Relaying Utilization Metrics of Diamond Cooperative Diversity Systems

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***Abstract*—We introduce relaying utilization statistical metrics to quantify the contribution of a relay node to the overall relaying process in cooperative diversity systems. Motivated by the second order statistics of the relay fading channel, the following metrics are defined: i) the average relaying selection rate, ii) the average relaying utilization time, and iii) the average relaying waiting time. These metrics are important for the cooperative systems' design, especially for applications where power consumption and delay are major concerns. As an application, we present exact analytical expressions of these parameters in an amplify-and-forward relaying system with selection diversity and diamond topology over Rayleigh fading channels.**

I. Introduction

Cooperative diversity, where the mobile users relay signals for each other to emulate an antenna array and exploit the benefits of spatial diversity, has gained great interest [1]–[6]. Scanning the literature, the performance analysis of cooperative diversity systems in terms of the outage probability, the average bit-error rate and the outage capacity for both amplify-and-forward (AF) and decode-and-forward (DF) relays, has been studied. However, these performance metrics do not show the time-varying nature of the fading relay channel. There are only few works dealt with the dynamic, time-varying of the underlying fading channel [7]–[10]. In [7], Yang *et al.* have studied the average level crossing rate (LCR) and the average fade duration of multihop communication systems employing DF relays over generalized fading channels. In [8], Patel *et al.* have studied for the first time the statistics of the relay fading channel and have derived useful exact analytical expressions for the AF channel's temporal statistical properties such as auto-correlation function and LCR. In [9], [10], Chau *et al.* have introduced the average LCR for a dual-hop selection cooperative diversity system and for dual-hop or triple-hop systems channels with fixed gain AF relays, respectively.

In this paper, new relaying utilization performance metrics, which quantify the contribution of a relay node to the relaying process in cooperative diversity systems, are introduced. These metrics, which are based on the second order statistics of the relay fading channel, are the average relaying selection rate (ARSR), the average relaying utilization time (ARUT), and the average relaying waiting time (ARWT). Furthermore, we study the aforementioned metrics in cooperative selection diversity systems with diamond topology [11] employing AF fixed gain relays over Rayleigh fading channels, presenting an exact mathematical analysis. Finally, the derived theoretical results are validated by Monte Carlo simulations, while numerical examples are further provided to collaborate on the presented analytical expressions.

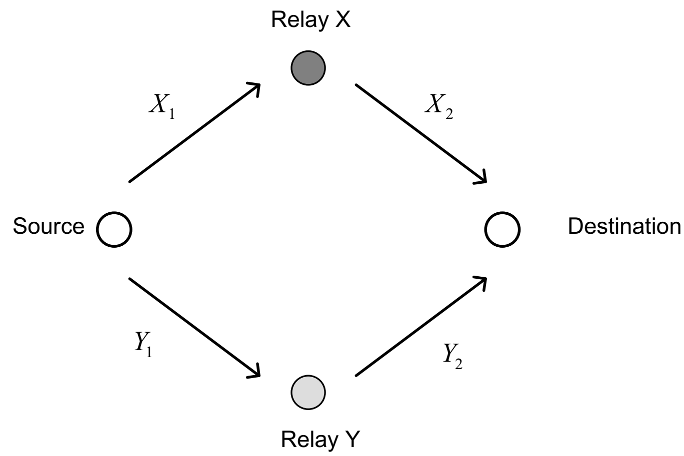

Fig. 1. A diamond cooperative diversity system.

II. Average Relaying Utilization Metrics

Next, we define new statistical performance metrics for cooperative diversity systems which quantify the utilization of a relay node to the overall relaying process in an average sense, as follows:

Definition 1: We define the *average relaying selection rate* (ARSR) as the average rate in which a relay is selected for cooperation in a cooperative diversity system.

Definition 2: The *average relaying utilization time* (ARUT) is defined as the average time in which a relay is utilized in a cooperative diversity system. The ARUT is the average time from the moment the relay is selected for participating in the cooperative process, until it loses this ability to some other relay.

Definition 3: The *average relaying waiting time* (ARWT) is defined as the average time in which a relay remains inactive in a cooperative diversity system. The ARWT indicates how long in average a relay has to wait for the next participation in the cooperative process.

The above metrics are important for the cooperative systems' design, especially for applications where power consumption and delay are concerns. Specifically, from a practical point-of-view, the ARUT can be used for the time slot length

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design while the ARWT is important for the time-out timer consideration in the upper layer protocols.

III. Application in Diamond Cooperative Diversity Systems

A. System and Channel Model

In Fig. 1, a diamond cooperative diversity system is depicted where two AF relays $R_k, k \in \{X, Y\}$, are available for forwarding the information signal received from the source node (S) to the destination (D), over independent but not-identically distributed (i.n.i.d.) flat Rayleigh fading channels with probability density function (PDF) of the signals' envelope given by

$$f(t_{k_j}) = \frac{2t_{k_j}}{\Omega_{k_j}} \exp(-\frac{t_{k_j}^2}{\Omega_{k_j}}) \tag{1}$$

where Ω_{k_j} with $j \in \{1, 2\}$, is the signal's mean power of the kth relay path and jth hop. Diamond topology assumes that there is no direct link between the source and the destination and between the relays since the corresponding channels are weak enough so as to be considered negligible [11].

The relaying process takes place in half-duplex channels and in a time-orthogonal fashion; according to this protocol, each transmission period is divided in two timeslots, corresponding to the $S-R_k$ and R_k-D communication interval, respectively. Each relay, R_k, amplifies the incoming signal by a certain fixed gain G_k without performing any sort of decoding. Fixed gain relays do not need instantaneous channel state information (CSI) of the incoming hop at the relay R_k, and consequently result in a variable power at the relay output. This type of relays have gained practical interest due to their low complex circuitry and yet comparable performance to that of CSI-assisted relays.

The selection of the appropriate dual-hop branch is based on a simple-path selection method which, in each timeslot, activates the relay (R_X or R_Y) with the maximum end-to-end signal-to-noise ratio (SNR) between the source and the destination (the so-called largest SNR criterion) [12].

Given a fixed gain, G_k, used in the relay R_k, the overall SNR, γ_k, of the signal transmitted from the source arriving at the destination via R_k, is [13]

$$\gamma_k = \frac{E_S \alpha_{S,k}^2 G_k^2 \alpha_{D,k}^2}{G_k^2 \alpha_{D,k}^2 N_{0,k} + N_{0,D}} = \frac{E_S}{N_{0,k}} \frac{\alpha_{S,k}^2 (G_k \alpha_{D,k})^2}{(G_k \alpha_{D,k})^2 + N_{0,D}/N_{0,k}} \tag{2}$$

where E_s is the average signal energy transmitted at the output of the source over one symbol period, $N_{0,k}$ is the one sided power spectral density (PSD) of the additive white Gaussian noise (AWGN) at the input of R_k, $N_{0,D}$ is the one sided PSD of the AWGN power at the input of the destination D, $\alpha_{S,k}$ is the Rayleigh fading envelope of the channel over hop $S \to R_k$ and $\alpha_{D,k}$ is the Rayleigh fading envelope of the channel over the hop $R_k \to D$.

According to the selection method assumed, the branch (relay path) selection is based on the largest SNR criterion, given by

$$\max \left\{ \frac{E_S}{N_{0,k}} \frac{\alpha_{S,k}^2 (G_k \alpha_{D,k})^2}{(G_k \alpha_{D,k})^2 + N_{0,D}/N_{0,k}} \right\} \quad k \in \{X, Y\} \tag{3}$$

or, equivalently,

$$\max \left\{ \frac{\alpha_{S,k} (G_k \alpha_{D,k})}{\sqrt{(G_k \alpha_{D,k})^2 + 1}} \right\} \quad k \in \{X, Y\}, \tag{4}$$

where without loss of generality, we assume equal AWGN power, $N_{0,k} = N_{0,D}$, at the input of the relays and the destination terminal. In this case, the selection criterion (4) is transformed to

$$\max \left\{ \frac{X_1 X_2}{\sqrt{1 + X_2^2}}, \frac{Y_1 Y_2}{\sqrt{1 + Y_2^2}} \right\} \tag{5}$$

where $X_1 = \alpha_{S,X}$, $X_2 = G_X \alpha_{D,X}$, $Y_1 = \alpha_{S,Y}$, $Y_2 = G_Y \alpha_{D,Y}$ are Rayleigh random variables (RVs) with mean power $E[X_1^2] = \Omega_{X_1} = \Omega_{S,X}$, $E[X_2^2] = \Omega_{X_2} = G_X^2 \Omega_{D,X}$, $E[Y_1^2] = \Omega_{Y_1} = \Omega_{S,Y}$, and $E[Y_2^2] = \Omega_{Y_2} = G_Y^2 \Omega_{D,Y}$, respectively, where $E[\cdot]$ stands for the expectation.

B. Average Relaying Selection Rate (ARSR)

Theorem 1: The ARSR of the relay R_X in the diamond selective relaying system is given by

$$\begin{aligned} N_X = & \frac{1}{\sqrt{2\pi}} \frac{2^4}{\Omega_{X_1} \Omega_{X_2} \Omega_{Y_1} \Omega_{Y_2}} \\ & \times \int_0^\infty \int_0^\infty \int_0^\infty \frac{y_1^2 y_2^2 \sqrt{1 + x_2^2}}{\sqrt{1 + y_2^2}} \\ & \times \left(\sigma_{\dot{X}_1}^2 + \frac{y_1^2 y_2^2}{x_2^4 (1 + x_2^2)(1 + y_2^2)} \sigma_{\dot{X}_2}^2 + \frac{y_2^2 (1 + x_2^2)}{x_2^2 (1 + y_2^2)} \sigma_{\dot{Y}_1}^2 \right. \\ & \left. + \frac{y_1^2 (1 + x_2^2)}{x_2^2 (1 + y_2^2)^3} \sigma_{\dot{Y}_2}^2 \right)^{\frac{1}{2}} \\ & \times \exp \left[- \left(\frac{y_1^2 y_2^2 (1 + x_2^2)}{\Omega_{X_1} x_2^2 (1 + y_2^2)} + \frac{x_2^2}{\Omega_{X_2}} + \frac{y_1^2}{\Omega_{Y_1}} + \frac{y_2^2}{\Omega_{Y_2}} \right) \right] \\ & dx_2 dy_1 dy_2 \end{aligned} \tag{6}$$

where $\sigma_{\dot{k}_j}^2 = (\pi f_{dk_j})^2 \Omega_{k_j}$, is the variance of the envelope's time derivative $\dot{k}_j$ and f_{dk_j} is the maximum Doppler frequency shift induced by the motion of the relay and/or the destination terminal in the kth relay path and jth hop.

Proof: The ARSR, N_X, can be mathematically expressed as the average number of times the process, R, defined as

$$R = \frac{X_1 X_2 \sqrt{1 + Y_2^2}}{Y_1 Y_2 \sqrt{1 + X_2^2}} \tag{7}$$

crosses the level one per unit time (e.g. seconds) in positive going direction. In (7), X_1, X_2, Y_1 and Y_2 follow the Rayleigh distribution and have average powers $\Omega_{X_1}, \Omega_{X_2}, \Omega_{Y_1}$ and Ω_{Y_2}, respectively. Therefore, the ARSR of the relay R_k can be evaluated by using the Rician formula for the LCR estimated for level 1 [14], [15], i.e.

$$N_X = \int_0^\infty \dot{R} f_{R,\dot{R}}(1,\dot{R}) d\dot{R} \tag{8}$$

where $f_{R\dot{R}}(\cdot,\cdot)$ represents the joint PDF of the process R and its time derivative $\dot{R}$. By applying the time derivative in both sides of (7), we obtain

$$\begin{aligned}\dot{R} &= \frac{R}{X_1}\dot{X}_1 + \frac{R}{X_2\left(1+X_2^2\right)}\dot{X}_2 - \frac{R}{Y_1}\dot{Y}_1 - \frac{R}{Y_2\left(1+Y_2^2\right)}\dot{Y}_2 \\ &= \frac{X_2\sqrt{1+Y_2^2}}{Y_1Y_2\sqrt{1+X_2^2}}\dot{X}_1 + \frac{R}{X_2\left(1+X_2^2\right)}\dot{X}_2 - \frac{R}{Y_1}\dot{Y}_1 \\ &\quad - \frac{R}{Y_2\left(1+Y_2^2\right)}\dot{Y}_2\end{aligned} \tag{9}$$

where $\dot{X}_1, \dot{X}_2, \dot{Y}_1, \dot{Y}_2$ are the time derivatives of the respective envelopes. It is well-known that these envelopes and their time derivatives are independent RVs, with the time derivatives following the Gaussian PDF with zero mean and variances $\sigma^2_{\dot{X}_1}, \sigma^2_{\dot{X}_2}, \sigma^2_{\dot{Y}_1}$ and $\sigma^2_{\dot{Y}_2}$, respectively, whose values will be specified later in the text [14], [15]. Then, the joint PDF of R and $\dot{R}$ is obtained as

$$\begin{aligned}f_{R\dot{R}}(r,\dot{r}) = \int_0^\infty\int_0^\infty\int_0^\infty & f_{R\dot{R}|X_2Y_1Y_2}\left(r,\dot{r}\left|x_2,y_1,y_2\right.\right) \\ & \times f_{X_2}(x_2) f_{Y_1}(y_1) f_{Y_2}(y_2)\, dx_2 dy_1 dy_2\end{aligned} \tag{10}$$

where $f_{X_2}(\cdot), f_{Y_1}(\cdot)$ and $f_{Y_2}(\cdot)$ are the pdfs of the envelopes X_2, Y_1 and Y_2, respectively, given by (1).

If we fix $X_2 = x_2$ and $Y_1 = y_1$, the conditional joint PDF of R and $\dot{R}$ is given by

$$\begin{aligned}f_{R\dot{R}|X_2Y_1Y_2}\left(r,\dot{r}\left|x_2,y_1,y_2\right.\right) =& f_{\dot{R}|RX_2Y_1Y_2}\left(\dot{r}\left|r,x_2,y_1,y_2\right.\right) \\ & \times f_{R|X_2Y_1Y_2}\left(r\left|x_2,y_1,y_2\right.\right)\end{aligned} \tag{11}$$

whereas the second multiplier of (11) is determined as

$$\begin{aligned}f_{R|X_2Y_1Y_2}\left(r\left|x_2,y_1,y_2\right.\right) =& \frac{y_1y_2\sqrt{1+x_2^2}}{x_2\sqrt{1+y_2^2}} \\ & \times f_{X_1}\left(r\frac{y_1y_2\sqrt{1+x_2^2}}{x_2\sqrt{1+y_2^2}}\right)\end{aligned} \tag{12}$$

where $f_{X_1}(\cdot)$ is the pdf of envelope X_1, given by (1). Substitution of (11) and (12) into (10), then (10) into (8) and changing the order of integration, yields

$$\begin{aligned}N_X = \int_0^\infty\int_0^\infty\int_0^\infty & \left[\int_0^\infty \dot{r} f_{\dot{R}|RX_2Y_1Y_2}\left(\dot{r}\left|1,x_2,y_1,y_2\right.\right) d\dot{r}\right] \\ & \times \frac{y_1y_2\sqrt{1+x_2^2}}{x_2\sqrt{1+y_2^2}} f_{X_1}\left(\frac{y_1y_2\sqrt{1+x_2^2}}{x_2\sqrt{1+y_2^2}}\right) \\ & \times f_{X_2}(x_2) f_{Y_1}(y_1) f_{Y_2}(y_2)\, dx_2 dy_1 dy_2\end{aligned} \tag{13}$$

When RVs R, X_2, Y_1 and Y_2 are fixed, $\dot{R}$ is a zero mean Gaussian RV, thus the bracket integral in (13) can be solved as

$$\begin{aligned}&\int_0^\infty \dot{r} f_{\dot{R}|RX_2Y_1Y_2}\left(\dot{r}\left|1,x_2,y_1,y_2\right.\right) d\dot{r} \\ &= \frac{1}{\sqrt{2\pi}}\frac{x_2\sqrt{1+y_2^2}}{y_1y_2\sqrt{1+x_2^2}} \times \left[\sigma^2_{\dot{X}_1} + \frac{y_1^2y_2^2}{x_2^4\left(1+x_2^2\right)\left(1+y_2^2\right)}\sigma^2_{\dot{X}_2}\right. \\ &\quad \left. + \frac{y_2^2\left(1+x_2^2\right)}{x_2^2\left(1+y_2^2\right)}\sigma^2_{\dot{Y}_1} + \frac{y_1^2\left(1+x_2^2\right)}{x_2^2\left(1+y_2^2\right)^3}\sigma^2_{\dot{Y}_2}\right]^{\frac{1}{2}}\end{aligned} \tag{14}$$

By introducing (1) and (14) into (13), one obtains the triple integral for the ARSR given in (6). ■

Note that the ARSR of R_Y, N_Y, is mathematically expressed as the rate at which the process $\tilde{R}$ defined by

$$\tilde{R} = \frac{1}{R} = \frac{Y_1Y_2\sqrt{1+X_2^2}}{X_1X_2\sqrt{1+Y_2^2}} \tag{15}$$

crosses the level 1 in positive going direction. Following a similar analysis as for *Theorem 1* and interchange the RVs X_j with Y_j in (6), N_Y can be easily derived.

Now, by using $x = x_2^2/\Omega_{X_2}$, $y = y_1^2/\Omega_{Y_1}$ and $z = y_2^2/\Omega_{Y_2}$ into (6), N_X can be expressed as

$$N_X = \int_0^\infty\int_0^\infty\int_0^\infty f(x,y,z)\, e^{-x}e^{-y}e^{-z} dx dy dz \tag{16}$$

where $f(x,y,z)$ is an exponentially decreasing function. Hence, N_x can be efficiently evaluated with sufficient accuracy using the Gauss-Laguerre Quadrature (GLQ) numerical integration method [16, p. 923] as

$$N_X \approx \sum_{\ell=1}^{t}\sum_{m=1}^{t}\sum_{n=1}^{t} w_\ell w_m w_n f(x_\ell, y_m, z_n) \tag{17}$$

where the abscissas x_ℓ, y_m, z_n are the ℓ^{th}, m^{th}, and n^{th} roots, respectively, of the Laguerre polynomial of order t and w_ℓ, w_m and w_n are the respective weights defined in [16, eq. (25.4.45)]. Both the roots and the weights are tabulated in [16, p. 923, Table (25.9)]. An excellent match between the results of the proposed numerical integration method and Mathematica software's ones, can be easily shown.

C. Average Relaying Utilization Time (ARUT)

Theorem 2: The ARUT of the relay R_X in the diamond selective relaying system is given by

$$\begin{aligned}T_X =& \frac{1}{N_X}\left[1 + \frac{1}{\Omega_{X_1}}\left(\Omega_{X_1} - \left(a\Omega_{X_1}\right)^2 e^{a^2\Omega_{X_1}}\Gamma\left(0, a^2\Omega_{X_1}\right)\right)\right. \\ & - \frac{4ab}{\Omega_{X_1}}\int_0^\infty z e^{-z\left(\frac{1}{\Omega_{X_1}}+\frac{1}{\Omega_{Y_1}}\right)} K_1\left(2\sqrt{az}\right)K_1\left(2\sqrt{bz}\right) dz \\ & \left. - 4a^2b\int_0^\infty z^{\frac{1}{2}} e^{-\frac{z}{\Omega_{Y_1}}} K_0\left(2\sqrt{az}\right)K_1\left(2\sqrt{bz}\right) dz\right]\end{aligned} \tag{18}$$

where $a = \sqrt{\frac{1}{\Omega_{X_1}\Omega_{X_2}}}$, $b = \sqrt{\frac{1}{\Omega_{Y_1}\Omega_{Y_2}}}$, $\Gamma(\cdot,\cdot)$ is the incomplete Gamma function defined in [17, eq. (8.350.1)] and $K_v(\cdot)$

is the υth order modified Bessel function of the second kind defined in [16, eq.(9.6.22)].

Proof: The ARUT for the relay R_k can be mathematically defined as the ratio

$$T_k = \frac{P_k}{N_k} \tag{19}$$

where P_k denotes the probability that R_k starts the relaying process.

Let us assume that R_X is the selected relay, then

$$\begin{aligned} P_X &= \Pr\{\gamma_X \geq \gamma_Y\} = \Pr\{R \geq 1\} \\ &= \Pr\left\{\frac{X_1 X_2}{\sqrt{1+X_2^2}} \geq \frac{Y_1 Y_2}{\sqrt{1+Y_2^2}}\right\} \\ &= \Pr\{Z \geq W\} \end{aligned} \tag{20}$$

where

$$Z = \frac{X_1^2 X_2^2}{1+X_2^2} \tag{21}$$

and

$$W = \frac{Y_1^2 Y_2^2}{1+Y_2^2} \tag{22}$$

Note that X_1^2, X_2^2, Y_1^2 and Y_2^2 are exponentially distributed RVs with means, Ω_{X_1}, Ω_{X_2}, Ω_{Y_1} and Ω_{Y_2}, respectively. Then, with the help of [18, eq. (6-42)], (20) can be expressed as

$$\begin{aligned} P_X = \Pr[Z \geq W] &= \int_0^\infty f_Z(z) \int_0^z f_W(w) dw dz \\ &= \int_0^\infty f_Z(z) F_W(z) dz \end{aligned} \tag{23}$$

where $f_Z(z)$ and $F_W(w)$ are the pdf and cdf of the RVs Z, W given by [13, eq. (10)]

$$\begin{aligned} f_Z(z) =& \frac{2}{\Omega_{X_1}} e^{-\frac{z}{\Omega_{X_1}}} \left[\sqrt{\frac{z}{\Omega_{X_1}\Omega_{X_2}}} K_1\left(2\sqrt{\frac{z}{\Omega_{X_1}\Omega_{X_2}}}\right) \right. \\ & \left. + \frac{1}{\Omega_{X_2}} K_0\left(2\sqrt{\frac{z}{\Omega_{X_1}\Omega_{X_2}}}\right)\right] \end{aligned} \tag{24}$$

$$F_W(w) = 1 - 2\sqrt{\frac{w}{\Omega_{Y_1}\Omega_{Y_2}}} e^{-\frac{w}{\Omega_{Y_1}}} K_1\left(2\sqrt{\frac{w}{\Omega_{Y_1}\Omega_{Y_2}}}\right), \tag{25}$$

respectively. By introducing (24) and (25) in (23) we have

$$\begin{aligned} P_X =& \frac{2}{\Omega_{X_1}} \int_0^\infty e^{-\frac{z}{\Omega_{X_1}}} \\ & \times \left[\sqrt{\frac{z}{\Omega_{X_1}\Omega_{X_2}}} K_1\left(2\sqrt{\frac{z}{\Omega_{X_1}\Omega_{X_2}}}\right) \right. \\ & \left. + \frac{1}{\Omega_{X_2}} K_0\left(2\sqrt{\frac{z}{\Omega_{X_1}\Omega_{X_2}}}\right)\right] \\ & \times \left(1 - 2\sqrt{\frac{z}{\Omega_{Y_1}\Omega_{Y_2}}} e^{-\frac{z}{\Omega_{Y_1}}} K_1\left(2\sqrt{\frac{z}{\Omega_{Y_1}\Omega_{Y_2}}}\right)\right) dz \end{aligned} \tag{26}$$

or after some simplifications, (26) is finally obtained as

$$P_X = I_1 + I_2 - I_3 - I_4 \tag{27}$$

where

$$\begin{aligned} I_1 &= \frac{2a}{\Omega_{X_1}} \int_0^\infty z^{\frac{1}{2}} e^{-\frac{z}{\Omega_{X_1}}} K_1\left(2\sqrt{az}\right) dz \\ &= \frac{1}{\Omega_{X_1}} \left[\Omega_{X_1} - \left(a\Omega_{X_1}\right)^2 e^{a^2\Omega_{X_1}} \Gamma\left(0, a^2\Omega_{X_1}\right)\right], \end{aligned} \tag{28}$$

$$I_2 = 2a^2 \int_0^\infty K_0\left(2\sqrt{az}\right) dz = 1, \tag{29}$$

$$I_3 = \frac{4ab}{\Omega_{X_1}} \int_0^\infty z e^{-z\left(\frac{1}{\Omega_{X_1}} + \frac{1}{\Omega_{Y_1}}\right)} K_1\left(2\sqrt{az}\right) K_1\left(2\sqrt{bz}\right) dz, \tag{30}$$

$$I_4 = 4a^2 b \int_0^\infty z^{\frac{1}{2}} e^{-\frac{z}{\Omega_{Y_1}}} K_0\left(2\sqrt{az}\right) K_1\left(2\sqrt{bz}\right) dz. \tag{31}$$

The integral I_1 can be solved with the help of [17, eq. (6.631.3)] and [16, eqs. (13.1.33) and (13.6.28)]. The integral I_2 is obtained using [17, eq. (6.561.16)]. As in *Theorem 1*, the integrals I_3 and I_4 can be evaluated via the GLQ numerical integration method. ■

The ARUT T_Y can be derived by following a similar procedure as in *Theorem 2* and interchange the RVs X_j with Y_j in (18).

D. Average Relaying Waiting Time (ARWT)

Corollary 1: The ARWT of the relay R_k in the diamond selective relaying system is given by

$$W_k = \frac{1 - P_k}{N_k} = \frac{1}{N_k} - T_k. \tag{32}$$

IV. NUMERICAL EXAMPLES AND SIMULATIONS

Figs. 2-3 illustrate the behavior of the introduced statistical performance metrics (ARSR, ARUT) for the diamond amplify-and-forward relaying systems under the following assumptions:

i) The source S and destination D are fixed, whereas the relays R_X and R_Y are mobile with same speeds. Thus, we have four fixed-to-mobile channels X_1, X_2, Y_1 and Y_2 with same maximum Doppler frequency spread, which we assume to be mutually equal, i.e. $f_{dX_1} = f_{dX_2} = f_{dY_1} = f_{dY_2} = f_{doppler}$. In such a case of all equal maximum Doppler spreads (all being equal to $f_{doppler}$), a linear dependence exists between the ARSR and $f_{doppler}$, which allows generalization of results by normalization of the utilization statistics. Therefore, the parameters ARSR and ARUT in the plots are normalized as $N_X/f_{doppler}$, $T_X \cdot f_{doppler}$, and $W_X \cdot f_{doppler}$, respectively,

ii) $\Omega_{X_1} = \Omega_{X_2} = \Omega_X$ and $\Omega_{Y_1} = \Omega_{Y_2} = \Omega_Y$, and

iii) $\Omega_Y = -10$ dB or $\Omega_Y = 0$ dB

These figures depict the dependence of the above normalized parameters from the mean power Ω_X expressed in dB, which helps to determine the effect of the unbalancing effect of the two hops on the above mentioned normalized parameters. Each figure consists of two sets (depending on the value of Ω_Y) of three curves: curve 1 - obtained by using the explained GLQ approximation approach with n = 30, curve 2 - obtained

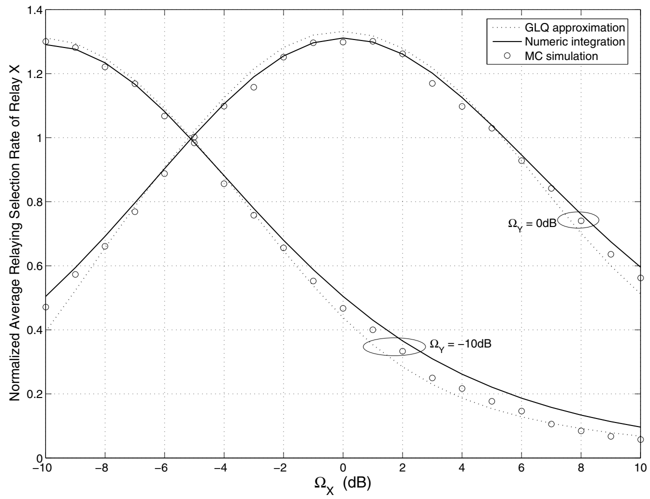

Fig. 2. Effect of the hops' unbalancing on the normalized N_X.

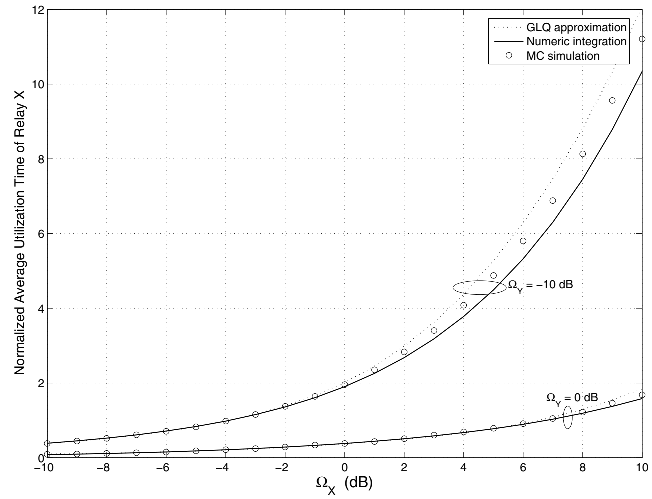

Fig. 3. Effect of the hops' unbalancing on the normalized T_X.

by numerical integration of (18) and (30) using software tools such as Mathematica, and curve 3 - obtained by Monte Carlo simulation.

In Fig. 2 the normalized N_X is illustrated versus the Ω_X. Obviously, for small values of Ω_X (lower than 0 dB), the selection of R_X does not happen very often and the system selects R_Y to relay the information data. The selection rate of R_X increases as the Ω_X increases and it is maximized when $\Omega_X = \Omega_Y$ i.e., the fading conditions in two branches is the same in an average sense and therefore the selection of each relay is equiprobable. Moreover, when Ω_X is greater than 0 dB, the normalized N_X decreases since the average channel conditions of branch X is better than Y and the system stays almost continuously with R_X to relay the information signal.

In Fig. 3 the normalized T_X versus the ratio Ω_X, is plotted. It can be easily observed that as the fading conditions of the branch X improved, the utilization time of R_X increases.

V. Conclusion

We introduced new relaying utilization metrics, the average relaying selection rate, the average relaying utilization time, and the average relaying waiting time, which describe the contribution of a relay node to the relaying process in cooperative diversity systems. These metrics are based on the second order statistics of the relay fading channel and quantify how often a relay is being selected or how long it remains active or not to convey the information signal from the source to the destination terminal. Useful results were also extracted by applying these metrics to dual branch cooperative systems with diamond topology over Rayleigh fading channels.

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