Optimized "Better Than" Raised-Cosine Pulse for Reduced ICI in OFDM Systems

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***Abstract*—The problem of inter-carrier interference (ICI) in orthogonal frequency-division multiplexing (OFDM) systems has a dramatic impact on their overall performance and robustness. For this reason, various techniques have been proposed in order to eliminate this undesired phenomenon with the most common being the use of transmitter pulse-shaping. Capitalizing on some previous investigations, we herein propose a class of optimized "better-than" raised cosine (BTRC) pulses which makes use of an additional parameter to suppress the ICI effects. More importantly, the optimal value of this new parameter is found to follow a quadratic relationship with the filter's roll-off factor; consequently, this important property can significantly facilitate the design of practical robust OFDM receivers for use in wireless communication systems.**

***Index Terms*—Carrier frequency offset, inter-carrier interference (ICI), orthogonal frequency-division multiplexing (OFDM).**

I. Introduction

It is widely known that over the last years, orthogonal frequency-division multiplexing (OFDM) has attracted an extensive amount of research and industry interest and has been deployed in diverse applications, spanning IEEE 802.11a to Digital Video Broadcasting (DVB). In addition, OFDM technology is expected to be incorporated in many emerging multiple-input multiple-output (MIMO) standards, such as IEEE 802.11n, IEEE 802.20 and IEEE 802.22 [1, Table I]. The main advantage of OFDM is its ability to decompose a frequency-selective fading channel into several nearly flat-fading channels. By doing so, an improved spectral efficiency is achieved along with a greater robustness against the effects of multipath fading and inter-symbol interference (ISI). One of the main disadvantages of OFDM though, is the sensitivity to carrier frequency offset which causes attenuation and rotation of subcarriers and ultimately leads to inter-carrier interference (ICI) between adjacent subcarriers [2]–[4]. The effects of the latter were examined in detail in [2].

A well established technique to overcome this undesired phenomenon is through the use of pulse-shaping filters [5], [6]. In general, there are three different types of filters that can be employed. The first category contains pulses of infinite time duration, normally implemented via the Weyl-Heisenberg frames [7], [8] or Hermite waveforms [9]. The second category includes pulses of finite duration whose length is longer than one OFDM symbol interval with the most seminal work being documented in [10]. The last category comprises pulses which have a predefined length of one OFDM symbol interval and are normally designed as Nyquist pulses in the time-domain (see for instance [11]–[16] and references therein). For a contemporary review of pulse shaping techniques for OFDM systems and its applications to wireless communications, the interested readers are referred to [17].

More specifically, the authors in [11] proposed the use of the so-called *better than raised-cosine* (BTRC) or *Beaulieu* pulse [18] which yields a superior performance compared to the conventional rectangular and raised-cosine (RC) pulses. The adaptive *second-order continuous window* (SOCW) pulses of [12], [13] may offer a marginally superior performance than BTRC but its optimal parameters are inherently dependent on the sample location, frequency offset and a series of elementary functions. Similarly, the *polynomial* pulse of [14] needs a set of properly chosen parameters whereas the *sinc power* pulse documented in [15], along with its extension in [16], require the numerical optimization of the filter degree and variance parameter, respectively.

The main goal of this contribution is to investigate the performance of the so-called optimized BTRC (OBTRC) pulses, originally devised in [19], with a view to ICI suppression in OFDM systems. These pulses employ an additional free parameter (FP) whose optimal value is shown to be in quadratic relationship with the filter's roll-off factor; thus, any member of the proposed family can be explicitly and *a priori* designed so that an enhanced performance of OFDM systems is attained. We highlight the fact that BTRC may not be the most robust alternative for ICI suppression [17] but its frequency response lends to tractable manipulations and, more importantly, can be efficiently optimized.

The remainder of this paper is organized as follows: In Section II, the OFDM system used throughout the paper is outlined along with the BTRC and OBTRC filters' responses. The ICI analysis is covered in Section III followed by the optimization of the OBTRC FP. A set of numerical results is given in Section IV. Finally, Section V concludes the paper and summarizes the key findings.

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II. System Model

We consider a single OFDM block consisting of N subcarriers with f_k being the frequency of the kth subcarrier and a_k the data symbol transmitted over it ($0 \leq k \leq N-1$). We also denote the minimum subcarrier frequency spacing as $1/T$. In the case of noiseless transmission, the received OFDM signal $r(t)$ after down-conversion is succinctly expressed as [2], [11]

$$r(t) = e^{j(2\pi \Delta f t + \theta)} \sum_{k=0}^{N-1} a_k p(t) e^{j2\pi f_k t} \tag{1}$$

where $p(t)$ is the time-limited pulse-shaping function, Δf is the frequency offset and θ the phase error. We note that the last two effects are usually caused by channel distortion, Doppler shifts or synchronization mismatches between the transmitter and receiver oscillators. As was previously mentioned, we are particularly interested in the BTRC and OBTRC pulses; the time-domain response of the former reads as [18]

$$p_{\text{BTRC}}(t) = \begin{cases} \frac{1}{T}, & 0 \leq |t| \leq \frac{T(1-\alpha)}{2} \\ \frac{1}{T} \exp\left(\gamma_1 \left(|t| - \frac{T(1-\alpha)}{2}\right)\right), & \frac{T(1-\alpha)}{2} \leq |t| \leq \frac{T}{2} \\ \frac{1}{T} \left\{1 - \exp\left(\gamma_1 \left(\frac{T(1+\alpha)}{2} - |t|\right)\right)\right\}, & \frac{T}{2} \leq |t| \leq \frac{T(1+\alpha)}{2} \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

where α corresponds to the roll-off factor ($0 \leq \alpha \leq 1$), determining the bandwidth occupied by the pulse and the time-domain tail suppression, while γ_1 is a constant defined as

$$\gamma_1 = -\frac{2 \ln 2}{\alpha T}. \tag{3}$$

Please note that the frequency response of the BTRC pulse was given in an explicit form in [18, Eq. (4)]. The time-response of the OBTRC pulse, as given in [19], is essentially a parametric version of (2), that is

$$p_{\text{OBTRC}}(t) = \begin{cases} \frac{1}{T}, & 0 \leq |t| \leq \frac{T(1-\alpha)}{2} \\ \frac{1}{T} \exp\left(\gamma_n \left(|t| - \frac{T(1-\alpha)}{2}\right)^n\right), & \frac{T(1-\alpha)}{2} \leq |t| \leq \frac{T}{2} \\ \frac{1}{T} \left\{1 - \exp\left(\gamma_n \left(\frac{T(1+\alpha)}{2} - |t|\right)^n\right)\right\}, & \frac{T}{2} \leq |t| \leq \frac{T(1+\alpha)}{2} \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

with n being the FP of the filter, which determines different pulses, and γ_n a constant defined as

$$\gamma_n = -\frac{2^n \ln 2}{(\alpha T)^n}. \tag{5}$$

To the best of our knowledge, no closed-form expression exists for the frequency response of the OBTRC pulse. However, the latter can be efficiently evaluated through a numerical inverse Fourier transform. We can easily infer that when $n = 1$ the two pulses coincide, whereas for $\alpha = 0$ they both degenerate into the rectangular pulse.

III. ICI Analysis and Performance Optimization

Using a similar line of reasoning as in [11]–[16], we can express the decision variable, $\hat{a}_m$, at the m-th subchannel demodulator according to

$$\hat{a}_m = a_m e^{j\theta} P(-\Delta f) + e^{j\theta} \sum_{\substack{k=0 \\ k \neq m}}^{N-1} a_k P\left(\frac{m-k}{T} + \Delta f\right) \tag{6}$$

with $P(f)$ being the Fourier transform of $p(t)$.[1] Then, the instantaneous power of the desired signal is $\sigma_m^2 = |a_m|^2 \left|P(\Delta f)\right|^2$, while the ICI power reads as

$$\sigma_{ICI}^m = \sum_{\substack{k=0 \\ k \neq m}}^{N-1} \sum_{\substack{n=0 \\ n \neq m}}^{N-1} a_k a_n^* P\left(\frac{k-m}{T} + \Delta f\right) P\left(\frac{n-m}{T} + \Delta f\right) \tag{7}$$

where $(\cdot)^*$ denotes the conjugate operation.

After introducing the common assumption that $\{a_n\}$ is a zero-mean independent sequence of unit variance, we can obtain the mean ICI power for the m-th subchannel, averaged across different data sequences, which for the considered case may be written as

$$\overline{\sigma_{ICI}^m} = \sum_{\substack{k=0 \\ k \neq m}}^{N-1} \left|P\left(\frac{k-m}{T} + \Delta f\right)\right|^2. \tag{8}$$

As seen in (8), the average ICI power depends on the number of subcarriers, the windowing function and the spectral magnitudes of $P(f)$ at the frequencies

$$\left(\frac{k-m}{T} + \Delta f\right), \quad k \neq m, \ k = 0, 1, \ldots, N-1. \tag{9}$$

For $\Delta f = 0$, no ICI occurs since the frequency spectra induce, by definition, nulls at the points $((k-m)/T, k \neq m)$ [11].

Another common figure of merit when assessing the performance of OFDM systems in the presence of frequency offset, is the so-called Signal-to-Interference ratio (SIR), normally defined as the ratio of the averaged signal power over the averaged ICI power

$$SIR = \frac{\left|P(\Delta f)\right|^2}{\sum_{\substack{k=0 \\ k \neq m}}^{N-1} \left|P\left(\frac{k-m}{T} + \Delta f\right)\right|^2}. \tag{10}$$

In the following analysis, our main goal is to maximize the SIR of the proposed OBTRC pulse for any value of the frequency offset. Hence, we settle to find the optimal FP of the OBTRC pulse, n_{opt}, which will yield the largest possible SIR compared to the conventional BTRC pulse. This sophisticated optimization problem aims to maximize the difference between the areas under the two SIR curves, across the entire range of

[1] A detailed mathematical derivation can be found in [11].

normalized frequency offsets ΔfT. Mathematically speaking, n_{opt} is the solution of

$$n_{\text{opt}}(\alpha) = \arg\max_{n} \left\{ \int_0^1 \left(\text{SIR}\left(\Delta fT|\alpha, n\right)_{\text{OBTRC}} - \text{SIR}\left(\Delta fT|\alpha\right)_{\text{BTRC}} \right) d(\Delta fT) \right\}. \tag{11}$$

The optimization problem in (11) can be efficiently solved by applying some of the well-known numerical optimization methods, like the computationally compact Nelder-Mead "simplex" technique [20], included in most software packages (e.g., Mathematica, MATLAB). Our analysis showed that the solution to this optimization problem indicates that a quadratic relationship does exist between α and n_{opt}

$$n_{\text{opt}} = 0.51263\alpha^2 + 0.60786\alpha - 0.04096. \tag{12}$$

From a practical perspective, we emphasize the fact that optimal pulse-shaping filters can be efficiently designed and implemented for any given value of the roll-off factor α. This fundamental property can potentially facilitate the design of robust OFDM receivers to the direction of combating the undesired implications of ICI.

In Fig. 1, the derived closed-form quadratic relationship is validated where we can easily observe the precise of the numerical results with the regression curve. The fact that for low values of the roll-off factor ($\alpha \leq 0.12$), n_{opt} assumes negative values is not surprising since, by definition, n can obtain any real value and, further, as $\alpha \to 0$ the sign of n has a diminishing impact on the filter's shape; this means that for small α both positive and negative values of n lead to almost identical characteristics [19].

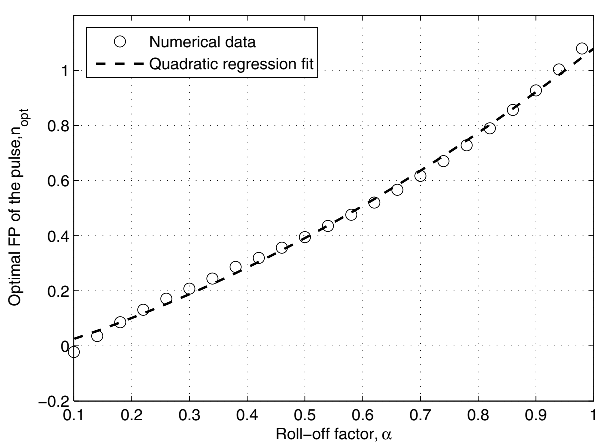

Fig. 1. Quadratic relationship between the optimal FP of the OBTRC pulse, n_{opt}, and the roll-off factor α.

IV. Numerical Results

In this section, by appropriately choosing the optimal FP of the OBTRC pulse, we investigate its performance compared to the original reference BTRC pulse. We begin with the frequency spectra which can provide a deeper insight into the decay rate and sidelobe suppression (see Fig. 2). Henceforth, an OFDM system with 64 subcarriers is considered while the roll-off factor is set equal to both $\alpha = 0.2$ and $\alpha = 0.6$. Substituting these values into (12), we can readily obtain $n_{\text{opt}} = 0.1011$ and $n_{\text{opt}} = 0.5083$, respectively. In Fig. 2, it is illustrated that OBTRC pulses offer a more narrow main lobe and a marginally smaller principal sidelobe (for $\alpha = 0.2$). As was originally pointed out in [11], these key characteristics, and especially the main lobe width, determine the extent to which each pulse-shaping filter is prone to frequency offset.

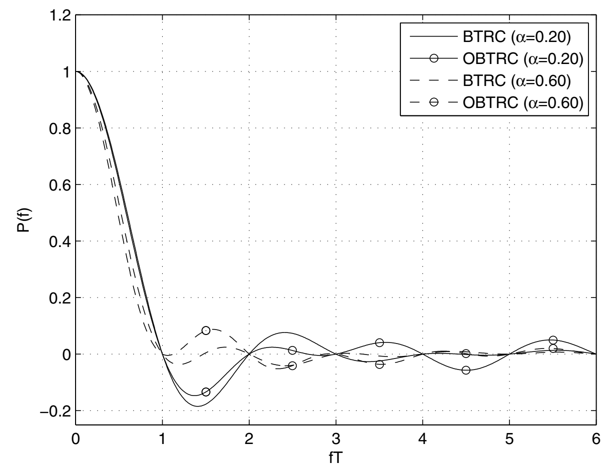

Fig. 2. Frequency spectra of the BTRC and OBTRC pulses ($\alpha = 0.2$ and $\alpha = 0.6$).

At a next stage, the ICI power evolution is explored as a function of the normalized frequency offset (c.f. Fig. 3). It is worth highlighting the systematic better performance of OBTRC over BTRC pulse which is more pronounced for low and moderate values of the frequency offset and higher values of α. For instance, for $\alpha = 0.60$ and $\Delta fT = 0.1$ an improvement of 2.07 dB is achieved whereas for $\alpha = 0.2$ this improvement reduces to 0.80 dB. Moreover, it can be seen that higher values of the roll-off factor lead to stronger ICI suppression; this trend is expected since, in general, increasing α leads to weaker sidelobes in the frequency spectrum of any pulse-shaping filter. On the other hand though, a larger α leads to higher spectral efficiency sacrifice, if the zero ISI prerequisite is to be maintained. We recall that these observations are consistent with the findings of [11], [13].

In Fig. 4, the averaged SIR of BTRC and OBTRC pulses is plotted against the frequency offset. Once more, the superiority of the proposed pulse is apparent across the entire range of the ΔfT values. More specifically, for an offset of $\Delta fT = 0.1$ and a roll-factor of $\alpha = 0.6$, the SIR difference is 2.02 dB; when $\alpha = 0.2$ the SIR improvement reduces to 0.79 dB. From inspection of Fig. 3 and 4, it is clear that as ΔfT gets large enough both filters asymptotically yield the same performance which implies that the critical characteristics of each filter

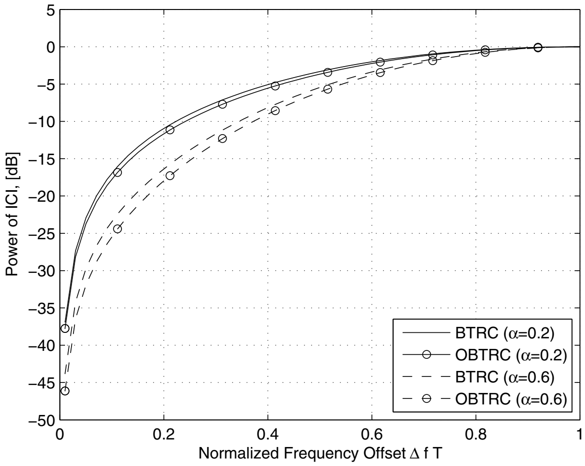

Fig. 3. Comparison of ICI power for the BTRC and OBTRC pulses in a 64-subcarrier OFDM system ($\alpha = 0.2$ and $\alpha = 0.6$).

are not significantly important in the high ΔfT regime. Referring back to (10), we can attribute this phenomenon to the increasing displacement of the mean ICI power from the zero-crossing points, as given in (9), with ΔfT.

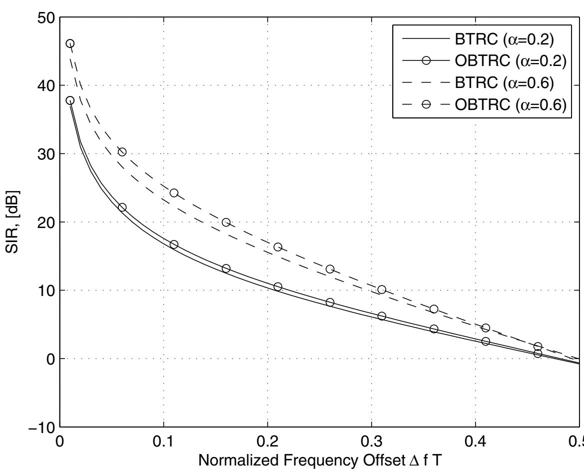

Fig. 4. Comparison of SIR for the BTRC and OBTRC pulses in a 64-subcarrier OFDM system ($\alpha = 0.2$ and $\alpha = 0.6$).

V. Conclusion

In this paper, we were mainly focused on optimizing the ICI performance of the BTRC pulse-shaping filters for use in OFDM systems. This was achieved with the aid of an additional free parameter whose optimal value was found to follow a quadratic relationship with the filter's roll-off factor, thereby implying that robust OFDM receivers can be easily designed. The effects of frequency offset were assessed in detail and it was demonstrated that the proposed OBTRC pulses yield a consistently enhanced performance in terms of ICI power reduction and SIR, especially for low and moderate values of the normalized frequency offset.

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