Channel Quality Estimation Index (CQEI): An Improved Performance Criterion for Wireless Communications Systems Over Fading Channels

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Abstract: **We introduce the Channel Quality Estimation Index (CQEI) as an alternative improved performance criterion of wireless communications systems operating over fading channels, directly related to the system's average error rate. CQEI is simple to evaluate, since it depends on the channel statistics, which vary much more slowly than the channel state itself, and thus can be obtained at the initialization state using a long training sequence and continuously improved during the whole communication period. As an application, we use CQEI to find the optimum number of branches in equal gain combining (EGC) receivers, operating over non-identical fading channels. Numerical and simulations results show that CQEI represents an efficient and easy to evaluate criterion to assess the performance of wireless systems.**

1. Introduction

Well known performance criteria used to describe the behavior of digital communications systems performing in fading channels, are the average signal-to-noise ratio (ASNR), the outage probability, the bit error rate (BER) and the amount of fading (AoF) [1]. The ASNR is the most simple to compute, since only the first moment of the instantaneous SNR is needed. However, it cannot be efficiently associated with the error performance, especially over fading channels, as the variance of the instantaneous SNR is totally ignored. Thus, in order to describe the error performance more effectively, higher moments of the instantaneous SNR have to be considered. Ensuing this concept, AoF was introduced in [2], [3], as a measure of the severity of the fading channel. It is the authors' suggestion that the AoF is often appropriate in the more general context of describing the behavior of systems with arbitrary combining techniques and channel statistics and thus can be used as an alternative performance criterion. Later AoF was used in [4] and [5] to assess the performance of wireless systems operating over correlated fading channels.

In this paper, we introduce the Channel Quality Estimation Index (CQEI), defined as the ratio of the variance of the SNR to the cubed average SNR, as an improved performance measure of wireless systems operating over fading channels, directly related to the BER. As an example, we apply CQEI in equal gain combining (EGC) receivers, to estimate the optimum number of branches, avoiding the so-called "combining loss" that contributes more in increasing the noise than the signal power during the combination stage.

The outline of this paper is as follows. In Section II, we introduce the CQEI and show that the error performance of a communications system, which operates over Nakagami-m fading channels, can be better estimated by CQEI compared to AoF or ASNR. In Section III, we apply the CQEI in EGC receivers operating over nonidentical fading channels. Some simulation results are provided in Section IV, and in Section V we present conclusions and final comments.

2. Channel Quality Estimation Index (CQEI)

Definition 1: For a given fading distribution of a received signal, the Channel Quality Estimation Index (CQEI) is defined by the ratio of the variance of the instantaneous received SNR to the cubed mean of the received SNR, γ, i.e.,

$$CQEI = \frac{Var\{\gamma\}}{[E\langle\gamma\rangle]^3} = \frac{AoF}{E\langle\gamma\rangle} \tag{1}$$

where γ is the instantaneous SNR of the received signal, defined as $\gamma = r^2 \frac{E_b}{N_0}$, with r being the fading envelope, E_b the energy per symbol, N_0 the Gaussian noise spectral density and $E\langle\cdot\rangle$ means expectation.

Next, we show that CQEI indicates significantly better the average error performance of a wireless system operating over Nakagami-m fading channel compared to other long-term criteria, namely, the AoF and the ASNR.

Let the fading envelope r be described by the Nakagami-m PDF, which is given by [6]

$$p_r(r) = \frac{2m^m r^{2m-1}}{\Omega^m \Gamma(m)} e^{-\frac{mr^2}{\Omega}}, \ \ r \geqslant 0 \tag{2}$$

where m is a fading parameter which ranges from 0.5 to infinity, Ω is the mean power of the fading envelope and $\Gamma(\cdot)$ the Gamma function. Then the SNR per symbol, γ, is distributed according to the gamma distribution as [1]

$$p_\gamma(\gamma) = \frac{m^m \gamma^{m-1}}{\overline{\gamma}^m \Gamma(m)} e^{-\frac{m\gamma}{\overline{\gamma}}}, \ \ \gamma \geqslant 0 \tag{3}$$

where $\overline{\gamma} = \Omega E_s/N_0$ is the average SNR per symbol. Taking into account that the moment generating function (MGF) is

$$M_\gamma(s) = \left(1 - \frac{s\overline{\gamma}}{m}\right)^{-m} \tag{4}$$

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the AoF and the CQEI are given by

$$AoF = \frac{1}{m} \quad (5)$$

and

$$CQEI = \frac{1}{m\overline{\gamma}}. \quad (6)$$

Lemma 1: Let us assume an M-PSK, or an M-QAM or a DPSK communications system, which operate over the non-identical Nakagami-m fading channels: $H_a \rightarrow (m_a,\overline{\gamma}_a)$ and $H_b \rightarrow (m_b,\overline{\gamma}_b)$. If $m_a > m_b$ and $\overline{\gamma}_a > \overline{\gamma}_b$, then $P_{e_a} < P_{e_b}$, where P_{e_a}, P_{e_b} are the average symbol error probabilities (ASEP) of the corresponding channels.

Proof. Using (4) and Appendix D, in order to prove that $P_{e_a} < P_{e_b}$, we have to prove that

$$\frac{1}{\pi}\left(1+\frac{\overline{\gamma}_a g}{m_a}\right)^{-m_a} < \frac{1}{\pi}\left(1+\frac{\overline{\gamma}_b g}{m_b}\right)^{-m_b} \quad (7)$$

where $g = \{g_{M-PSK}, g_{M-QAM}, g_{DPSK}\}$, depend on the modulation scheme and are given in Appendix D. The above is directly proved, employing the results deduced in Appendix A, considering $\overline{\gamma} g$ as the variable x and m as the variable y.

■

Lemma 2: Let us assume an M-PSK, or an M-QAM or a DPSK communications system, which operate over the non-identical fading channels: $H_a \rightarrow (m_a,\overline{\gamma}_a)$ and $H_b \rightarrow (m_b,\overline{\gamma}_b)$. The uncertainty region of the error performance estimation significantly decreases when CQEI is used, compared to AoF and ASNR.

Proof.

Case I: $m_a > m_b$ and $\overline{\gamma}_a > \overline{\gamma}_b$.

In this case it will be $AoF_a < AoF_b$ and $CQEI_a < CQEI_b$ and as mentioned in Theorem 1, it will be $P_{e_a} < P_{e_b}$.

Case II: $m_a > m_b$ and $\overline{\gamma}_a < \overline{\gamma}_b$.

In that case it will be $AoF_a < AoF_b$, while $CQEI_a$ may be greater or less than $CQEI_b$, depending on the values of m_a, m_b and $\overline{\gamma}_a$, $\overline{\gamma}_b$. Let us assume that

$$\frac{1}{\pi}\left(1+\frac{\overline{\gamma}_a g}{m_a}\right)^{-m_a} > \frac{1}{\pi}\left(1+\frac{\overline{\gamma}_b g}{m_b}\right)^{-m_b} \quad (8)$$

which equivalently gives

$$\overline{\gamma}_b > \frac{m_b}{g}\left[\left(1+\frac{\overline{\gamma}_a g}{m_a}\right)^{\frac{m_a}{m_b}} - 1\right]. \quad (9)$$

Since both parts of (8) are monotonic functions of constant sign, the inequality will also hold after integration of the two parts over the same integration interval, i.e.

$$\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(1+\frac{\overline{\gamma}_a \sin^2\left(\frac{\pi}{M}\right)}{m_a \sin^2(\varphi)}\right)^{-m_a} d\varphi > \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(1+\frac{\overline{\gamma}_b \sin^2\left(\frac{\pi}{M}\right)}{m_b \sin^2(\varphi)}\right)^{-m_b} d\varphi \quad (10)$$

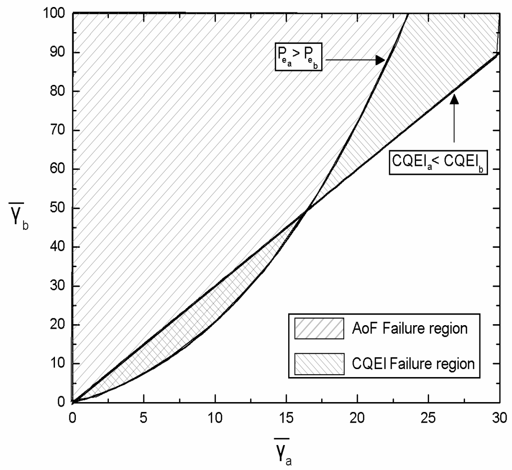

Figure 1: The failure region of the AoF, compared with the failure region of the CQEI, assuming two Nakagami-m channels ($m_a = 4$, $m_b = 2$) and a 4-PSK communication system.

or $P_{e_a} > P_{e_b}$. Thus, the AoF will fail to correctly estimate the error performance of the two channels for those $\overline{\gamma}_b, \overline{\gamma}_a$ satisfying (9).

On the contrary if $CQEI_a < CQEI_b$, or equivalently $\overline{\gamma}_a m_a > \overline{\gamma}_b / m_b$, it will be $P_{e_a} > P_{e_b}$, only for those $\overline{\gamma}_b, \overline{\gamma}_a$, satisfying

$$\frac{m_a\overline{\gamma}_a}{m_b} > \overline{\gamma}_b > \frac{m_b}{g}\left[\left(1+\frac{\overline{\gamma}_a g}{m_a}\right)^{\frac{m_a}{m_b}} - 1\right], \quad (11)$$

and if $CQEI_a > CQEI_b$, it will be $P_{e_a} < P_{e_b}$, only for those $\overline{\gamma}_b, \overline{\gamma}_a$, satisfying

$$\frac{m_a\overline{\gamma}_a}{m_b} < \overline{\gamma}_b < \frac{m_b}{g}\left[\left(1+\frac{\overline{\gamma}_a g}{m_a}\right)^{\frac{m_a}{m_b}} - 1\right]. \quad (12)$$

Obviously, same results could be derived assuming $m_a < m_b$ and $\overline{\gamma}_a > \overline{\gamma}_b$. ■

Note, that the ASNR is not evidently a reliable criterion for the error performance of a wireless system operating over non-identical fading channels, since it totally ignores the fading severity. Therefore, comparing the ASNRs of channels with different m parameters, is the same as comparing the ASNRs of different distributions, which does not lead in safe results or decisions. For example, a channel with higher ASNR, but with more severe fading conditions than an other channel, may result in worse error performance.

An example for the above theoretic results is shown in Fig. 1, where a 4-PSK communication system is assumed, operating over two Nakagami-m fading channels. In this figure we compare the failure regions of the AoF and the CQEI, which were found using (9)-(11) for $m_a = 4$, $m_b = 2$ and $ASNR < 20\ dB$. It is observed that the values of $\overline{\gamma}_a$ and $\overline{\gamma}_b$ for which is $P_{e_a} > P_{e_b}$ and $AoF_a < AoF_b$, are significantly

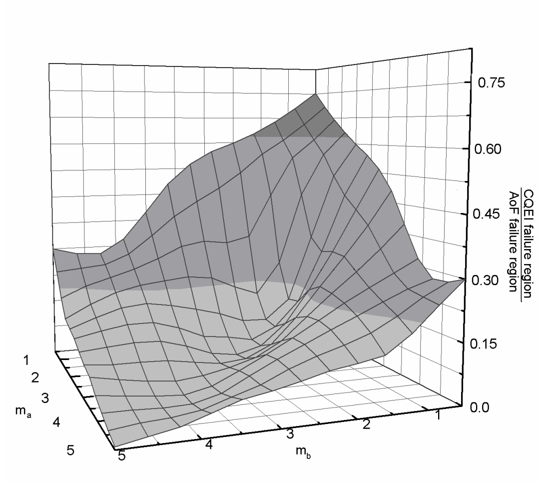

Figure 2: The ratio of the AoF failure region to the CQEI failure region, as a function of the Nakagami-m parameters, for a 4-PSK communication system.

greater than the range of values of $\overline{\gamma}_a$ and $\overline{\gamma}_b$ for which $P_{e_a} > P_{e_b}$ and $CQEI_a < CQEI_b$ or $P_{e_a} < P_{e_b}$ and $CQEI_a > CQEI_b$. Similar results can be obtained for any values of m_a and m_b, as shown in Fig. 2, which depicts the ratio of the CQEI failure region, to the AoF failure region, versus the m-parameters of the Nakagami fading channels. We observe that the CQEI failure region is smaller than the corresponding region of the AoF, for any values of the m parameters. Moreover, for channels with comparable fading parameters, (true in most practical applications), and values larger than 2, the above ratio falls even below 0.1.

3. A CQEI Application: Near-optimum Number of Branches Estimation for EGC Receivers

3.1. System and Channel Model

We consider an L-branch predetection EGC diversity receiver, operating in a fading environment. The received signal at the ith antenna at timing instant t is

$$z_i(t) = r_i(t)s(t)\cos[2\pi f_c t + \theta_i(t)] + n_i(t) \qquad (13)$$

where f_c is the carrier frequency, $n_i(t)$ is the additive white Gaussian noise (AWGN) with a two-sided power spectral density $N_0/2$, *s(t)* is the transmitted signal, $\theta_i(t)$ is the uniformly distributed random phase due to Doppler shift and oscillators' frequency mismatch, and $r_i(t)$ is the fading envelope. The receiver equally weights all input signals and then sums them to produce the decision statistic. For equally likely transmitted symbols, the instantaneous output SNR per symbol is given by

$$\gamma_{out} = \frac{E_s}{LN_0}(r_1 + r_2 + \ldots + r_L)^2 \qquad (14)$$

with E_s being the energy per symbol. In a Nakagami-m fading environment, $r_1, r_2, \ldots, r_L$ are RVs with pdf's given by (2). Here, it is assumed that the m parameter is *not* the same for all branches, which is true in several practical applications, and the power delay profile of the input paths could be uniform ($\overline{\gamma}_i = \overline{\gamma}$) or nonuniform, representing antenna diversity or multipath diversity over frequency-selective fading channels, respectively.

3.2. Moments of the Output SNR

Using (14), the nth moment of the EGC output SNR is [7], by definition

$$\begin{aligned}\mu_n &= E\langle\gamma_{oui}^n\rangle = E\langle \left[\frac{E_s}{LN_0}(r_1 + r_2 \ldots + r_L)^n\right]\rangle \\ &= \left(\frac{E_s}{LN_0}\right)^n E\langle(r_1 + r_2 \ldots + r_L)^{2n}\rangle \qquad (15)\end{aligned}$$

where $E\langle\bullet\rangle$ means expectation. Using the multinomial theorem [8] and $\gamma_i = E_s r_i^2/N_0$, $i = 1, \ldots, L$, with γ_i, being the instantaneous SNR at the ith input path of the combiner, results in

$$\mu_n = \frac{(2n)!}{L^n} \sum_{\substack{n_1,\ldots,n_L=0 \\ n_1+,\ldots,+n_L=2n}}^{2n} \left[\frac{E\langle\gamma_1^{n_1/2}\ldots\gamma_L^{n_L/2}\rangle}{\prod_{i=1}^{L} n_i!}\right]. \qquad (16)$$

For independent but not necessarily identically distributed (i.d) branches, $E\langle\gamma_1^{n_1/2}\ldots\gamma_L^{n_L/2}\rangle$ is expressed as

$$E\langle\gamma_1^{n_1/2}\ldots\gamma_L^{n_L/2}\rangle = E\langle\gamma_1^{n_1/2}\rangle \ldots E\langle\gamma_1^{n_L/2}\rangle. \qquad (17)$$

Now, using the well-known expression for the nth moment of the SNR of a single Nakagami-m channel [1]

$$E\langle\gamma_i^n\rangle = \frac{\Gamma(m+n)}{\Gamma(m)m^n}\overline{\gamma}_i^n \qquad (18)$$

which holds also for noninteger values of n, (16) can be expressed as

$$\mu_n = \frac{(2n)!}{L^n \prod_{i=1}^{L}\Gamma(m_i)} \sum_{\substack{n_1,\ldots,n_L=0 \\ n_1+,\ldots,+n_L=2n}}^{2n} \prod_{i=1}^{L} \frac{\Gamma(m_i + \frac{n_i}{2})\overline{\gamma}_i^{n_i/2}}{n_i! m_i^{n_i/2}}. \qquad (19)$$

Knowing the raw moments given by (19), the k-central moment of output SNR, $E\langle(\gamma_{out,L} - \overline{\gamma}_{out,L})^k\rangle$ can be written after using the binomial theorem, as

$$\begin{aligned} &E\langle(\gamma_{out,L} - \overline{\gamma}_{out,L})^k\rangle = \\ &= \sum_{n=0}^{k} \frac{k!}{n!(k-n)!} E\langle\gamma_{out,L}^n(-\overline{\gamma}_{out,L})^{k-n}\rangle \\ &= \sum_{n=0}^{k} \frac{k!(-1)^{k-n}(\overline{\gamma}_{out,L})^{k-n}}{n!(k-n)!}\mu_n \qquad (20)\end{aligned}$$

where $E\langle\gamma_{out,L}^n\rangle$ is the nth moment of $\gamma_{out,L}$ given by (19).

3.2.1. Amount of Fading

The Amount of Fading for an EGC receiver is

$$AoF = \frac{Var\langle\gamma_{out,L}\rangle}{\overline{\gamma}_{out,L}^2}. \qquad (21)$$

where $Var\langle\gamma_{out,L}\rangle = E\langle(\gamma_{out,L} - \overline{\gamma}_{out,L})^2\rangle$ is the second central moment of the average SNR and $\overline{\gamma}_{out,L}$ is the averaged SNR, and can be directly calculated using the closed forms of (19) and (20).

3.2.2. Channel Quality Estimation Index

The Channel Quality Estimation Index for an EGC receiver is

$$CQEI = \frac{Var\langle\gamma_{out,L}\rangle}{\overline{\gamma}^3_{out,L}}. \qquad (22)$$

and can be directly calculated using the closed forms of (19) and (20).

3.3. An Algorithm for the Estimation of the Near-optimum Number of Branches for EGC Receivers

It is well known that in EGC receivers, increment of the used branches does not always leads to performance improvement, especially when nonidentical branches are considered. This phenomenon occurs because some branches may receive a highly attenuated replica of the transmitted signal or even only noise. Thus, the optimum performance would be achieved by the selection of the number of branches that would minimize the BER. Such a technique requires either a continuous estimation of the BER, leading in high complexity, or closed forms for the BER calculation in nonidentical fading channels. Here, we propose a near optimum algorithm for the number of branches used at the EGC receiver, which is described below.

The algorithm assumes that the parameters m_i and $\overline{\gamma}_i$ of the channel at each branch are known. The steps of the proposed algorithm are:

1. *Estimation of* $\overline{\gamma}_i$, for $i = 1, \ldots, L$, *using a long training sequence at the initialization stage.*

2. *Calculation of the second central moment of the output SNR* $Var\langle\gamma_{out,i}\rangle$ for $i = 2, \ldots, L$, *using the closed form of (20):*

3. *Calculation of the Channel Quality Estimation Index as:*

$$CQEI_i = \frac{Var\langle\gamma_{out,i}\rangle}{\mu^3_{1,i}}, \; i = 2, \ldots, L. \qquad (23)$$

4. *Estimation of the number of branches as:*

$$L_{opt} = \arg\min_{i=2\ldots L}\{CQEI_i\} \qquad (24)$$

4. Numerical Results

In this section, we provide numerical representative curves and tables illustrating the performance of the proposed algorithm in an EGC receiver over Nakagami-m fading channels and the efficiency of the ASNR, AoF and CQEI to estimate the optimum diversity branches. The maximum available branches are assumed to be $L = 5$ with each branch to carry signal from a propagation path with different fading parameters and the receiver to operate with an exponentially power delay profile (pdp), i.e. $\overline{\gamma}_i = \overline{\gamma}e^{-di}$, where d is the decaying factor. The diversity branches to be used are determined by the following decisions for each of the criteria.

$L_{SNR} = \arg\max_{i=2\ldots L}\{SNR_i\}$,
$L_{AoF} = \arg\min_{i=2\ldots L}\{AoF_i\}$,
$L_{CQEI} = \arg\min_{i=2\ldots L}\{CQEI_i\}$ and
$L_{BER} = \arg\min_{i=2\ldots L}\{BER_i\}$.

The optimum criterion is obviously the expected BER, which is evaluated via simulations. In Fig. 3 we compare the error performance of the EGC receiver when various criteria employed, over Nakagami-m fading and assuming a decay factor $d = 1$. It is observed that when the CQEI is used the combining loss is almost eliminated, while the other criteria, as the ASNR and the AoF, fail to estimate the optimum diversity branches. For a decay factor of $d = 2$ the above comparisons are shown in Fig.4, where similar results can be derived, with the difference that the performance of the ASNR concerning the branches estimation has improved, while the AoF totally fails. This behavior is expected, as high values of the decay factor result in an error performance which is determined only by the first one or two branches. The CQEI, in both cases, seems to successfully determine the near optimum diversity branches.

In Table I, we present the ability of the ASNR, AoF and CQEI in estimating the optimum diversity branches for various values of the decay factor and for fading parameters that follow a truncated Gaussian distribution with

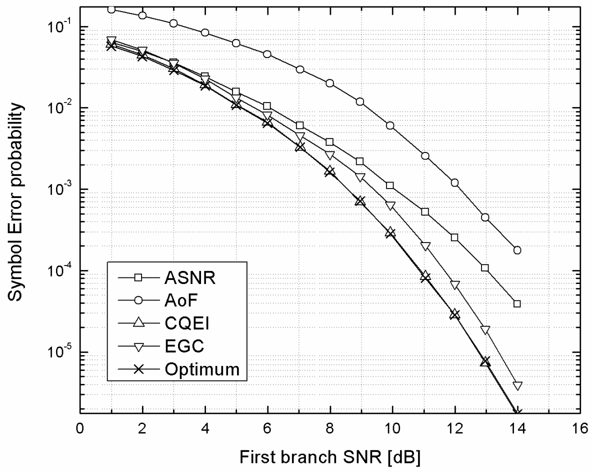

Figure 3: The performance of the EGC receiver when various criteria employed, compared to an $L = 5$ EGC receiver for a BPSK communication system performing over Nakagami-m fading, assuming non-i.i.d branches (d =1, $m_1 = 1.5$, $m_2 = 2$, $m_3 = 2.5$, $m_4 = 3.2$, $m_5 = 1.3$).

Table 1: Percentage (%) of successfully selecting the optimum number of branches, when different criteria applied, for various values of the average snr and the decay factor at the receiver over Nakagami-m fading.

	d	$\overline{\gamma}$	0	5	10	15	20
$ASNR$	0.5		71	78	77	78	82
	1		65	26	15	8	8
	1.5		90	82	65	36	16
	2		91	92	91	65	62
AoF	0.5		92	91	89	91	90
	1		23	24	38	60	65
	1.5		2	2	3	2	8
	2		6	7	5	2	2
$CQEI$	0.5		95	93	94	95	92
	1		68	71	92	84	78
	1.5		80	79	88	85	79
	2		94	96	96	94	86

mean, $\mu = 2$ and variance, $\sigma^2 = 1.5$. The latter assumption has been adopted in the channel modeling standards for Ultra Wideband Systems [9]. We note, that failure in estimating the optimum diversity branches does not always lead to dramatic performance decrement, as the difference from the optimum may not be significant. Thus, from Table I, we can see that the CQEI estimates the optimum diversity branches better that the other criteria for almost any case. In general, we observe that the AoF constitutes a reliable criterion only for small decay factors, while the ASNR for high power delay profiles. We note that the decision on the optimum number of branches is based on the CQEI of the output SNR, which distribution is unknown. Taking into account the above remarks and the efficiency of the CQEI in estimating the optimum number of branches, we could describe the CQEI as an error performance criterion for generalized fading channels, even with unknown statistics.

5. Conclusions

In this paper, a novel error performance criterion for wireless digital communications systems operating over generalized fading channels was presented. Comparatively to other classical performance measures, such as ASNR, or AoF, the CQEI can describe significantly better the system's behavior. Theoretical study of the CQEI in Nakagami-m fading channels for M-PSK, M-QAM and DPSK modulation schemes, showed its efficiency towards AoF. CQEI was applied in the EGC receiver as an optimum number of branches selection criterion, improving the overall error performance. Finally numerical results and simulations were evaluated to validate the conclusions derived above.

Appendix A

Let us consider the function

$$f(x,y) = \left(1+\frac{x}{y}\right)^{-y},\ x,\ y \in \mathbb{R}^{+}. \qquad \text{(A1)}$$

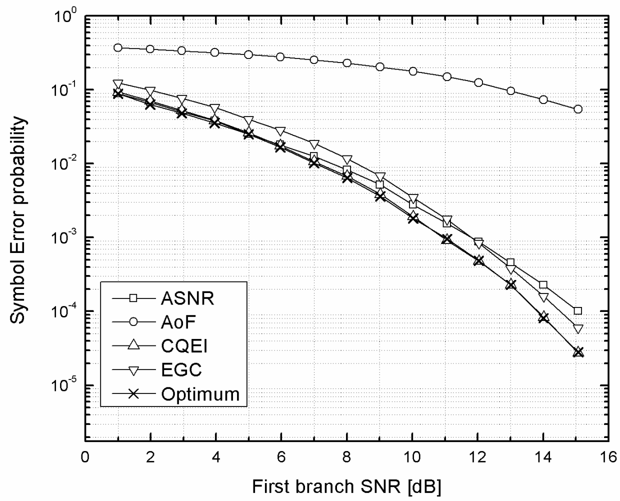

Figure 4: The performance of the EGC receiver when various criteria employed, compared to an $L = 5$ EGC receiver for a BPSK communication system performing over Nakagami-m fading, assuming non-i.i.d branches (d =2, $m_1 = 1.2$, $m_2 = 2$, $m_3 = 3.1$, $m_4 = 2.2$, $m_5 = 3.0$).

The partial derivative $\frac{\partial}{\partial y} f(x,y)$ is given by

$$\frac{\partial}{\partial y} f(x,y) = \left(1+\frac{x}{y}\right)^{-y} \left(\frac{x}{y+x} - \ln\left(1+\frac{x}{y}\right)\right). \qquad \text{(A2)}$$

Using the well known inequality

$$\frac{x}{x+1} < \ln(1+x), \qquad \text{(A3)}$$

it is deduced that

$$\left(\frac{x}{y+x} - \ln\left(1+\frac{x}{y}\right)\right) < 0 \qquad \text{(A4)}$$

and since

$$\left(1+\frac{y}{x}\right)^{-x} \geqslant 0 \qquad \text{(A5)}$$

we derive that

$$\frac{\partial}{\partial y} f(x,y) \leqslant 0. \qquad \text{(A6)}$$

The partial derivative $\frac{\partial}{\partial x} f(x,y)$ is given by

$$\frac{\partial}{\partial x} f(x,y) = -\left(1+\frac{x}{y}\right)^{-1-y} \qquad \text{(A7)}$$

which is negative $\forall\, y > 0, x > 0$.

Thus the function $f(x,y)$ is monotonically decreasing, which means that $f(x_1,y_1) < f(x_2,y_2)$, $\forall$ $x_1 > x_2$, $y_1 > y_2$, .

Appendix B

Let us consider the function

$$f(x,y)=\left(\frac{1+x^2}{1+x^2+y}\right)\exp\left(\frac{x^2y}{1+x^2+y}\right), x,y\in\mathbb{R}^+ \tag{B1}$$

The partial derivative $\frac{\partial}{\partial y}f(x,y)$ is given by

$$\frac{\partial}{\partial y}f(x,y)= \tag{B2}$$

$$-\frac{\exp\left(\frac{x^2y}{1+x^2+y}\right)(1+x^2)(1+2x^2+x^4+y)}{(1+x^2+y)^3}<0. \tag{25}$$

The partial derivative $\frac{\partial}{\partial x}f(x,y)$ is given by

$$\frac{\partial}{\partial x}f(x,y)=-\frac{2\exp\left(\frac{x^2y}{1+x^2+y}\right)x^3y^2}{(1+x^2+y)^3}<0 \tag{B3}$$

Thus the function $f(x,y)$ is monotonically decreasing, which means that

$$f(x_1,y_1)<f(x_2,y_2),\ \forall\, x_1>x_2,\ y_1>y_{2,}. \tag{B4}$$

Appendix C

Let us consider the function

$$f(x,y)=\left(1+2y+\frac{(2y)^2x^2}{(1+x^2)^2}\right)^{-\frac{1}{2}},\ x,\ y\in\mathbb{R}^+. \tag{C1}$$

The partial derivative $\frac{\partial}{\partial y}f(x,y)$ is given by

$$\frac{\partial}{\partial y}f(x,y)=-\frac{2+\frac{8x^2y}{(1+x)^2}}{2\left(1+2y+\frac{(2y)^2x^2}{(1+x)^2}\right)^{\frac{3}{2}}}<0. \tag{C2}$$

The partial derivative $\frac{\partial}{\partial x}f(x,y)$ is given by

$$\frac{\partial}{\partial x}f(x,y)=\frac{4x(-1+x^2)y^2}{(1+x^2)^3\left(1+2y+\frac{(2y)^2x^2}{(1+x^2)^2}\right)^{\frac{3}{2}}} \tag{C3}$$

which is negative for $0<x<1$.

Thus the function $f(x,y)$ is monotonically decreasing for $0<x<1$, $y>0$, which means that

$$f(x_1,y_1)<f(x_2,y_2),\ \forall\, x_1>x_2,\ y_1>y_{2,}. \tag{C4}$$

Appendix D

The average symbol error probabilities for $M-$PSK, $M-$QAM and DPSK communications systems are given by

$$M-\text{PSK:}\quad \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} M_\gamma\left(-\frac{g_{MPSK}}{\sin^2(\varphi)}\right)d\varphi$$

$$M-\text{QAM}:\ \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\int_0^{\frac{\pi}{2}} M_\gamma\left(-\frac{g_{MQAM}}{\sin^2(\varphi)}\right)d\varphi$$
$$-\frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)^2\int_0^{\frac{\pi}{4}} M_\gamma\left(-\frac{g_{MQAM}}{\sin^2(\varphi)}\right)d\varphi$$

$$\text{DPSK}:\quad \frac{1}{2}M_\gamma\left(-g_{DPSK}\right).$$

where $g_{MPSK}=\sin^2(\pi/M)$, $g_{MQAM}=3/2(M-1)$ and $g_{DPSK}=1$.

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