This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the ICC 2008 proceedings.

FSO Links with Spatial Diversity Over Strong Atmospheric Turbulence Channels

Theodoros A. Tsiftsis* , Harilaos G. Sandalidis†, George K. Karagiannidis*, and Murat Uysal‡

*Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece, Email: {thtsif, geokarag}@auth.gr
†Department of Informatics with Biomedical Applications, University of Central Greece, 35100 Lamia, Greece, Email: sandalidis@hotmail.com
‡Department of Electrical and Computer Engineering of University of Waterloo, Waterloo, Ontario, Canada, N2L 3G1, Email: muysal@ece.waterloo.ca

***Abstract*—Free-space optical (FSO) communication has received much attention in recent years as a cost-effective, license-free and wide-bandwidth access technique for high data rates applications. The performance of FSO communication, however, severely suffers from turbulence-induced fading caused by atmospheric conditions. Multiple laser transmitters and/or receivers can be placed at both ends to mitigate the turbulence fading and exploit the advantages of spatial diversity. Spatial diversity is particularly crucial for strong turbulence channels in which single-input single-output (SISO) link performs extremely poor. Atmospheric-induced strong turbulence fading in outdoor FSO systems can be modeled as a multiplicative random process which follows the K distribution. In this paper, we investigate the error rate performance of FSO systems for K-distributed atmospheric turbulence channels and potential advantages of spatial diversity deployments at the transmitter and/or receiver. Our results demonstrate significant diversity gains of multiple transmitter/receivers deployment in FSO channels. We further present efficient approximated closed-form expressions for the average bit-error rate (BER) of multiple-input single-output (MISO) and single-input multiple-output (SIMO) FSO systems. These analytical tools are reliable alternatives to time-consuming Monte Carlo simulation of FSO systems where BER targets as low as 10^{-9} are typically aimed to achieve.**

I. Introduction

Free-Space Optical (FSO) communication is a license-free and cost-effective access technique, which has attracted significant attention recently for a variety of applications [1], [2]. Channels in FSO systems have wider bandwidth and therefore are able to support more users compared to radio frequency (RF) counterparts. Through relaying techniques, outdoor FSO optical transceivers can also cover large distances [3]. With its high-data-rate capacity and wide bandwidth on unregulated spectrum, FSO communication is a promising solution for the "last mile" problem, however its performance is highly vulnerable to adverse atmospheric conditions. Atmospheric turbulence occurs as a result of the variations in the refractive index due to inhomogeneities in temperature and pressure changes. This results in rapid fluctuations at the received signal, i.e. known as fading or scintillation, impairing the system performance particulary for link ranges for 1 km and above.

Over the years, a number of statistical channel models have been proposed to describe weak or strong atmospheric-induced turbulence fading [1]. For strong turbulence conditions, the K distribution has been found to be a suitable model since it provides an excellent agreement between theoretical and experimental data [4]. In [5], Uysal and Li have used this channel model to evaluate the performance of coded FSO systems in terms of the pairwise error probability and bit-error rate (BER). In [6], they have extended their results for a correlated K turbulence model where an exponential correlation profile is adopted. In [7], Kiasaleh has studied the BER performance of a FSO heterodyne system over the K channel. The results in these papers demonstrate that the performance of single-input single-output (SISO) FSO links severely suffers from strong turbulence and is far away from satisfying the typical BER targets for FSO applications within the practical ranges of signal-to-noise ratio (SNR). This necessitates the deployment of powerful fading-mitigation techniques. In the existing literature on FSO communication, two techniques have been proposed to mitigate the degrading effects of atmospheric turbulence: Error control coding in conjunction with interleaving [6], [8] and maximum likelihood sequence detection (MLSD) [9]. However, both approaches come with some practical limitations. The first one requires large-size interleavers whereas the later suffers from high computational complexity.

Another promising solution is the use of spatial diversity, a well known diversity technique in RF systems. By using multiple apertures at the transmitter and/or the receiver, the inherent redundancy of spatial diversity has the potential to significantly enhance the performance. The possibility for temporal blockage of the laser beams by obstructions is further reduced and longer distances can be covered through heavier weather conditions. The use of space diversity in FSO systems has been first proposed in [10]. In [11], [12], Shin and Chan have investigated the outage probability of multiple-input multiple-output (MIMO) FSO systems over log-normal turbulence channels. In [13], [14] Wilson *et. al* have studied MIMO FSO transmissions assuming pulse-position-modulation (PPM) [13] and Q-ary PPM [14] both in log-normal and Rayleigh fading regimes. In [15], Navidpour *et al.* have studied the BER performance of MIMO FSO links for both independent and correlated log-normal atmospheric turbulence channels.

In this paper, we investigate the performance of MIMO FSO links over K turbulence channels. We assume intensity-modulation/direct-detection (IM/DD) with on-off keying

978-1-4244-2075-9/08/$25.00 ©2008 IEEE

(OOK). First, as a benchmark, we derive a closed-form expression for the BER of SISO case. Then, we present highly accurate approximated closed-form BER expressions for FSO links with multiple apertures at the receive and/or transmitter. All the derived expressions are given in terms of the well-known Meijer's G-functions which are available as built-in functions of many commercial mathematical software packages. These expressions are highly efficient analytical tools and stand out as reliable alternatives to time-consuming Monte Carlo simulation of FSO systems where very low BER targets (from 10^{-6} to 10^{-9}) are aimed to achieve.

II. System and Channel Model

A. System Model

A FSO system is considered where the information signal is transmitted via M apertures and received by N apertures over a discrete-time ergodic channel with additive white Gaussian noise (AWGN). We assume binary-input and continuous output and IM/DD with OOK. The received signal at the nth receive aperture is given by

$$r_n = x\eta \sum_{m=1}^{M} I_{mn} + v_n,\ n = 1, ..., N \tag{1}$$

where $x \in \{0, 1\}$ represents the information bits, η is the optical-to-electrical conversion coefficient, I_{mn} denotes the irradiance from the mth transmitter to the nth receiver, and v_n is the AWGN with zero mean and variance $\sigma_v = N_0/2$. Under the Gaussian noise approximation, it has been implicitly assumed that the presence of ambient light in photodetectors can be ignored. Although it is a major source of interference particularly during daylight, it can be significantly reduced using infrared filters over the photodiodes in practical FSO implementations. Considering that the coherence length of the optical beams is of the order of centimeters, this can be easily justified if the transmitters and/or receivers are placed a few centimeters apart.

B. Channel Statistics

Strong atmospheric turbulence is modeled using a widely accepted distribution, the K distribution [4]. K turbulence model can be considered as a product of two independent models [6], (i.e., exponential distribution $*$ gamma distribution) and its probability density function (pdf) of the normalized irradiance is given by

$$f_{I_{mn}}(I_{mn}) = \frac{2\alpha^{(\alpha+1)/2}}{\Gamma(\alpha)} I_{mn}^{(\alpha-1)/2} K_{\alpha-1}\left(2\sqrt{\alpha I_{mn}}\right),\ I_{mn} > 0 \tag{2}$$

where α is a channel parameter related to the effective number of discrete scatterers, $\Gamma(\cdot)$ is the well-known Gamma function [16, eq. (8.310.1)], and $K_\nu(\cdot)$ is the νth-order modified Bessel function of the second kind [16, eq. (8.432.2)]. When $\alpha \to \infty$, (2) approaches the negative exponential (NE) distribution.

The n-th order moment represented by $\mu_{I_{mn}}(n) = \int_0^\infty I_{mn}^n f_{I_{mn}}(I_{mn}) dI_{mn}$ is given in a closed form using [17, eq. (24)] as

$$\mu_{I_{mn}}(n) = \frac{\Gamma(n+1)\Gamma(n+\alpha)}{\alpha^n \Gamma(\alpha)}. \tag{3}$$

From the above equation we can define the *scintillation index* (SI) as

$$SI \triangleq \frac{E\left[I_{mn}^2\right] - E^2\left[I_{mn}\right]}{E^2\left[I_{mn}\right]} = \frac{\alpha+2}{\alpha} \tag{4}$$

where $E[\cdot]$ denotes the expected value of the enclosed. Since SI depends only on the parameter α, one can see that the turbulence is stronger (SI is high) for lower values of α and gets weaker as α increases.

C. Electrical SNR Statistics

The instantaneous electrical SNR can be defined as $\gamma_{mn} = (\eta I_{mn})^2 / N_0$. The average electrical SNR is defined as, $\mu_{mn} = (\eta E\left[I_{mn}\right])^2 / N_0$[1] [18]. After a simple power transformation of the random variable (rv) I_{mn}, the pdf of the electrical SNR, γ_{mn}, can be derived as

$$f_{\gamma_{mn}}(\gamma_{mn}) = \frac{\alpha^{\frac{\alpha+1}{2}} \gamma_{mn}^{\frac{\alpha-3}{4}}}{\Gamma(\alpha) \mu_{mn}^{\frac{\alpha+1}{4}}} K_{\alpha-1}\left(2\sqrt{\alpha\sqrt{\frac{\gamma_{mn}}{\mu_{mn}}}}\right), \gamma_{mn} > 0. \tag{5}$$

III. SISO FSO Links

The BER of IM/DD with OOK in the presence of AWGN and perfect CSI at the receiver side is given by $P_b(e)$=$P(1)P(e|1)$+$P(0)P(e|0)$ where $P(1)$ and $P(0)$ are the probabilities of sending 1 and 0 bits, respectively, and $P(e|1)$ and $P(e|0)$ denote the conditional bit-error probabilities when the transmitted bit is 1 and 0. We consider that $P(1) = P(0)$ =0.5 and $P(e|1) = P(e|0)$. It is easy to show that conditioned on I (the indexes m, n are omitted for brevity) [15]

$$P_b(e|I) = P(e|1, I) = P(e|0, I) = Q\left(\frac{\eta I}{\sqrt{2N_0}}\right) \tag{6}$$

where $Q(\cdot)$ is the Gaussian Q-function defined as $Q(x) = \left(1/\sqrt{2\pi}\right) \int_x^\infty exp^{-t^2/2} dt$ and also related to the complementary error function erfc$(\cdot)$ by erfc$(x) = 2Q(\sqrt{2}x)$.

The average BER, $P_b(e)$, over the K channel can be obtained by averaging (6) over the normalized irradiance I, i.e.,

$$P_b(e) = \int_0^\infty f_I(I) \left[\frac{1}{2}\text{erfc}\left(\frac{\eta I}{2\sqrt{N_0}}\right)\right] dI. \tag{7}$$

The above integral can be evaluated by expressing the $K_\nu(\cdot)$ and the erfc$(\cdot)$ integrands as Meijer's G-functions $(K_\nu(x) = \frac{1}{2} G_{0,2}^{2,0}\left[x^2/4\middle|_{\frac{\nu}{2}, -\frac{\nu}{2}}^{-}\right]$ [17, eq. (14)], erfc$\left(\sqrt{x}\right) = \frac{1}{\sqrt{\pi}} G_{1,2}^{2,0}\left[x\middle|_{0,1/2}^{1}\right]$ [19, eq. (06.27.26.0006.01)]) and using [17, eq. (21)]. Therefore, a closed-form solution yields as

$$P_{SISO}(e) = \frac{2^{\alpha-2}}{\sqrt{\pi^3}\Gamma(\alpha)} G_{5,2}^{2,4}\left[\frac{4\eta^2}{N_0\alpha^2} \middle| \begin{array}{c} \frac{1-\alpha}{2}, \frac{2-\alpha}{2}, 0, \frac{1}{2}, 1 \\ 0, \frac{1}{2} \end{array}\right]. \tag{8}$$

[1]Note that $E[I] = 1$ since I_{mn} is normalized. Also μ is different than $\overline{\gamma} = E[\gamma]$ since the latter quantity is defined as $\overline{\gamma} = \eta^2 E\left[I^2\right]/N_0$.

Alternatively, if we express (6) in terms of γ, i.e., $Q\left(\frac{nI}{\sqrt{2No}}\right)=Q\left(\sqrt{\frac{\gamma}{2}}\right)=\frac{1}{2}\text{erfc}\left(\frac{\sqrt{\gamma}}{2}\right)$, and average over the pdf of γ, the above average BER can be expressed as

$$P_{SISO}(e)=\frac{2^{\alpha-2}}{\sqrt{\pi^3}\Gamma(\alpha)}G_{5,2}^{2,4}\left[\frac{4\mu}{\alpha^2}\middle| \begin{matrix}\frac{1-\alpha}{2},\frac{2-\alpha}{2},0,\frac{1}{2},1\\ 0,\frac{1}{2}\end{matrix}\right]. \quad (9)$$

Note that using [16, eq. (9.303], the Meijers G-function can be written in terms of the more familiar generalized hypergeometric functions [16, eq. (9.14.1)].

IV. MIMO FSO Links

Since the BER performance of SISO FSO link is quite poor (i.e., higher than 10^{-3} in the SNR range of 30-50 dB) as expected over strong turbulence, the use of diversity techniques is absolutely necessary. The use of spatial diversity can be implemented either at the transmitter (MISO) or at the receiver (SIMO) or at both of them (MIMO). The optimum decision metric for OOK is given by [15, eq. (16)]

$$P\left(\mathbf{r}|\text{on},I_{mn}\right)\underset{\text{off}}{\overset{\text{on}}{\gtrless}}P\left(\mathbf{r}|\text{off},I_{mn}\right) \quad (10)$$

where $\mathbf{r}=(r_1,r_2,...r_n)$ is the received signal vector. By following the same analysis as the one presented in [15] for the conditional probabilities of the received vector being in On or in Off state, the average error rate can be calculated from the integral

$$P_{MIMO}(e)=\int_{\mathbf{I}}f_{\mathbf{I}}\left(\mathbf{I}\right)\times Q\left(\frac{\eta}{MN\sqrt{2N_0}}\sqrt{\sum_{n=1}^{N}\left(\sum_{m=1}^{M}I_{mn}\right)^2}\right)d\mathbf{I} \quad (11)$$

where $f_{\mathbf{I}}(\mathbf{I})$ is the joint pdf vector $\mathbf{I}=(I_{11},I_{12},...I_{MN})$ of length MN. The average BER in (11) can be calculated through multi-dimensional numerical integration and with the help of mathematical software packages. In order to fairly compare MIMO links with SISO one, the factor M is used in (11) to ensure that the total transmit power of the MISO FSO system is the same as the power of the SISO link. Moreover, the factor N ensures that the area of the receive aperture in SISO links has the same size with the sum of N receive aperture areas of SIMO links [11]. To have further insight into the performance analysis of FSO with spatial diversity, we investigate the transmit and receive diversity as special cases.

V. MISO FSO Links

A. Independent and Not Necessarily Identically Distributed K Channels

When transmit diversity is used, i.e., $N=1$, (11) is written as

$$P_{MISO}(e)=\int_{\mathbf{I}}f_{\mathbf{I}}\left(\mathbf{I}\right)Q\left(\frac{\eta}{M\sqrt{2N_0}}\sum_{m=1}^{M}I_m\right)d\mathbf{I}. \quad (12)$$

For independent and not necessarily identically distributed (i.n.i.d.) K atmospheric turbulence channels, the average BER can be expressed by

$$P_{MISO}(e)=\prod_{m=1}^{M}\int_0^\infty f_{I_m}\left(I_m\right)\times\left[Q\left(\frac{\eta}{M\sqrt{2N_0}}\sum_{m=1}^{M}I_m\right)\right]^{\frac{1}{M}}dI_m. \quad (13)$$

The derived formulae in (13) can not be solved directly and requires multidimensional numerical integration.

B. Independent and Identically Distributed K Channels

For independent and identically distributed (i.i.d.) turbulence channels in transmit diversity scenario, (12) can be written as

$$P_{MISO}(e)=\int_{\mathbf{I}}f_{\mathbf{I}}\left(\mathbf{I}\right)Q\left(\frac{\eta}{\sqrt{2N_0}}I\right)d\mathbf{I}. \quad (14)$$

The integral presented in (14) is very difficult, if not impossible, to be evaluated in closed-form. For that reason we use the approximation for the Q-function presented in [20] (i.e., $Q(x)\approx\frac{1}{12}\,\text{e}^{-\frac{x^2}{2}}+\frac{1}{4}\,\text{e}^{-\frac{2x^2}{3}}$) and thus the average BER for i.i.d. MISO FSO links can be calculated as

$$P_{MISO}(e)\approx\frac{1}{12}\left(\int_0^\infty f_I(I)e^{-\frac{\eta^2}{4MN_0}I^2}dI\right)^M+\frac{1}{4}\left(\int_0^\infty f_I(I)e^{-\frac{\eta^2}{3MN_0}I^2}dI\right)^M. \quad (15)$$

By applying [19, eq. (07.34.21.0093.01)] in (15), a closed-form for the approximated average BER is derived as

$$P_{MISO}(e)\approx\frac{1}{4}\left(\frac{2^{\alpha-1}}{\pi\Gamma(\alpha)}\right)^M\times\left[\frac{1}{3}\left(G_{4,1}^{1,4}\left[\frac{4\eta^2}{\alpha^2MN_0}\middle| \begin{matrix}\frac{1-\alpha}{2},\frac{2-\alpha}{2},0,\frac{1}{2}\\ 0\end{matrix}\right]\right)^M+\left(G_{4,1}^{1,4}\left[\frac{16\eta^2}{3\alpha^2MN_0}\middle| \begin{matrix}\frac{1-\alpha}{2},\frac{2-\alpha}{2},0,\frac{1}{2}\\ 0\end{matrix}\right]\right)^M\right] \quad (16)$$

Equation (16) can be rewritten also in terms of average electrical SNR as

$$P_{MISO}(e)\approx\frac{1}{4}\left(\frac{2^{\alpha-1}}{\pi\Gamma(\alpha)}\right)^M\times\left[\frac{1}{3}\left(G_{4,1}^{1,4}\left[\frac{4\mu}{\alpha^2M}\middle| \begin{matrix}\frac{1-\alpha}{2},\frac{2-\alpha}{2},0,\frac{1}{2}\\ 0\end{matrix}\right]\right)^M+\left(G_{4,1}^{1,4}\left[\frac{16\mu}{3\alpha^2M}\middle| \begin{matrix}\frac{1-\alpha}{2},\frac{2-\alpha}{2},0,\frac{1}{2}\\ 0\end{matrix}\right]\right)^M\right] \quad (17)$$

VI. SIMO FSO Links

A. Optimal Combining (OC)

1) Independent and Not Necessarily Identically Distributed K Channels: When receive diversity is applied, the variance of the noise in each aperture is N times smaller since the variance of the noise in each receiver is $\sigma_v^2 = \frac{N_0}{2N}$. Therefore, for $M = 1$ and OC implementation at the receiver with perfect CSI, (11) is written as

$$P_{SIMO}(e) = \int_{\mathbf{I}} f_{\mathbf{I}}(\mathbf{I})\, Q\left(\frac{\eta}{\sqrt{2N\,N_0}}\sqrt{\sum_{n=1}^{N} I_n^2}\right) d\mathbf{I}. \quad (18)$$

By applying the approximation for the Q-function to (18), the average BER can be evaluated as

$$P_{SIMO}(e) \approx \frac{1}{12}\prod_{n=1}^{N}\int_0^\infty f_{I_n}(I_n) e^{-\frac{\eta^2}{4NN_0}\sum_{n=1}^{N} I_n^2} dI_n + \frac{1}{4}\prod_{n=1}^{N}\int_0^\infty f_{I_n}(I_n) e^{-\frac{\eta^2}{3NN_0}\sum_{n=1}^{N} I_n^2} dI_n \quad (19)$$

and finally a closed-form expression is derived given by

$$P_{SIMO}(e) \approx \frac{1}{12}\prod_{n=1}^{N}\frac{2^{\alpha_n-1}}{\pi\Gamma(\alpha_n)} G_{4,1}^{1,4}\left[\frac{4\mu_n}{\alpha_n^2 N}\left|\begin{matrix}\frac{1-\alpha_n}{2},\frac{2-\alpha_n}{2},0,\frac{1}{2}\\0\end{matrix}\right.\right] + \frac{1}{4}\prod_{n=1}^{N}\frac{2^{\alpha_n-1}}{\pi\Gamma(\alpha_n)} G_{4,1}^{1,4}\left[\frac{16\mu_n}{3\alpha_n^2 N}\left|\begin{matrix}\frac{1-\alpha_n}{2},\frac{2-\alpha_n}{2},0,\frac{1}{2}\\0\end{matrix}\right.\right] \quad (20)$$

2) Independent and Identically Distributed K Channels: For i.i.d. I_n rvs, we have

$$P_{SIMO}(e) = \int_{\mathbf{I}} f_{\mathbf{I}}(\mathbf{I})\, Q\left(\frac{\eta\, I}{\sqrt{2N_0}}\right) d\mathbf{I} \quad (21)$$

where it is observed that is exactly the same as for the MISO deployments. Hence, the average BER for SIMO FSO links can be approximated by

$$P_{SIMO}(e) \approx \frac{1}{4}\left(\frac{2^{\alpha-1}}{\pi\Gamma(\alpha)}\right)^N \times \left[\frac{1}{3}\left(G_{4,1}^{1,4}\left[\frac{4\mu}{\alpha^2 N}\left|\begin{matrix}\frac{1-\alpha}{2},\frac{2-\alpha}{2},0,\frac{1}{2}\\0\end{matrix}\right.\right]\right)^N + \left(G_{4,1}^{1,4}\left[\frac{16\mu}{3\alpha^2 N}\left|\begin{matrix}\frac{1-\alpha}{2},\frac{2-\alpha}{2},0,\frac{1}{2}\\0\end{matrix}\right.\right]\right)^N\right] \quad (22)$$

B. Equal gain Combining (EGC)

For the case where EGC is implemented at the receiver side (i.e., the receiver adds the receiver branches) the average error rate can be expressed as

$$P_{SIMO}(e) = \int_{\mathbf{I}} f_{\mathbf{I}}(\mathbf{I})\, Q\left(\frac{\eta}{N\sqrt{2N_0}}\sum_{n=1}^{N} I_N\right) d\mathbf{I} \quad (23)$$

It should be emphasized here, that the resulting expression is equivalent to the one obtained for the MISO FSO links (12) assuming EGC at the receiver side. Therefore, the derived expressions are similar to MISO ones for both i.i.d. and i.n.i.d. fading channels. Also, it is interesting to note that although EGC is used at the receiver, the knowledge for CSI is still needed for threshold calculation on the decision rule as comprehensively described in [15, eqs. (31) & (32)]. Furthermore, it can be observed from (14), (18) and (23) that the BER performance of MISO and SIMO deployments is exactly the same both for OC and EGC diversity receivers for i.i.d. turbulence-induced fading channels.

VII. Asymptotic Expressions for High Average Electrical SNR

For high average electrical SNRs, the arguments of the Meijer's G-function in the average BER expressions tend to infinity. Hence, following an asymptotic expansion of the Meijer's G-function [19, eq. (07.34.06.0018.01)]

$$G_{p,q}^{m,n}\left[z\left|\begin{matrix}a_1,\ldots,a_n,a_{n+1},\ldots,a_p\\b_1,\ldots,b_m,b_{m+1},\ldots,b_q\end{matrix}\right.\right] = \sum_{k=1}^{n}\frac{\prod_{j=1,j\neq k}^{n}\Gamma(a_k-a_j)\prod_{j=1}^{m}\Gamma(1-a_k+b_j)}{\prod_{j=m+1}^{q}\Gamma(a_k-b_j)\prod_{j=n+1}^{p}\Gamma(a_j-a_k+1)} z^{a_k-1} \quad (24)$$

where a_i, b_i, and $z > 0$ are arbitrary real values and m, n, p, and q are arbitrary positive integers, any of the derived BER expression may be used in conjunction with (24) to derive corresponding simple closed-form expressions for both MISO and SIMO aperture deployments, operating in the high SNR region. Specifically, since all the presented closed-form expressions for BER incorporate $G_{4,1}^{1,4}\left[z\left|\begin{matrix}\frac{1-\alpha}{2},\frac{2-\alpha}{2},0,\frac{1}{2}\\0\end{matrix}\right.\right]$, using (24) it can be easily simplified as $\sum_{k=1}^{4}\prod_{j=1,j\neq k}^{4}\Gamma(a_k-a_j)\Gamma(1-a_k) z^{a_k-1}$. By applying the simplified sum of the Meijer's G-function into (17) or (22) for i.i.d. channels, it can be easily shown that the BER decay at a rate μ^{-M} or μ^{-N}, i.e., diversity gain M or N, respectively

VIII. Numerical Examples & Discussion

In this section, the error performance of MISO and SIMO deployment of apertures is investigated. In Fig. 1, the average BER in terms of μ for various parameters of the scintillation index, is depicted. We particularly examine the performance when SI takes values between 1 and 4. Note that the SI in (4) is invalid for $SI \leq 1$. We observe that as SI increases the turbulence effect is getting stronger and thus the BER increases. This is expected since α decreases as it is inversely proportional to SI. In the limiting case of $SI = 1$, $\alpha \to \infty$ and hence a low BER bound exists. It is obvious that even for high values of average electrical SNR (i.e, 30-50 dB) BER is not exceeding 10^{-3} dB which is not an acceptable BER for practical FSO systems. This fully justifies the use of spatial diversity.

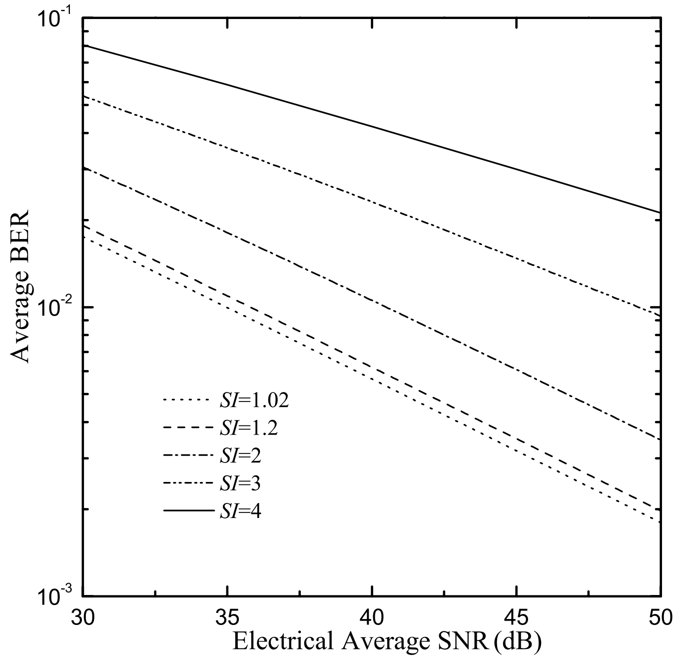

Fig. 1. Average BER of SISO FSO links as a function of SI.

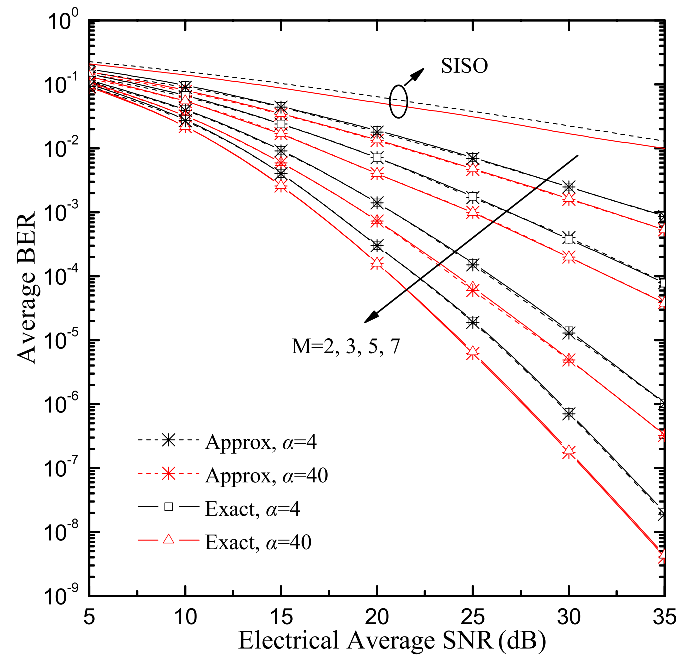

Fig. 2. Comparison of the exact and approximate average BER for MISO FSO links assuming perfect CSI.

In Fig. 2 the average BER performance of MISO FSO links with $M = 2,3,5,7$ transmit apertures over strong turbulence channels with α=4 or α=40, is depicted. Both the exact (i.e, (14)) and its approximation (i.e, (17)) are illustrated for i.i.d. K turbulence channels. It is observed that there is an excellent match between approximations (i.e., Meijer's G-function) and exact expression which requires numerical integration. It is also clearly depicted that the average BER is significantly improved as the number of transmit antennas increases compared to the SISO deployment which is also depicted. Indeed, it can be easily derived that with $M = 5$ transmit apertures it can be obtained an SNR improvement of about 65 dB with respect to SISO at a target BER=10^{-5}.

Finally, in Fig. 3 the error performance of SIMO FSO links with $N = 2,3$ receive apertures employing EGC and OC over i.n.i.d. atmospheric turbulence channels, is illustrated. It is shown that the performance of EGC receivers is close to OC receivers. Specifically, for $N = 2$ there is only a 1.2 dB difference at BER=10^{-5}. The difference in the performance between EGC and OC receivers is expected to be similar for more receive apertures, as also presented in [15] for weak turbulence. However, it is not plotted here since the results are difficult if not impossible to be extracted for EGC. This result (i.e., similar error performance of EGC and OC receive apertures) demonstrates the aperture averaging effect i.e., separate receive aperture provide a similar performance with the deployment of large receive aperture. Note that Fig. 3 has been plotted using the approximation for the Q-function both for OC and EGC.

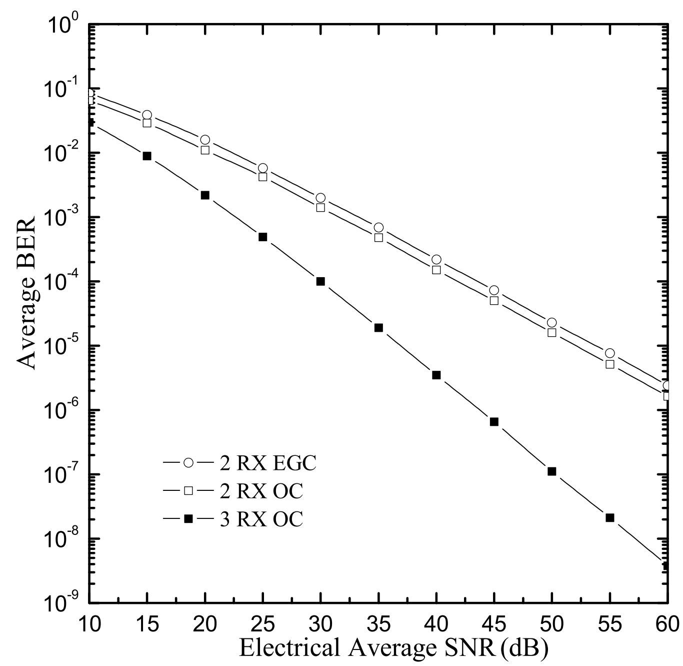

Fig. 3. Comparison of the OC and EGC receivers for SIMO FSO links for $M = 2$ ($\alpha_1 = 4$, $\alpha 2 = 40$, $\mu_1 = \mu$, $\mu_2 = 2\mu$) and $M = 3$ ($\alpha_1 = 4$, $\alpha 2 = 10$, $\alpha_3 = 50$, $\mu_1 = \mu$, $\mu_2 = 2\mu$, $\mu_3 = 4\mu$).

IX. Conclusions

In this paper, we have studied the error rate performance of FSO communication systems using spatial diversity over K distributed atmospheric turbulence channels. We have obtained highly accurate approximated closed-form expressions for the average BER of MISO/SIMO FSO systems in terms of

Meijer's G-function. Our results demonstrate that the use of multiple apertures at the transmitter and/or receiver enhance the quality of FSO systems similar to RF ones where the diversity order is equal to the number of transmit/receive apertures. In comparison to SISO case, a performance improvement of 65 dB is obtained at a target BER rate of 10^{-5} using 5 transmit apertures. Moreover, it is shown that the required number of apertures over i.i.d. strong turbulence channels for transmit/receive diversity FSO systems in order to have a meaningful performance at a practical SNR value is more than 5.

REFERENCES

[1] L. Andrews, R. L. Philips, and C. Y. Hopen, *Laser Beam Scintillation with Applications*. SPIE Press, 2001.

[2] D. Kedar and S. Arnon, "Urban optical wireless communications networks: The main challenges and possible solutions," *IEEE Communications Magazine*, vol. 42, no. 5, pp. 2–7, Feb. 2003.

[3] T. A. Tsiftsis, H. G. Sandalidis, G. K. Karagiannidis, and N. C. Sagias, "Multihop free-space optical communications over strong turbulence channels," in *Proc. IEEE Int. Conf. on Commun. (ICC'06)*, Istanbul, Turkey, June 2006, pp. 2755–2759.

[4] E. Jakeman and P. N. Pusey, "A model for non-Rayleigh sea echo," vol. 24, pp. 806–814, Nov. 1976.

[5] M. Uysal and J. T. Li, "BER performance of coded free-space optical links over strong turbulence channels," in *Proc. IEEE Veh. Tech. Conf. (VTC spring)*, Milan, Italy, May 2004, pp. 168–172.

[6] M. Uysal, S. M. Navidpour, and J. T. Li, "Error rate performance of coded free-space optical links over strong turbulence channels," *IEEE Commun. Lett.*, vol. 8, pp. 635–637, Oct. 2004.

[7] K. Kiasaleh, "Performance of coherent DPSK free-space optical communication systems in K-distributed turbulence," *IEEE Trans. Commun.*, vol. 54, no. 4, pp. 604–607, Apr. 2006.

[8] X. Zhu and J. M. Kahn, "Performance bounds for coded free-space optical communications through atmospheric turbulence channels," *IEEE Trans. Commun.*, vol. 51, pp. 1233–1239, Aug. 2003.

[9] ——, "Markov chain model in maximum-likelihood sequence detection-for free-space optical communication through atmospheric turbulence channels," *IEEE Trans. Commun.*, no. 3, 2003.

[10] M. M. Ibrahim and A. M. Ibrahim, "Performance analysis of optical receivers with space diversity reception," *Proc. IEE-Commun.*, vol. 143, no. 6, pp. 369–372, Dec. 1996.

[11] E. J. Shin and V. W. S. Chan, "Optical communication over the turbulent atmospheric channel using spatial diversity," in *IEEE GLOBECOM '02*, Nov. 2002.

[12] ——, "Part1: Optical communication over the clear turbulent atmospheric channel using diversity," vol. 22, no. 9, Nov. 2004, pp. 1896–1906.

[13] S. G. Wilson, M. Brandt-Pearce, Q. Cao, and M. Baedke, "Optical repetition MIMO transmission with multipulse PPM," *IEEE Trans. J. Sel. Areas Commun.*, vol. 9, no. 23, pp. 1901–1910, Sep. 2005.

[14] S. G. Wilson, M. Brandt-Pearce, Q. Cao, and J. H. Leveque-III, "Free-space optical MIMO transmission with Q-ary PPM," *IEEE Trans. Commun.*, vol. 53, no. 8, pp. 1402–1412, Aug. 2005.

[15] S. M. Navidpour, M. Uysal, and M. Kavehrad, "BER performance of free-space optical transmission with spatial diversity," *IEEE Trans.on Wireless Commun.*, vol. 6, no. 8, pp. 2813–2819, Aug. 2007.

[16] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. New York: Academic, 2000.

[17] V. S. Adamchik and O. I. Marichev, "The algorithm for calculating integrals of hypergeometric type functions and its realization in REDUCE system," in *Proc. International Conference on Symbolic and Algebraic Computation*, Tokyo, Japan, 1990, pp. 212–224.

[18] X. Zhu and J. M. Kahn, "Free-space optical communication through atmospheric turbulence channels," *IEEE Trans. Commun.*, vol. 50, pp. 1293–1300, Aug. 2002.

[19] Wolfram. (2004) The Wolfram functions site. Internet. [Online]. Available: http://functions.wolfram.com

[20] M. Chiani, D. Dardari, and M. K. Simon, "New exponential bounds and approximations for the computation of error probability in fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, pp. 840–845, Jul. 2003.