Power allocation for Quasi-Orthogonal Space-Time Block Codes with 1 or 2 bits feedback

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***Abstract*—In this paper, a closed-loop wireless communication system employing Quasi-Orthogonal Space-Time Block Codes (QOSTBCs) is presented, where the total available power can be suitably allocated to the four transmit antennas, considering a very low-complexity decoding scheme at the receiver side. After deriving a particular optimization problem concerning the optimal choice of the power weights, we prove that the best strategy in terms of error probability is to switch-off three out of the four transmit antennas under the requirement of symbol-by-symbol maximum-likelihood decoding (MLD). Our proposed scheme requires a very low-data rate dedicated feedback link, since just one or two bits per block are sufficient for outperforming other existing closed-loop systems. The technique applies to all the well-known QOSTBCs with arbitrary number of receive antennas, while some special cases are taken indicatively for illustration purposes.**

I. Introduction

In [1] a closed-loop system requiring 2 bits feedback from the receiver to the transmitter side by switching-off two transmit antennas is presented. In this paper we prove that this power allocation strategy is not optimal in terms of system performance. Additionally, a phase-rotation technique for orthogonalizing the quasi-orthogonal equivalent channel matrix under the requirement of 1 real quantity to be fed back to the transmitter, is developed in [2].

We conjecture that 2 bits per block is a sufficient information for the transmitter in order to achieve full-diversity and extra gain. In our analysis we emphasize on the limited nature of the feedback channel data rate, as this reflects the majority of practical cases. Hence, though we start with the assumption of continuous power control through quantized feedback, the solution of the derived optimization problem reveals that the best power allocation strategy is to inform the transmitter with one or two bits feedback, thus without introducing any quantization errors. We have to note that the proposed closed-loop system applies to all the well-known QOSTBCs, including for example the large family of [3] or the distinct codes found in [4] and [5].

In the sequel, $*, T, \dagger$ denote complex conjugation, matrix transpose and Hermitian transpose. Additionally, $\otimes, \hbar, \mathbb{E}, \dot{\vee}$ define the Kronecker product, Hermitian inner product, expectation and exclusive disjunction, respectively.

II. System Model

We consider a wireless communication system operating in a rich scattering environment, with N_t antennas simultaneously transmitting one signal each and a single or more receive antennas combining the multipath replicas of these signals. Space-time block coding (STBC) is performed at the transmitter, whereby n_s information symbols $\{s_1, \ldots, s_{n_s}\}$, drawn generally from a two-dimensional constellation, are encoded into a transmit matrix $\mathbf{S} \in \mathbb{C}^{N \times N_t}$ with code rate equal to $R_c = n_s/N$, where N is the block length or number of time slots per block. If $\mathbf{s} = [s_1 \cdots s_{n_s}]^T$ is the symbol vector conveying all the codeword information, the average symbol energy can be given by $E_s = \mathbb{E}\left[\mathbf{s}^\dagger \mathbf{s}\right]/n_s$, implying that $E_s = \mathbb{E}\left[|s_k|^2\right]$, for all $k = 1, 2, \ldots, n_s$. We further mark that we focus on STBC where at each time slot all n_s symbols are transmitted through the $N_t = n_s$ different transmit antennas. Hence, each row of the transmit matrix $\mathbf{S}$ can be considered as signed/unsigned permutation of either $\mathbf{s}$ or $\mathbf{s}^*$ entries. This consideration permits a general approach to the problem formulation, while it covers the majority of quasi-orthogonal space-time block codes (QOSTBCs). Hence, at each time slot the total radiated power from all antennas is $\mathcal{P} = N_t E_s$, which will define the power constraint for the power allocation strategy adopted in the sequel.

We assume a quasi-static frequency-flat fading channel and enough separation within transmit/receive antennas, such that the fades for each transmission path can be considered as independent. For the sake of clarity, the system model defined here considers only one receive antenna, while the generalization of the proposed methodology to an arbitrary number of receive antennas is handled in Section III. Hence, the complex channel gain h_i from transmit antenna i to the single receive antenna remains constant for N time instants before changing to an independent realization [6], thus resulting in a random channel vector $\mathbf{h} \in \mathbb{C}^{N_t}$ with independent, identically distributed (iid) circularly-symmetric zero-mean complex Gaussian entries, i.e. $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}_{N_t}, \Omega \mathbf{I}_{N_t})$, where Ω is the average fading power being equal for all paths (flat power delay profile). Full channel state information (CSI) is considered at the receiver, while part of this is conveyed back to the transmitter via a low-rate feedback link. Due to the slowly fading environment any feedback delay is considered negligible.

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Let now define the vector $\mathbf{p} = [p_1 \cdots p_{N_t}]^T$ consisting of N_t nonnegative weights that will be appropriately utilized by the radio system in order to allocate the available transmit power among the different antennas towards a target fulfillment that will arise during our analysis. Each power load p_i, $i = 1, 2, \ldots, N_t$, is assigned to the corresponding transmit antenna at the beginning of a block transmission and remains fixed for N time instants, before the next block starts and new weights p_i allocate the total power differently. In such a way, the power amplifier gain of each radio frequency (RF) chain at the transmitter –being in general equal to the number of transmit antenna elements– does not need to fluctuate from one time slot to another, especially when equal energy constellation is employed in the modulation procedure. The resulting transmit matrix gets the form now $\mathbf{S}_p = \mathbf{SP}$, where the diagonal matrix $\mathbf{P} = \text{diag}(\sqrt{p_1}, \ldots, \sqrt{p_{N_t}})$ acts as a linear transformation on the column vectors of $\mathbf{S}$. In order to avoid penalty in terms of power radiation, we restrict the loading factors to satisfy the condition $\mathbb{E}\left[\|\mathbf{Ps}\|^2\right] = \mathcal{P}$, which is equivalent to $\sum_{i=1}^{N_t} p_i = N_t$ with p_i taking values in the interval $[0, N_t]$. Obviously, the equal power allocation scheme imposes $p_1 = p_2 = \ldots = p_{N_t} = 1$, rendering the weighting matrix $\mathbf{P}$ an $N_t \times N_t$ identity matrix.

The complex baseband representation of the received vector $\mathbf{r} \in \mathbb{C}^N$, according to [7], can be written as

$$\mathbf{r} = \mathbf{S}_p\mathbf{h} + \mathbf{n} \tag{1}$$

where $\mathbf{n} \in \mathbb{C}^N$ denotes the additive white noise vector consisting of iid circularly-symmetric complex Gaussian random variables with zero-mean and variance $N_o/2$ per dimension, i.e. $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}_N, N_o\mathbf{I}_N)$.

To proceed further, (1) can be expressed alternatively as

$$\mathbf{r} = \mathbf{S}\mathbf{h}_p + \mathbf{n} \tag{2}$$

where $\mathbf{h}_p = \mathbf{Ph}$ defines a linear transformed version of the channel vector. Following a similar approach to that presented in [8], particular processing of the received vector $\mathbf{r}$ can enable one to rewrite (2) as

$$\tilde{\mathbf{r}} = \mathbf{H}_p\mathbf{s} + \tilde{\mathbf{n}} \tag{3}$$

with $\mathbf{H}_p \in \mathbb{C}^{N \times N_t}$ representing the equivalent channel matrix (ECM), yielding

$$\mathbf{H}_p = \Lambda_S\left(\mathbf{I}_N \otimes \mathbf{h}_p^T\right)\mathbf{G}^T. \tag{4}$$

In the above equation, $\mathbf{G}$ denotes an $N_t \times NN_t$ appended matrix with real entries within the set $\{0, \pm 1\}$, consisting of the generalized permutation matrices (GPMs) $\mathbf{G}_i \in \mathbb{R}^{N_t \times N_t}$, $i = 1, 2, \ldots, N$. Also, the application of Λ_S operator on a complex $m \times n$ matrix $\mathbf{Z}$, induces conjugation on those rows of $\mathbf{Z}$, whose indices are entries of the set $S \subseteq \{i : i \in \mathbb{N}; i \leq m\}$. Adopting this notation, the vectors $\tilde{\mathbf{r}}$ and $\tilde{\mathbf{n}}$, introduced in (3), can be given by $\tilde{\mathbf{r}} = \Lambda_S(\mathbf{r})$ and $\tilde{\mathbf{n}} = \Lambda_S(\mathbf{n})$, respectively. Obviously, all $\mathbf{G}_i$ and S are related to the inherent structure of the given QOSTBC, that is the location of $\{\pm s_k, \pm s_k^*\}$, $k = 1, 2, \ldots, n_s$, in the transmit matrix $\mathbf{S}$.

After channel-matched filtering, the received signal vector can be written as

$$\mathbf{r}_m = \mathbf{H}_p^\dagger\tilde{\mathbf{r}} = \mathbf{H}_m\mathbf{s} + \mathbf{n}_m$$

where $\mathbf{n}_m = \mathbf{H}_p^\dagger\tilde{\mathbf{n}}$ is the random noise vector at the matched filter output and $\mathbf{H}_m = \mathbf{H}_p^\dagger\mathbf{H}_p$ is a square sparse matrix of order N_t, given by

$$\mathbf{H}_m = \mathbf{G}\Lambda_S^b\left(\mathbf{I}_N \otimes \mathbf{h}_p^*\mathbf{h}_p^T\right)\mathbf{G}^T \tag{5}$$

with Λ_S^b denoting an extension of the Λ_S operator to apply on block matrices in a similar way.[1]

As proved in [8], given that the code matrix $\mathbf{S}$ is a QOSTBC, then $\mathbf{H}_p$ will stem from a quasi-orthogonal design (QOD) as well. Therefore, $\mathbf{H}_m$ can be rearranged into a block diagonal matrix, implying that the receiver can retrieve the information symbols by means of joint rather than symbol-by-symbol detection that arises in the case of orthogonal STBCs. Hence, if n is the number of submatrices $\mathbf{H}_m^b$, $b = 1, 2, \ldots, n$, that can be extracted after block diagonalizing $\mathbf{H}_m$, the decoder can obtain separately a maximum-likelihood (ML) estimate of the symbol vector $\mathbf{s}_b \in \mathbb{C}^{N_t/n}$ according to[2]

$$\hat{\mathbf{s}}_b = \underset{\mathbf{s}_b \in \mathcal{S}_b}{\arg\min}\|\mathbf{r}_m^b - \mathbf{H}_m^b\mathbf{s}_b\|^2 \tag{6}$$

where $\mathbf{r}_m^b \in \mathbb{C}^{N_t/n}$ is the corresponding received subvector and $\mathcal{S}_b$ is the signal space under search.

III. Power Allocation Strategy

For the sake of clarity, we will present the optimum power control assuming $N_t = 4$ transmit antennas. In this case, the ECM is written as $\mathbf{H}_p = [\mathbf{c}_1\ \mathbf{c}_2\ \mathbf{c}_3\ \mathbf{c}_4]$, where $\mathbf{c}_j \in \mathbb{C}^N$, $j = 1, 2, \ldots, 4$, represents its jth column vector. The quasi-orthogonal structure of $\mathbf{H}_p$ enables the partitioning of its columns into two complex vector subsets, $S_a, S_b \subseteq \mathbb{C}^N$, both with the same cardinality, i.e. $|S_a| = |S_b| = 2$, fulfilling also the conditions

$$\hbar(\mathbf{c}_i^a, \mathbf{c}_j^b) = \hbar(\mathbf{c}_j^b, \mathbf{c}_i^a) = 0 \tag{7}$$

$$\hbar(\mathbf{c}_i^a, \mathbf{c}_j^a) = \begin{cases} \|\mathbf{c}_i^a\|^2 & \text{if } i = j \\ \hbar^*(\mathbf{c}_j^a, \mathbf{c}_i^a) & \text{if } i \neq j \end{cases} \tag{8}$$

$$\hbar(\mathbf{c}_i^b, \mathbf{c}_j^b) = \begin{cases} \|\mathbf{c}_i^b\|^2 & \text{if } i = j \\ \hbar^*(\mathbf{c}_j^b, \mathbf{c}_i^b) & \text{if } i \neq j \end{cases} \tag{9}$$

where $\mathbf{c}_i^a \in S_a, \mathbf{c}_j^b \in S_b$ and $i, j \in \{1, 2\}$, implying that vector elements from different subsets are orthogonal to each other. From the structure of $\mathbf{H}_p$ shown in (4) and particular utilization of the GPMs involved [9], we can also derive the extra relations

$$\|\mathbf{c}_i^a\|^2 = \|\mathbf{c}_j^b\|^2 = \|\mathbf{h}_p\|^2 \quad \forall\ i, j \tag{10}$$

[1]The entries of set S include now the indices of the partitioned (block) rows that are to be complex conjugated.

[2]For the QOSTBC case, the condition $N_t \pmod{n} = 0$ is obviously satisfied.

$$\hbar(\mathbf{c}_i^a, \mathbf{c}_j^a) = \pm\hbar(\mathbf{c}_i^b, \mathbf{c}_j^b) \quad i \neq j. \tag{11}$$

Particular observation of (7)–(11) gives evidence that the 4×4 Hermitian matrix $\mathbf{H}_m$ can be written only in terms of a real α and a complex γ. For instance, suppose that $S_a = \{\mathbf{c}_1, \mathbf{c}_4\}$ and $S_b = \{\mathbf{c}_2, \mathbf{c}_3\}$. Then (5) takes the simplified form

$$\mathbf{H}_m = \begin{pmatrix} \alpha & 0 & 0 & \gamma \\ 0 & \alpha & \pm\gamma & 0 \\ 0 & \pm\gamma^* & \alpha & 0 \\ \gamma^* & 0 & 0 & \alpha \end{pmatrix} \tag{12}$$

where the real diagonal elements are given by

$$\alpha = \sum_{i=1}^{4} p_i |h_i|^2 \tag{13}$$

and the nonzero off-diagonal entries can be written with respect to γ according to the two possible cases

$$\gamma = \pm 2 \left(\sqrt{p_1 p_4}\, \mathrm{Re}\{h_1 h_4^*\} \pm \sqrt{p_2 p_3}\, \mathrm{Re}\{h_2 h_3^*\}\right) \quad \text{or}$$
$$\gamma = \pm 2j \left(\sqrt{p_1 p_4}\, \mathrm{Im}\{h_1 h_4^*\} \pm \sqrt{p_2 p_3}\, \mathrm{Im}\{h_2 h_3^*\}\right). \tag{14}$$

By substituting (12) into (6), it is easy to notice that the receiver will have to perform a joint detection for each one of the symbol pairs $\{s_1, s_4\}$ and $\{s_2, s_3\}$, with α being the diversity gain and γ playing the role of interference.

The proposed scheme aims to fulfill two specific goals through the desired power control: (a) To eliminate the interference γ in order to achieve symbol-by-symbol detection, and (b) to maximize the gain α, thus improving the system performance. From both formulas given in (14), we can observe that the problem of γ elimination can be reduced to the optimal choice of p_1, p_2, p_3, p_4 that satisfy the constraint $a\sqrt{p_1 p_4} = b\sqrt{p_2 p_3}$, where a, b are real. If the four columns of $\mathbf{H}_m$ are partitioned differently into subgroups, then the location of the interference terms in the right-hand side of (12) will change according to the new column vector indices within each subgroup. Hence, in all possible cases, the off-diagonal entries of $\mathbf{H}_m$ will vanish, converting the latter into a scaled identity matrix.

Taking into account all the previous considerations, the optimal power allocation strategy decides in favor of the weighting vector $\mathbf{p}$ whose entries maximize (13), under the linear power constraints indicated in Section II and the non-linear one resulting from the zero interference requirement. This optimization problem is non-convex and its solution will be derived directly through the establishment of the following theorem.

Theorem 1: Let $\mathbf{p}$ be a vector containing the real variables p_1, p_2, p_3, p_4 that lie in the interval $[0, 4]$ and w_1, w_2, w_3, w_4 are given positive constants. Then the maximum of the function

$$f(\mathbf{p}) = \sum_{i=1}^{4} w_i p_i \tag{15}$$

under the constraints

$$p_1 + p_2 + p_3 + p_4 = 4 \tag{16}$$
$$a\sqrt{p_1 p_4} = b\sqrt{p_2 p_3} \tag{17}$$

where the coefficients a, b are arbitrary real numbers, occurs at

$$p_i = 4\delta_{ij}, \quad j = \arg\max_{j=1,\ldots,4}\{w_j\} \tag{18}$$

with the objective function taking value at that point

$$\hat{f}(\mathbf{p}) = 4 \max_{i=1,\ldots,4}\{w_i\}.$$

Proof: Let us consider the case $ab > 0$. Then, condition (17) can be replaced by

$$a^2 p_1 p_4 = b^2 p_2 p_3 \tag{19}$$

First of all we are going to prove that there is no interior point $\mathbf{p}$ of the hypercube $[0, 4]^4$ which yields a global maximum. Let us reformulate the optimization problem with the help of the Lagrange multipliers. The Lagrangian reads

$$L(\mathbf{p}, \lambda_1, \lambda_2) = \sum_{i=1}^{4} w_i p_i - \lambda_1\Big(\sum_{i=1}^{4} p_i - 4\Big) - \lambda_2(a^2 p_1 p_4 - b^2 p_2 p_3).$$

An interior stationary point satisfies $\partial L/\partial p_i = 0$. Thus we get the system of equations

$$\begin{cases} w_1 - \lambda_1 - \lambda_2 a^2 p_4 = 0 \\ w_2 - \lambda_1 + \lambda_2 b^2 p_3 = 0 \\ w_3 - \lambda_1 + \lambda_2 b^2 p_2 = 0 \\ w_4 - \lambda_1 - \lambda_2 a^2 p_1 = 0 \end{cases} \Leftrightarrow \begin{cases} p_4 = \frac{w_1 - \lambda_1}{a^2 \lambda_2} \\ p_3 = -\frac{w_2 - \lambda_1}{b^2 \lambda_2} \\ p_2 = -\frac{w_3 - \lambda_1}{b^2 \lambda_2} \\ p_1 = \frac{w_4 - \lambda_1}{a^2 \lambda_2} \end{cases} \tag{20}$$

where we have assumed that $\lambda_2 \neq 0$. By multiplying the first four equations in (20) with $p_1, \ldots, p_4$ respectively and adding them, then, by making use of (16) and (19), we get $f(\mathbf{p}) = 4\lambda_1$. Since $p_i \in (0, 4)$, if we examine the second set of equations in (20), we arrive at the conclusion that the quantities

$$\lambda_1 - w_1,\ -(\lambda_1 - w_2),\ -(\lambda_1 - w_3),\ \lambda_1 - w_4$$

have the same sign. Thus λ_1 satisfies one of the two inequalities

$$\max(w_1, w_4) < \lambda_1 < \min(w_2, w_3)$$
$$\max(w_2, w_3) < \lambda_1 < \min(w_1, w_4).$$

However, at the point $p_i = 4\delta_{ij}$, $j = \arg\max_{j=1,\ldots,4}\{w_j\}$, the objective function, according to (15), takes the value $f(\mathbf{p}) = 4\max_{i=1,\ldots,4}\{w_i\}$, which is clearly greater than $4\lambda_1$. Thus the value $4\lambda_1$, even if it corresponds to a local maximum, cannot be a global maximum. It remains to examine whether $\lambda_2 = 0$ could yield an interior maximum. By returning to the first set of (20) we gather that this could happen only in the exceptional case of $w_1 = w_2 = w_3 = w_4 \equiv w$. But this would mean that $f(\mathbf{p}) = 4w$ (constant), which is trivial.

Having proved that there is no global maximum in the interior of the cube $[0,4]^4$, we conclude that $f(\mathbf{p})$ attains its maximum at the boundary of this set. This means that at least one of p_i equals 4 or 0. In the first case, restriction (16) yields that all other values are zero. Thus in this case the maximum is attained at the point $p_i = 4\delta_{ij}$, where $j = \arg\max_{j=1,\ldots,4}\{w_j\}$. In the other case (i.e. one of p_i is zero), the restriction (17) yields that at least another p_j $(i \neq j)$ is zero too. Without loss of generality, let us assume that $p_1 = p_2 = 0$. Then $p_4 = 4-p_3$ and $f(\mathbf{p}) = p_3(w_3-w_4)+4w_4$. This linear function takes its maximum when p_3 is either 0 or 4. Thus, we conclude again that the maximum is attained when one and only one p_i equals 4 and the rest vanish.

The case $ab < 0$ is much simpler, since then (17) implies that $p_1p_4 = p_2p_3 = 0$. Therefore, at least two of p_i vanish, and using the same argument as before we can prove that, in fact, three of p_i must vanish and one must equal 4. This concludes the proof. ■

Before utilizing the result of Theorem 1 it is straightforward to derive the generalized versions of (13) and (14), considering N_r antennas at the receiver side, i.e.

$$\alpha = \sum_{i=1}^{4}\sum_{j=1}^{N_r} p_i |h_{ij}|^2 \tag{21}$$

and

$$\gamma = \pm 2\Big(\sqrt{p_1p_4}\sum_{j=1}^{N_r}\mathrm{Re}\{h_{1j}h_{4j}^*\} \pm \sqrt{p_2p_3}\sum_{j=1}^{N_r}\mathrm{Re}\{h_{2j}h_{3j}^*\}\Big)$$

or

$$\gamma = \pm 2j\Big(\sqrt{p_1p_4}\sum_{j=1}^{N_r}\mathrm{Im}\{h_{1j}h_{4j}^*\} \pm \sqrt{p_2p_3}\sum_{j=1}^{N_r}\mathrm{Im}\{h_{2j}h_{3j}^*\}\Big). \tag{22}$$

We can easily observe now that the optimization problem under concern is a special case of Theorem 1, with

$$w_i = \sum_{j=1}^{N_r}|h_{ij}|^2, \quad i = 1,2,3,4$$

where the coefficients a, b can take real values in the following distinct pairs

$$\{a,b\} = \{\sum_{j=1}^{N_r}\mathrm{Re}\{h_{1j}h_{4j}^*\}, \pm\sum_{j=1}^{N_r}\mathrm{Re}\{h_{2j}h_{3j}^*\}\} \tag{23}$$

or

$$\{a,b\} = \{\sum_{j=1}^{N_r}\mathrm{Im}\{h_{1j}h_{4j}^*\}, \pm\sum_{j=1}^{N_r}\mathrm{Im}\{h_{2j}h_{3j}^*\}\}. \tag{24}$$

Theorem 1 implies that the optimal power allocation strategy is to feed the antenna which corresponds to the best channel[3] with all the available power. This solution is very desirable in terms of the limited feedback channel bandwidth, since just two information bits are adequate for successfully updating the information at the transmitter. However, one can infer that these scheme supersedes the need for multiple transmit antennas. Nevertheless, we have to underline that this result has been derived under the requirement of vanishing all the interference terms, $\gamma, \pm\gamma, \gamma^*, \pm\gamma^*$, appearing at the receiver after the matched filtering processing in order to achieve symbol-by-symbol ML decoding. If alternative decoupling/decoding techniques are considered, then another solution –maybe more desirable in terms of symbol error performance– can be obtained for the same size of feedback information.

[3]As "best" is considered the channel with the maximum amplitude gain.

Finally, the outcome of Theorem 1 has to be assessed from a generalized point of view, since the objective function in (15), the linear power constrained in (16) and the limitation of the power weights inside the interval $[0,4]$, hold the same for all QOSTBC cases. Additionally, the orthogonality constraint in (17) is a representative sample of all the possible 2-subsets of $\{p_1,p_2,p_3,p_4\}$, namely $\binom{4}{2} = 6$ in number. Thus, the result of Theorem 1 is applicable in all QOSTBCs with four transmit antennas.

An alternative (non-optimal) power allocation strategy can be derived at this point. The structure of matrix (12) implies that the column space of the initial QOSTBC can be divided into two orthogonal groups, with each element within a group interfering with the other according to the non-diagonal entries of (12). Therefore, one can use just one bit to choose one pair of strong channels out of the two possible in such a way that the interference is assured to be zero. This case will be illustrated graphically later.

IV. Applications and Discussion

In this section, we apply the proposed power allocation strategy to three well-known QOSTBCs and compare their performance with other low-rate feedback schemes found in the literature.

A. The Jafarkhani code

For the Jafarkhani code $\mathbf{S}_p^J$ in [10], the matrix $\mathbf{H}_m^J$ in (5) obtains the form

$$\mathbf{H}_m^J = \begin{pmatrix} \alpha & 0 & 0 & \gamma \\ 0 & \alpha & -\gamma & 0 \\ 0 & -\gamma & \alpha & 0 \\ \gamma & 0 & 0 & \alpha \end{pmatrix}$$

where the gain and the interference parameter read

$$\alpha = p_1|h_1|^2 + p_2|h_2|^2 + p_3|h_3|^2 + p_4|h_4|^2$$
$$\gamma = 2\left(\sqrt{p_1p_4}\,\mathrm{Re}\{h_1h_4^*\} - \sqrt{p_2p_3}\,\mathrm{Re}\{h_2h_3^*\}\right).$$

The orthogonality constraint (17) becomes

$$\mathrm{Re}\{h_1h_4^*\}\sqrt{p_1p_4} = \mathrm{Re}\{h_2h_3^*\}\sqrt{p_2p_3}$$

and hence the optimal power allocation solution can be derived by application of Theorem 1, as dictated by (18).

B. The Tirkkonen-Boariu-Hottinen code

Similarly, for the Tirkkonen-Boariu-Hottinen (or ABBA) code $\mathbf{S}_p^{TBH}$ in [11], the Hermitian matrix $\mathbf{H}_m^{TBH}$ reads

$$\mathbf{H}_m^{TBH} = \begin{pmatrix} \alpha & 0 & \gamma & 0 \\ 0 & \alpha & 0 & \gamma \\ \gamma & 0 & \alpha & 0 \\ 0 & \gamma & 0 & \alpha \end{pmatrix}$$

where the gain and the interference parameter are given by

$$\begin{aligned} \alpha &= p_1|h_1|^2 + p_2|h_2|^2 + p_3|h_3|^2 + p_4|h_4|^2 \\ \gamma &= 2\left(\sqrt{p_1p_3}\ \mathrm{Re}\{h_1h_3^*\} + \sqrt{p_2p_4}\ \mathrm{Re}\{h_2h_4^*\}\right). \end{aligned}$$

The orthogonality constraint (17) becomes

$$\mathrm{Re}\{h_1h_3^*\}\sqrt{p_1p_3} = -\mathrm{Re}\{h_2h_4^*\}\sqrt{p_2p_4}$$

with the optimal power loads also given by (18).

C. The Papadias-Foschini code

Finally, for the Papadias-Foschini code $\mathbf{S}_p^{PF}$ in [12], (5) can be rewritten as

$$\mathbf{H}_m^{PF} = \begin{pmatrix} \alpha & 0 & \gamma & 0 \\ 0 & \alpha & 0 & -\gamma \\ -\gamma & 0 & \alpha & 0 \\ 0 & \gamma & 0 & \alpha \end{pmatrix}$$

where the gain and the interference read

$$\begin{aligned} \alpha &= p_1|h_1|^2 + p_2|h_2|^2 + p_3|h_3|^2 + p_4|h_4|^2 \\ \gamma &= 2j\left(\sqrt{p_1p_3}\ \mathrm{Im}\{h_1h_3^*\} + \sqrt{p_2p_4}\ \mathrm{Im}\{h_2h_4^*\}\right). \end{aligned}$$

The orthogonality constraint (17) becomes

$$\mathrm{Im}\{h_1h_3^*\}\sqrt{p_1p_3} = -\mathrm{Im}\{h_2h_4^*\}\sqrt{p_2p_4}$$

with the optimal power control given again by (18).

D. Simulation Results

In Figs. 1-4 the bit error rate (BER) curves versus the average signal-to-noise ratio (SNR) per symbol are illustrated for the proposed power allocated (PA) scheme, considering the three aforementioned QOSTBCs (PA-J, PA-TBH, PA-PF) with one and two bits feedback. Their performance is compared with their equal power allocated open-loop (OL) counterparts (OL-J, OL-TBH, OL-PF), considering availability of the same total power $\mathcal{P}$ at the transmitter, Gray coding and an equal spectral efficiency for all the codes on the same graph. We also include the performance of the closed-loop scheme proposed in [1], where 2 bits are sent back to the transmitter in order to switch-off two out of the four antennas and then apply equal power allocation in the rest (CL-J, CL-TBH, CL-PF). On the same axis, the BER curve of the channel-orthogonalized scheme proposed in [2], that requires 1 real feedback for phase rotation, is also plotted (CO-J, CO-TBH, CO-PF). Finally, the BER curves of the half-rate orthogonal code (OSTBC1/2) [13] and the 3/4 orthogonal code (OSTBC3/4) [14] are used as a reference, for spectral efficiencies equal to 2 and 3 b/s/Hz, respectively.

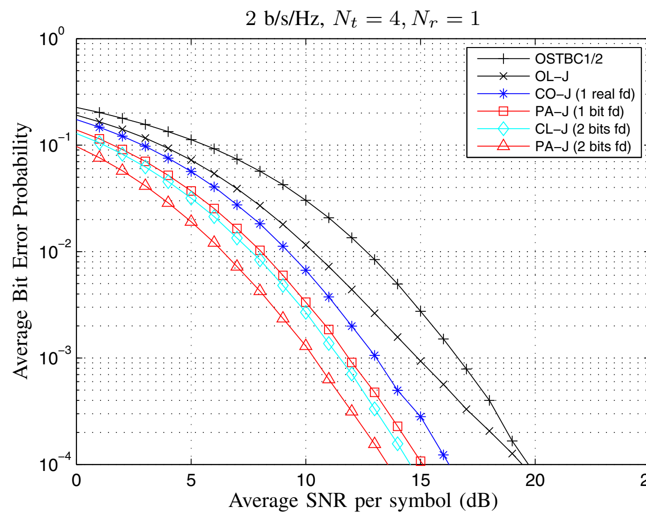

Fig. 1. BEP performance comparison for the $\mathbf{S}_p^J$ QOSTBC

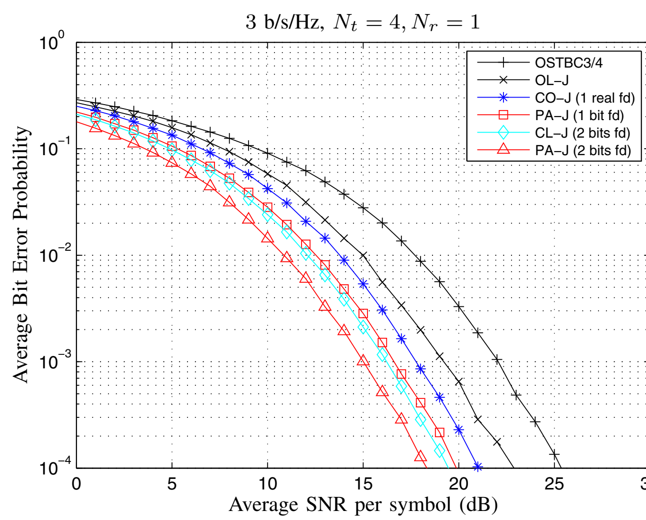

Fig. 2. BEP performance comparison for the $\mathbf{S}_p^J$ QOSTBC

In Fig. 1, where 16-QAM modulation is considered for the OSTBC1/2 code and quadrature phase-shift keying (QPSK) for the rest, a performance gap of about 3 dB (for the 1-bit feedback case) and 5 dB (for the 2-bit feedback case) at BER $= 10^{-3}$ can be observed with respect to the OL-J code. Also, the PA schemes perform better (nearly 1 dB for the 1-bit feedback case and 3 dB for the 2-bit feedback case) than the CO-J for the whole SNR region. Concerning the 1-bit feedback PA-J scheme, we have to highlight its superior performance over the CO-J scheme, though the latter requires a real feedback value (or equivalently at least 3 bits for its sufficient quantization). Finally, the 2-bit PA-J scheme outperforms the CL-J, providing a gain of about 1.5 dB for all SNR values. Similar conclusions can be drawn from Fig. 2 where 16-PSK is considered for the OSTBC3/4 code and 8-PSK for the rest, as well from Figs. 3 and 4 where two antennas are utilized at the receiver side.

All figures, clearly indicate that the proposed scheme uti-

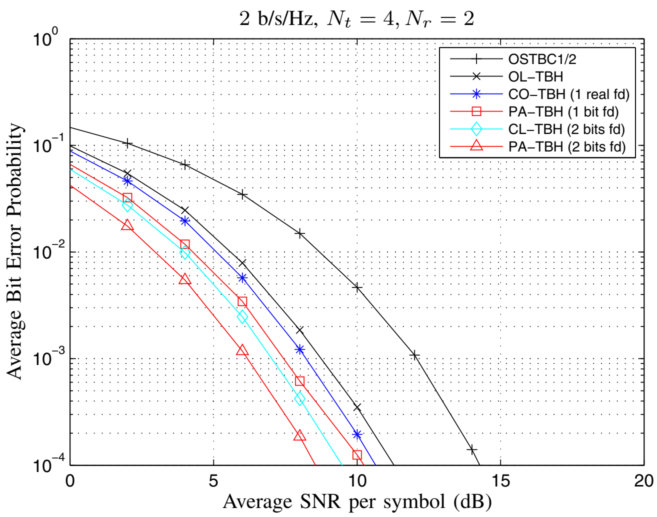

Fig. 3. BEP performance comparison for the $\mathbf{S}_p^{TBH}$ QOSTBC

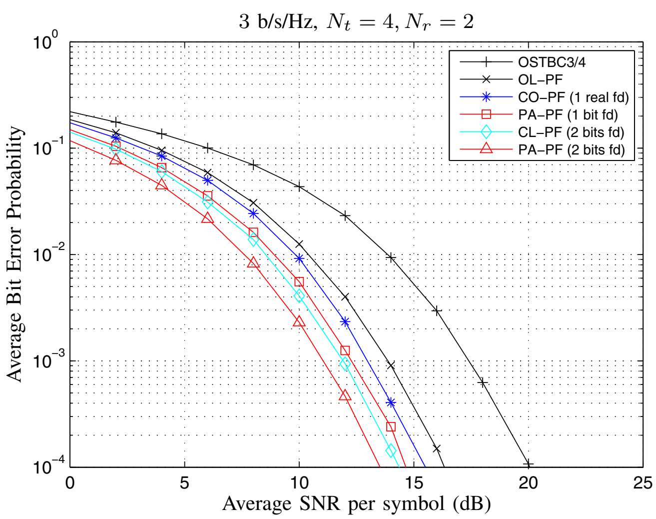

Fig. 4. BEP performance comparison for the $\mathbf{S}_p^{PF}$ QOSTBC

lizes in a more appropriate way the 2-bit information feedback when compared to the CL system. This can be easily realized if we consider the result of Theorem 1, which implies that all solutions of the form $p_i = p_j = 0,\ p_k = p_l = 2$ for $\{i, j, k, l\} = \{1, 2, 3, 4\}\dot{\vee}\{1, 3, 2, 4\}\dot{\vee}\{2, 4, 1, 3\}\dot{\vee}\{3, 4, 1, 2\}$ are not optimal, since although the orthogonality constraint (19) is satisfied, the objective function (15) is locally maximized, thus failing to attain its global maximum. Furthermore, from the slope of the plotted curves, it can be deduced that the CO, CL, PA schemes achieve all the same diversity gain, with the CL and 2-bit PA schemes obtaining an extra coding gain of 1.5 dB and approximately 3 dB, respectively.

E. Conclusion

For any QOSTBC system with four transmit and an arbitrary number of receive antennas, we proved that an optimum power allocation solution in terms of BER performance and reduced decoding complexity (symbol-by-symbol MLD) is the antenna selection strategy, which can be achieved by a 2-bit dedicated feedback channel. Another solution that validates the tradeoff between feedback data rate vs decoding complexity is the $4 \times N_r$ architecture QOSTBC system working with an 1-bit feedback channel. Further research on this work is ongoing with the case of more than four transmit antennas under consideration. Similar works reported in the literature, including [15] and [16], will be compared to the proposed technique.

ACKNOWLEDGMENT

This work was performed within the framework of the Satellite Communications Network of Excellence (SatNex) project (IST-507052) and its Phase-II, SatNex-II (IST- 27393), funded by the European Commission (EC) under its FP6 program.

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