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Average Spectral Efficiency of Opportunistic QRD-based Cyclic Prefixed Single-Carrier Cooperative Diversity Systems with Power Allocation

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***Abstract*—Due to higher peak-to-average power ratio and higher power backing-off in OFDM relaying systems, a cyclic prefixed single-carrier cooperative diversity system is considered in this paper. Under the transmission power constraint, the joint optimal power allocation to the source and relay node is investigated. For a two-hop decode-and-forward relaying protocol, the optimal power allocation is first obtained in the considered system, and then the opportunistic destination terminal selection is applied to improve the achievable average spectral efficiency (ASE). Based on the proposed QR decomposition (QRD)-based receiver in the destination terminal, closed-form expressions for the maximum achievable ASE are derived. Simulation results verify the derived closed-form analytical expressions.**

I. Introduction

For relaying transmission, several power allocation approaches have been proposed in [1] over flat fading channel environments. One particular approach is to apply a joint optimal power allocation [2]–[4] to the source and relay in the decode-and-forward (DF) relaying protocol [5], [6]. In [2], an unified optimal power allocation has been proposed for different relaying protocols. Considering a large peak-to-average power ratio (PAPR) and large power-backing off in the OFDM transmission, the cyclic prefixed (CP) single-carrier (SC) transmission [7] has been started to be adopted in mmWave Wireless Personal Area Networks (WPANs) [8], [9] systems. To the best of our knowledge, very few researches [10], [11] consider the CP SC-based relaying system without considering a joint power allocation in the system.

In this paper, we first apply the joint power allocation or standard max-min problem [4] to the source and relay in the CP SC-based relaying transmission system using the non-diversity DF relaying protocol that has no direct path between the source and the destination terminal. After then, we apply the opportunistic destination terminal selection mechanism [12] to select one desired terminal being used in the actual transmission among a set of destination terminals. As shown in [12], the opportunistic user selection can improve the effective spectral efficiency by virtue of the multiuser diversity. Moreover, we will show that the improved spectral efficiency resulted from the multiuser diversity can be maintained by the multipath diversity in the CP SC-based transmission. To exploit the multipath diversity gain and apply the time-domain equalization, we use the QR decomposition (QRD)-based receiver in the destination [13]. Using properties of the circulant channel matrices in the relay links, we are able to compute the effective source-to-destination signal-to-noise ratio (ESNR) efficiently and then select the desired terminal that has the strongest ESNR among a set of terminals. After deciding the selected user terminal at the source, the actual transmission can be made from the source to the desired terminal via the relay node. To verify the performance of the proposed CP SC system, we use the statistical properties of the derived source-to-destination ESNRs, which make us to obtain closed-form expressions for the average spectral efficiency (ASE).

Notation. The superscripts ^{T} and ^{H} stand for transposition and conjugate transposition, respectively. $E\{\cdot\}$ denotes statistical expectation; $\boldsymbol{I}_N$ is the $N \times N$ identity matrix; $\mathbf{0}$ stands for an all zeros matrix of appropriate dimensions; $\mathcal{CN}\left(\mu, \sigma^2\right)$ denotes circularly symmetric complex Gaussian distribution with mean μ and variance σ^2; $\|\boldsymbol{x}\|$ denotes the norm of the vector $\boldsymbol{x}$; and the (i, j)-th element of the matrix $\boldsymbol{A}$ is denoted by $(\boldsymbol{A})_{i,j}$.

II. System and Channel Model

In the employed dual-hop relay system under consideration, the source node (S) communicates with the destination node (D) via one relay node (R). The CP SC-based is used as a transmission technique whereas we assume a two-hop DF relaying transmission as a relay protocol [14]. The symbol block size is N. In the first hop, the source node broadcasts its transmission symbol block $\boldsymbol{d}(2n) \in \mathbb{C}^N$, $\boldsymbol{d}(2n) = [d_1(2n), .., d_N(2n)]^T$, to the relay and destination through corresponding channels. In the second hop, the relay node first decodes the transmitted symbol from the received vector signal and transmits the message to the destination after re-encoding. A CP of N_g symbols is appended to the front of the transmission block to prevent inter-symbol interference (ISI). The instantaneous channel between the source and the relay node is denoted by $\boldsymbol{h}^{S-R}(2n) \triangleq [h_0^{S-R}(2n), .., h_{N_f-1}^{S-R}(2n)]^T$ with N_f being the channel order. Similarly, the instantaneous channel between the source and the destination is denoted by $\boldsymbol{h}^{S-D}(2n) \triangleq [h_0^{S-D}(2n), .., h_{N_f-1}^{S-D}(2n)]^T$. After eliminating CP, the received vector signal at the relay node becomes

$$\tilde{\boldsymbol{y}}^{S-R}(2n) = \sqrt{P_s}\alpha^{S-R}\boldsymbol{H}_{cir}^{S-R}(2n)\boldsymbol{d}(2n) + \boldsymbol{n}^{S-R}(2n), \quad (1)$$

where P_s is the power allocated to the source, $\boldsymbol{H}_{cir}^{S-R}(2n) \in \mathbb{C}^{N\times N}$ is a time variant circulant matrix such that its (j,l)-th element is given by $(\boldsymbol{H}_{cir}^{S-R}(2n))_{j,l} = h_{<j-l>_N}^{S-R}(2n)$, and $\boldsymbol{n}^{S-R}(2n) \sim \mathcal{CN}(\boldsymbol{0}, \sigma_n^2 \boldsymbol{I}_N)$. In addition, the factor α^{S-R} comprises the effects of pathloss for the channel between the source and relay, and is normalized with respect to the distance between the source and destination. Let α^{S-D} be a constant reflecting the path loss between the source and destination. The received vector signal at the destination is given by

$$\tilde{\boldsymbol{y}}^{S-D}(2n) = \sqrt{P_s}\alpha^{S-D}\boldsymbol{H}_{cir}^{S-D}(2n)\boldsymbol{d}(2n) + \boldsymbol{n}^{S-D}(2n), \quad (2)$$

where $(\boldsymbol{H}_{cir}^{S-D}(2n))_{j,l} = h_{<j-l>_N}^{S-D}(2n)$ and $\boldsymbol{n}^{S-D}(2n) \sim \mathcal{CN}(\boldsymbol{0}, \sigma_n^2 \boldsymbol{I}_{N\times N})$.

After receiving $\tilde{\boldsymbol{y}}^{S-R}(2n)$, the relay node applies its own decoding and re-encoding, and then transmits to the destination, such that the received vector signal in the second hop at the destination is expressed as follows

$$\begin{aligned}\tilde{\boldsymbol{y}}^{R-D}(2n+1) = \; & \sqrt{P_r}\alpha^{R-D}\boldsymbol{H}_{cir}^{R-D}(2n+1)\boldsymbol{d}(2n) \\ & +\boldsymbol{n}^{R-D}(2n+1), \end{aligned} \quad (3)$$

where $(\boldsymbol{H}_{cir}^{R-D}(2n+1))_{j,l} = h_{<j-l>_N}^{R-D}(2n+1)$ with $\boldsymbol{h}^{R-D}(2n+1) \triangleq [h_0^{R-D}(2n+1), .., h_{N_f-1}^{R-D}(2n+1)]^T$, $\boldsymbol{n}^{R-D}(2n+1) \sim \mathcal{CN}(\boldsymbol{0}, \sigma_n^2 \boldsymbol{I}_{N\times N})$ and P_r is the power allocated to the relay. Also, α^{R-D} is similarly defined to take into account the path loss between the relay and destination. Since when the relay is fail to decode $\boldsymbol{d}(2n)$ correctly, there will be no transmission, we assume that the symbol block is decoded correctly in the relay. All the channels are assumed to be remain static over $N+N_g$ symbols and the length of the CP, N_g, is also assumed to be bigger than N_f.

Observing that $\boldsymbol{H}_{cir}^{S-D}(2n)$, $\boldsymbol{H}_{cir}^{S-R}(2n)$, and $\boldsymbol{H}_{cir}^{R-D}(2n+1)$ are all circulant matrices, we obtain the followings

$$\begin{aligned}\boldsymbol{H}_{cir}^{S-D}(2n) &= \boldsymbol{Q}^{S-D}\boldsymbol{R}^{S-D}, \\ \boldsymbol{H}_{cir}^{S-R}(2n) &= \boldsymbol{Q}^{S-R}\boldsymbol{R}^{S-R}, \\ \boldsymbol{H}_{cir}^{R-D}(2n+1) &= \boldsymbol{Q}^{R-D}\boldsymbol{R}^{R-D}, \end{aligned} \quad (4)$$

where $\{\boldsymbol{Q}^{S-D}, \boldsymbol{Q}^{S-R}, \boldsymbol{Q}^{R-D}\}$ are unitary matrices, whereas $\{\boldsymbol{R}^{S-D}, \boldsymbol{R}^{S-R}, \boldsymbol{R}^{R-D}\}$ are upper triangular matrices. With available channel state information (CSI), the QRD is employed. Without loss of generality, we suppress symbol time indices $(2n)$ and $(2n+1)$ to simplify our notations.

Lemma 1: Using (4), ESNR over $S \rightarrow D$ link is defined by

$$\begin{aligned}\gamma^{S-D} &\triangleq \frac{P_s(\alpha^{S-D})^2 E\{\|\boldsymbol{H}_{cir}^{S-D}(2n)\boldsymbol{d}(2n)\|^2\}}{\sigma_D^2} \\ &= \frac{P_s(\alpha^{S-D})^2 N r_1}{\sigma_n^2}, \end{aligned} \quad (5)$$

where $r_1 \triangleq ((\boldsymbol{R}^{S-D})_{1,1})^2$, $r_2 \triangleq ((\boldsymbol{R}^{S-R})_{1,1})^2$, and $r_3 \triangleq ((\boldsymbol{R}^{R-D})_{1,1})^2$.

Proof: It can be easily shown that

$$\begin{aligned}E\{\|\boldsymbol{H}_{cir}^{S-D}(2n)\boldsymbol{d}(2n)\|^2\} &= \text{Trace}\{(\boldsymbol{R}^{S-D})^H\boldsymbol{R}^{S-D}\} \\ &= N((\boldsymbol{R}^{S-D})_{1,1})^2 \triangleq N r_1, \end{aligned} \quad (6)$$

where $E\{\boldsymbol{d}(2n)\boldsymbol{d}(2n)^H\} = \boldsymbol{I}_{N\times N}$ and the classical Gram-Schmidt (CGS) orthogonalization [15] is used to the circulant matrix. This is true for all circulant matrices when $N_g \geq N_f$. Similarly, $\gamma^{S-R} = \frac{P_s(\alpha^{S-R})^2 N r_2}{\sigma_n^2}$ and $\gamma^{R-D} = \frac{P_r(\alpha^{R-D})^2 N r_3}{\sigma_n^2}$ for $S \rightarrow R$, and $R \rightarrow D$ links, respectively, can be defined. ■

III. Optimal Power Allocation

To simplify our approach, we assume that any combining between two hops is not employed in the destination. Under this condition, the spectral efficiency is given by [2], [4], [5]

$$R = \frac{1}{2}\log_2(1 + \min(\gamma^{S-R}, \gamma^{R-D})). \quad (7)$$

Note that from (7), we find that there is no diversity gain. Now the optimization problem (or min-max optimization [4]) under the transmission power constraint $P_T = P_s + P_r$ can be formulated as follows:

$$\begin{aligned}\max_{P_s, P_r} \quad & \frac{1}{2}\log_2(1 + \min(\gamma^{S-R}, \gamma^{R-D})) \\ & s.t. \; P_s + P_r = P_T. \end{aligned} \quad (8)$$

According to [2], the maximum spectral efficiency is obtained when $\gamma^{S-R} = \gamma^{R-D}$. Using *Lemma 1*, this is equivalent to

$$\frac{P_s(\alpha^{S-R})^2 N r_2}{\sigma_n^2} = \frac{P_r(\alpha^{R-D})^2 N r_3}{\sigma_n^2}, \quad (9)$$

from which the optimal power allocations to the source and relay node are respectively given by

$$P_s = \frac{P_T(\alpha^{R-D})^2 r_3}{r_2(\alpha^{S-R})^2 + r_3(\alpha^{R-D})^2}, \quad (10)$$

$$P_r = \frac{P_T(\alpha^{S-R})^2 r_2}{r_2(\alpha^{S-R})^2 + r_3(\alpha^{R-D})^2}. \quad (11)$$

Substituting (10) and (11) into (7) yields the maximum achievable spectral efficiency as follows:

$$R_o = \frac{1}{2}\log_2\left(1 + \frac{NP_T}{\sigma_n^2}\frac{(\alpha^{S-R})^2 r_2(\alpha^{R-D})^2 r_3}{(\alpha^{S-R})^2 r_2 + (\alpha^{R-D})^2 r_3}\right). \quad (12)$$

The upper bound of (12) can be readily verified that

$$R_o^{up} = \frac{1}{2}\log_2\left(1 + \beta\min\left((\alpha^{S-R})^2 r_2, (\alpha^{R-D})^2 r_3\right)\right), \quad (13)$$

which means that all the power P_T is allocated to the link that has the minimum ESNR. In (13), $\beta \triangleq \frac{NP_T}{\sigma_n^2}$.

Theorem 1: When the entries of $\boldsymbol{H}_{cir}^{S-R}(2n)$ and $\boldsymbol{H}_{cir}^{R-D}(2n+1)$ are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance, the probability density function (PDF) and the cumulative distribution function (CDF) of $\gamma^{up} \triangleq \beta \min(\alpha_2 r_2, \alpha_3 r_3)$, with introducing $\alpha_2 \triangleq (\alpha^{S-R})^2$ and $\alpha_3 \triangleq (\alpha^{R-D})^2$, are respectively given by

$$f_{\gamma^{up}}(x) = \frac{1}{\beta^{N_f}(N_f-1)!(x)^{N_f-1}} e^{-(\frac{1}{\alpha_2}+\frac{1}{\alpha_3})\frac{x}{\beta}} \times \left(\frac{1}{\alpha_2^{N_f}} \sum_{l=0}^{N_f-1} \frac{1}{l!} \left(\frac{x}{\beta\alpha_3}\right)^l + \frac{1}{\alpha_3^{N_f}} \sum_{l=0}^{N_f-1} \frac{1}{l!} \left(\frac{x}{\beta\alpha_2}\right)^l \right), \quad (14)$$

$$F_{\gamma^{up}}(x) = 1 - e^{-(\frac{1}{\alpha_2}+\frac{1}{\alpha_3})\frac{x}{\beta}} \sum_{l=0}^{N_f-1} \frac{1}{l!} \left(\frac{x}{\beta\alpha_2}\right)^l \times \sum_{u=0}^{N_f-1} \frac{1}{u!} \left(\frac{x}{\beta\alpha_3}\right)^u. \quad (15)$$

Proof: Imposing the property of the circulant matrix, we first find from γ^{S-R} and γ^{R-D} that

$$r_2 \triangleq \left((\boldsymbol{R}^{S-R})_{1,1}\right)^2 = \sum_{l=0}^{N_f-1} \left|h_l^{S-R}(2n)\right|^2, \quad (16)$$

$$r_3 \triangleq \left((\boldsymbol{R}^{R-D})_{1,1}\right)^2 = \sum_{l=0}^{N_f-1} \left|h_l^{R-D}(2n+1)\right|^2. \quad (17)$$

Now from (16) and (17), r_2 and r_3 are distributed by the chi-square distribution with $2N_f$ degrees of freedom, such that their PDFs are given by

$$r_2 = r_3 \sim f_{r_k}(x) = \frac{1}{(N_f-1)!} x^{N_f-1} e^{-x} u(x),\ k=2,3. \quad (18)$$

The corresponding CDFs are given by [16]

$$F_{r_k}(x) \triangleq P_r(r_k \le x) = 1 - e^{-x} \sum_{l=0}^{N_f-1} \frac{1}{l!} x^l,\ \ k=2,3. \quad (19)$$

Using (18), (19), and [17, eqs. (6-56) and (6-58)], we obtain

$$\begin{aligned} f_{\gamma^{up}}(x) &= \frac{1}{\beta\alpha_2} f_{r_2}\left(\frac{x}{\beta\alpha_2}\right)\left(1 - F_{r_3}\left(\frac{x}{\beta\alpha_3}\right)\right) \\ &\quad + \frac{1}{\beta\alpha_3} f_{r_3}\left(\frac{x}{\beta\alpha_3}\right)\left(1 - F_{r_2}\left(\frac{x}{\beta\alpha_2}\right)\right), \\ &= \frac{1}{\beta^{N_f}(N_f-1)!}(x)^{N_f-1} e^{-\frac{x}{\beta}\left(\frac{1}{\alpha_2}+\frac{1}{\alpha_3}\right)} \\ &\quad \times \left(\frac{1}{\alpha_2^{N_f}} \sum_{l=0}^{N_f-1} \frac{1}{l!} \left(\frac{x}{\beta\alpha_3}\right)^l + \frac{1}{\alpha_3^{N_f}} \sum_{l=0}^{N_f-1} \frac{1}{l!} \left(\frac{x}{\beta\alpha_2}\right)^l \right), \end{aligned} \quad (20)$$

$$\begin{aligned} F_{\gamma^{up}}(x) &= F_{r_2}\left(\frac{x}{\beta\alpha_2}\right) + F_{r_3}\left(\frac{x}{\beta\alpha_3}\right) \\ &\quad - F_{r_2}\left(\frac{x}{\beta\alpha_2}\right) F_{r_3}\left(\frac{x}{\beta\alpha_3}\right), \\ &= 1 - e^{-\frac{x}{\beta}(\frac{1}{\alpha_2}+\frac{1}{\alpha_3})} \sum_{l=0}^{N_f-1} \frac{1}{l!} \left(\frac{x}{\beta\alpha_2}\right)^l \\ &\quad \times \sum_{u=0}^{N_f-1} \frac{1}{u!} \left(\frac{x}{\beta\alpha_3}\right)^u. \end{aligned} \quad (21)$$

■

When $\alpha \triangleq \alpha_2 = \alpha_3$, (14) and (15) result in

$$f_{\gamma^{up}}(x) = \frac{2}{\beta^{N_f}(N_f-1)!}(x)^{N_f-1} e^{-2\frac{x}{\beta}\left(\frac{1}{\alpha}\right)} \times \left(\frac{1}{\alpha^{N_f}} \sum_{l=0}^{N_f-1} \frac{1}{l!} \left(\frac{x}{\beta\alpha}\right)^l u(x) \right), \quad (22)$$

$$F_{\gamma^{up}}(x) = 1 - e^{-2\frac{x}{\beta}(\frac{1}{\alpha})} \left(\sum_{l=0}^{N_f-1} \frac{1}{l!} \left(\frac{x}{\beta\alpha}\right)^l \right)^2. \quad (23)$$

Having derived the expressions for the PDF and CDF of γ^{up} after applying the optimal power allocation, the ASE with the opportunistic destination terminal selection can be exactly derived in the following section.

IV. ASE for Opportunistic Destination Terminal Selection

In a multi-destination system, each destination is considered as a terminal for a distinct user. The opportunistic destination terminal selection allocates the channel at each given transmission instants $((2n), (2n+1))$ only to the destination terminal that has the best channel condition that is described by the ESNR in the proposed relaying system [18]. As shown in [12], a considerable total throughput can be increased as the number of terminals increases when there is no feeding back delay between the destination terminals and the source. In the considered scheme, the total transmission power P_T is first optimally allocated to the source and relay of each terminal taking into account $\alpha_2 r_2$ and $\alpha_3 r_3$ in their individual links and then feed backs to the source. After receiving them, the source selects the desired terminal which has the maximum magnitude of $\beta \min(\alpha_2 r_2, \alpha_3 r_3)$. It is assumed that α_2 and α_3 are identical to all terminals in the system. Let $\gamma^{up,max} = \max\{\gamma_1^{up}, .., \gamma_K^{up}\}$ be the maximum ESNR among a set of K terminals' ESNRs. The ESNR for the k-th terminal is defined by γ_k^{up}. To compute the ASE $\bar{R}_o^{up}$ after applying the cooperative opportunistic terminal selection, the statistical knowledge of $\gamma^{up,max}$ is necessary. For this reason, we provide the PDF of $\gamma^{up,max}$ in the following theorem.

Theorem 2: When there are K destination terminals and all the channels between the source and distinct terminals are i.i.d. complex Gaussian random variables with zero mean and unit variance, the PDF of $\gamma^{up,max}$ is given by

$$f_{\gamma^{up,max}}(x) = \frac{K}{\beta^{N_f}(N_f-1)!} \sum_{i=0}^{K-1} \binom{K-1}{i} (-1)^i e^{-\frac{(i+1)}{\beta}x} \times \left(\left(\frac{1}{\alpha_3}\right)^{N_f} c_2^i + \left(\frac{1}{\alpha_2}\right)^{N_f} c_3^i \right), \quad (24)$$

where $\tilde{\beta} \triangleq = \frac{\beta}{\frac{1}{\alpha_2}+\frac{1}{\alpha_3}}$ and

$$c_2^i \triangleq \sum_{l_{1:N_f}}^{i+1} \sum_{u_{1:N_f}}^{i} \beta_2(i+1, l_{1:N_f}, i, u_{1:N_f}) \times x^{N_f+\left(\sum_{t=0}^{N_f-1} tl_{t+1}\right)+\left(\sum_{t=0}^{N_f-1} tu_{t+1}\right)-1}, \quad (25)$$

where

$$l_{1:N_f} \triangleq (l_1, .., l_{N_f}), \ u_{1:N_f} \triangleq (u_1, .., u_{N_f}), \quad (26)$$

$$\beta_2(i+1, l_{1:N_f}, i, u_{1:N_f}) \triangleq \left(\frac{(i+1)!}{l_1! l_2! ... l_{N_f}!}\right)\left(\frac{(i)!}{u_1! u_2! ... u_{N_f}!}\right) \times \frac{1}{\prod_{t=0}^{N_f-1}(t!(\beta\alpha_2)^t)^{l_{t+1}}} \frac{1}{\prod_{t=0}^{N_f-1}(t!(\beta\alpha_3)^t)^{u_{t+1}}}. \quad (27)$$

The nonnegative integers $[l_1, l_2, ..., l_{N_f}]$ and $[u_1, u_2, ..., u_{N_f}]$ are satisfying $\sum_{t=1}^{N_f} l_t = (i+1)$ and $\sum_{t=1}^{N_f} u_t = (i)$, respectively. In addition,

$$c_3^i = \sum_{l_{1:N_f}}^{i} \sum_{u_{1:N_f}}^{i+1} \beta_3(i, \tilde{l}_{1:N_f}, i+1, \tilde{u}_{1:N_f}) \times x^{N_f+\left(\sum_{t=0}^{N_f-1} t\tilde{l}_{t+1}\right)+\left(\sum_{t=0}^{N_f-1} t\tilde{u}_{t+1}\right)-1}, \quad (28)$$

where

$$\beta_3(i, \tilde{l}_{1:N_f}, i+1, \tilde{u}_{1:N_f}) \triangleq \left(\frac{(i)!}{\tilde{l}_1! \tilde{l}_2! ... \tilde{l}_{N_f}!}\right)\left(\frac{(i+1)!}{\tilde{u}_1! \tilde{u}_2! ... \tilde{u}_{N_f}!}\right) \frac{1}{\prod_{t=0}^{N_f-1}(t!(\beta\alpha_2)^t)^{\tilde{l}_{t+1}}} \times \frac{1}{\prod_{t=0}^{N_f-1}(t!(\beta\alpha_3)^t)^{\tilde{u}_{t+1}}}. \quad (29)$$

In this case, $\sum_{t=1}^{N_f} \tilde{l}_t = (i)$ and $\sum_{t=1}^{N_f} \tilde{u}_t = (i+1)$.

Proof: Proofs of (25) and (28) can be derived by substituting (14) and (15) into the following equation

$$f_{\gamma^{up,max}}(x) = K F_{\gamma^{up}}(x)^{K-1} f_{\gamma^{up}}(x) \quad (30)$$

and then applying both the binomial and multinomial theorem. ■

Using (30), the ASE of (13) can be written as

$$\bar{R}_o^{up} = \frac{1}{2}\int_0^\infty \log_2(1+x) f_{\gamma^{up,max}}(x)dx \quad (31)$$

$$= \frac{K}{2\ln(2)\beta^{N_f}(N_f-1)!}\sum_{i=0}^{K-1}\binom{K-1}{i}(-1)^i \times \left[\left(\frac{1}{\alpha_3}\right)^{N_f}\sum_{l_{1:N_f}}^{i+1}\sum_{u_{1:N_f}}^{i}\beta_2(i+1, l_{1:N_f}, i, u_{1:N_f}) \times G_{2,3}^{3,1}\left(d(i)\middle| \begin{matrix} -m, 1-m \\ 0, -m, -m \end{matrix}\right) + \left(\frac{1}{\alpha_2}\right)^{N_f}\sum_{\tilde{l}_{1:N_f}}^{i}\sum_{\tilde{u}_{1:N_f}}^{i+1}\beta_3(i, \tilde{l}_{1:N_f}, i+1, \tilde{u}_{1:N_f}) \times G_{2,3}^{3,1}\left(d(i)\middle| \begin{matrix} -\tilde{m}, 1-\tilde{m} \\ 0, -\tilde{m}, -\tilde{m} \end{matrix}\right)\right], \quad (32)$$

where $G_{p,q}^{m,n}$ is the Meijer G-function defined in [19, eq. (9.301)]. Then by applying [20, eq. (21)] in (31) results in (32). In (32), we introduced the following definitions: $d(i) \triangleq \frac{i+1}{\tilde{\beta}}$, $m = N_f + \left(\sum_{t=0}^{N_f-1} tl_{t+1}\right) + \left(\sum_{t=0}^{N_f-1} tu_{t+1}\right)$, and $\tilde{m} = N_f + \left(\sum_{t=0}^{N_f-1} t\tilde{l}_{t+1}\right) + \left(\sum_{t=0}^{N_f-1} t\tilde{u}_{t+1}\right)$.

Employing an equal power allocation to the source and relay, that is, $P_s = P_r = P_T/2$, the spectral efficiency without the optimal power allocation can be given by

$$R_{no} = \frac{1}{2}\log_2\left(1+\min\left(\gamma^{S-R}, \gamma^{R-D}\right)\right) = \frac{1}{2}\log_2\left(1+\min\left(\frac{P_s\alpha_2 Nr_2}{\sigma_n^2}, \frac{P_r\alpha_3 Nr_3}{\sigma_n^2}\right)\right), = \frac{1}{2}\log_2\left(1+\frac{\beta}{2}\min\left(\alpha_2 r_2, \alpha_3 r_3\right)\right). \quad (33)$$

Since the form of (33) is very similar to that of (13), its upper ASE bound $\bar{R}_{no}^{up}$ can be obtained by substituting $\beta_{no} = \beta/2$ into (32).

V. Simulation Results & Discussion

The ASEs in terms of (α_2, α_3) are plotted in Fig. 1, where the theoretical $\bar{R}_{o,theo}^{up}$s are first compared with the empirical $\bar{R}_{o,em}^{up}$s at $(\alpha_2 = 0.1, \alpha_3 = 0.9)$ and $(\alpha_2 = 0.5, \alpha_3 = 0.5)$. In the system, $N_f = 2$ and $K = 2$ are assumed. The derived $\bar{R}_{o,theo}^{up}$ is shown to be matched to $\bar{R}_{o,em}^{up}$ exactly. As α_2 approaches to 0.5, the higher $\bar{R}_o^{up}$ can be obtained since two relay links have the same effect on the performance. When we assume a unique path loss exponent and the distance between the source and relay is same as the distance between the relay and destination, this condition can be satisfied. Fig. 2 is the plot for $\bar{R}_o^{up}$ as a function of α_2. Since the links $S \to R$ and $R \to D$ are assumed to have the same channel characteristics in the simulations, $\bar{R}_o^{up}$ is symmetric with respect to $\alpha_2 = \alpha_3 = 0.5$, where the maximum $\bar{R}_o^{up}$ can be obtained. This figure also proves the comparisons with $\bar{R}_o$, $\bar{R}_{no}$, and $\bar{R}_{no}^{up}$ that apply the uniform power allocation. From (32) and (33), we can easily anticipate the improved ASE with applying the optimal power allocation. For a general case of (N_f, K), we find the trend of $\bar{R}_o^{up}$ from Fig. 3. In the first case, we fixed $K = 2$ while varying N_f to show the multipath gain effect on the ASE, which also shows we can maintain the improved ASE obtained from the opportunistic user terminal selection mechanism. In the second case, we fixed $N_f = 2$, but we use a different number of user terminals in the system to show the multiuser diversity. As the number of terminals increases, the ASE can be improved. From these two cases, the multiuser diversity and multipath gain simultaneously play important role in determining the ASE. Without using the optimal power allocation, performance loss can be observed in the ASE.

VI. Conclusions

In this paper, we have derived the optimal power allocation for the two-hop DF relay protocol employing CP SC transmission. After then, we have applied the opportunistic user terminal selection to achieve a better spectral efficiency. Based on the derived source-to-destination ESNR, we have derived closed-form expressions for the ASE. Simulation results verified that the proposed power allocation improve the spectral efficiency. Moreover, it is shown that the multipath gain is shown to be an important parameter to maintain the performance obtained from the power allocation and the opportunistic user selection.

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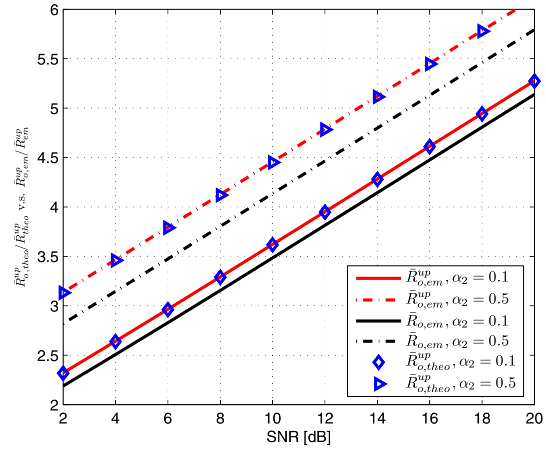

Fig. 1. Average spectral efficiency.

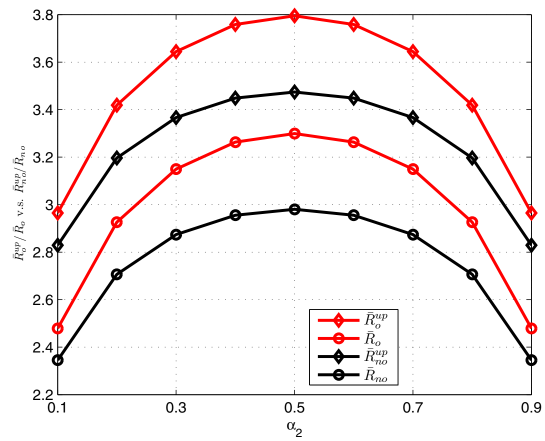

Fig. 2. Average spectral efficiency in terms of α_2.

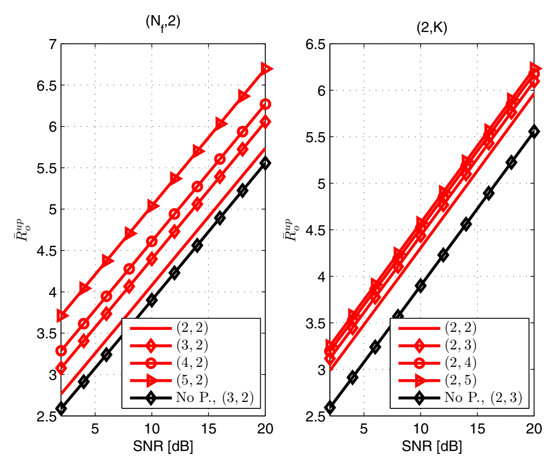

Fig. 3. Average spectral efficiency at various (N_f, K).