals S, R_1, R_2 and R_3, hence the overall SNR in such case will be the combination of $\gamma_{eq,2}$ and $P_{D,R_3}/P_N$, i.e.,

$$\gamma_{eq,3,fG} = \mathcal{D}\left(\gamma_{eq,2,fG}, \frac{P_{D,R_3}}{P_N}\right). \quad (11)$$

The terminal R_3 combines the signal transmitted from terminals S, R_1, R_2, thus the equivalent SNR in its input can be regarded as the end-to-end SNR in a two-class multihop diversity system, denoted by $\gamma'_{eq,2,fG}$, where the source and destination nodes are the nodes S, R_3 respectively, and R_1, R_2 are the system relays. Thus, the ratio $P_{D,R_3}/P_N$ is the equivalent SNR of a simple dual-hop system where the SNR in the input of the relay is $\gamma'_{eq,2,fG}$, and the SNR at the hop between the relay and the destination terminal is γ_{R_3D}., expressed by

$$\frac{P_{D,R_3}}{P_N} = \frac{\gamma'_{eq,2,fG}\gamma_{R_3D}}{C_3 + \gamma_{R_3D}}. \quad (12)$$

Using (11) and (12),

$$\gamma_{eq,3,fG} = \mathcal{D}\left(\gamma_{eq,2,fG}, \frac{\gamma'_{eq,2,fG}\gamma_{R_3D}}{C_3 + \gamma_{R_3D}}\right). \quad (13)$$

In general, when a new relay, R_N, is added to the existing set of $N-1$ relays in a fixed gain multihop diversity system with known overall SNR $\gamma_{eq,N-1,fG}$, the overall SNR in the resultant N-class multihop diversity system can be evaluated recursively as follows

$$\gamma_{eq,N,fG} = \mathcal{D}\left(\gamma_{eq,N-1,fG}, \frac{\gamma'_{eq,N-1}\gamma_{R_ND}}{C_N + \gamma_{R_ND}}\right) \quad (14)$$

where $\gamma'_{eq,N-1}$ is the end-to-end SNR in a $(N-1)$-class multihop diversity system which differs from the original $(N-1)$-class system in the fact that the destination terminal of the former is the R_{N-1} relay of the latter.

3.2. General Case

Considering the above, it is easy to provide the instantaneous end-to-end SNR in a N-class multihop diversity system using CSI or modified CSI-assisted relays. The end-to-end SNR in a CSI-assisted and modified CSI-assisted dual-hop system is given in [2], and equals respectively to

$$\gamma_{dualhop,CSI} = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 + 1} \quad (15)$$

$$\gamma_{dualhop,M-CSI} = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2} \quad (16)$$

Therefore, the end-to-end SNR in an N-class multihop diversity system, when CSI-assisted and modified CSI-assisted gain relays are considered, can be expressed respectively as

$$\gamma_{eq,N,CSI} = \mathcal{D}\left(\gamma_{eq,N-1,CSI}, \frac{\gamma'_{eq,N-1}\gamma_{R_ND}}{\gamma'_{eq,N-1} + \gamma_{R_ND} + 1}\right) \quad (17)$$

$$\gamma_{eq,N,M-CSI} = \mathcal{D}\left(\gamma_{eq,N-1,M-CSI}, \frac{\gamma'_{eq,N-1}\gamma_{R_ND}}{\gamma'_{eq,N-1} + \gamma_{R_ND}}\right) \quad (18)$$

To summarize, the instantaneous end-to-end SNR in a N-class multihop diversity system can be given using the following expression

$$\gamma_{eq,N} = \mathcal{D}\left(\gamma_{eq,N-1}, \mathcal{G}\left(\gamma'_{eq,N-1}, \gamma_{R_ND}\right)\right) \quad (19)$$

where

$$\mathcal{G}(x,y) = \begin{cases} \frac{xy}{C_N+y} & \text{fixed-gain relays} \\ \frac{xy}{x+y+1} & \text{CSI-assisted relays} \\ \frac{xy}{x+y} & \text{modified CSI-assisted relays} \end{cases} \quad (20)$$

is a function describing the type of each relay gain.

4. Performance Analysis of the Two-Class Multihop Diversity System in Fading Environment

Assume that the fading in all intermediate channels in Fig. 1 is Rayleigh distributed, with probability density function (PDF) given by

$$f_{a_{ij}}(a_{ij}) = \frac{2a_{ij}}{\Omega_{ij}} \exp\left(-\frac{a_{ij}^2}{\Omega_{ij}}\right), \; a_{ij} \geq 0 \; \forall i,j \quad (21)$$

where $\Omega_{ij} = E\left[a_{ij}^2\right] = \overline{a^2}_{ij}$ and $E[\cdot]$ stands for the expectation value. It is known that the SNR in a Rayleigh fading environment is exponentially distributed, hence the PDF of γ_{ij} can be expressed as

$$f_{\gamma_{ij}}(\gamma_{ij}) = \frac{1}{\overline{\gamma}_{ij}} \exp\left(-\frac{\gamma_{ij}}{\overline{\gamma}_{ij}}\right) \quad \forall i,j \quad (22)$$

where $\overline{\gamma}_{ij} = E\left[\gamma_{ij}\right]$.

4.1. Average End-To-End SNR

Next, we derive a closed form expression for the average end-to-end SNR with respect to the intermediate channels' average SNRs. From (10), the average end-to-end SNR in a two-class non-regenerative multihop diversity system with fixed gain relays can be expressed as

$$E\left[\gamma_{eq,2}\right] = E\left[\mathcal{D}\left(\gamma_{SD}, \frac{\gamma_{SR_1}\gamma_{R_1D}}{C_1 + \gamma_{R_1D}}, \frac{\left[\mathcal{D}\left(\gamma_{SR_2}, \frac{\gamma_{SR_1}\gamma_{R_1R_2}}{C_1+\gamma_{R_1R_2}}\right)\right]\gamma_{R_2D}}{C_2 + \gamma_{R_2D}}\right)\right] \quad (23)$$

Assuming MRC at all terminals, (23) can be written as

$$\begin{aligned}\overline{\gamma}_{eq,2} &= E\left[\gamma_{eq,2}\right] = E\left[\gamma_{SD}\right] + E\left[\frac{\gamma_{SR_1}\gamma_{R_1D}}{C_1 + \gamma_{R_1D}}\right] \\ &+ E\left[\frac{\left(\gamma_{SR_2} + \frac{\gamma_{SR_1}\gamma_{R_1R_2}}{C_1+\gamma_{R_1R_2}}\right)\gamma_{R_2D}}{C_2 + \gamma_{R_2D}}\right] \\ &= I + J + K \end{aligned} \quad (24)$$

where

$$I = E\left[\gamma_{SD}\right] = \overline{\gamma}_{SD}, \tag{25}$$

$$J = E\left[\frac{\gamma_{SR_1}\gamma_{R_1D}}{C_1+\gamma_{R_1D}}\right] \tag{26}$$

and

$$K = E\left[\frac{\left(\gamma_{SR_2}+\frac{\gamma_{SR_1}\gamma_{R_1R_2}}{C_1+\gamma_{R_1R_2}}\right)\gamma_{R_2D}}{C_2+\gamma_{R_2D}}\right]. \tag{27}$$

Assuming mutually independent fading in each pair of intermediate channels, (26) and (27) can be rewritten as

$$\begin{aligned} J &= \overline{\gamma}_{SR_1} E\left[\frac{\gamma_{R_1D}}{C_1+\gamma_{R_1D}}\right] \\ &= \overline{\gamma}_{SR_1}\int_0^\infty \frac{\gamma_{R_1D}}{C_1+\gamma_{R_1D}}\frac{1}{\overline{\gamma}_{R_1D}}\exp\left(-\frac{\gamma_{R_1D}}{\overline{\gamma}_{R_1D}}\right)d\gamma_{R_1D} \end{aligned} \tag{28}$$

$$\begin{aligned} K &= \left(\overline{\gamma}_{SR_2}+\overline{\gamma}_{SR_1}E\left[\frac{\gamma_{R_1R_2}}{C_1+\gamma_{R_1R_2}}\right]\right)E\left[\frac{\gamma_{R_2D}}{C_2+\gamma_{R_2D}}\right] \\ &= \left[\overline{\gamma}_{SR_2}+\frac{\overline{\gamma}_{SR_1}}{\overline{\gamma}_{R_1R_2}}\int_0^\infty \frac{\gamma_{R_1R_2}}{C_1+\gamma_{R_1R_2}}e^{-\frac{\gamma_{R_1R_2}}{\overline{\gamma}_{R_1R_2}}}d\gamma_{R_1R_2}\right] \\ &\times \int_0^\infty \frac{\gamma_{R_2D}}{C_2+\gamma_{R_2D}}\frac{1}{\overline{\gamma}_{R_2D}}\exp\left(-\frac{\gamma_{R_2D}}{\overline{\gamma}_{R_2D}}\right)d\gamma_{R_2D} \end{aligned} \tag{29}$$

Note that each integral in (28) and (29) has the general form $\int_0^\infty \frac{x}{\beta+x}e^{-\frac{x}{\mu}}dx$, a solution of which is given in [13, eq. (3.353.5)]

$$\begin{aligned} \int_0^\infty \frac{x}{c+x}e^{-x/\mu}dx &= \mu + c\,\mathrm{Ei}\left(-\frac{c}{\mu}\right)e^{\frac{c}{\mu}} \\ &= \mu - ce^{\frac{c}{\mu}}\Gamma\left(0,\frac{c}{\mu}\right) \end{aligned} \tag{30}$$

where $\mathrm{Ei}(\cdot)$ and $\Gamma(\cdot,\cdot)$ denote the exponential integral function and incomplete gamma function defined respectively in [13, eq. (8.211.1)] and [13, eq. (8.350.2)]. By substituting (30) to (28) and to (29), J and K can be rewritten as

$$J = \overline{\gamma}_{SR_1}\left(\overline{\gamma}_{R_1D} - C_1\exp\left(\frac{C_1}{\overline{\gamma}_{R_1D}}\right)\Gamma\left(0,\frac{C_1}{\overline{\gamma}_{R_1D}}\right)\right) \tag{31}$$

$$\begin{aligned} K &= \left[\overline{\gamma}_{SR_2}+\overline{\gamma}_{SR_1}\left(\overline{\gamma}_{R_1R_2}-C_1e^{\frac{C_1}{\overline{\gamma}_{R_1R_2}}}\Gamma\left(0,\frac{C_1}{\overline{\gamma}_{R_1R_2}}\right)\right)\right] \\ &\times\left(\overline{\gamma}_{R_2D}-C_2\exp\left(\frac{C_2}{\overline{\gamma}_{R_2D}}\right)\Gamma\left(0,\frac{C_2}{\overline{\gamma}_{R_2D}}\right)\right) \end{aligned} \tag{32}$$

Therefore, from (24), (25), (31), (32) and after some simplifications we obtain

$$\begin{aligned} \overline{\gamma}_{eq,2} &= \overline{\gamma}_{SD}+\overline{\gamma}_{SR_1} - \frac{C_1\overline{\gamma}_{SR_1}\exp\left(\frac{C_1}{\overline{\gamma}_{R_1R_2}}\right)\Gamma\left(0,\frac{C_1}{\overline{\gamma}_{R_1D}}\right)}{\overline{\gamma}_{R_1D}} \\ &+ \frac{\left(\overline{\gamma}_{SR_1}+\overline{\gamma}_{SR_2}-\frac{C_1\overline{\gamma}_{SR_1}\exp\left(\frac{C_1}{\overline{\gamma}_{R_1R_2}}\right)\Gamma\left(0,\frac{C_1}{\overline{\gamma}_{R_1R_2}}\right)}{\overline{\gamma}_{R_1R_2}}\right)}{\overline{\gamma}_{R_2D}} \\ &\times\left(\overline{\gamma}_{R_2D}-C_2\exp\left(\frac{C_2}{\overline{\gamma}_{R_2D}}\right)\Gamma\left(0,\frac{C_2}{\overline{\gamma}_{R_2D}}\right)\right) \end{aligned} \tag{33}$$

One choice for the fixed gains is set to be equal to the average of modified CSI-assisted gains shown in (3), specifically

$$G_k^2 = E\left[\frac{1}{a_{eqk}^2}\right] = \frac{1}{\overline{a^2}_{eqk}}, \tag{34}$$

which represents an upper bound for the "semi-blind" fixed relay gain as defined in [2] by

$$G_k^2 = E\left[\frac{1}{a_{eqk}^2+N_0}\right]. \tag{35}$$

Then, from (1),

$$C_k = \frac{\overline{a^2}_{eqk}}{N_0}. \tag{36}$$

It is easy to notice that $\overline{a^2}_{eq1} = \overline{a^2}_{SR_1}$, hence

$$C_1 = \frac{\overline{a^2}_{eqk}}{N_0} = E\left[\frac{1}{P_S}\right]\overline{\gamma}_{SR_1} = \frac{1}{\overline{P}_S}\overline{\gamma}_{SR_1} \tag{37}$$

Considering MRC receivers in all relays, (5) yields

$$\overline{\gamma}_{eq,1,fG} = \frac{\overline{P}_S}{N_0}\overline{a^2}_{eqk} = E\left[\gamma_{SR_2}+\frac{\gamma_{SR_1}\gamma_{R_1R_2}}{C_1+\gamma_{R_1R_2}}\right]. \tag{38}$$

Using (30) and (38), (36) yields

$$C_2 = \frac{1}{\overline{P}_S}\left[\overline{\gamma}_{SR_2}+\frac{\overline{\gamma}_{SR_1}}{\overline{\gamma}_{R_1R_2}}\left(\overline{\gamma}_{R_1R_2}-\overline{\gamma}_{SR_1}e^{\frac{\overline{\gamma}_{SR_1}}{\overline{\gamma}_{R_1R_2}}}\Gamma\left(0,\frac{\overline{\gamma}_{SR_1}}{\overline{\gamma}_{R_1R_2}}\right)\right)\right] \tag{39}$$

If the average power of the source relay is set equal to unity for simplicity, (37) and (39) can be expressed as

$$C_1 = \overline{\gamma}_{SR_1} \tag{40}$$

$$C_2 = \overline{\gamma}_{SR_2}+\frac{\overline{\gamma}_{SR_1}}{\overline{\gamma}_{R_1R_2}}\left(\overline{\gamma}_{R_1R_2}-\overline{\gamma}_{SR_1}e^{\frac{\overline{\gamma}_{SR_1}}{\overline{\gamma}_{R_1R_2}}}\Gamma\left(0,\frac{\overline{\gamma}_{SR_1}}{\overline{\gamma}_{R_1R_2}}\right)\right) \tag{41}$$

By substituting (40) and (41) in (33), an expression of the average end-to-end SNR with respect to each average intermediate hop SNR is derived.

4.2. ASER

Having found expressions for the instantaneous overall SNR in a N-class multihop diversity system we can analytically evaluate the ASER by averaging specific functions of the overall SNR, depending on the modulation scheme used, with respect to each intermediate channel fading distribution, e.g., for BPSK modulation scheme the ASER can be evaluated as:

$$P_e = \frac{1}{2}\int_0^\infty\int_0^\infty ... \int_0^\infty \text{erfc}\left(\sqrt{\gamma_{eq,N}}\right) \times f_{\gamma_{SD}}\left(\gamma_{SD}\right) d\gamma_{SD}\left(\gamma_{SD}\right) \prod_{j \in \mathcal{N}} f_{\gamma_{Sj}}\left(\gamma_{Sj}\right) d\gamma_{Sj}\left(\gamma_{Sj}\right) \times \prod_{i \in \mathcal{N}} f_{\gamma_{iD}}\left(\gamma_{iD}\right) d\gamma_{iD}\left(\gamma_{iD}\right) \prod_{\substack{i,j \in \mathcal{N} \\ i<j}} f_{\gamma_{ij}}\left(\gamma_{ij}\right) d\gamma_{ij}\left(\gamma_{ij}\right) \tag{42}$$

where $\gamma_{eq,N}$ is given in (19), erfc$(\cdot)$ is the complementary error function and $\mathcal{N}$ is the set consisting of the system relays $\{R_1, ..., R_N\}$. For Rayleigh fading distribution, the average bit error rate (BER) can be evaluated by substituting (22) into (42).

5. Numerical Examples and Discussion

In this section we provide numerical examples of the results presented so far, concerning the overall system performance and the dependence on the intermediate channels' average SNR. The theoretic results are compared to the corresponding ones derived from simulations. In simulations, a BPSK modulation scheme is used; the interference among the nodes is assumed to be negligible, and the relaying is taking place in separate orthogonal channels. The normalized average SNR of the intermediate channels with respect to $\overline{\gamma}_{SD}$ is set as follows: $\overline{\gamma}_{SR_1}/\overline{\gamma}_{SD} = 3$, $\overline{\gamma}_{R_1R_2}/\overline{\gamma}_{SD} = 3$, $\overline{\gamma}_{R_2D}/\overline{\gamma}_{SD} = 3$, $\overline{\gamma}_{SR_2}/\overline{\gamma}_{SD} = 2$, $\overline{\gamma}_{R_1D}/\overline{\gamma}_{SD} = 2$.

5.1. Overall System Performance

5.1.1. ASER

From (42), assuming fixed gain relays according to (34), a comparison can be derived between the two-class multihop diversity system and the direct reference channel, on the basis of ASER. As direct reference channel we denote the direct source-destination channel with instantaneous SNR given by

$$SNR_{SD,ref} = \frac{P_{eq}}{N_0} a_{SD}^2, \tag{43}$$

where P_{eq} represents the instantaneous power transmitted by all system nodes. For computing the average value of P_{eq}, we used the approximation

$$\overline{P}_{eq} \simeq (N+1)\,\overline{P}_S, \tag{44}$$

assuming uniform transmitting power allocation among the nodes. In fact, $(N+1)\,\overline{P}_S$ represents an upper bound of $\overline{P}_{eq}$, due to the fact that (34) is an upper bound of (35), where by definition the average power transmitted by each node is identical[2]. In cases when the signal degradation due to noise is small compared to the degradation due to multipath fading, (44) gives a tight

[2]This holds under the assumption that the gain of each relay does not exceed its saturation point.

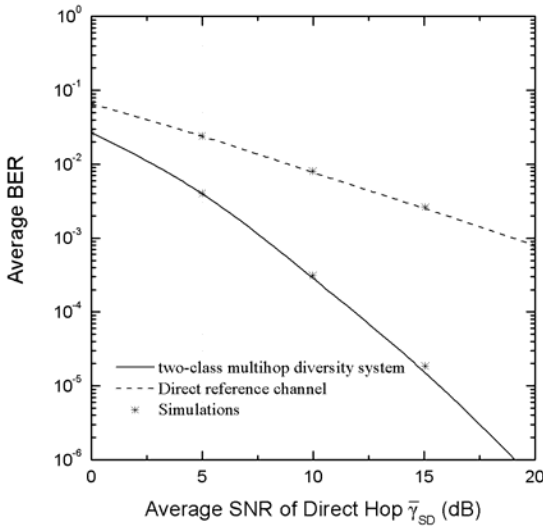

Figure 3: Comparison between the two-class multihop diversity system and the direct reference channel in terms of average BER.

approximation of $\overline{P}_{eq}$. Fig. 3 depicts the average BER versus the average direct channel SNR ($\overline{\gamma}_{SD}$). The comparison between multihop diversity system and the direct reference channel shows a noteworthy improvement on system performance, which becomes wider as $\overline{\gamma}_{SD}$ increases. In Fig. 4, the comparison between MRC and SC performance is presented, in terms of ASER. In both cases, C_1 is evaluated directly from (40). For the MRC case, C_2 is calculated from (41), while in SC case, C_2 is evaluated from simulation results, due to the fact that a closed form of which does not exist. The results show a constant, approximately 3 dB improvement on system performance when using MRC instead of SC.

5.1.2. Average End-To-End SNR

In Fig. 5 the average overall SNR versus $\overline{\gamma}_{SD}$ is plotted, for various values of the ratio $\kappa = \overline{\gamma}_{SR_1}/\overline{\gamma}_{SD} = \overline{\gamma}_{R_1R_2}/\overline{\gamma}_{SD} = \overline{\gamma}_{R_2D}/\overline{\gamma}_{SD}$ and $\lambda = \overline{\gamma}_{SR_2}/\overline{\gamma}_{SD} = \overline{\gamma}_{R_1D}/\overline{\gamma}_{SD}$. The diversity type used in this example is MRC, and each relay gain is set fixed according to (34). As it was expected, the results show a linear increase on $\overline{\gamma}_{eq,2}$ with respect to $\overline{\gamma}_{SD}$, which becomes greater as the ratios κ and λ increase. Note that for high values of $\overline{\gamma}_{SD}$, the cases when $(\kappa, \lambda) = (3, 2)$ and $(2, 3)$ result in approximately the same $\overline{\gamma}_{eq,2}$.

5.2. Dependence on the Intermediate Channels' Average SNR

In Fig. 6, the overall average BER is depicted, when $\overline{\gamma}_{SD} = 16$ dB and the normalized average SNR of a single hop takes values in the interval $[0 \text{ dB}, 10 \text{ dB}]$, while all other intermediate average SNRs remain constant to their original values. Observing these figures, we conclude to the following:

1. A possible change on the fading state of a single intermediate link have a slight or none impact on

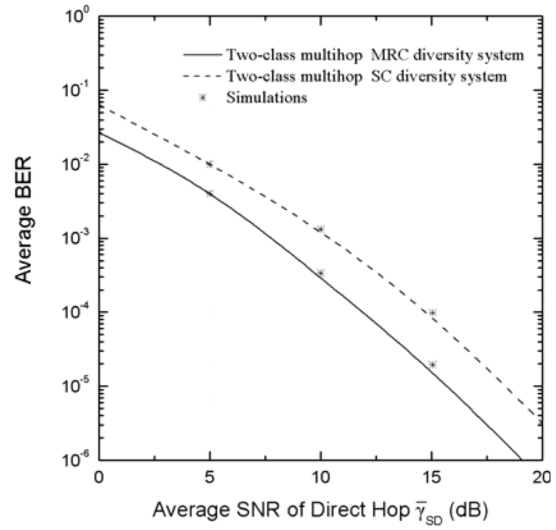

Figure 4: Comparison of the average BER using MRC and SC receivers.

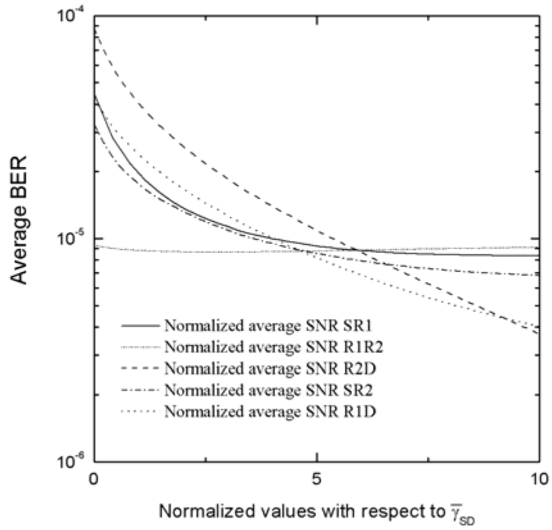

Figure 6: ASER dependence on average intermediate channels' SNR

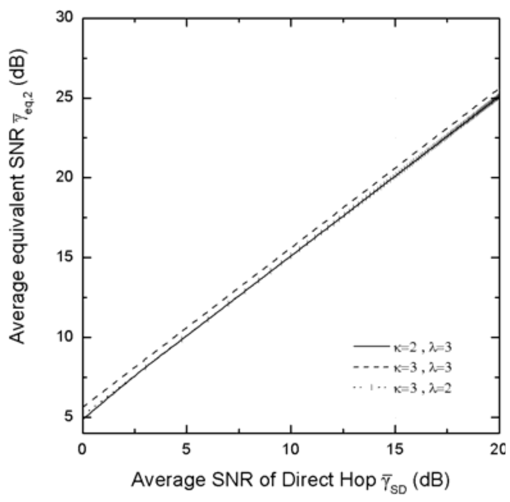

Figure 5: Average overall SNR dependence on average intermediate channels' SNR

the total system performance. An exception to that occurs when the normalized average single hop values lies in the interval [0 dB, 2 dB], which however occurs in rare practical cases.

2. The intermediate links that impact our system the most are the hops i-D, where $i \in \mathcal{N}$, specifically the hops R_1-D and R_2-D. This is due to the fact that, according to (34), the relay gain G_k is set to counterbalance any possible improvement on the attenuation factor a_{eqk}. Therefore, the only intermediate channels that do not suffer such counterbalance and thus impacting the system the most are the hops R_1-D and R_2-D, and generally the hops i-D.[3]

3. The R_1-R_2 link has approximately none impact on total system performance. The explanation is intuitive, claiming that the overall performance does not relatively depend on the intermediate channels that connect two system relays, due to the existence of diversity.

[3]This result concerns multihop diversity systems where the relay gains have the form of (34) or (35). It is obvious that the network administrator can determine which channels will impact the system the most, by modifying each relay gain according to system's needs.

6. Conclusion

In this paper, we studied the performance of non-regenerative wireless multihop diversity systems. We showed that these systems outperform the conventional ones, in terms of ASER, and that the intermediate channels' dependence is greater on the channels ending at the destination terminal D, in the case where each relay gain is given by (34). However, the main drawback of those systems is their high complexity, and this is the reason why efficient protocols and an in-depth cross-layer analysis are needed, in order to efficiently operate.

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