

Level Crossing Rate and Average Fade Duration of the Multihop Rayleigh Fading Channel

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Abstract—We present a novel analytical framework for the evaluation of important second order statistical parameters, as the level crossing rate (LCR) and the average fade duration (AFD) of the amplify-and-forward multihop Rayleigh fading channel. More specifically, motivated by the fact that this channel is a cascaded one, which can be modelled as the product of N fading amplitudes, we derive novel analytical expressions for the average LCR and AFD of the product of N Rayleigh fading envelopes, or of the recently so-called N *Rayleigh channel. Furthermore, we derive simple and efficient closed-form approximations to the aforementioned parameters, using the multivariate Laplace approximation theorem. It is shown that our general results reduce to the specific dual-hop case, previously published. Numerical and computer simulation examples verify the accuracy of the presented mathematical analysis and show the tightness of the proposed approximations.

I. INTRODUCTION

Multihop communications, a viable option for providing broader and more efficient coverage, can be categorized as either non-regenerative (amplify-and-forward, AF) or regenerative decode-and-forward, DF) depending on the relay functionality [1]-[9]. In DF systems, each relay decodes its received signal and then re-transmits this decoded version. In AF systems, the relays just amplify and re-transmit their received signal. Furthermore, a system with AF relays can use channel state information (CSI)-assisted relays [1] or fixed-gain relays [2] (also known as blind or semi-blind relays [6]). A (CSI)-assisted relay uses instantaneous CSI of the channel between the transmitting terminal and the receiving relay to adjust its gain, whereas a fixed-gain relay just amplifies its received signal by a fixed gain [2][6]. Systems with fixed-gain relays perform close to systems with (CSI)-assisted relays [2], while their easy deployment and low complexity make them attractive from a practical point of view.

Several works in the open literature have provided performance analysis of AF or DF systems in terms of bit error rate (BER) and outage probability under different assumptions of the amplifier gain [1]-[9]. Among them, only two works dealt with the dynamic, time-varying nature of the underlying fading channel, [8], [9], despite the fact that it is necessary for the system's design or rigorous testing. In [8], the level crossing rate (LCR) and the average fade duration (AFD) of multihop DF communication systems over generalized fading

channels was studied, both for noise-limited and interference-limited systems, while Patel et. al in [9] provide useful exact analytical expressions for the AF channel's temporal statistical parameters such as the auto-correlation and the LCR. However, the approach presented in [9] is limited only to the dual-hop fixed-gain AF Rayleigh fading channel.

In this paper, we study the second order statistics of the fixed-gain AF multihop Rayleigh fading channel. More specifically, motivated by the fact that this channel is a cascaded one, which can be modeled as the product of N fading amplitudes, we derive a novel analytical framework for the evaluation of the average LCR and the AFD of the product of N Rayleigh fading envelopes. Furthermore, we derive simple and efficient closed-form approximations using the multivariate Laplace approximation theorem [16, Chapter IX.5], [17]. These important theoretical results are then applied to investigate the second order statistics of the multihop Rayleigh fading channel. Numerical and computer simulation examples verify the accuracy of presented mathematical analysis and show the tightness of the proposed approximations.

II. LEVEL CROSSING RATE AND AVERAGE FADE DURATION OF THE PRODUCT OF N RAYLEIGH ENVELOPES

Let $\{X_i(t)\}_{i=1}^N$ be N independent and not necessarily identically distributed (i.n.i.d.) Rayleigh random processes, each distributed according to [10]-[11],

$$f_{X_i}(x) = \frac{2x}{\Omega_i} \exp\left(-\frac{x^2}{\Omega_i}\right), \quad x \geq 0, \quad (1)$$

in an arbitrary moment t , where $\Omega_i = E\{X_i^2(t)\}$ is the mean power of the i -th random process ($1 \leq i \leq N$).

If $\{X_i(t)\}_{i=1}^N$ represent received signal envelopes in an isotropic scattering radio channel exposed to the Doppler Effect, they must be considered as time-correlated random processes with some resulting Doppler spectrum. This Doppler spectrum differs depending on whether fixed-to-mobile channel [10]-[11] or mobile-to-mobile channel [12]-[13] appears in the wireless communications system. In both cases, it was found that time derivative of i -th envelope is independent from the envelope itself, and follows the Gaussian PDF [10]-[13]

$$f_{\dot{X}_i}(\dot{x}) = \frac{1}{\sqrt{2\pi}\sigma_{\dot{X}_i}} \exp\left(-\frac{\dot{x}^2}{2\sigma_{\dot{X}_i}^2}\right), \quad (2)$$

with variance calculated as

$$\sigma_{\dot{X}_i}^2 = \pi^2 \Omega_i f_i^2. \quad (3)$$

If envelope X_i is formed on a fixed-to-mobile channel, then $f_i = f_{mi}$ where f_{mi} is the maximum Doppler frequency shift induced by the motion of the mobile station [10]-[11]. If envelope X_i is formed on a mobile-to-mobile channel, then

$$f_i = \sqrt{f_{mi}^2 + f_{mi}^{\prime 2}}. \quad (4)$$

where f_{mi}^{\prime} and $f_{mi}^{\prime\prime}$ are the maximum Doppler frequency shifts induced by the motion of both mobile stations (i.e., the transmitting and the receiving stations, respectively) [13]. It is important to underline that the maximum Doppler frequency in a fixed-to-mobile channel is $f_{d\max} = f_{mi}$, whereas the maximum Doppler frequency in a mobile-to-mobile channel is $f_{d\max} = f_{mi} + f_{mi}^{\prime}$. The above results are essential in deriving the second-order statistical parameters of individual envelopes, as the LCR and the AFD [10], [11], [13].

Below, we derive exact and approximate solutions for both of the above parameters for product of N Rayleigh envelopes,

$$Y(t) = \prod_{i=1}^N X_i(t). \quad (5)$$

We denote $Y(t)$ as N *Rayleigh random process or, at any given moment t , N *Rayleigh random variable, following the definition given in [14].

For some specified value $\{X_i\}_{i=1}^N = \{x_i\}_{i=1}^N$, the product Y is fixed to the specific value $y = \prod_{i=1}^N x_i$. The LCR of Y at threshold y is defined as the rate at which the random process crosses level y in the negative direction [10]. To extract LCR, we need to determine the joint probability density function (PDF) between Y and \dot{Y} , $f_{Y\dot{Y}}(y, \dot{y})$, and to apply the Rice's formula [11, Eq. (2.106)],

$$N_Y(y) = \int_0^\infty \dot{y} f_{Y\dot{Y}}(y, \dot{y}) d\dot{y}. \quad (6)$$

Our method does not require explicit determination of $f_{Y\dot{Y}}(y, \dot{y})$ in order to determine analytically the LCR of the N *Rayleigh random process, as presented below.

First, we need to find the time derivative of (5), which is

$$\dot{Y} = Y \sum_{i=1}^N \frac{\dot{X}_i}{X_i}. \quad (7)$$

Conditioning on the first $N - 1$ envelopes $\{X_i\}_{i=1}^{N-1} = \{x_i\}_{i=1}^{N-1}$, we have the conditional joint PDF Y and \dot{Y} written as $f_{Y\dot{Y}|X_1 \dots X_{N-1}}(y, \dot{y}|x_1, \dots, x_{N-1})$. This conditional joint PDF can be averaged with respect to the joint PDF of the $N - 1$ envelopes $\{X_i\}_{i=1}^{N-1}$ to produce the required joint PDF,

$$\begin{aligned} & f_{Y\dot{Y}}(y, \dot{y}) \\ &= \int_{x_1=0}^\infty \dots \int_{x_{N-1}=0}^\infty f_{Y\dot{Y}|X_1 \dots X_{N-1}}(y, \dot{y}|x_1, \dots, x_{N-1}) \\ & \quad \times f_{X_1}(x_1) \dots f_{X_{N-1}}(x_{N-1}) dx_1 \dots dx_{N-1} \quad (8) \end{aligned}$$

where to derive (8) the mutual independence of the $N - 1$ envelopes is used.

The conditional joint PDF $f_{Y\dot{Y}|X_1 \dots X_{N-1}}(y, \dot{y}|x_1, \dots, x_{N-1})$ can be further simplified by setting $Y = y$ and using the total probability theorem,

$$\begin{aligned} & f_{Y\dot{Y}|X_1 \dots X_{N-1}}(y, \dot{y}|x_1, \dots, x_{N-1}) \\ &= f_{\dot{Y}|Y X_1 \dots X_{N-1}}(\dot{y}|y, x_1, \dots, x_{N-1}) \\ & \quad \times f_{Y|X_1 \dots X_{N-1}}(y|x_1, \dots, x_{N-1}), \quad (9) \end{aligned}$$

where each of the two multipliers in (9) can be determined from the above defined individual PDFs and their parameters.

Based on (7), the conditional PDF $f_{\dot{Y}|Y X_1 \dots X_{N-1}}(\dot{y}|y, x_1, \dots, x_{N-1})$ can be easily established to follow the Gaussian PDF with zero mean and variance

$$\begin{aligned} \sigma_{\dot{Y}|Y X_1 \dots X_{N-1}}^2 &= \left(y^2 \sum_{i=1}^{N-1} \frac{\sigma_{\dot{X}_i}^2}{x_i^2} + \sigma_{\dot{X}_N}^2 \prod_{i=1}^{N-1} x_i^2 \right) \\ &= \sigma_{\dot{X}_N}^2 \left[1 + y^2 \left(\prod_{i=1}^{N-1} \frac{1}{x_i^2} \right) \sum_{i=1}^{N-1} \frac{\sigma_{\dot{X}_i}^2}{\sigma_{\dot{X}_N}^2} \frac{1}{x_i^2} \right] \prod_{i=1}^{N-1} x_i^2. \quad (10) \end{aligned}$$

The conditional PDF of Y given $\{X_i\}_{i=1}^{N-1} = \{x_i\}_{i=1}^{N-1}$ that appears in (9) is easily determined in terms of the PDF of the remaining N -th envelope,

$$\begin{aligned} & f_{Y|X_1 \dots X_{N-1}}(y|x_1, \dots, x_{N-1}) \\ &= f_{X_N} \left(y \prod_{i=1}^{N-1} \frac{1}{x_i} \right) \prod_{i=1}^{N-1} \frac{1}{x_i} \quad (11) \end{aligned}$$

Introducing (9) and (11) into (8), then (8) into (6) and changing the orders of the integration, we obtain

$$\begin{aligned} N_Y(y) &= \int_{x_1=0}^\infty \dots \int_{x_{N-1}=0}^\infty \left(\int_{\dot{y}=0}^\infty \dot{y} f_{Y\dot{Y}|Y X_1 \dots X_{N-1}}(\dot{y}|y, x_1, \dots, x_{N-1}) d\dot{y} \right) \prod_{i=1}^{N-1} \frac{1}{x_i} \\ & f_{X_N} \left(y \prod_{i=1}^{N-1} \frac{1}{x_i} \right) f_{X_1}(x_1) \dots f_{X_{N-1}}(x_{N-1}) dx_1 \dots dx_{N-1} \quad (12) \end{aligned}$$

The bracketed integral in (12) is found using (10) as

$$\int_0^\infty \dot{y} f_{Y\dot{Y}|Y X_1 \dots X_{N-1}}(\dot{y}|y, x_1, \dots, x_{N-1}) d\dot{y} = \frac{\sigma_{\dot{Y}|Y X_1 \dots X_{N-1}}}{\sqrt{2\pi}} \quad (13)$$

By substituting (1) and (13) into (12), we obtain the exact formula for the LCR as

$$\begin{aligned} N_Y(y) &= \frac{\sigma_{\dot{X}_N} 2^N y}{\sqrt{2\pi} \Phi} \\ & \times \int_{x_1=0}^\infty \dots \int_{x_{N-1}=0}^\infty \left[1 + y^2 \left(\prod_{i=1}^{N-1} \frac{1}{x_i^2} \right) \sum_{i=1}^{N-1} \frac{\sigma_{\dot{X}_i}^2}{\sigma_{\dot{X}_N}^2} \frac{1}{x_i^2} \right]^{1/2} \\ & \times \exp \left[- \left(\frac{y^2}{\Omega_N} \prod_{i=1}^{N-1} \frac{1}{x_i^2} + \sum_{i=1}^{N-1} \frac{x_i^2}{\Omega_i} \right) \right] dx_1 \dots dx_{N-1}, \quad (14) \end{aligned}$$

where

$$\Phi = \prod_{k=1}^N \Omega_k \quad (15)$$

In principle, (14) together with (15) provide an exact analytical expression for the LCR of the product of the product of N Rayleigh envelopes (i.e., N *Rayleigh random process [14]). However, (14) becomes computationally attractive only for small values of N , where it is possible to apply a numerical computation method (as Gaussian-Hermite quadrature).

Note that, (14) is transformed into a single integral when $N = 2$, which, after introducing (3) for $i = 1, 2$ and changing integration variable x with new variable t according $x = y/t$, reduces to the known result [9, Eq. (17)].

The AFD of Y at threshold y is defined as the average time that the N *Rayleigh random process remains below level y after crossing that level in the downward direction,

$$T_Y(y) = \frac{F_Y(y)}{N_Y(y)}, \quad (16)$$

where $F_Y(\cdot)$ denotes the cumulative distribution function (CDF) of Y . Fortunately, $F_Y(\cdot)$ was derived recently in closed-form [14, Eq. (7)], as

$$F_Y(y) = G_{1,N+1}^{N,1} \left[\frac{y^2}{\Phi} \middle| \underbrace{1, 1, \dots, 1}_N, 0 \right], \quad (17)$$

where $G[\cdot]$ is the Meijer's G -function [15, Eq. (9.301)].

A. An Approximate Solution for the LCR

Next, we present a tight closed-form approximation of (14) using the multivariate Laplace approximation theorem [16, Chapter IX.5], [17] for the Laplace-type integral

$$J(\lambda) = \int_{\mathbf{x} \in D} u(\mathbf{x}) \exp(-\lambda h(\mathbf{x})) d\mathbf{x}, \quad (18)$$

where u and h are real-valued multivariate functions of $\mathbf{x} = [x_1, \dots, x_{N-1}]$, λ is a real parameter and D is unbounded domain in the multidimensional space R^{N-1} .

A comparison of (14) and (18) yields

$$u(\mathbf{x}) = \left[1 + y^2 \left(\prod_{i=1}^{N-1} \frac{1}{x_i^2} \right) \sum_{i=1}^{N-1} \frac{\sigma_{\dot{X}_i}^2}{\sigma_{\dot{X}_N}^2} \frac{1}{x_i^2} \right]^{1/2}, \quad (19)$$

$$h(\mathbf{x}) = \frac{y^2}{\Omega_N} \prod_{i=1}^{N-1} \frac{1}{x_i^2} + \sum_{i=1}^{N-1} \frac{x_i^2}{\Omega_i}, \quad (20)$$

and $\lambda = 1$. Note, that in the case of (14), all the applicability conditions of the theorem are fulfilled. Namely, within the domain of interest D , the function $h(\mathbf{x})$ has a single interior critical point $\tilde{\mathbf{x}} = [\tilde{x}_1, \dots, \tilde{x}_{N-1}]$, where

$$\tilde{x}_i = y^{1/N} \frac{\Omega_i^{1/2}}{\Phi^{1/(2N)}}, \quad 1 \leq i \leq N-1, \quad (21)$$

which is obtained from solving the set of equations $\partial h / \partial x_i = 0$, where $1 \leq i \leq N-1$. The Hessian $(N-1) \times (N-1)$ square matrix \mathbf{A} , defined by [15, Eq. (14.314)], is written as

$$\mathbf{A} = \begin{bmatrix} 8/\Omega_1 & 4/\sqrt{\Omega_1\Omega_2} & \cdots & 4/\sqrt{\Omega_1\Omega_{N-1}} \\ 4/\sqrt{\Omega_2\Omega_1} & 8/\Omega_2 & \cdots & 4/\sqrt{\Omega_2\Omega_{N-1}} \\ \vdots & \vdots & \ddots & \vdots \\ 4/\sqrt{\Omega_{N-1}\Omega_1} & 4/\sqrt{\Omega_{N-1}\Omega_2} & \cdots & 8/\Omega_{N-1} \end{bmatrix} \quad (22)$$

By using induction, it is easy to determine that the $N-1$ eigenvalues of \mathbf{A} are calculated as $\mu_i = 4/\Omega_i$ for $1 \leq i \leq N-2$, and $\mu_{N-1} = 4N/\Omega_{N-1}$. Thus, all eigenvalues of \mathbf{A} are positive, which, by definition, means that the matrix \mathbf{A} is positive definite. By means of the second derivative test, since the Hessian matrix \mathbf{A} is positive definite at point $\tilde{\mathbf{x}}$, $h(\mathbf{x})$ attains a local minimum at this point (which in this case is the absolute minimum in the entire domain D).

At this interior critical point $\tilde{\mathbf{x}}$,

$$u(\tilde{\mathbf{x}}) = \left(1 + \sum_{i=1}^{N-1} \frac{\sigma_{\dot{X}_i}^2}{\sigma_{\dot{X}_N}^2} \frac{\Omega_N}{\Omega_i} \right)^{1/2} = \left(1 + \sum_{i=1}^{N-1} \frac{f_i^2}{f_N^2} \right)^{1/2}, \quad (23)$$

$$h(\tilde{\mathbf{x}}) = N \left(\frac{y^2}{\Phi} \right)^{1/N}, \quad (24)$$

where (23) is obtained using (3). Now, it is possible to approximate (18) for large λ as

$$J(\lambda) \approx \left(\frac{2\pi}{\lambda} \right)^{(N-1)/2} \left[\frac{1}{\det(\mathbf{A})} \left(1 + \sum_{i=1}^{N-1} \frac{f_i^2}{f_N^2} \right) \right]^{1/2} \times \exp \left(-\lambda N \frac{y^{2/N}}{\Phi^{1/N}} \right). \quad (25)$$

It is well-know that the determinant of the square matrix is equal to the product of its eigenvalues. Hence, \mathbf{A} can be written as

$$\det(\mathbf{A}) = \frac{N2^{2(N-1)}}{\prod_{k=1}^{N-1} \Omega_k} = \frac{\Omega_N N 2^{2(N-1)}}{\Phi}. \quad (26)$$

Although approximation (25) is proven for large λ [16]-[17], it is often applied when λ is small and is observed to be very accurate as well. Similarly to [18], we apply the theorem for $\lambda = 1$. Therefore, the approximate closed-form solution for the LCR of N *Rayleigh random process Y at threshold y is

$$\begin{aligned} N_Y(y) &\approx \frac{\sigma_{\dot{X}_N} 2^N y}{\sqrt{2\pi} \Phi} J(1) = \frac{2y(2\pi)^{N/2-1} \sigma_{\dot{X}_N}}{\Omega_N^{1/2} \Phi^{1/2}} \\ &\times \left[\frac{1}{N} \left(1 + \sum_{i=1}^{N-1} \frac{f_i^2}{f_N^2} \right) \right]^{1/2} \exp \left(-N \frac{y^{2/N}}{\Phi^{1/N}} \right) \\ &= \left(\frac{1}{N} \sum_{i=1}^N f_i^2 \right)^{1/2} \frac{(2\pi)^{N/2} y}{\Phi^{1/2}} \exp \left(-N \frac{y^{2/N}}{\Phi^{1/N}} \right). \end{aligned} \quad (27)$$

The numerical results presented in Section IV validate the high accuracy of the Laplace approximation applied for our particular case.

Combining (17) and (27) into (16), the AFD of the N *Rayleigh random process Y at threshold y is approximated as

$$\begin{aligned} T_Y(y) &\approx \left(\frac{1}{N} \sum_{i=1}^N f_i^2 \right)^{-1/2} \frac{\Phi^{1/2}}{(2\pi)^{N/2}} \frac{1}{y} \\ &\times G_{1,N+1}^{N,1} \left[\frac{y^2}{\Phi} \middle| \underbrace{1, 1, \dots, 1}_N, 0 \right] \exp \left(N \frac{y^{2/N}}{\Phi^{1/N}} \right). \end{aligned} \quad (28)$$

III. SECOND ORDER STATISTICS OF MULTIHOP TRANSMISSION

Next, we apply the important theoretical result of the previous Section to analyze the second order statistics of the multihop relay fading channel.

A. System Model

We now consider a multihop wireless communications system, operating over i.n.i.d flat fading channels. Source station S communicates with destination station D through $N - 1$ relays T_1, T_2, \dots, T_{N-1} , which act as intermediate stations from one hop to the next. These intermediate stations are employed with non-regenerative relays with fixed gain G_i given by

$$G_i^2 = \frac{1}{C_i W_{0,i}} \quad (29)$$

with $G_0 = 1$ and $C_0 = 1$ for the source S . In (29), $W_{0,i}$ is the variance of the Additive White Gaussian Noise (AWGN) at the output of the i -th relay, and C_i is a constant for the fixed gain G_i .

Assume that terminal S is transmitting a signal $s(t)$ with an average power normalized to unity. Then, the received signal at the first relay, T_1 , at moment t , can be written as

$$r_1(t) = \alpha_1(t)s(t) + w_1(t), \quad (30)$$

where $\alpha_1(t)$ is the fading amplitude between S and T_1 , and $w_1(t)$ is the AWGN at the input of T_1 with variance $W_{0,1}$. The signal r_1 is then multiplied by the gain G_1 of the relay T_1 and re-transmitted to relay T_2 . Generally, the received signal at the k -th relay T_k ($k = 1, 2, \dots, N - 1$) is given by

$$r_k(t) = G_{k-1}\alpha_k(t)r_{k-1}(t) + w_k(t), \quad (31)$$

resulting in a total fading amplitude at the destination node D , given by

$$\alpha(t) = \prod_{i=1}^N \alpha_i(t)G_{i-1}. \quad (32)$$

B. LCR and AFD of Multihop Transmissions

If the fading amplitude received at node T_i , $\alpha_i(t)$, is a time-correlated (due to mobility of T_{i-1} and/or T_i) Rayleigh random process, distributed according to (1) with mean power $\hat{\Omega}_i = E\{\alpha_i^2(t)\}$, then the i -th element of the product in (32), $X_i(t) = \alpha_i(t)G_{i-1}$, is again a time-correlated Rayleigh random process, distributed according to (1) with mean power $\Omega_i = \hat{\Omega}_i G_{i-1}^2$.

Comparing (5) and (32), we realize that the total fading amplitude at the destination station D (i.e., the received desired signal without the AWGN) is described as the N *Rayleigh random process $Y(t) = \alpha(t)$, whose average LCR and AFD are determined in the previous Section.

If all stations are assumed mobile with maximum Doppler frequency shifts f_{mS}, f_{mD}, f_{mi} ($1 \leq i \leq N - 1$) for the source S , destination D and relays, respectively, then for the i -th hop $f_i^2 = f_{m(i-1)}^2 + f_{mi}^2$ with $f_{m0} = f_{mS}$ and $f_{mN} = f_{mD}$, and

$$\sum_{i=1}^N f_i^2 = f_{mS}^2 + 2 \sum_{i=1}^{N-1} f_{mi}^2 + f_{mD}^2. \quad (33)$$

Combining (27) and (33), we obtain approximate solution for the average LCR of the total fading amplitude α at the output of a multihop non-regenerative relay transmission system,

$$N_\alpha(\alpha) \approx \left[\frac{1}{N} \left(f_{mS}^2 + 2 \sum_{i=1}^{N-1} f_{mi}^2 + f_{mD}^2 \right) \right]^{1/2} \times \frac{(2\pi)^{N/2} \alpha}{\Phi^{1/2}} \exp \left(-N \frac{\alpha^{2/N}}{\Phi^{1/N}} \right), \quad (34)$$

where Φ is given by (15). We see that (34) approximates the average LCR of the total fading amplitude for arbitrary mean power of the fading amplitudes $\hat{\Omega}_i$, arbitrary relay gains G_i and arbitrary maximal Doppler shifts f_{mi} .

Note that, for $N = 2$, (34) is an efficient closed-form alternative to the corresponding one [9, Eq. (17)] for the dual-hop case, which is shown in next section to be highly accurate.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we provide some illustrative examples for the average LCR and AFD of the fading gain process of the received desired signal at the destination of the multihop non-regenerative relay transmission system model from Section III. The numeric examples obtained from the derived approximate solutions are validated by extensive Monte-Carlo simulations.

We considered a multihop system consisted of a source terminal S , 4 relays, and a destination terminal D . The fixed-gain relays are assumed semi-blind with gains in Rayleigh fading channel calculated as [2, Eq. (15)] and [6, Eq. (19)]

$$G_{i, sb}^2 = \frac{1}{\hat{\Omega}_i} \exp \left(\frac{1}{\bar{\gamma}_i} \right) \Gamma \left(0, \frac{1}{\bar{\gamma}_i} \right), \quad (35)$$

where $\bar{\gamma}_i = \hat{\Omega}_i / W_{0,j}$ is the mean SNR on the i -th hop, and $\Gamma(\cdot, \cdot)$ is the incomplete Gamma function. Relay gain calculated according to (35) assures mean power consumption equal to that of a CSI-assisted relay, whose gain inverts the fading effect of the previous hop while limiting the output power at moments with deep fading.

Depending on the stations' mobility, we used two different 2D isotropic scattering models for the Rayleigh radio channel on each hop of the multihop transmission system. For the fixed-to-mobile channel (hop), we used the classic Jakes channel model [10]-[11]. For the mobile-to-mobile channel (hop), we used the Akki and Habber's channel model [12]-[13]. The Monte-Carlo simulations of the latter were realized by using the sum-of-sinusoids method proposed in [19]-[20].

More precisely, all mobile stations are assumed to induce same maximal Doppler shifts f_m , while the destination D is fixed. For all hops, $\hat{\Omega}_i = \hat{\Omega}$ and $W_{0,i} = W_0$. Thus, $\bar{\gamma}_i = \bar{\gamma}$, $G_{i, sb} = G_{sb}$, and the mean of Rayleigh random process $X_i(t) = \alpha_i(t)G_{i-1, sb}$ is calculated as

$$\Omega_i = \exp \left(\frac{1}{\bar{\gamma}} \right) \Gamma \left(0, \frac{1}{\bar{\gamma}} \right) = \Omega, \quad 2 \leq i \leq N \quad (36)$$

whereas $\Omega_1 = \hat{\Omega}$ is selected independently from the AWGN, since $G_0 = 1$. In this case,

$$\Phi = \hat{\Omega} \exp \left(\frac{N-1}{\bar{\gamma}} \right) \left[\Gamma \left(0, \frac{1}{\bar{\gamma}} \right) \right]^{N-1} \quad (37)$$

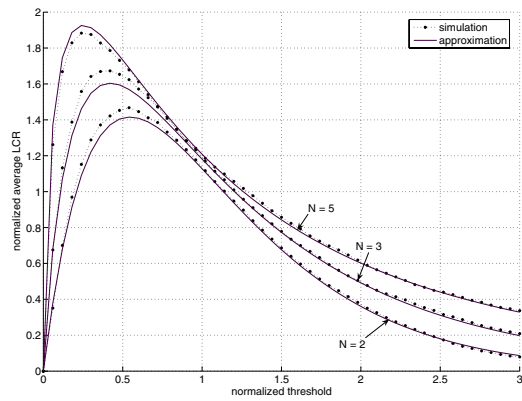


Fig. 1. Average LCR, $\hat{\gamma}_i = \hat{\gamma} = 5$ dB

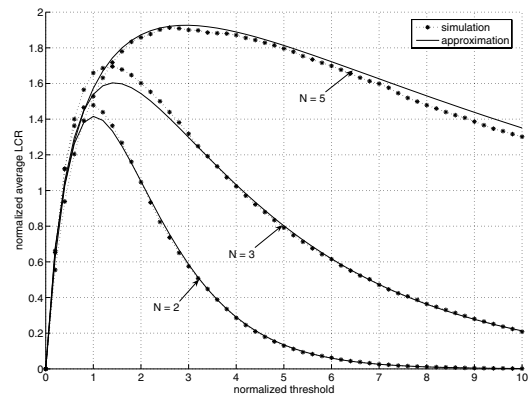


Fig. 3. Average LCR, $\hat{\gamma}_i = \hat{\gamma} = 20$ dB

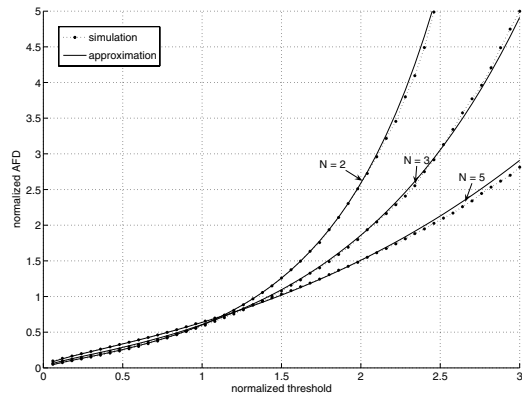


Fig. 2. AFD, $\hat{\gamma}_i = \hat{\gamma} = 5$ dB

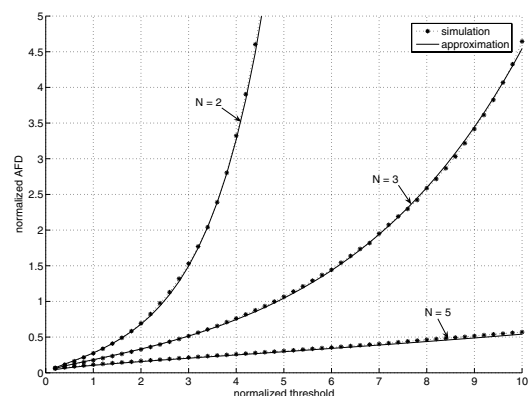


Fig. 4. AFD, $\hat{\gamma}_i = \hat{\gamma} = 20$ dB

Note that, when introducing above scenario into (34), α and $\hat{\Omega}$ appear together as $\alpha/\sqrt{\hat{\Omega}}$.

Figs. 1-4 depict the received signal's normalized LCR (N_α/f_m) or normalized AFD ($T_\alpha f_m$) versus the normalized threshold ($\alpha/\sqrt{\hat{\Omega}}$) at 3 different stations along the multihop transmission system: at relay T_2 (curve denoted by $N = 2$), at relay T_3 (curve denoted by $N = 3$) and at the destination D (curve denoted by $N = 5$). All comparative curves show an excellent match between the approximate solution and the Monte-Carlo simulations.

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