Average Outage and Non-Outage Duration of Selective Decode-and-Forward Relaying

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Abstract—In this paper, assuming transmission with fixed rate and fixed power, we derive the average duration of capacity outage and non-outage events of selective decode-and-forward relaying with repetition coding over slow Rayleigh fading channels. Furthermore, we develop high signal-to-noise ratio (SNR) approximations for both durations which provide significant insight into the impact of various system and channel parameters. For high SNR, on a double logarithmic scale, both the average outage duration (AOD) and the average non-outage duration (ANOD) become straight lines when plotted as functions of the SNR. However, while the slope of the ANOD improves with increasing diversity order, the slope of the AOD is $-1/2$ independent of the diversity order.

I. INTRODUCTION

Cooperative diversity is an efficient technique to exploit the inherent spatial diversity of wireless networks [1]-[4]. In particular, opportunistic relaying, where only one out of the set of available relays is selected, extracts the full diversity of the channel and minimizes the loss in spectral efficiency caused by the repetition at the relays [1], [3]. For fixed rate transmission, the random nature of the propagation environment may lead to capacity outage events if the transmission rate exceeds the capacity of the end-to-end channel. Under these conditions, the performance of cooperative diversity systems is generally characterized by the outage probability. However, in systems with mobile nodes, the channel gains are slowly time varying and the outage events in neighboring coding blocks become correlated. This phenomenon is not captured by the outage probability itself but can be characterized by the average outage duration (AOD) and the average non-outage duration (ANOD). The AOD and ANOD are useful measures for the design of systems which exploit the temporal changes of the channel such as systems using automatic repeat request (ARQ) and multi-user scheduling.

In this paper, we derived the AOD and ANOD of a simple selective repetition coding (SRC) based decode-and-forward (DF) relaying protocol introduced in [1]. Besides the accurate results, we develop high signal-to-noise ratio (SNR) approximations which provide significant insight into the impact of the various system and channel parameters. Related work includes [5] and [6]. Compared to this paper, in [5], the direct link between the source and the destination was ignored and a different relay selection policy was considered. In [6], cooperative protocols with only one relay were studied. Furthermore, to the best of our knowledge, the ANOD and its asymptotic behavior have not been considered in any other work before.

Organization: In Section II, the system model and some definitions are presented. The AOD and ANOD for the considered SRC DF protocol are derived in Section III, and illustrated based on a numerical example in Section IV. Conclusions are drawn in Section V.

II. PRELIMINARIES

In this section, we present the adopted system and channel models as well as the definitions of the AOD and ANOD.

A. System Model

We consider a cooperative network consisting of a source S, a destination D, and M relays denoted as R_i, $i \in \{1, 2, \ldots, M\}$, operating in a Rayleigh fading enviroment. The channel gain of the direct link ($S \to D$) is denoted by $X(t)$, the channel gain of the link between the source and relay R_i ($S \to R_i$) is denoted by $Y_i(t)$, and the channel gain of the link between relay R_i and the destination ($R_i \to D$) is denoted by $Z_i(t)$. The average channel gain powers are defined as $E[X^2] \triangleq \Omega_X$, $E[Y_i^2] \triangleq \Omega_Y$, and $E[Z_i^2] \triangleq \Omega_Z$, where $E[\cdot]$ denotes expectation[1]. All nodes are assumed to transmit with equal power P_T, and $\Gamma_0 \triangleq P_T/N_0$ denotes the transmit SNR, where N_0 is the power spectral density of the underlying additive white Gaussian noise.

We assume two-dimensional isotropic scattering around all nodes. Furthermore, S, R_i, $i \in \{1, 2, \ldots, M\}$, and D are assumed to be mobile, thus introducing maximum Doppler shifts f_{mS}, f_{mR}, and f_{mD}, respectively. Under these conditions, the channel gains $X(t)$, $Y_i(t)$, and $Z_i(t)$ are time-correlated Rayleigh random processes and their auto-covariance functions and Doppler spectra are given in [7]. The time derivative of the channel gain $\alpha(t)$, $\alpha \in \{X, Y, Z\}$, is denoted by $\dot{\alpha}(t)$. This derivative is independent of the channel gain itself and is a zero-mean Gaussian random variable (RV) with variance [8]

$$\sigma_{\dot{\alpha}} \triangleq \pi f_{m\alpha} \sqrt{\Omega_\alpha}, \tag{1}$$

where $f_{mX} \triangleq \sqrt{f_{mS}^2 + f_{mD}^2}$, $f_{mY} \triangleq \sqrt{f_{mS}^2 + f_{mR}^2}$, and $f_{mZ} \triangleq \sqrt{f_{mR}^2 + f_{mD}^2}$.

B. Relay Protocol

The considered relaying protocol was introduced in [1]. The transmission is performed in two phases. In the first phase, the source broadcasts a codeword to an a priori selected best relay and to the destination. The relay decodes the codeword and forwards it to the destination in the second phase using repetition coding. The destination coherently combines the signals received in both phases and attempts to decode the transmitted message.

The single best relay for this SRC DF protocol is chosen as follows. Assume that for a relay to decode successfully its source-relay channel, Y_i, has to exceed some threshold Y_0. We collect the indices i of all relays R_i, $i \in \{1, 2, \ldots, M\}$, with $Y_i > Y_0$ in set $\mathcal{D}(t)$. The selected best relay, R_j, has the maximum relay-destination channel gain among all relays with indices in $\mathcal{D}(t)$, i.e., $Z_j(t) = \max \{Z_i(t)\}_{i \in \mathcal{D}(t)}$. We note

[1]Throughout this paper we omit reference to time t whenever possible without giving rise to ambiguity.

that since the relay-destination gains $Z_i(t)$ are independent, identically distributed (i.i.d.) Rayleigh RVs, the probability density function (PDF) of the gain of the best relay, $Z_j(t)$, depends only on the cardinality $k \triangleq |\mathcal{D}(t)|$ of set $\mathcal{D}(t)$. Hence, assuming $\mathcal{D}(t)$ contains k elements, the PDF of $Z_j(t)$ is given by [9]

$$f_{Z_j,k}(z) = k\left[1 - \exp\left(-\frac{z^2}{\Omega_Z}\right)\right]^{k-1} \frac{2z}{\Omega_Z} \exp\left(-\frac{z^2}{\Omega_Z}\right). \quad (2)$$

The instantaneous SNR of the end-to-end channel can be expressed as $\gamma(t) = \Gamma_0 G^2(t)$, where $G(t)$ is given by

$$G(t) = \begin{cases} \sqrt{X^2(t) + Z_j^2(t)} & \text{if } \mathcal{D}(t) \neq \emptyset \\ X(t) & \text{if } \mathcal{D}(t) = \emptyset \end{cases} \quad (3)$$

C. Capacity Outage and Non-Outage Durations

We assume the source transmits at a fixed rate of R_0 b/s/Hz. Hence, the relays can decode if $\log_2(1+\Gamma_0 Y_i^2(t))/2 \geq R_0$ and the decoding threshold at the relays is $Y_0 \triangleq \sqrt{(2^{2R_0}-1)/\Gamma_0}$. The mutual information of the cooperative end-to-end S-D channel is given by [10]

$$I(t) = \frac{1}{2}\log_2\left(1 + \Gamma_0 G^2(t)\right), \quad (4)$$

and reliable decoding at the destination is possible as long as $I(t) \geq R_0$. On the other hand, decoding will fail (outage event) if $I(t) < R_0$ or equivalently

$$G(t) < G_0, \quad (5)$$

where $G_0 \triangleq \sqrt{(2^{2R_0}-1)/\Gamma_0} = Y_0$.[2] Hence, the outage probability of the system is given by

$$P_{\text{out}} = P(I(t) < R_0) = P(G(t) < G_0), \quad (6)$$

where $P(A)$ denotes the probability of event A. We emphasize that for (6) to bear relevance for a practical system two conditions have to be satisfied [10], [11]. First, a coding block has to comprise a sufficiently large number of symbols such that capacity approaching codes exist. Second, the duration of a coding block, T, has to be sufficiently short such that the channel $G(t)$ is practically constant during the transmission of one codeword, i.e., T has to be much smaller than the coherence time, T_{coh}, of the channel.

Although $G(t)$ is approximately constant within one coding block, it changes slowly from one coding block to the next. Thus, both the average duration of a capacity outage event (AOD) and the average duration of a capacity non-outage event (ANOD) are of interest for system design. Because of the relation between $I(t)$ and $G(t)$, the AOD is identical to the average duration that the random process $G(t)$ remains below G_0 after crossing that level in the downward direction. Thus, the AOD can be obtained as [12]

$$O_I(R_0) = O_G(G_0) = P(G < G_0)/L_G(G_0), \quad (7)$$

where $L_G(G_0)$ denotes the level crossing rate (LCR) of random process $G(t)$ at threshold G_0. Similarly, the ANOD is identical to the average duration that the random process $G(t)$ remains above G_0, after crossing that level in the upward direction. Thus, the ANOD is given by

$$N_I(R_0) = N_G(G_0) = (1 - P\{G < G_0\})/L_G(G_0). \quad (8)$$

Based on (7) and (8) we can express the outage probability in terms of the AOD and the ANOD

$$P(I < R_0) = P(G < G_0) = \frac{O_G(G_0)}{O_G(G_0) + N_G(G_0)}. \quad (9)$$

In a practical system, the minimum duration of an outage event is equal to the duration of one coding block. In general, on average approximately $\lfloor O_I(R_0)/T \rfloor$ and $\lfloor N_I(R_0)/T \rfloor$ consecutive codewords will be decoded unsuccessfully and successfully, respectively. As a result, for example, in ARQ systems, the interval between two retransmission attempts should be chosen on the order of $O_I(R_0)$.

III. AOD AND ANOD

In this section, the exact and asymptotic expressions for the AOD and ANOD of the considered SRC cooperative protocol are derived. As can be observed from (7) and (8), the AOD and ANOD can be obtained easily if the LCR, $L_G(G_0)$, and the outage probability, $P(G \leq G_0)$, are known.

A. LCR - Smooth Part of $G(t)$

Random process $G(t)$ as a function of time t is a function with discontinuities, i.e, there are points at which $\lim_{\epsilon \to 0} |G(t-\epsilon) - G(t+\epsilon)| > 0$. Thus, $G(t)$ is not differentiable at every point and the well-known Rice's formula [12] cannot be directly applied for obtaining the LCR. To circumvent this problem, we decompose the level crossings of $G(t)$ into two types. The first type corresponds to the time period when the cardinality k of $\mathcal{D}(t)$ is fixed. During this time period, $G(t)$ is a smooth function. The second type of crossings is caused by changes in set $\mathcal{D}(t)$.

We first consider the level crossings that are caused by the smooth part of $G(t)$. To this end, it is convenient to denote the end-to-end gain for a set $\mathcal{D}(t)$ of cardinality k by $G_k(t)$. Thus, by the total probability theorem the joint PDF of $G(t)$ and its derivative, $\dot{G}(t)$, can be expressed as

$$f_{G\dot{G}}(g,\dot{g}) = \sum_{k=0}^{M} f_{G_k\dot{G}_k}(g,\dot{g})P(|\mathcal{D}| = k) \quad (10)$$

where $f_{G_k\dot{G}_k}(g,\dot{g})$ is the joint PDF of $G_k(t)$ and its derivative, $\dot{G}_k(t)$, and

$$P(|\mathcal{D}| = k) = \binom{M}{k}P(Y \geq Y_0)^k P(Y < Y_0)^{M-k} \quad (11)$$

with $P(Y \geq Y_0) = 1 - P(Y < Y_0)$ and $P(Y < Y_0) = 1 - \exp(-Y_0^2/\Omega_Y)$. Since for a given k, $G(t)$ is a smooth function, we can apply Rice's formula [12]

$$L_{G_S}(G_0) = \int_0^\infty \dot{g} f_{G\dot{G}}(G_0,\dot{g})d\dot{g} \quad (12)$$

to obtain the corresponding LCR,

$$L_{G_S}(G_0) = \sum_{k=0}^{M} L_{G_k}(G_0)P(|\mathcal{D}| = k), \quad (13)$$

where $L_{G_k}(G_0)$ denotes the LCR of G_k,

$$L_{G_k}(G_0) = \int_0^\infty \dot{g} f_{G_k\dot{G}_k}(G_0,\dot{g})d\dot{g}. \quad (14)$$

For a given cardinality k, $Z_j(t)$ and its derivative $\dot{Z}_j(t)$ are independent and $\dot{Z}_j(t)$ is a zero mean Gaussian RV with

variance $\sigma_{\dot{Z}}^2$ [13]. Thus, exploiting (2) and following the method in [14], we obtain the LCR of G_k as

$$
L_{G_k}(G_0) = \sqrt{\frac{2}{\pi}} \frac{2k}{\Omega_X \Omega_Z} \int_0^{G_0} \sqrt{(G_0^2 - z^2)\sigma_{\dot{X}}^2 + z^2\sigma_{\dot{Z}}^2}
$$

$$
\exp\left(-\frac{G_0^2 - z^2}{\Omega_X}\right) \left[1 - \exp\left(-\frac{z^2}{\Omega_Z}\right)\right]^{k-1} z \exp\left(-\frac{z^2}{\Omega_Z}\right) dz,
\tag{15}
$$

for $k > 0$. For $k = 0$, $L_{G_k}(G_0)$ is equal to the LCR of $X(t)$ which is given by [13]

$$
L_\alpha(G_0) = \frac{\sigma_{\dot{\alpha}}}{\sqrt{2\pi}} \frac{2G_0}{\Omega_\alpha} \exp\left(-\frac{G_0^2}{\Omega_\alpha}\right), \quad \alpha \in \{X, Y, Z\}. \tag{16}
$$

The LCR of the smooth part of $G(t)$ can be obtained by combining (13)-(16).

B. LCR - Discontinuities of $G(t)$

Let us define $t_- \triangleq \lim_{\epsilon \to 0}(t-\epsilon)$ and $t_+ \triangleq \lim_{\epsilon \to 0}(t+\epsilon)$. A discontinuity of $G(t)$ at time t can happen if and only if a relay was removed from or added to set $\mathcal{D}(t)$, i.e., $\mathcal{D}(t_-) \neq \mathcal{D}(t_+)$. In particular, upward crossings of G_0 can happen only if a relay is added to $\mathcal{D}(t)$. Thus, denoting the cardinality of the set at time t_- by $k-1$, we have $k - 1 = |\mathcal{D}(t_-)| < |\mathcal{D}(t_+)| = k$. A upward level crossing happens if we have for the corresponding channel gains

$$
G_{k-1}(t_-) < G_0 \text{ AND } G_k(t_+) \geq G_0. \tag{17}
$$

For computation of the LCR, we first count the average number of changes of the cardinality from $k-1$ to k. The fraction of these changes that lead to level crossings is obtained by a simple multiplication with the probability of the event in (17). A subsequent summation over all $k \in \{1, 2, \ldots M\}$ yields the LCR due to discontinuities of $G(t)$.

A new relay is added to $\mathcal{D}(t)$ if the source-relay gain, $Y_i(t)$, of relay R_i that is not included in $\mathcal{D}(t)$, yet, crosses Y_0. Let us put all $k-1$ source-relay gains, $Y_i(t)$, with $Y_i(t) \geq Y_0$ into set $\mathcal{A}$ and all $M - k + 1$ remaining source-relay gains into set $\bar{\mathcal{A}}$. $|\mathcal{D}(t)|$ will change from $k-1$ to k if one of the elements of $\bar{\mathcal{A}}$ crosses the threshold Y_0 in an upward direction. Let the element of $\bar{\mathcal{A}}$, out of the $M - k + 1$ possible elements, that crosses Y_0 in an upward direction be Y_i. Furthermore, let $\bar{\mathcal{A}}_i$ denote a set equal to the set $\bar{\mathcal{A}}$ but with Y_i removed, i.e., $\bar{\mathcal{A}}_i = \bar{\mathcal{A}} \setminus Y_i$. The average number of upward crossings of Y_i is $L_Y(Y_0)$, cf. (16), and the average number of upward crossings made by Y_i when the elements of $\bar{\mathcal{A}}_i$ are smaller than Y_0 and the elements of $\mathcal{A}$ are larger than Y_0 is

$$
L_Y(Y_0) \prod_{Y_k \in \mathcal{A}} P(Y_k \geq Y_0) \prod_{Y_l \in \bar{\mathcal{A}}_i} P(Y_l < Y_0)
$$

$$
= L_Y(Y_0) P(Y \geq Y_0)^{k-1} P(Y < Y_0)^{M-k}. \tag{18}
$$

Since the two events that (a) Y_i crosses Y_0 when $Y_j \in \bar{\mathcal{A}}_i$ and (b) $Y_j(t)$ crosses Y_0 when $Y_i \in \bar{\mathcal{A}}_j$, $i \neq j$, are mutually exclusive, the total number of upward crossings made by all elements of $\bar{\mathcal{A}}$ is the sum of the crossings made by the individual elements of $\bar{\mathcal{A}}$ given by

$$
\sum_{Y_i \in \bar{\mathcal{A}}} L_Y(Y_0) P(Y \geq Y_0)^{k-1} P(Y < Y_0)^{M-k}
$$

$$
= (M - k + 1) L_Y(Y_0) P(Y \geq Y_0)^{k-1} P(Y < Y_0)^{M-k}. \tag{19}
$$

Eq. (19) multiplied with the probability of event (17) gives the average number of upward crossings of G_0 due to the discontinuities of $G(t)$ for one specific $\bar{\mathcal{A}}$. Since $\bar{\mathcal{A}}$ contains $M - k + 1$ of the M gains $Y_i(t)$, $i \in \{1, 2, \ldots, M\}$, there are $\binom{M}{M-k+1}$ different sets $\bar{\mathcal{A}}$. Hence, using $(M-k+1)\binom{M}{M-k+1} = k\binom{M}{k}$ and summing over $k \in \{1, 2, \ldots, M\}$, based on (19), the LCR caused by the discontinuities of $G(t)$, $L_{G_D}(G_0)$, is obtained as

$$
L_{G_D}(G_0) = L_Y(Y_0) \sum_{k=1}^M \left[k\binom{M}{k} P(Y \geq Y_0)^{k-1} \right.
$$

$$
\left. \times P(Y < Y_0)^{M-k} P(G_{k-1} < G_0 \text{ AND } G_k \geq G_0) \right]. \tag{20}
$$

For the term $P(G_{k-1} < G_0 \text{ AND } G_k \geq G_0)$, we obtain for $k = 1$

$$
P(G_{k-1} < G_0 \text{ AND } G_k \geq G_0)
$$

$$
= \frac{\Omega_Z}{(\Omega_X - \Omega_Z)} \left[\exp\left(-\frac{G_0^2}{\Omega_X}\right) - \exp\left(-\frac{G_0^2}{\Omega_Z}\right) \right] \tag{21}
$$

and for $k > 1$

$$
P(G_{k-1} < G_0 \text{ AND } G_k \geq G_0)
$$

$$
= \frac{(k-1)e^{-G_0^2/\Omega_Z}}{(\Omega_X - \Omega_Z)} \int_0^{G_0^2} (1 - e^{-z/\Omega_Z})^{k-2} e^{-z/\Omega_Z}
$$

$$
\times \left[\exp\left(-(G_0^2 - z)\frac{\Omega_Z - \Omega_X}{\Omega_X \Omega_Z}\right) - 1 \right] dz. \tag{22}
$$

C. Exact AOD and ANOD

As mentioned before, for computation of both the AOD and ANOD, we require the LCR and the outage probability. Based on the results of the previous two subsections, the LCR, $L_G(G_0)$, of random process $G(t)$ is obtained as

$$
L_G(G_0) = L_{G_S}(G_0) + L_{G_D}(G_0), \tag{23}
$$

where we have added the crossings due to the smooth and discontinuous parts of $G(t)$.

The outage probability of $G(t)$ is obtained as

$$
P(G(t) < G_0) = \sum_{k=0}^M P(G_k(t) < G_0) P(|\mathcal{D}| = k), \tag{24}
$$

where for $k = 0$, $P(G_0(t) < G_0) = P(X(t) < G_0) = 1 - \exp(-G_0^2/\Omega_X)$, and for $k > 0$,

$$
P(G_k(t) < G_0) = \int_0^{G_0^2} \left[1 - \exp\left(-\frac{G_0^2 - x}{\Omega_Z}\right)\right]^k \frac{e^{-x/\Omega_X}}{\Omega_X} dx \tag{25}
$$

Finally, the AOD and ANOD are obtained by inserting (23)-(25) in (7) and (8), respectively. We note that all remaining integrals can be solved in closed form by using the binomial expansion. This is omitted here for brevity.

D. Asymptotic AOD and ANOD

To gain more insight, it is interesting to consider the high SNR regime, i.e., $\Gamma_0 \to \infty$, or equivalently, $G_0 \to 0$ and $Y_0 \to 0$. The results in this section were obtained by exploiting the expansions $e^{-t} \sim 1 - t$ and $\Gamma(k, t) \sim \Gamma(k) - t^k/k$ as $t \to 0$ for the exponential and incomplete Gamma functions [15], respectively. Here, the notation $g(x) \sim h(x)$ as $x \to x_0$ means that $g(x)$ and $h(x)$ are asymptotically equivalent, i.e., $\lim_{x \to x_0} g(x)/h(x) = 1$.

Applying the aforementioned expansions, for high SNR, we obtain from (23) for the asymptotic LCR

$$L_G(G_0) \sim \sqrt{\frac{2}{\pi}} \frac{\sqrt{2^{2R_0}-1}^{2M+1}}{\Omega_X \Omega_Y^M \Gamma_0^{M+1/2}} \left(\sigma_{\dot{X}} + \sum_{k=1}^{M} \binom{M}{k} \left(\frac{\Omega_Y}{\Omega_Z}\right)^k \right.$$
$$\left. \times \left[\sigma_{\dot{X}} {}_2F_1\left(-\frac{1}{2}, k; k+1; 1-\frac{\sigma_{\dot{Z}}^2}{\sigma_{\dot{X}}^2}\right) + \sigma_{\dot{Y}}\frac{\Omega_Z}{\Omega_Y} \right] \right), \quad (26)$$

where ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$ denotes the Gauss' hypergeometric function [15, (15.1)]. Similarly, we obtain for the asymptotic outage probability

$$P(G < G_0) \sim \frac{(2^{2R_0}-1)^{M+1}}{\Omega_X \Omega_Y^M \Gamma_0^{M+1}} \sum_{k=0}^{M} \binom{M}{k} \frac{1}{k+1} \left(\frac{\Omega_Y}{\Omega_Z}\right)^k. \quad (27)$$

The asymptotic AOD and ANOD are obtained by inserting (26), (27) in (7) and (8), respectively.

To gain more insight, we consider the case of a symmetric network with $\Omega_X = \Omega_Y = \Omega_Z = \Omega$ and $\sigma_{\dot{X}} = \sigma_{\dot{Y}} = \sigma_{\dot{Z}} = \pi f_m \sqrt{\Omega}$. In this case, we obtain for the AOD

$$O_G \sim \frac{\sqrt{2^{2R_0}-1}}{\sqrt{2\pi} f_m (M+1)\sqrt{\Gamma_0}} \quad (28)$$

and for the ANOD

$$N_G \sim \frac{\Omega^{M+1/2} \Gamma_0^{M+1/2}}{\sqrt{2\pi} f_m (2^{M+1}-1)\sqrt{2^{2R_0}-1}^{2M+1}}. \quad (29)$$

From (28) and (29) we observe that for high SNR, on a double-logarithmic scale, the AOD and ANOD decrease and increase linearly with the SNR, respectively. However, while the slope of the AOD is equal to $-1/2$ independent of M, the slope of the ANOD is equal to $M + 1/2$ and thus, similar to the outage probability, strongly benefits from an increasing number of relays. Hence, while the number of available relays has a relatively minor effect on the average number of successive codewords affected by an outage, the effect on the average number of successive correctly decoded codewords is major. This will be illustrated by an example in the next section.

IV. NUMERICAL EXAMPLE

In Fig. 1, we show the AOD and ANOD vs. SNR, Γ_0, for $R_0 = 1$ b/s/Hz, $\Omega_X = 1.3$, $\Omega_Y = 0.9$, $\Omega_Z = 0.8$, and $f_{mX} = f_{mY} = f_{mZ} = 50$ Hz. The exact analytical results from Section III-C are confirmed by simulation results and the high SNR approximations from Section III-D become tight for $\Gamma_0 > 18$ dB. The fundamentally different impact of the number of relays on the AOD and ANOD is confirmed by Fig. 1. For example, assuming an SNR of 20 dB and a codeword duration of $T = 10^{-5}$ s, for $M = 1$, 3, and 7 on average $O_I(G_0)/T = 67$, 35, and 18 consecutive codewords are in outage. In contrast, for the same numbers of relays on average $N_I(G_0)/T = 5.8 \times 10^4$, 1.07×10^7, and 4.7×10^{11} consecutive codewords are not in outage.

V. CONCLUSIONS

In this paper, we derived the AOD and ANOD of an SRC DF cooperative diversity system. Besides the exact expressions for the AOD and ANOD, we developed insightful high SNR approximations. This asymptotic analysis revealed that while increasing the number of relays has a moderate positive effect on the AOD, it has a much larger positive effect on the ANOD.
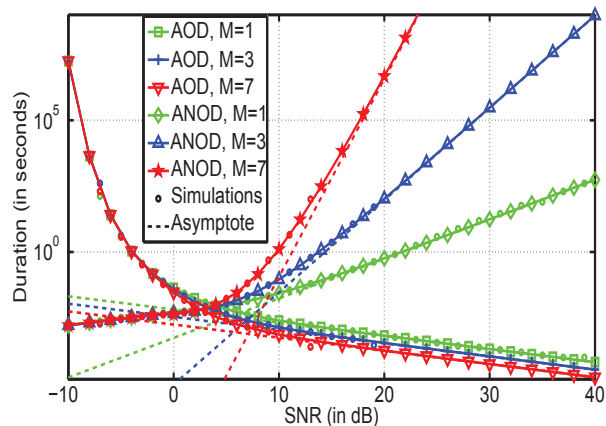

Fig. 1. AOD and ANOD vs. SNR, Γ_0. Solid lines with markers: Exact results, cf. Section III-C. Dashed lines: Asymptotic results, cf. Section III-D. Circle markers: Simulations.

In a practical system, the AOD may be used as a design guideline for the minimum interval between two retransmission attempts in an ARQ system or the duration of a scheduling slot in a multi-user system. The ANOD on the other hand can serve as an estimate for the duration of interruption free transmission.

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