Optical Wireless Communications with Adaptive Subcarrier PSK Intensity Modulation

Nestor D. Chatzidiamantis*, Athanasios S. Lioumpas*, George K. Karagiannidis* and Shlomi Arnon†
*Department of Electrical & Computer Engineering, Aristotle University of Thessaloniki, Thessaloniki, Greece, Emails: {nestoras, alioumpa, geokarag}@auth.gr
†Department of Electrical and Computer Engineering, Ben-Gurion University of the Negev, Beer-Sheva, Israel (e-mail: shlomi@ee.bgu.ac.il).

***Abstract*—We propose an adaptive transmission technique for optical wireless (OW) systems, operating in atmospheric turbulence and employing subcarrier phase shift keying (S-PSK) intensity modulation. Exploiting the constant envelope characteristics of S-PSK, the proposed technique offers efficient utilization of the OW channel capacity by adapting the modulation order of S-PSK, according to the instantaneous state of turbulence induced fading and a pre-defined bit error rate (BER) requirement. Novel expressions for the spectral efficiency and average BER of the proposed adaptive OW system are presented and performance investigations under various turbulence conditions and target BER requirements are carried out. Numerical results indicate that significant spectral efficiency gains are offered without increasing the transmitted average optical power or sacrificing BER requirements in moderate-to-strong turbulence conditions.**

I. Introduction

Optical wireless (OW), also known as Free Space Optical, communication is a wireless technology, which has recently attracted considerable interest within the research community, since it can be advantageous for a variety of applications [1], [2]. However, despite its significant advantages, the widespread deployment of OW systems is limited by their high vulnerability to adverse atmospheric conditions [3]. Even in a clear sky, due to inhomogeneities in temperature and pressure changes, the refractive index of the atmosphere varies stochastically and results in atmospheric turbulence. This causes rapid fluctuations at the intensity of the received optical signal, known as turbulence-induced fading, that severely affect the reliability and/or communication rate provided by the OW link.

A promising solution for mitigating the degrading effects of turbulence induced fading is the employment of adaptive transmission, a well known technique employed in radio frequency (RF) wireless systems [4]. By varying basic transmission parameters according to the channel's fading intensity, adaptive transmission takes advantage of the time varying nature of turbulence, allowing higher data rates to be transmitted under favorable turbulence conditions. Thus, the spectral efficiency of the OW link can be improved without wasting additional optical power or sacrificing performance requirements.

The concept of adaptive transmission was first introduced in the context of OW systems in [5], where an adaptive scheme that varied the period of the transmitted binary Pulse Position modulated (BPPM) symbol was studied. Since then, various adaptive OW systems have been proposed. In [6], a variable rate OW system employing adaptive Turbo-based coding schemes in conjunction with On-Off keying modulation was investigated, while in [7] an adaptive power scheme was suggested for reducing the average power consumption in constant rate optical satellite-to-earth links. Recently, in [8], an adaptive transmission scheme that varied both the power and the modulation order of a OW system with pulse amplitude modulation (PAM), has been studied.

In this work, we propose an adaptive modulation scheme for OW systems operating in turbulence, using an alternative type of modulation; subcarrier phase shift keying intensity modulation (S-PSK). S-PSK refers to the transmission of PSK modulated RF signals, after being properly biased[1], through intensity modulation direct detection (IM/DD) optical systems and its employment can be advantageous, since:

- in the presence of turbulence, it offers increased demodulation performance compared to PAM signalling [9],
- due to its constant envelope characteristic, both the average and peak optical power are constant in every symbol transmitted, and,
- it allows RF signals to be directly transmitted through OW links providing protocol transparency in heterogeneous wireless networks [10]- [12].

Taking into consideration that the bias signal required by S-PSK in order to satisfy the non-negativity requirement is independent of the modulation order, we present a novel variable rate transmission strategy that is implemented through the modification of the modulation order of S-PSK, according to the instantaneous turbulence induced fading and a pre-defined value of Bit Error Rate (BER). The performance of the proposed variable rate transmission scheme is investigated, in terms of spectral efficiency and BER, under different degrees of turbulence strength, and is further compared to non-adaptive modulation and the upper capacity bound provided by [13].

The remainder of the paper is organized as follows. In Section II, the non-adaptive S-PSK OW system model is described and its performance in the presence of turbulence induced fading is investigated. In section III, the adaptive S-PSK strategy is presented, deriving expressions for its spectral

[1]Since optical intensity must satisfy the non-negativity constraint, a proper DC bias must be added to the RF electrical signal in order to prevent clipping and distortion in the optical domain.

efficiency and BER performance. Section IV discusses some numerical results and useful concluding remarks are drawn in section V.

Notations: $\mathrm{E}\{\cdot\}$ denotes statistical expectation; $N(\mu, \sigma^2)$ denotes Gaussian distribution with mean μ and variance σ^2.

II. Non-Adaptive Subcarrier PSK Intensity Modulation

We consider an IM/DD OW system which uses a subcarrier signal for the modulation of the optical carrier's intensity and operates over the atmospheric turbulence induced fading channel.

A. System and Channel Model

On the transmitter end, we assume that the RF subcarrier signal is modulated by the data sequence using PSK. Moreover a proper DC bias is added in order to ensure that the transmitted waveform always satisfies the non-negativity input constraint. Hence, the transmitted optical power can be expressed as

$$P_t(t) = P[1 + \mu s(t)] \tag{1}$$

where P is the average transmitted optical power and μ is the modulation index $(0 < \mu < 1)$ which ensures that the laser operates in its linear region and avoids over-modulation induced clipping. Further, $s(t)$ is the output of the electrical PSK modulator which can be written as

$$s(t) = \sum_k g(t - kT)\cos(2\pi f_c t + \phi_k) \tag{2}$$

where f_c is the frequency of the RF subcarrier signal, T is the symbol's period, $g(t)$ is the shaping pulse, $\phi_k \in \left[0, ..., (M-1)\frac{\pi}{M}\right]$ is the phase of the kth transmitted symbol and M is the modulation order.

On the receiver's end, the optical power which is incident on the photodetector is converted into an electrical signal through direct detection. We assume operation in the high signal-to-noise ratio (SNR) regime where the shot noise caused by ambient light is dominant and therefore Gaussian noise model is used as a good approximation of the Poisson photon counting detection model [14].

After removing the DC bias and demodulating through a standard RF PSK demodulator, the electrical signal sampled during the kth symbol interval, which is obtained at the output of the receiver, is expressed as

$$r[k] = \mu\eta\sqrt{\frac{E_g}{2}}PI[k]\,s[k] + n[k] \tag{3}$$

where η corresponds to the receiver's optical-to-electrical efficiency, $s[k] = \cos\phi_k - j\sin\phi_k$, E_g is the energy of the shaping pulse and $n[k]$ is the zero mean circularly symmetric complex Gaussian noise component with $\mathrm{E}\{n[k]\,n^*[k]\} = 2\sigma_n^2 = N_o$. Furthermore, $I[k]$ represents turbulence-induced fading coefficient during the kth symbol interval and is given by

$$I[k] = I_o \exp(2x[k]) \tag{4}$$

where I_o denotes the signal light intensity without turbulence and $x[k]$ is a normally distributed random variable with mean m_x and variance σ_x^2, i.e. $f_{x[k]}(x) = N(m_x, \sigma_x^2)$. Hence $I[k]$ follows a lognormal distribution with probability density function (PDF) provided by

$$f_{I[k]}(I) = \frac{1}{2I}\frac{1}{\sqrt{2\pi\sigma_x^2}}\exp\left(-\frac{\left(\ln\left(\frac{I}{I_o}\right) - 2m_x\right)^2}{8\sigma_x^2}\right) \tag{5}$$

To ensure that the fading does not attenuate or amplify the average power, we normalize the fading coefficients such that $\mathrm{E}\left\{\left|\frac{I[k]}{I_o}\right|\right\} = 1$. Doing so requires the choice of $m_x = -\sigma_x^2$ [15]. Moreover, without loss of generality, it is assumed that $I_o = 1$.

Atmospheric turbulence results in a very slowly-varying fading in OW systems. For the signalling rates of interest ranging from hundreds to thousands of Mbps, the fading coefficient can be considered as constant over hundred of thousand or millions of consecutive symbols, since the coherence time of the channel is about 1-100ms [16]. Hence, it is assumed that turbulence induced fading remains constant over a block of K symbols (block fading channel), and therefore we drop the time index k, i.e.

$$I = I[k], \quad k = 1, ...K \tag{6}$$

It should be noted that in the analysis that follows, it is further assumed that the information message is long enough to reveal the long-term ergodic properties of the turbulence process.

The instantaneous electrical SNR is defined as

$$\gamma = \frac{\mu^2\eta^2P^2E_sI^2}{N_o} \tag{7}$$

while the average electrical SNR is given by

$$\bar{\gamma} = \frac{\mu^2\eta^2P^2E_s}{N_o} \tag{8}$$

with $E_s = \frac{E_g}{2}$.

B. BER Performance

The BER performance of the OW system under consideration depends on the statistics of atmospheric turbulence and the modulation order.

When $M = 2$ (BPSK), the conditioned on the fading coefficient, I, BER is given by [9]

$$P_b(2, I) = Q\left(I\sqrt{2\bar{\gamma}}\right) \tag{9}$$

while for $M > 2$ the following approximation can be used [17]

$$P_b(M, I) \approx \frac{2}{\log_2 M}Q\left(I\sqrt{2\bar{\gamma}}\sin\frac{\pi}{M}\right) \tag{10}$$

where $Q(\cdot)$ is the Gaussian Q-function defined as $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{t^2}{2}}dx$. Hence, the average BER will be obtained by averaging over the turbulence PDF, i.e.

$$\bar{P}_b(M) = \int_0^\infty P_b(M, I)\,f_I(I)\,dI \tag{11}$$

which can be efficiently evaluated using the Gauss-Hermitte quadrature formula [15].

C. High-SNR Capacity Upper Bound

Using the trigonometric moment space method presented in [13], an upper bound for the capacity of optical intensity channels when multiple subcarrier modulation is employed has been derived. By applying these results to the OW system under consideration (one subcarrier), the conditioned on the fading coefficient, I, capacity can be upper bounded by

$$C_{up}(I) = \frac{W}{2}\left[\log_2 \pi + \log_2\left(\frac{\mu^2\eta^2P^2E_gI^2}{\pi e N_o}\right) + o(\sigma_n)\right] \quad (12)$$

where W denotes the electrical bandwidth and $o(\sigma_n)$ represents the capacity residue which vanishes exponentially as $\sigma_n \to 0$. Hence at high values of electrical SNR, (12) can be approximated by

$$C_{up}(I) \approx \frac{W}{2}\log_2\left(\frac{\bar{\gamma}I^2}{e}\right). \quad (13)$$

The unconditional high-SNR capacity upper bound will be obtained by averaging (13) over the fading distribution, i.e.

$$C_{up} \approx \frac{W}{2}\int_0^\infty \log_2\left(\frac{\bar{\gamma}I^2}{e}\right) f_I(I)\,dI, \quad (14)$$

which, according to the Appendix, can be written in closed form as

$$C_{up} \approx \frac{W}{2}\left(\log_2\left(\frac{\bar{\gamma}}{e}\right) - \frac{4\sigma_x^2}{\ln 2}\right) \quad (15)$$

and will be used as a benchmark in the analysis that follows.

III. Adaptive Modulation Strategy

In this section we introduce an adaptive transmission strategy that improves the spectral efficiency of S-PSK OW systems, without increasing the transmitted average optical power or sacrificing the performance requirements.

A. Mode of Operation

By inserting pilot symbols at the beginning of a block of symbols[2], the receiver accurately estimates the instantaneous channel's fading state, I, which is experienced by the remaining symbols of the block. Based on this estimation, a decision device at the receiver selects the modulation order to be used for transmitting the non-pilot symbols of the block, configures the electrical demodulator accordingly and informs the adaptive PSK transmitter about that decision via a reliable RF feed back path.

The objective of the above described transmission technique is to maximize the number of bits transmitted per symbol interval, by using the largest possible modulation order under the target BER requirement P_o. Hence the problem is formulated as

$$\begin{aligned} &\max_{M}\ \log_2 M \\ &\text{s.t. } P_b(M,I) \le P_o \end{aligned} \quad (16)$$

In practice, the modulation order will be selected from N available ones, i.e. $\{M_1, M_2, ..., M_N\}$, depending on the values of I and P_o. Specifically, the range of the values of the fading term is divided in $(N+1)$ regions and each region is associated with the modulation order, M_j, according to the rule

$$M = M_j = 2^j \text{ if } I_j \le I < I_{j+1},\ j = 1, ...N \quad (17)$$

The region boundaries $\{I_j\}$ are set to the required values of turbulence-induced fading required to achieve the target P_o. Hence, according to (9) and (10), they are obtained by

$$I_1 = \sqrt{\frac{1}{2\bar{\gamma}}}Q^{-1}(P_o), \quad (18)$$

$$I_j = \frac{1}{\sin\frac{\pi}{M_j}}\sqrt{\frac{1}{2\bar{\gamma}}}Q^{-1}\left(\frac{\log_2 M_j}{2}P_o\right),\ j = 2, ...N \quad (19)$$

and

$$I_{N+1} = R, \quad (20)$$

where $R \to \infty$ and $Q^{-1}(\cdot)$ denotes the inverse Q-function, which is a standard built-in function in most of the well-known mathematical software packages. It should be noted that in the case of $I < I_1$, the proposed transmission technique stops transmission.

B. Performance Evaluation

1) Achievable Spectral Efficiency: The achievable spectral efficiency is defined as the information rate transmitted in a given bandwidth, and for the communication system under consideration is given by[3]

$$S = \frac{C}{W} = \frac{\bar{n}}{2} \quad (21)$$

with C representing the capacity measured in bit/s and $\bar{n}$ the average number of transmitted bits.

The average number of transmitted bits in the adaptive S-PSK scheme is obtained by

$$\bar{n} = \sum_{j=1}^{N+1} a_j \log_2 M_j \quad (22)$$

where

$$a_j = \Pr\{I_j \le I < I_{j+1}\} = \int_{I_j}^{I_{j+1}} f_I(I)\,dI. \quad (23)$$

Taking into consideration that the CDF of the LN fading distribution is given by

$$F_I(I_{th}) = \int_0^{I_{th}} f_I(I)\,dI = 1 - Q\left(\frac{\ln(I_{th}) + 2\sigma_x^2}{2\sigma_x}\right), \quad (24)$$

[2]Taking into consideration the signalling rates of interest and the channel's coherence time, the insertion of pilot symbols will not cause significant overhead.

[3]Note that since the subcarrier PSK modulation requires twice the bandwidth than PAM signalling, the average number of bits transmitted in a symbol's interval will be divided by two.

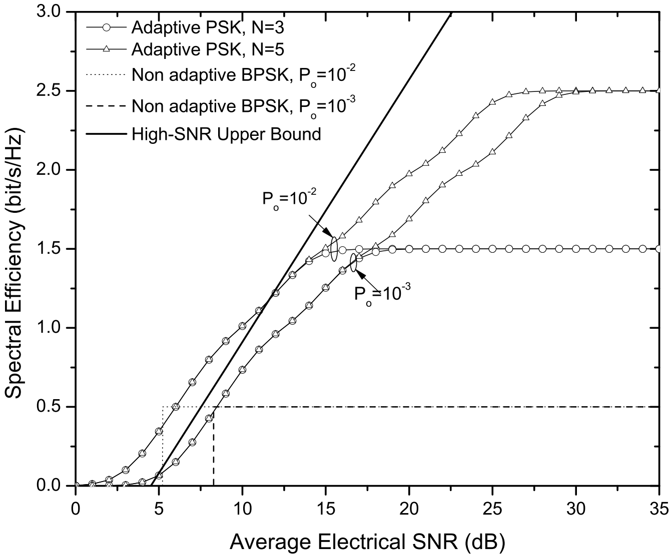

Fig. 1. Spectral efficiency of the adaptive subcarrier PSK scheme when $\sigma_x = 0.1$.

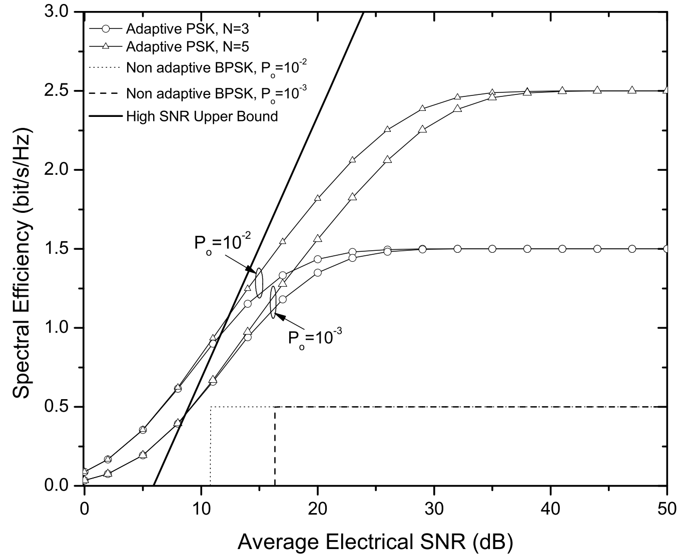

Fig. 2. Spectral efficiency of the adaptive subcarrier PSK scheme when $\sigma_x = 0.3$.

eq. (23) can be equivalently written as

$$a_j = F_I(I_{j+1}) - F_I(I_j) = Q(x_j) - Q(x_{j+1}) \qquad (25)$$

with $x_j = \frac{\ln(I_j)+2\sigma_x^2}{2\sigma_x}$. Hence, the achievable spectral efficiency can be written as

$$S = \frac{\sum_{j=1}^{N+1} a_j \log_2 M_j}{2} = \frac{\sum_{j=1}^{N} Q(x_j) - (N+1)\, Q(x_{N+1})}{2}, \qquad (26)$$

which is simplified to

$$S = \frac{\sum_{j=1}^{N} Q(x_j)}{2} \qquad (27)$$

since $Q(x_{N+1}) \to 0$, according to (20).

2) Average Bit Error Rate: The average BER of the proposed variable rate OW system can be calculated as the ratio of the average number of bits in error over the total number of transmitted bits [4]. The average number of bits in error can be obtained by

$$\bar{n}_{err} = \sum_{j=1}^{N+1} \langle P_b \rangle_j \log_2 M_j \qquad (28)$$

where

$$\langle P_b \rangle_j = \int_{I_j}^{I_{j+1}} P_b(M_j, I)\, f_I(I)\, dI. \qquad (29)$$

Hence the average BER is given by

$$\bar{P}_b = \frac{\bar{n}_{err}}{\bar{n}}. \qquad (30)$$

IV. Results & Discussion

In this section, we present numerical results for the performance of the adaptive S-PSK scheme in various turbulence conditions and for different target BERs.

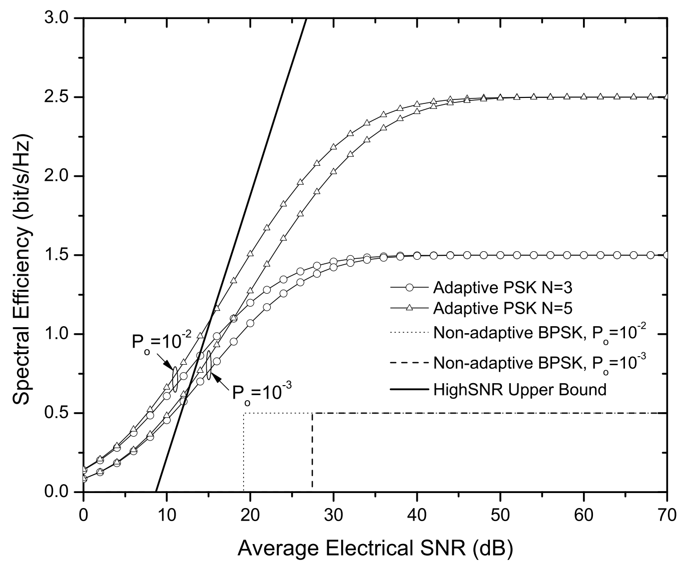

Fig. 3. Spectral efficiency of the adaptive subcarrier PSK scheme when $\sigma_x = 0.5$.

Figs. 1-3 depict the spectral efficiency of the adaptive subcarrier PSK transmission scheme at different degrees of turbulence strength, i.e. $\sigma_x = 0.1$, $\sigma_x = 0.3$ and $\sigma_x = 0.5$. Specifically, numerical results obtained by (27) for two different target BER requirements, $P_o = 10^{-2}$ and $P_o = 10^{-3}$, are plotted as a function of the average electrical SNR, along with the upper bound provided by (15) and the spectral efficiency of the non-adaptive subcarrier BPSK ($M = 2$). The latter is found by determining the value of the average electrical SNR for which the BER performance of the non-adaptive BPSK, as given by (11), equals P_o. It is obvious from the figures that the spectral efficiency of the adaptive transmission scheme increases and comes closer the upper bound by increasing

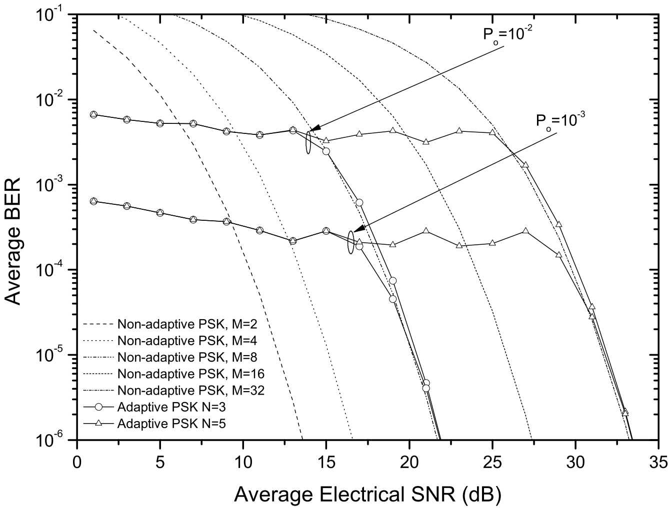

Fig. 4. Average BER of the adaptive subcarrier PSK scheme when $\sigma_x = 0.1$.

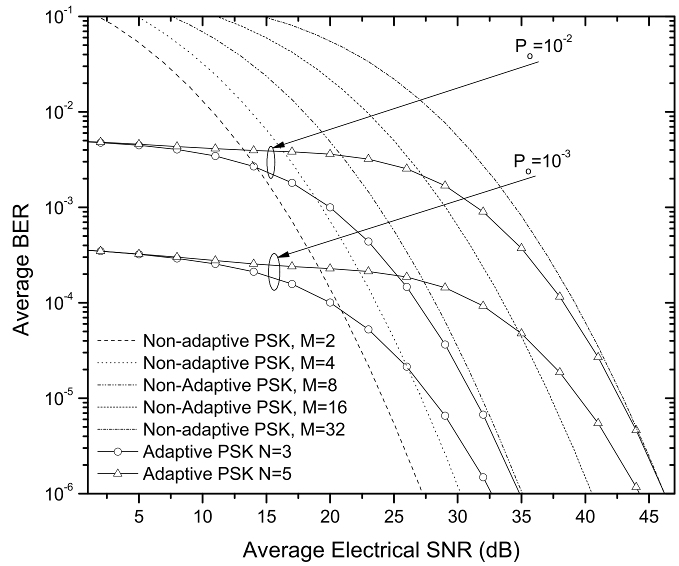

Fig. 5. Average BER of the adaptive subcarrier PSK scheme when $\sigma_x = 0.3$.

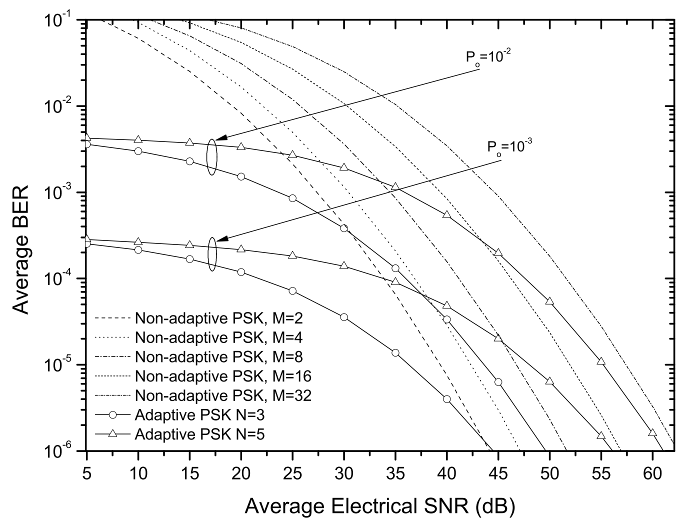

Fig. 6. Average BER of the adaptive subcarrier PSK scheme when $\sigma_x = 0.5$.

the target P_o. Moreover, when compared to the non-adaptive BPSK, it is observed that adaptive PSK offers large spectral efficiency gains (14dB when $P_o = 10^{-3}$) at strong turbulence conditions ($\sigma_x = 0.5$); however, these gains are reduced as σ_x reduces. For low turbulence ($\sigma_x = 0.1$), it is observed that non-adaptive BPSK reached its maximum spectral efficiency ($S = 0.5$) at lower values of SNR than the proposed adaptive scheme, indicating that in these conditions it is more effective to modify the modulation order based on the value of the average SNR rather than the instant value of fading intensity. Hence, the proposed adaptive PSK technique, which is based on the estimation of the instantaneous value of the channel's fading intensity, can be considered particularly effecient only in the moderate to strong turbulence regime.

Figs. 4-6 illustrate the average BER performance of the adaptive transmission technique for the same target BER requirements and turbulence conditions. It is clearly depicted that the average BER of the adaptive system is lower than the target P_o in all cases examined, satisfying the basic design requirement of (16). Moreover, it can be easily observed that the performance of the adaptive system approaches the performance of the non-adaptive system with the largest modulation order, at high values of average SNR; this was expected, since in this SNR regime the adaptive scheme chooses to transmit with the largest available modulation order.

V. Conclusions

We have presented a novel adaptive transmission technique for OW systems operating in atmospheric turbulence and employing S-PSK intensity modulation. The described technique was implemented through the modification of the modulation order of S-PSK according to the instantaneous fading state and a pre-defined BER requirement. Novel expressions for the spectral efficiency and average BER of the adaptive OW system were derived and investigations over various turbulence conditions and target BER requirements were performed. Numerical results indicated that adaptive transmission offers significant spectral efficiency gains, compared to the non-adaptive modulation, at the moderate-to-strong turbulence regime (14dB at $S = 0.5$, when $P_o = 10^{-3}$ and $\sigma_x = 0.5$); however, it was observed that in low turbulence, it is more efficient to perform adaptation based on the average electrical SNR, instead of the instantaneous fading state.

Appendix

This appendix provides a closed-form expression for (14). Using the pdf of turbulence induced fading, (14) can be written as

$$C_{up} \approx \frac{W}{4\sqrt{2\pi\sigma_x^2}} \int_0^\infty \frac{\log_2\left(\frac{\bar{\gamma}I^2}{e}\right)}{I} \exp\left(-\frac{\left(\ln I + 2\sigma_x^2\right)^2}{8\sigma_x^2}\right) dI. \tag{31}$$

To simplify (31), we substitute $\ln I$ by y and hence

$$C_{up} \approx K_1 + K_2 \tag{32}$$

where

$$K_1 = \frac{W}{4\sqrt{2\pi\sigma_x^2}} \log_2\left(\frac{\bar{\gamma}}{e}\right) \int_{-\infty}^{\infty} \exp\left(-\frac{\left(y+2\sigma_x^2\right)^2}{8\sigma_x^2}\right) dy \tag{33}$$

and

$$K_2 = \frac{W}{2\ln 2\sqrt{2\pi\sigma_x^2}} \int_{-\infty}^{\infty} y \exp\left(-\frac{\left(y+2\sigma_x^2\right)^2}{8\sigma_x^2}\right) dy. \tag{34}$$

Using [18, Eq. (3.321/3)], (33) is reduced to

$$K_1 = \frac{W}{2} \log_2\left(\frac{\bar{\gamma}}{e}\right) \tag{35}$$

while, using [18, Eq. (3.461/2)], (34) is reduced to

$$K_2 = -\frac{2W\sigma_x^2}{\ln 2}. \tag{36}$$

Hence, by substituting in (32), Eq. (15) is obtained.

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