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Spectral Efficient Cooperative Communications via Spatial Signal Separation

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***Abstract*— We propose a communication system that utilizes two relays and multiple receiving antennas, in order to spatially separate the signals arriving concurrently at the destination and thus avoid the need for orthogonal transmissions in either the frequency or the time domain. The whole concept is based upon two key elements: a) To combat the half-duplex constraint by having two relays transmitting alternatively, i.e., the one receiving while the other transmitting and vice versa, and b) to separate the signals arriving concurrently at the destination using the well-known space division multiple access (SDMA) technique. Numerical results manifest that the proposed model outperforms conventional relaying in terms of outage probability, due to the former's advantage of higher spectral efficiency.**

I. Introduction

Cooperative diversity [1] has been recently proposed as a promising method that achieves the beneficial effects of diversity by employing a number of relaying terminals that are willing to forward the information received by the source terminal to a specified destination. (see e.g., [2]- [5]). Nonetheless, the advantages of cooperative diversity come at the expense of a reduction in spectral efficiency due to a) the half-duplex constraint [1] and b) the orthogonal relaying transmissions at either the frequency or the time domain, since the relays must transmit on orthogonal channels in order to take full advantage of the multiple available paths. This drawback of spectral inefficiency becomes more evident in cases where the destination terminal is capable of employing multiple receiving antennas (for example, in the uplink of cellular communications or wireless LANs), since for those cases one may argue that the extra degrees of freedom induced by this multiple-antenna usage may be utilized for achieving higher multiplexing gains, instead of diversity ones. To the best of the authors' knowledge, although there exist some remarkable works in the literature where protocols that mitigate the spectral efficiency loss are proposed [6]- [5], none of them takes advantage of the extra resource that multiple-antenna reception entails, leaving thus the potential for spatial signal separation unexploited.

To this end, in this paper we propose a novel relaying protocol that employs multiple antennas at the destination so as to spatially separate the received signals, thus letting the source and the relays transmit in the same band at the same time, while still being able to attain diversity. The whole concept is based on two key ideas: a) To combat the half-duplex constraint by having two relays transmitting alternatively, i.e., as long as the one operates at the transmitting mode the other remains at the receiving mode and vice versa, and b) to separate the signals arriving concurrently at the destination (i.e., one directly from the source and one from one of the relays) using the well-known space division multiple access (SDMA) technique. As a result, this enables cooperative communications to provide diversity and/or robustness against large-scale fading without paying the price of spectral inefficiency.

II. System Model

We consider a single-antenna source node S, communicating with an L-antenna destination node, denoted by D. Two single antenna relays, namely R_1 and R_2, are willing to assist this communication, by decoding the message received from S and forward the re-encoded data to D, operating thus in the well-known decode and forward (DF) mode. In order to protect the system from error propagations the relays are assumed to decode only if the signal-to-noise-ratio (SNR) at their inputs is greater than a pre-specified threshold value, denoted by T, following thus the so-called threshold DF protocol (see e.g., [9]). Moreover, half-duplex relaying is assumed, implying that the relays cannot transmit and receive simultaneously, but on different timeslots, so as to satisfy causality. The whole system is operating in flat, Rayleigh fading environment, that is supposed to be slowly varying; that is, the channel coherence time is assumed to be much larger than the symbol duration.

A. Protocol Description

Contrary to the typical multi-relay cooperative diversity systems, the two relays in the proposed scheme are assumed not to decode at the same time, nor to transmit the same symbol to the destination in orthogonal channels. Instead, they are activated alternatively, i.e., the one receives while the other transmits and vice versa. In particular, the whole transmission period is divided into a series of frames, each one consisting of two subframes: During the first subframe, R_1 transmits to the destination what it previously received, while R_2 listens to the source; during the second subframe, the roles of R_1 and R_2 are inverted, i.e., R_2 transmits while R_1 listens.

Let us assume that the symbols x_1 and x_2 are transmitted by the source at the first and second subframe, respectively. The received signal at the destination at these subframes consists of the one received directly from the source and that received from one of the relays, that was actually received by the corresponding relay one subframe earlier. It is further

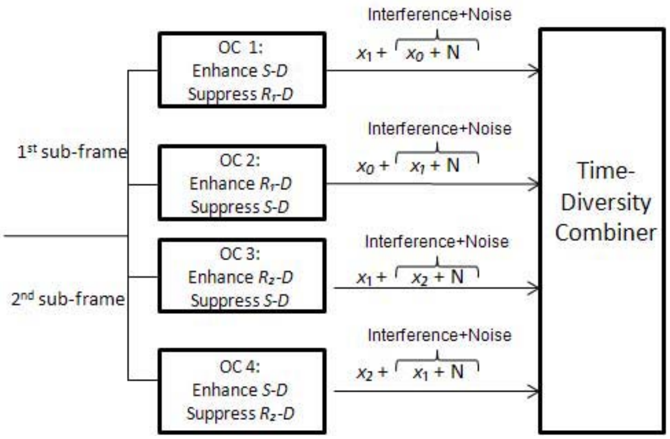

Fig. 1. First and second combining stage

assumed that the relaying transmissions do not interfere with one another, i.e., R_2 does not listen to R_1 at the first subframe, while R_1 does not listen to R_2 at the second subframe. This can be achieved by using, for example, infrastructure based relays with directive antennas towards the fixed destination terminal. Hence, denoting with $\mathbf{r}_i$ the L-dimensional vector received at the ith subframe, we have $\mathbf{r}_1 = x_1\mathbf{u}_{SD} + x_0\mathbf{u}_{R_1D} + \mathbf{n}$; $\mathbf{r}_2 = x_2\mathbf{u}_{SD} + x_1\mathbf{u}_{R_2D} + \mathbf{n}$, where the column vectors $\mathbf{n}$, $\mathbf{u}_{SD}$ and $\mathbf{u}_{R_iD}$ $(i = 1, 2)$ are mutually independent L-dimensional zero-mean complex Gaussian random vectors that represent the noise vector and the spatial signatures of the S-D and the R_i-D channel, respectively. The mean powers of the components of $\mathbf{u}_{SD}$ are identical with each other, denoted by Ω_{SD}; likewise, $\Omega_{R_1D} = \Omega_{R_2D} = \Omega_{RD}$ are the mean powers of the components of $\mathbf{u}_{R_1D}$ and $\mathbf{u}_{R_2D}$, implying independent and identically distributed (i.i.d.) fading between the relays and the destination. For the sake of simplicity of the analysis, unitary source and relay transmitting powers are assumed; the analysis however can be easily extended to the case of non-identical transmitting powers between the source and the relays, as well as non-identically distributed fading on the R_1-D and R_2-D channels. The AWGN noise powers in each receiving antenna are identical with one another, denoted by σ^2.

It is important to note that the transmissions of the source and the relays require the usage of a single channel (i.e., they occur concurrently using the same frequency). This implies that *the proposed scheme offers no loss in spectral efficiency compared to conventional point-to-point communications*, while still achieving diversity since two independent replicas of each symbol are received by D.

III. On the Overall Signal-to-Interference-plus-Noise Ratio (SINR)

A. First Combining Stage: Signal Separation through SDMA

According to the proposed scheme, in each subframe the destination receives two co-channel signals, one incident from the source an another from either R_1 or R_2. Taking advantage of the multiple antennas at the destination, these two signals are separated with one another by utilizing the well-known concept of SDMA. In particular, in each subframe the combined signal received by the multiple antennas passes through two space diversity combiners, that weight each of the L diversity branches by a complex weight. These weights are appropriately chosen so that the two combiners perform the so-called *optimum combining* [10], that leads to the maximum SINR in wireless systems in the presence of cochannel interference and thermal noise. For example, during the first subframe the first optimum combiner OC1 considers the S-D signal as the desired one, and the R_1-D as the interfering, whereas the second combiner OC2 considers these signals as the interference and the desired one, respectively. During the second subframe, the two combiners involved (namely OC3 and OC4) extract the S-D signal out of the "interfering" R_2-D one, and the R_2-D out of the "interfering" S-D signal, respectively. Fig. 1 summarizes the way these combiners are allocated in each subframe, as well as the resulting signals at their outputs.

Let us adopt the general notation of A representing the terminal transmitting the desired signal, and B representing the interfering one, with $A, B \in \{S, R_1, R_2\}$ (e.g., during the first subframe $A = S,\ B = R_1$). Define

$$\gamma_{AD} \triangleq \mathbf{u}_{AD}^H \mathbf{u}_{AD} / \sigma^2, \tag{1}$$

i.e., γ_{AD} represents the instantaneous SNR of the A-D link at the output of the combiner given that no interference takes place. This is the case when the relay supposed to transmit at a given subframe (e.g., R_1 referring to the first subframe) is inactive due to the previously mentioned thresholding mechanism, i.e., the source-relay SNR at the previous subframe was below the threshold T. Moreover, we denote by $\overline{\gamma}_{AD} = \Omega_{AD}/\sigma^2$ the average SNR per antenna associated with the A-D link. Due to the fact that $\mathbf{u}_{AD}$ is comprised of L complex Gaussian components, γ_{AD} is an L-order Gamma distributed random variable (RV); its probability density function (PDF) is thus given by

$$f_{\gamma_{AD}}(\gamma_{AD}) = \frac{\gamma_{AD}^{L-1}}{\overline{\gamma}_{AD}^{L}\Gamma(L)} \exp\left(-\frac{\gamma_{AD}}{\overline{\gamma}_{AD}}\right), \tag{2}$$

where $\Gamma(\cdot)$ is the gamma function defined in [11, eq. (8.310.1)].

Because of the slow fading assumption, constant spatial signatures in each subframe are considered. Therefore, the $L \times L$-dimensional interference-plus-noise covariance matrices are constructed as $\mathbf{\Phi}_B = \mathbf{u}_{BD}\mathbf{u}_{BD}^H + \sigma^2\mathbf{I}_L$, where $\mathbf{I}_L$ is the L-dimensional identity matrix and $(\cdot)^H$ represents the Hermitian operator. Let $\mathbf{w}_{A,B}$ represent the complex weight vectors associated with the combiner that filters the A-D and nulls the B-D channel. With optimum combining, $\mathbf{w}_{A,B}$ are given as $\mathbf{w}_{A,B} = \mathbf{\Phi}_B^{-1}\mathbf{u}_{AD}$ [10]. Consequently, denoting with P_A and P_{B+N} the signal power at the output of the combiner associated with the signal incident from A, and the signal-plus-noise power associated with terminal B, respectively, the resulting SINR is given by [10], [12]

$$\gamma_b^{A,B} = \frac{P_A}{P_{B+N}} = \frac{\left|\mathbf{w}_{A,B}^H \mathbf{u}_{AD}\right|^2}{\mathbf{w}_{A,B}^H \mathbf{\Phi}_B \mathbf{w}_{A,B}} = \frac{\left(\mathbf{u}_{AD}^H \mathbf{\Phi}_B^{-1} \mathbf{u}_{AD}\right)^2}{\mathbf{u}_{AD}^H \mathbf{\Phi}_B^{-1} \mathbf{u}_{AD}} = \mathbf{u}_{AD}^H \mathbf{\Phi}_B^{-1} \mathbf{u}_{AD}. \quad (3)$$

B. Second Combining Stage: Time Diversity

As it can be observed from Fig. 1, the destination collects two independent replicas of each of the transmitting symbols within a window of two consecutive subframes. Hence, the attained diversity order can be increased if the signals issuing from the first combining stage are combined again into a proper time-diversity combiner. Next, the operation of this second-stage combiner that maximizes the overall SINR is analyzed.

Without loss of generality, we focus henceforth on the reception of x_1. The resulting analysis thus refers to the performance associated with that symbol, and actually represents the overall performance due to the symmetry assumption between the S-R_1-D and S-R_2-D links. Symbol x_1 is extracted once at the first subframe from S with a suppressed version of that transmitted on R_1-D being the interference; it is also extracted at the second subframe from R_2, where the S-D is interfering. Hence, the instantaneous signal powers at output of the first-stage combiners associated with these two versions of x_1 are taken from (3) as

$$P_S^{(x_1)} = \left(\mathbf{u}_{SD}^H \mathbf{\Phi}_{R_1}^{-1} \mathbf{u}_{SD}\right)^2, \; P_{R_2}^{(x_1)} = \left(\mathbf{u}_{R_2D}^H \mathbf{\Phi}_S^{-1} \mathbf{u}_{R_2D}\right)^2,$$

whereas the corresponding interference-plus-noise powers as

$$P_{R_1+N}^{(x_0)} = \mathbf{u}_{SD}^H \mathbf{\Phi}_{R_1}^{-1} \mathbf{u}_{SD}, \; P_{S+N}^{(x_2)} = \mathbf{u}_{R_2D}^H \mathbf{\Phi}_S^{-1} \mathbf{u}_{R_2D}.$$

The two versions of x_1 together with the interference components are co-phased and added into the second-stage time diversity coherent equal gain combiner (EGC). The resulting overall SINR, namely γ, is the ratio of the total signal power over the total interference plus noise power, i.e.,

$$\gamma = \frac{\left(\sqrt{P_S^{(x_1)}} + \sqrt{P_{R_2}^{(x_1)}}\right)^2}{P_{R_1+N}^{(x_0)} + P_{S+N}^{(x_2)}} = \mathbf{u}_{SD}^H \mathbf{\Phi}_{R_1}^{-1} \mathbf{u}_{SD} + \mathbf{u}_{R_2D}^H \mathbf{\Phi}_S^{-1} \mathbf{u}_{R_2D} = \gamma_b^{S,R_1} + \gamma_b^{R_2,S}. \quad (4)$$

Notice that γ is the sum of SINRs in the first and second subframe; this means that the performance of the second-stage EGC is identical with that of a maximal ratio combiner (MRC). This can be also verified by noting that the weights of a MRC in this particular case would be identical, namely $\omega_1 = \sqrt{P_S^{(x_1)}}/P_{R_1+N}^{(x_0)} = \sqrt{P_{R_2}^{(x_1)}}/P_{S+N}^{(x_2)} = \omega_2 = 1$[1]. The reader should also note that the overall SINR expression shown in (4) is general enough so as to also hold for the case where one of the relays is inactive due the aforementioned thresholding mechanism. Should R_1 be inactive, for example, then the corresponding interference-plus-noise covariance matrix becomes $\mathbf{\Phi}_{R_1} = \sigma^2 \mathbf{I}_L$, yielding $\gamma_b^{S,R_1} = \mathbf{u}_{SD}^H \mathbf{u}_{SD}/\sigma^2 = \gamma_{SD}$; should R_2 be inactive, then the time-diversity EGC does not take the second subframe into account, hence $\gamma_b^{R_2,S} = 0$.

[1]We note that a second-stage MRC in our case would differ from the conventional one used in noise limited systems in the sense that the denominator in our case is the instantaneous interference plus noise power, whereas in conventional systems is the average noise power; the resulting SINRs, however, are the same.

IV. Performance Analysis

A. Exact Moment Generating Function (MGF) of the overall SINR

The MGF of γ is defined as

$$\mathcal{M}_\gamma(s) = E_\gamma\left\langle e^{-s\gamma}\right\rangle = \sum_{i=1}^{4} \rho_i E_\gamma \left\langle \exp\left(-s\gamma \left|\Theta_i\right.\right)\right\rangle, \quad (5)$$

where $E_X\langle\cdot\rangle$ denotes expectation over the random variable X; $\Theta_1, ..., \Theta_4$ are the four cases that may be considered, corresponding to each of the possible combinations of whether or not R_1 and R_2 are active; $\rho_1, ..., \rho_4$ are the corresponding occurrence probabilities. The above cases are further analyzed below.

1) Case Θ_1: Both R_1 and R_2 active: Should this be the case, the received signal at both subframes contains the symbol x_1 together with an interference component. Under the i.i.d Rayleigh fading assumption on S-R_1 and S-R_2, this case occurs with probability $\rho_1 = \Pr\{\gamma_{SR_1}, \gamma_{SR_2} > T\} = e^{-2T/\overline{\gamma}_{SR}}$. Let us keep the notation of A representing the desired and B the interfering terminal. The eigen decomposition of the covariance matrix $\mathbf{\Phi}_B$ yields

$$\mathbf{\Phi}_B = \mathbf{V}_B \mathbf{\Lambda}_B \mathbf{V}_B^H, \quad (6)$$

where $\mathbf{V}_B$ is the eigenvector matrix and $\mathbf{\Lambda}_B = \mathrm{diag}\left(\lambda_1^B, ..., \lambda_L^B\right)$, with λ_i^B representing the ith eigenvalue of $\mathbf{\Phi}_B$, which can be easily evaluated as

$$\lambda_i^B = \begin{cases} \mathbf{u}_{BD}^H \mathbf{u}_{BD} + \sigma^2, & i = 1 \\ \sigma^2, & i = 2, ..., L \end{cases}. \quad (7)$$

Therefore, combining (5), (3) and (6) we obtain the MGF of $\gamma_b^{A,B}$ as

$$\mathcal{M}_{\gamma_b^{A,B}}(s) = E_{v_{A_i} v_{A_i}^*} \left\langle \exp\left(-\sum_{i=1}^{L} \frac{s v_{A_i} v_{A_i}^*}{\lambda_i^B}\right)\right\rangle, \quad (8)$$

where $(\cdot)^*$ stands for the complex conjugate of its argument and v_{A_i} is the ith component of the vector $\mathbf{v}_{AD} = \mathbf{V}_B \mathbf{u}_{AD}$. Note that the Hermitian nature of $\mathbf{\Phi}_B$ implies that the latter transformation is unitary, hence $E\left\langle v_{A_i} v_{A_i}^*\right\rangle = \Omega_{AD}$. Consequently, averaging over the exponential distribution of $v_{A_i} v_{A_i}^*$ and then using (7), (1) and (8), we may obtain the conditional MGF of $\gamma_b^{A,B}$ given γ_{BD} as [13]

$$\mathcal{M}_{\gamma_b^{A,B}}(s\left|\gamma_{BD}\right.) = \prod_{i=1}^{L} \frac{\lambda_i^B}{\lambda_i^B + s\Omega_{AD}} = \frac{(\gamma_{BD}+1)/\overline{\gamma}_{AD}}{(\gamma_{BD}+1)/\overline{\gamma}_{AD} + s} \left(\frac{1/\overline{\gamma}_{AD}}{1/\overline{\gamma}_{AD} + s}\right)^{L-1}. \quad (9)$$

Then, $\mathcal{M}_{\gamma_b^{A,B}}(s)$ is obtained by averaging (9) over the PDF given in (2), which using [11, eq. (3.353.5)], [11, eq. (8.352.3)] and [14, eq. (6.5.9)] yields

$$\mathcal{M}_{\gamma_b^{A,B}}(s) = \int_0^\infty \mathcal{M}_{\gamma_b^{A,B}}(s|\gamma_{BD}) f_{\gamma_{BD}}(\gamma_{BD}) d\gamma_{BD} = \frac{\left[\mathcal{E}_L\left(\frac{1+s\overline{\gamma}_{AD}}{\overline{\gamma}_{BD}}\right) + L\overline{\gamma}_{BD}\mathcal{E}_{L+1}\left(\frac{1+s\overline{\gamma}_{AD}}{\overline{\gamma}_{BD}}\right)\right] e^{\frac{1+s\overline{\gamma}_{AD}}{\overline{\gamma}_{BD}}}}{\overline{\gamma}_{BD}(1+s\overline{\gamma}_{AD})^{L-1}} \quad (10)$$

where $\mathcal{E}_n(\cdot)$ is the exponential integral function of order n, defined in [14, eq. (5.1.4)]. It follows then from (4) that the overall MGF for this case is $\mathcal{M}_\gamma(s|\Theta_1) = \mathcal{M}_{\gamma_b^{S,R}}(s)\mathcal{M}_{\gamma_b^{R,S}}(s)$, where due to symmetry it holds $\overline{\gamma}_{R_1D} = \overline{\gamma}_{R_2D} = \overline{\gamma}_{RD}$.

2) Case Θ_2: R_1 active; R_2 inactive: The occurrence probability for this case is evaluated as $\rho_2 = e^{-\frac{T}{\overline{\gamma}_{SR}}}\left(1 - e^{-\frac{T}{\overline{\gamma}_{SR}}}\right) = e^{-\frac{T}{\overline{\gamma}_{SR}}} - e^{-\frac{2T}{\overline{\gamma}_{SR}}}$. The overall SINR is $\gamma = \gamma_b^{S,R_1}$, hence the resulting MGF $\mathcal{M}_\gamma(s|\Theta_2)$ is that given in (10), with $A = S, B = R_1$.

3) Case Θ_3: R_1 inactive; R_2 active: Because of the symmetry between the S-R_1 and S-R_2 links, it holds $\rho_3 = \rho_2$. Due to the absence of interference at the first subframe, the SINR now becomes $\gamma = \gamma_{SD} + \gamma_b^{R_2,S}$. The MGF of γ for this case is therefore derived as $\mathcal{M}_\gamma(s|\Theta_3) = \mathcal{M}_{\gamma_{SD}}(s)\mathcal{M}_{\gamma_b^{R_2,S}}(s) = \mathcal{M}_{\gamma_b^{R_2,S}}(s)/(1+s\overline{\gamma}_{SD})^L$, where $\mathcal{M}_{\gamma_{SD}}(s) = (1+s\overline{\gamma}_{SD})^{-L}$, obtained by averaging $\exp(-s\gamma_{SD})$ over the PDF of γ_{SD} given in (2).

4) Case Θ_4: Both R_1 and R_2 inactive: This case occurs with probability $\rho_4 = \left(1 - e^{-\frac{T}{\overline{\gamma}_{SR}}}\right)^2$. Since the SINR now reduces to $\gamma = \gamma_{SD}$, the resulting MGF for Θ_4 is $\mathcal{M}_\gamma(s|\Theta_4) = \mathcal{M}_{\gamma_{SD}}(s) = (1+s\overline{\gamma}_{SD})^{-L}$.

B. An Approximate MGF Expression

The above analysis provides an exact closed-form expression for the overall MGF, by substituting in (5) the conditional MGF expressions, conditioned on the cases $\Theta_1, ..., \Theta_4$. Unfortunately, however, this expression is not easily tractable in the sense that it cannot easily yield expressions for the overall performance through the MGF-based approach (see e.g., [15, ch. 5]). Nonetheless, an approximate yet much easier to process closed-form expression for $\mathcal{M}_{\gamma_b^{A,B}}(s)$ (and thus for $\mathcal{M}_\gamma(s)$) can be derived, by using a first-order Taylor approximation on the conditional MGF $\mathcal{M}_{\gamma_b^{A,B}}(s|\gamma_{BD})$ [16, ch. 7], from which $\mathcal{M}_{\gamma_b^{A,B}}(s)$ is derived in terms of $E_{\gamma_{BD}}\langle\gamma_{BD}\rangle = L\overline{\gamma}_{BD}$ as

$$\mathcal{M}_{\gamma_b^{A,B}}(s) \simeq \frac{(L\overline{\gamma}_{BD}+1)/\overline{\gamma}_{AD}}{(L\overline{\gamma}_{BD}+1)/\overline{\gamma}_{AD}+s}\left(\frac{1/\overline{\gamma}_{AD}}{1/\overline{\gamma}_{AD}+s}\right)^{L-1}. \quad (11)$$

It turns out that this approximation on the MGF of optimum combining SINR is accurate in most cases, as it also noted in [17].

Consequently, the MGF of the overall SINR can be approximated by

$$\mathcal{M}_\gamma(s) \simeq \rho_1\mathcal{M}_{\gamma_b^{S,R}}(s)\mathcal{M}_{\gamma_b^{R,S}}(s) + \rho_2\mathcal{M}_{\gamma_b^{S,R}}(s) + \rho_3(1+s\overline{\gamma}_{SD})^{-L}\mathcal{M}_{\gamma_b^{R,S}}(s) + \rho_4(1+s\overline{\gamma}_{SD})^{-L} \quad (12)$$

where $\mathcal{M}_{\gamma_b^{S,R}}(s)$, $\mathcal{M}_{\gamma_b^{R,S}}(s)$ are taken from (11).

C. Outage Analysis

The outage probability of the proposed scheme is actually the cumulative distribution function (CDF) of γ, evaluated at the outage threshold SNR γ_{th}, that is related to the target rate r through $\gamma_{th} = 2^r - 1$. The CDF of γ is derived through its probability density function (PDF), $f_\gamma(\cdot)$, that is actually the inverse Laplace transformation of $\mathcal{M}_\gamma(s)$. Specifically, $f_\gamma(\cdot)$ can be evaluated in closed form by expanding $\mathcal{M}_\gamma(s)$ in partial fractions and then using the fact that $\mathcal{L}^{-1}\left\{(s+\alpha)^{-n}\right\}(x) = x^{n-1}\exp(-\alpha x)/(n-1)!$, where $\mathcal{L}^{-1}\{\cdot\}$ denotes the inverse Laplace transform operator. We may then derive the desired outage probability in terms of γ_{th} using [11, eq. (3.381.3)] and [11, eq. (3.381.4)] as it shown in eq. (13) shown at the top of next page, where Ξ_i, Ψ_i, Δ_i, F_i, Υ_i are the fraction nominators derived by expanding the terms of (12) in partial fractions, as it is shown in [18]; $\Gamma(\cdot,\cdot)$ is the incomplete Gamma function defined in [11, eq. (8.350.2)].

V. Numerical Examples and Discussion

The outage performance of the proposed model for the case of $L = 3$ is illustrated in Fig. 2, where the outage probabilities of three different schemes are also depicted, for comparison purposes. These schemes are a) threshold-based DF relaying with a single relay satisfying the half-duplex constraint, b) threshold-based DF relaying with two relays transmitting in time-divisioned orthogonal channels, and c) classical point-to-point communications where no relaying takes place. The x-axis refers to the average channel conditions on the relay-destination channels, which are assumed identical with the source-relay ones for the sake of simplicity. Moreover, the direct S-D channel is assumed relatively shadowed with respect to the relaying ones, so that $\overline{\gamma}_{SD} = \overline{\gamma}_{SR}/10 = \overline{\gamma}_{RD}/10$. The reasoning behind this assumption lies in the fact that in cases where multiple receiving antennas are employed, space diversity can combat small-scale fading but not the large-scale one; generally and loosely speaking, this is the case where relaying transmissions are expected to be employed, so as to tackle both small-scale fading and shadowing.

As it can be noted from Fig. 2, the proposed model outperforms the compared ones within the region of practical outage probabilities. Moreover, it is important to note that for high spectral efficiency requirements (e.g., for high data rate applications) the difference on the outage probability between the proposed and the conventional relaying systems increases. This is because of the orthogonality assumption on conventional relaying schemes, leading to a capacity decrease by a factor of $M+1$, where M is the total number of participating relays, yielding $C_{CR} = [1/(M+1)]\log_2(1+SNR)$. This results in an outage threshold SNR having the form of $SNR_{th,CR} = 2^{(M+1)r} - 1$, i.e., higher than the proposed model's threshold SINR $\gamma_{th} = 2^r - 1$, which is the same as the SNR threshold of point-to-point communications. This implies that the beneficial effects of diversity that conventional relaying offers

$$
\begin{aligned}
P_o\left(\gamma_{th}\right) = {} & \rho_1 \left[\frac{\Xi_1 \overline{\gamma}_{SD}}{L\overline{\gamma}_{RD}+1} \left(1 - e^{-\frac{L\overline{\gamma}_{RD}+1}{\overline{\gamma}_{SD}}\gamma_{th}}\right) + \sum_{i=2}^{L} \Xi_i \overline{\gamma}_{SD}^{i-1} \left(1 - \frac{\Gamma\left(i-1, \frac{\gamma_{th}}{\overline{\gamma}_{SD}}\right)}{\Gamma\left(i-1\right)}\right) + \frac{\Psi_1 \overline{\gamma}_{RD}}{L\overline{\gamma}_{SD}+1} \left(1 - e^{-\frac{L\overline{\gamma}_{SD}+1}{\overline{\gamma}_{RD}}\gamma_{th}}\right) \right. \\
& \left. + \sum_{i=2}^{L} \Psi_i \overline{\gamma}_{RD}^{i-1} \left(1 - \frac{\Gamma\left(i-1, \frac{\gamma_{th}}{\overline{\gamma}_{RD}}\right)}{\Gamma\left(i-1\right)}\right) \right] + \rho_2 \left[\frac{\Delta_1 \overline{\gamma}_{SD}}{L\overline{\gamma}_{RD}+1} \left(1 - e^{-\frac{L\overline{\gamma}_{RD}+1}{\overline{\gamma}_{SD}}\gamma_{th}}\right) + \sum_{i=2}^{L} \Delta_i \overline{\gamma}_{SD}^{i-1} \left(1 - \frac{\Gamma\left(i-1, \frac{\gamma_{th}}{\overline{\gamma}_{SD}}\right)}{\Gamma\left(i-1\right)}\right) \right] \\
& + \rho_3 \left[\frac{\digamma_1 \overline{\gamma}_{RD}}{L\overline{\gamma}_{SD}+1} \left(1 - e^{-\frac{L\overline{\gamma}_{SD}+1}{\overline{\gamma}_{RD}}\gamma_{th}}\right) + \sum_{i=2}^{L} \digamma_i \overline{\gamma}_{RD}^{i-1} \left(1 - \frac{\Gamma\left(i-1, \frac{\gamma_{th}}{\overline{\gamma}_{RD}}\right)}{\Gamma\left(i-1\right)}\right) + \sum_{i=1}^{L} \Upsilon_i \overline{\gamma}_{SD}^{i} \left(1 - \frac{\Gamma\left(i, \frac{\gamma_{th}}{\overline{\gamma}_{SD}}\right)}{\Gamma\left(i\right)}\right) \right] \\
& + \rho_4 \left(1 - \frac{\Gamma\left(L, \frac{\gamma_{th}}{\overline{\gamma}_{SD}}\right)}{\Gamma\left(L\right)}\right) \qquad (13)
\end{aligned}
$$

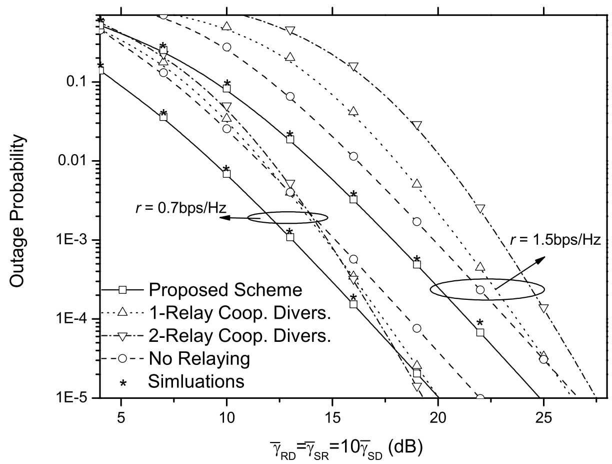

Fig. 2. Outage probability of the proposed scheme, as compared to conventional relaying with a single or two relays and the no-relaying case, for $L = 3$.

seem insufficient to support high spectral efficiencies without a large SNR increase*, resulting in higher outage probabilities in conventional relaying compared to the proposed model*. We note that in Fig. 2 the relay activation threshold, T, was set equal to the outage one, i.e., $T = 2^r - 1$ for the proposed scheme; $T = 2^{(M+1)r} - 1$ for conventional relaying, so as to follow the outage definition of relaying channels, where an outage occurs if either the S-R or the R-D channels cannot support the desired spectral efficiency. The reader may also notice the tightness of the outage probability approximation shown in (13), as compared to simulation results.

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