Outage Rate and Outage Duration of Decode-and-Forward Cooperative Diversity Systems

Nikola Zlatanov†, Zoran Hadzi-Velkov*, George K. Karagiannidis‡, and Robert Schober†

†Department of Electrical and Computer Engineering, University of British Columbia, Vancouver, Canada
*Faculty of Electrical Engineering and Information Technologies, Ss. Cyril and Methodius University, Skopje
‡Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, Thessaloniki

***Abstract*—A complete evaluation of the benefits of cooperative diversity schemes should not only include the outage and error rate performance but also the second-order statistics of the achievable information-theoretic capacity. In a non-ergodic fading channel, the system is said to be in outage when the destination cannot decode the fixed-rate transmitted signal with negligible error probability. Because of the Doppler effect, which is induced by the mobility of the wireless nodes, these capacity outage events are correlated. In this paper, we derive the average outage rate (AOR) and the average outage duration (AOD) of two well-known cooperative diversity protocols, decode-and-forward relaying and selection decode-and-forward relaying, operating in slow Rayleigh fading channels. We also analyze the asymptotic behavior of these statistical parameters for high SNRs, where it is shown that the AOR exhibits a similar behavior as the outage probability.**

I. Introduction

Cooperative diversity is an efficient technique which employs spatially distributed nodes (also referred to as relays) to effectively synthesize a virtual array that emulates the operation of a multi-antenna transceiver [1] - [3]. Sendonaris et al. [1] showed that cooperative diversity enlarges the Shannon capacity region for ergodic fading channels (when channel state information (CSI) is available to the transmitters), and leads to higher system robustness against fading (even though the interuser channel is noisy). Laneman et al. showed in [2] that, compared to non-cooperative transmission, cooperative diversity does not increase the maximum sum-rate when CSI is available only to the receivers. For non-ergodic or delay-constrained scenarios, [2] developed several simple repetition-based cooperative protocols: fixed amplify-and-forward relaying (AF), fixed decode-and-forward relaying (DF), selection decode-and-forward relaying (SR), and incremental relaying. The performance of these protocols was characterized in [2] in terms of the outage probability (OP), and it was shown analytically that, except for fixed DF relaying, each of the cooperative protocols achieves full diversity. A form of DF relaying that achieves full diversity was studied in [3] and [4].

Another important cooperative relay mechanism is relay selection, which achieves a diversity order equal to the number of available relays [5]. The effect of time-varying fading in selective cooperative networks was studied in [6], where the relay switching rate as a function of the average channel gains and the maximum Doppler frequency of each of the source-relay and relay-destination links was derived. Besides repetition of the overheard signal, the cooperation can also significantly benefit from using more sophisticated cooperative strategies, such as cooperative coding and distributed space-time codes [3].

Since wireless channels are time varying, due to the random nature of the propagation environment and the mobility of the nodes (inducing the Doppler effect), the underlying channel capacity randomly varies with time. The time-varying capacity therefore suffers from the random occurrence of capacity fades, during which the channel is unable to support a specific data rate. The statistical characteristics of this fundamental limitation of cooperative systems has to be investigated in order to better understand the behavior of such systems. In analogy to the level crossing rate (LCR) and the average fade duration (AFD) of mobile fading channels, in this paper, we describe the temporal variations of the capacity of the channel in cooperative systems by the average outage rate (AOR) and the average outage duration (AOD).

The AOR and the AOD describe the second-order statistical behavior of the supportable data rate of a cooperative diversity system. In a non-ergodic fading channel, a capacity outage event occurs when the destination can no longer decode the fixed-rate transmitted signal with negligible error probability [7] [8]. The notions of AOR and AOD have been first introduced for opportunistic relaying systems in [9]. Previously, a similar definition of second-order outage statistics was used for multiple-input multiple-output (MIMO) systems in [10]. The notion of AOD also appears in [11], which studies the second-order outage statistics of multihop communication systems, but only in terms of the classic understanding of envelope AFD, without relating it to the system capacity outage.

Apart from the OP and the average error performance, the design and the configuration of cooperative diversity systems also requires proper knowledge and understanding of second-order statistical parameters, such as the AOR and the AOD. In the system layer, these parameters may assist in the proper selection of the transmission rate. In the physical layer, they are important for selecting appropriate (adaptive) modulation and coding schemes. In the link layer, AOR and AOD may help to determine the transmitter buffer lengths for delay sensitive applications and the retransmission periods of automatic repeat request (ARQ) protocols.

In this paper, we present an analytical framework for the evaluation of the second-order capacity statistics of cooperative diversity networks. Specifically, we derive exact closed-form expressions for the AOR and the AOD for two well-known cooperative protocols introduced in [2] - namely the

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DF and SR protocols, assumed to operate at fixed transmission rates over slowly time-varying Rayleigh fading channels. We also analyze the asymptotic behavior of these protocols for high signal-to-noise ratios (SNRs).

II. Cooperative diversity with mobile nodes

A. System and Channel Model

We consider the same scenario as in [2], where the cooperative diversity network consists of source S, destination D, and relay R. Source S and destination D communicate by one of the two considered cooperative diversity protocols, DF or SR, over two half-duplex sub-channels: the direct sub-channel ($S \rightarrow D$) and the relayed sub-channel, consisting of the $S \rightarrow R$ link and the $R \rightarrow D$ link. The transmissions over the sub-channels are made orthogonal by dividing each transmission slot into two consecutive independent sub-slots, one for each sub-channel. During the first sub-slot, S transmits and D and R listen. In the second sub-slot, R transmits and D listens. Without loss of generality, source S and relay R transmit with equal power P_T, so $\Gamma_0 = P_T/N_0$ denotes the SNR without fading (also referred to as the transmit SNR).

The three links are subject to frequency-nonselective slow fading, which means that their channel coefficients are approximately constant for the entire slot duration (so called *quasi-static* scenario). The three channel coefficients are modelled as zero-mean, independent, circularly symmetric complex Gaussian random processes. Their amplitudes follow the Rayleigh probability density function (PDF) and their phases are uniformly distributed over $[0, 2\pi)$. The channel coefficient of each link (i.e., the CSI) is available at the respective receiver but not at the transmitter.

In each slot t, the channel amplitudes of the $S \rightarrow D$ link, $S \rightarrow R$ link, and $R \rightarrow D$ link are denoted by $X(t)$, $Y(t)$, and $Z(t)$, respectively, and are distributed according to the Rayleigh PDF,

$$f_\alpha(r) = \frac{2r}{\Omega_\alpha} \exp\left(-\frac{r^2}{\Omega_\alpha}\right) \quad , \quad r \geq 0, \quad \alpha \in \{X, Y, Z\} \qquad (1)$$

with cumulative distribution function (CDF) $F_\alpha(r) = \Pr\{\alpha \leq r\} = 1 - \exp(-r^2/\Omega_\alpha)$. The average squared amplitudes are defined as $E[X^2] = \Omega_{SD} = \Omega_X$, $E[Y^2] = \Omega_{SR} = \Omega_Y$, and $E[Z^2] = \Omega_{RD} = \Omega_Z$, where $E[\cdot]$ denotes expectation.

In case of the DF protocol, in the first sub-slot, R attempts to decode the full information from S; in the second sub-slot, R re-encodes and re-transmits the information estimate, whereas D combines the two replicas in an attempt to decode. In case of the SR protocol, if the measured channel amplitude $Y(t)$ falls below a certain threshold Y_0, then, in the second sub-slot, S simply re-transmits the same packet and R remains silent; otherwise, R re-encodes and re-transmits the information estimate.

B. Mobility of the Nodes

We consider 2-dimensional isotropic scattering around source S, relay R, and destination D, which are all assumed to be mobile. Thus, S, R, and D introduce maximum Doppler rates of f_{mS}, f_{mR}, and f_{mD}, respectively. The links $S \rightarrow D$, $S \rightarrow R$, and $R \rightarrow D$ behave as independent mobile-to-mobile Rayleigh channels, whose channel amplitudes, $X(t)$, $Y(t)$, and $Z(t)$, are time-correlated Rayleigh random processes with known statistical parameters, such as the auto-covariance function and the Doppler spectrum [14]. If a station at one end of a link is fixed, the mobile-to-mobile channel model simplifies to the "classic" Jake's fading channel model [13]. For both channel types, the time derivative of the channel amplitude $\dot{\alpha}$ is independent from the amplitude α itself, and follows a zero-mean Gaussian PDF with the respective variances given by

$$\sigma_{\dot{X}}^2 = \pi^2 \Omega_X \left(f_{mS}^2 + f_{mD}^2\right) \qquad (2a)$$

$$\sigma_{\dot{Y}}^2 = \pi^2 \Omega_Y \left(f_{mS}^2 + f_{mR}^2\right) \qquad (2b)$$

$$\sigma_{\dot{Z}}^2 = \pi^2 \Omega_Z \left(f_{mR}^2 + f_{mD}^2\right) \qquad (2c)$$

III. Average Outage Rate and Outage Duration

The end-to-end link between S and D can be modelled in form of an equivalent channel. If, in a given time slot t, the fading amplitude of that equivalent channel is denoted by $G(t)$, then the instantaneous received SNR at D is $G^2(t)\Gamma_0$ and the mutual information is $I(t) = \frac{1}{2}\log_2(1+G^2(t)\Gamma_0)$. For the cooperative network considered in this paper, the capacity reduction factor $1/2$ appears in the definition of $I(t)$ because of the repetition-coded transmission over the two successive sub-slots [2]. The random event that this mutual information drops below some target spectral efficiency R_0,

$$I(t) = \frac{1}{2}\log_2\left[1 + G^2(t)\,\Gamma_0\right] \leq R_0, \qquad (3)$$

is referred to as the system outage event [7] [12]. Reliable decoding is possible as long as $I(t)$ exceeds the transmitter encoding rate R_0.

From (3), we observe that a capacity outage event in a given slot t occurs if

$$G(t) \leq G_0\,, \qquad (4)$$

where G_0 is the *outage threshold* defined as

$$G_0 = \sqrt{\frac{2^{2R_0} - 1}{\Gamma_0}}\,. \qquad (5)$$

Depending on the particular cooperative protocol, $G(t)$ is a random process that depends on the channel amplitudes, $X(t)$, $Y(t)$, and $Z(t)$, i.e.,

$$G(t) = f\left[X(t), Y(t), Z(t)\right]. \qquad (6)$$

Note that $\Pr\{I \leq R_0\} = \Pr\{G \leq G_0\}$ determines the OP of the cooperative diversity network. Since $X(t)$, $Y(t)$, and $Z(t)$ are time-correlated Rayleigh random processes, $G(t)$, is also a time-correlated random process whose second-order statistical parameters - the LCR and the AFD evaluated at threshold G_0 - correspond to the AOR and the AOD evaluated at threshold R_0 of the considered cooperative system. Thus, to determine the AOR of the cooperative diversity system, N_I, we can apply Rice's formula, which defines the LCR of random process

$G(t)$ as the rate at which that random process crosses level G_0 in the positive (or negative) direction [13, Eq. (2.101)],

$$N_I(R_0) = N_G(G_0) = \int_0^\infty \dot{g} f_{G\dot{G}}(G_0, \dot{g})\, d\dot{g}\,, \tag{7}$$

where $\dot{G}$ denotes the time derivative of random process G, and $f_{G\dot{G}}(g,\dot{g})$ is the joint PDF of G and $\dot{G}$. The AOD of the cooperative diversity system, T_I, is determined from the average time that random process $G(t)$ remains below G_0 after crossing that level in downward direction, and is defined as

$$T_I(R_0) = T_G(G_0) = \frac{\Pr(G \le G_0)}{N_G(G_0)}\,. \tag{8}$$

A. Direct Transmission

The non-cooperative case (i.e., the direct transmission between S and D) is used to establish a reference for our subsequent study of the AOR and the AOD of cooperative diversity systems. The maximum average mutual information between the input and the output, achieved by independent and identically distributed (i.i.d.) zero-mean, circularly symmetric complex Gaussian inputs, is given by $I_D(t) = \log_2(1 + X^2(t)\Gamma_0)$. A capacity outage event occurs when the mutual information, I_D, drops below some target spectral efficiency R_0 [12, Eq. (5.54)], or, equivalently, if

$$X(t) \le X_0 \tag{9}$$

for the outage threshold $X_0 = \sqrt{(2^{R_0}-1)/\Gamma_0}$. The OP for this non-cooperative diversity system is equal to

$$\Pr\{X \le X_0\} = 1 - \exp\left(-\frac{X_0^2}{\Omega_X}\right)\,. \tag{10}$$

The AOR N_I can be expressed in terms of the LCR of $X(t)$, and is given by [13, Eq. (2.101)]

$$N_I(R_0) = N_X(X_0) = \sqrt{\frac{2\sigma_{\dot{X}}^2}{\pi}}\frac{X_0}{\Omega_X}\exp\left(-\frac{X_0^2}{\Omega_X}\right)\,. \tag{11}$$

The AOD T_I is obtained by using (10) and (11) in (8).

B. Decode-and-Forward (DF) Relaying

For DF relaying, an exact expression for the maximum average mutual information can be obtained only under assumption of full decoding of the source message by the relay R [8] [2, Eq. (15)]. In this case, the amplitude $G(t)$ of the equivalent channel in (6) is determined as

$$G(t) = \min\{Y(t), U(t)\}\,, \tag{12}$$

where $U(t)$ is an auxiliary random process defined as

$$U(t) = \sqrt{X^2(t) + Z^2(t)}\,. \tag{13}$$

The OP for this cooperative diversity system is equal to $\Pr\{G \le G_0\}$, which is well known to be given by

$$\Pr\{G \le G_0\} = 1 - \Pr\{Y > G_0\}\Pr\{U > G_0\}\,, \tag{14}$$

where [15]

$$\Pr\{U > G_0\} = \begin{cases} \frac{\Omega_X}{\Omega_X - \Omega_Z} e^{-G_0^2/\Omega_X} + \frac{\Omega_Z}{\Omega_Z - \Omega_X} e^{-G_0^2/\Omega_Z}, & \Omega_X \ne \Omega_Z \\ e^{-G_0^2/\Omega_X}\left(1 + G_0^2/\Omega_X\right), & \Omega_X = \Omega_Z \end{cases} \tag{15}$$

and

$$\Pr\{Y > G_0\} = \exp\left(-\frac{G_0^2}{\Omega_Y}\right)\,. \tag{16}$$

Based on the time derivative of both sides of (12),

$$\dot{G} = \begin{cases} \dot{Y}, & Y \le U \\ \dot{U}, & Y > U \end{cases}\,, \tag{17}$$

and the independence of the channel amplitudes Y and U, the joint PDF of G and $\dot{G}$ is given by

$$f_{G\dot{G}}(g,\dot{g}) = f_{Y\dot{Y}}(g,\dot{g})\Pr\{U > g\} + f_{U\dot{U}}(g,\dot{g})\Pr\{Y > g\}\,. \tag{18}$$

Introducing (18) into (7) yields

$$\begin{aligned} N_I(R_0) &= N_G(G_0) \\ &= N_Y(G_0)\Pr\{U > G_0\} + N_U(G_0)\Pr\{Y > G_0\}\,, \end{aligned} \tag{19}$$

where $\Pr\{U > G_0\}$ and $\Pr\{Y > G_0\}$ are given by (15) and (16), respectively. In (19), $N_Y(G_0)$ is the LCR of Rayleigh random process $Y(t)$, which is well known and given by [13, Eq. (2.101)]

$$N_Y(G_0) = \sqrt{\frac{2\sigma_{\dot{Y}}^2}{\pi}}\frac{G_0}{\Omega_Y}\exp\left(-\frac{G_0^2}{\Omega_Y}\right)\,, \tag{20}$$

whereas $N_U(G_0)$ is the LCR of random process $U(t)$.

For $\Omega_X \ne \Omega_Z$ and arbitrary f_{mS}, f_{mR}, and f_{mD}, we derive in Appendix A the LCR of random process $U(t)$ as

$$\begin{aligned} N_U(G_0) = &\sqrt{\frac{2}{\pi}}\frac{\sigma_{\dot{X}}^3}{\sigma_{\dot{Z}}^2 - \sigma_{\dot{X}}^2}\frac{\exp\left(-G_0^2/\Omega_X\right)}{\Omega_X\Omega_Z}\left(\frac{G_0}{\sqrt{W(G_0)}}\right)^3 \\ &\times \exp\Big(W(G_0)\Big)\left[\Gamma\left(\frac{3}{2}, W(G_0)\right) - \Gamma\left(\frac{3}{2}, \frac{\sigma_{\dot{Z}}^2}{\sigma_{\dot{X}}^2}W(G_0)\right)\right] \end{aligned} \tag{21}$$

where $\Gamma(\cdot,\cdot)$ is the incomplete Gamma function [16, Eq. (6.5.3)], and

$$W(G_0) = \frac{G_0^2(\Omega_X - \Omega_Z)}{\Omega_X\Omega_Z}\frac{\sigma_{\dot{X}}^2}{\sigma_{\dot{Z}}^2 - \sigma_{\dot{X}}^2}\,,$$

where $\sigma_{\dot{X}}$ and $\sigma_{\dot{Z}}$ are given by (2). When $\Omega_X = \Omega_Z$, after applying the results presented in Appendix A, we obtain

$$N_U(G_0) = \frac{4G_0^3}{3\sqrt{2\pi}}\frac{\exp\left(-G_0^2/\Omega_X\right)}{(\Omega_X)^2}\frac{\sigma_{\dot{Z}}^3 - \sigma_{\dot{X}}^3}{\sigma_{\dot{Z}}^2 - \sigma_{\dot{X}}^2}\,. \tag{22}$$

When $\Omega_X = \Omega_Z$ and $f_{mS} = f_{mR} = f_{mD}$ then $\sigma_{\dot{X}} = \sigma_{\dot{Z}}$, in which case one can apply the limit operation $\sigma_{\dot{X}} \to \sigma_{\dot{Z}}$ in (22) and obtain

$$N_U(G_0) = \frac{\sqrt{2}\sigma_{\dot{X}}G_0^3}{\sqrt{\pi}}\frac{\exp\left(-G_0^2/\Omega_X\right)}{(\Omega_X)^2}\,. \tag{23}$$

The closed-form expression for the AOR N_I of the fixed DF cooperative diversity protocol is obtained by inserting (15), (16), (20), and (21) [or (22) or (23)] into (19), which is omitted here for brevity. The AOR is obtained by inserting (14) and (19) into (8).

C. Selection Decode-and-Forward Relaying (SR)

The SR protocol activates the relay in the second sub-slot of a given slot t only if the measured channel amplitude $Y(t)$ is above a given selection threshold Y_0 [1] and have S retransmit the packet/message otherwise, thus doubling the total combined receive power compared to the case when R is activated. Based on [2, Eq. (19)], the amplitude $G(t)$ of the equivalent fading channel in (6) is determined as

$$G(t) = \begin{cases} \sqrt{2}X(t), & Y(t) \le Y_0 \\ U(t), & Y(t) > Y_0 \end{cases}, \tag{24}$$

where $U(t)$ is as in (13). The OP for this cooperative diversity system is simply expressed as

$$\Pr\{G \le G_0\} = \Pr\left\{\sqrt{2}X \le G_0\right\}\Pr\{Y \le Y_0\} + \Pr\{U \le G_0\}\Pr\{Y > Y_0\}, \tag{25}$$

where

$$\Pr\{Y \le Y_0\} = 1 - \Pr\{Y > Y_0\} = 1 - \exp\left(-\frac{Y_0^2}{\Omega_Y}\right), \tag{26}$$

$$\Pr\left\{\sqrt{2}X \le G_0\right\} = 1 - \exp\left(-\frac{G_0^2}{2\Omega_X}\right), \tag{27}$$

and $\Pr\{U \le G_0\} = 1 - \Pr\{U > G_0\}$ is calculated from (15).

By carefully analyzing the conditions for downward crossings of $G(t)$ of the outage threshold G_0, one can observe the following four independent downward crossing events:

(i) downward crossing of $G(t) = X(t)$, if $Y(t) \le Y_0$,
(ii) downward crossing of $G(t) = U(t)$, if $Y(t) > Y_0$,
(iii) downward crossing of $G(t)$ when $Y(t)$ switches from condition $Y(t) \le Y_0$ to condition $Y(t) > Y_0$, but only if $\sqrt{2}X(t) > G_0$ and $U(t) < G_0$, and
(iv) downward crossing of $G(t)$ when $Y(t)$ switches from condition $Y(t) > Y_0$ to condition $Y(t) \le Y_0$, but only if $U(t) > G_0$ and $\sqrt{2}X(t) < G_0$.

Note that event (i) occurs with probability $\Pr\{Y \le Y_0\}$, event (ii) occurs with probability $\Pr\{Y > Y_0\}$, event (iii) occurs with probability $\Pr\{\sqrt{2}X > G_0 \cap U < G_0\}$, and event (iv) occurs with probability $\Pr\{\sqrt{2}X < G_0 \cap U > G_0\}$. Thus, the AOR N_I at threshold R_0 is determined from the LCR of random process $G(t)$ evaluated at threshold G_0,

$$\begin{aligned} N_I(R_0) &= N_G(G_0) \\ &= N_{\sqrt{2}X}(G_0)\Pr\{Y \le Y_0\} + N_U(G_0)\Pr\{Y > Y_0\} \\ &\quad + N_Y(Y_0)\Pr\left\{\sqrt{2}X(t) > G_0 \cap U(t) < G_0\right\} \\ &\quad + N_Y(Y_0)\Pr\left\{\sqrt{2}X(t) < G_0 \cap U(t) > G_0\right\}, \end{aligned} \tag{28}$$

[1] Although $Y_0 = G_0$ holds, it is convenient to keep both variables in the following since they indicate the considered link (source-relay link vs. end-to-end link).

where, using (11), $N_{\sqrt{2}X}(G_0) = N_X(G_0/\sqrt{2})$. Based on Appendix B, we obtain

$$\Pr\left\{\sqrt{2}X > G_0 \cap U < G_0\right\} = \begin{cases} \frac{\Omega_X}{\Omega_Z - \Omega_X} e^{-G_0^2/\Omega_X} \\ \quad + e^{-G_0^2/(2\Omega_X)}\left(1 - \frac{\Omega_Z}{\Omega_Z - \Omega_X} e^{-G_0^2/(2\Omega_Z)}\right), & \Omega_X \ne \Omega_Z \\ e^{-G_0^2/(2\Omega_X)} - e^{-G_0^2/\Omega_X}\left(1 + G_0^2/(2\Omega_X)\right), & \Omega_X = \Omega_Z \end{cases} \tag{29}$$

whereas

$$\Pr\left\{\sqrt{2}X < G_0 \cap U > G_0\right\} = \begin{cases} \frac{\Omega_Z}{\Omega_X - \Omega_Z} \\ \quad \times \left(e^{-G_0^2/(2\Omega_X)} e^{-G_0^2/(2\Omega_Z)} - e^{-G_0^2/\Omega_Z}\right), & \Omega_X \ne \Omega_Z \\ G_0^2 e^{-G_0^2/\Omega_X} \big/ (2\Omega_X), & \Omega_X = \Omega_Z \end{cases} \tag{30}$$

The closed-form expression for the AOR N_I of the selection DF cooperative diversity protocol is obtained by inserting (20), (21), (26), (29), (30) into (28), but is omitted here for brevity. Typically, $Y_0 = G_0$. The AOD is obtained by inserting (25) and (28) into (8).

IV. Asymptotic Behavior of AOR and AOD

In this section, we study the asymptotic (in the high SNR region) behavior of the AOR and the AOD of cooperative diversity networks utilizing either the DF or the SR protocol. We assume $f_{mS} = f_{mD} = f_{mR} = f_m$, in which case we can conveniently consider the normalized parameters N_I/f_m and $T_I \cdot f_m$.

For the AOR, we obtain approximations of (11), (19), and (28) when $\Gamma_0 \to \infty$ by using the Mclaurin series, $\exp(-x) \sim 1 - x + x^2/2 + \cdots$ as $x \to 0$, since conditions $\Gamma_0 \to \infty$ and $G_0 \to 0$ are equivalent for given R_0. Note, $f_1(x) \sim f_2(x)$ as $x \to x_0$ implies $\lim_{x \to x_0} f_1(x)/f_2(x) = 1$. We also used the expansion of the incomplete Gamma function $\Gamma(\cdot,\cdot)$ [16, Eqs. (6.5.3), (6.5.4) and (6.5.29)]. Omitting the derivations for brevity, the normalized AOR and AOD of the DF and SR protocols for high SNR can be expressed in a simple general form as

$$\frac{N_I}{f_m} \sim \left(\frac{2^{2R_0} - 1}{\Gamma_0}\right)^{d_k - 1/2} g_k(\Omega_X, \Omega_Y, \Omega_Z) \quad \text{as } \Gamma_0 \to \infty, \tag{31}$$

$$T_I \cdot f_m \sim \left(\frac{2^{2R_0} - 1}{\Gamma_0}\right)^{1/2} u_k(\Omega_X, \Omega_Y, \Omega_Z) \quad \text{as } \Gamma_0 \to \infty, \tag{32}$$

where d_k (k = DF or SR) represents the diversity gain of the DF and SR cooperative protocols, i.e., $d_{DF} = 1$ and $d_{SR} = 2$, respectively [2]. Clearly, for direct transmission, $d_{DT} = 1$. For each of the two protocols, the functions $g_{DF}(\cdot)$ and $g_{SR}(\cdot)$ from (31) are respectively given by

$$g_{DF} = \sqrt{\frac{4\pi}{\Omega_Y}}, \tag{33}$$

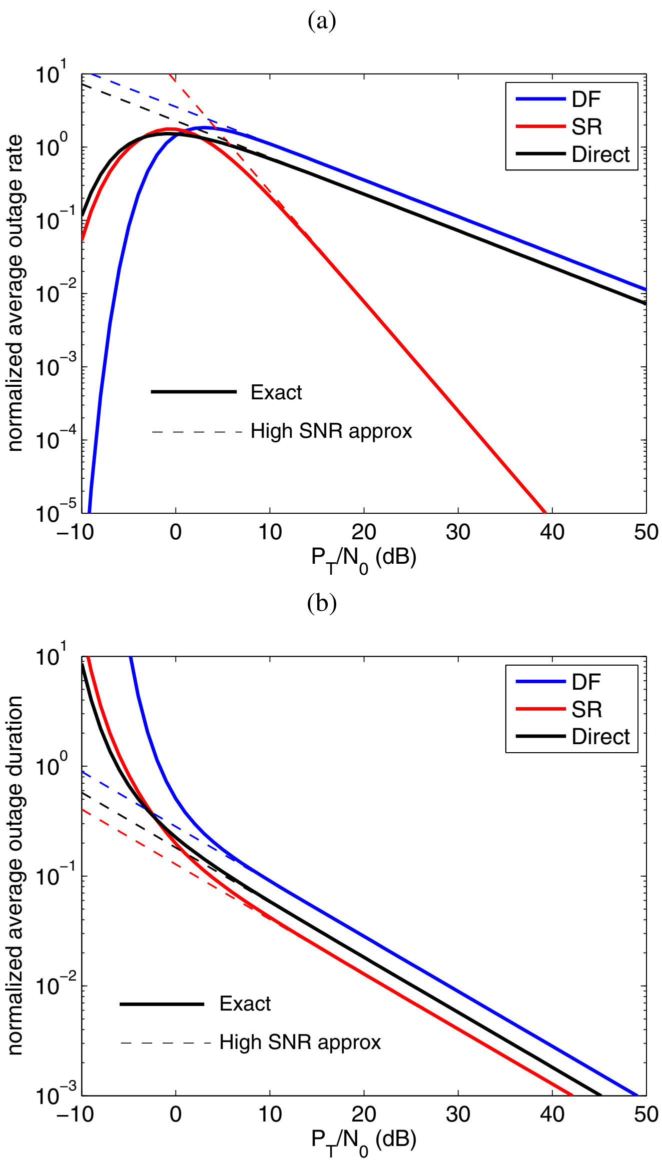

Fig. 1. Normalized AOR and AOD of DF and SR in statistically symmetric cooperative diversity networks ($\Omega_X = \Omega_Y = \Omega_Z = 1$) for R_0 =0.5 bps/Hz

$$g_{SR} = \frac{\sqrt{2\pi}}{\sqrt{\Omega_X \Omega_Y}} + \frac{\sqrt{\pi}}{\Omega_X \sqrt{\Omega_Y}} + \frac{4\sqrt{\pi}}{3\Omega_X \Omega_Z} \frac{\Omega_X + \sqrt{\Omega_X \Omega_Z} + \Omega_Z}{\sqrt{\Omega_X} + \sqrt{\Omega_Z}} . \quad (34)$$

For the AOD, functions $u_{DF}(\cdot)$ and $u_{SR}(\cdot)$ in (32) can be determined using the OP asymptotic results [2, Eq. (18)] and [2, Eq. (22)] and combining them with (31) and (8).

V. Numerical Examples

Table I presents the asymptotic expressions for the AOR and AOD for symmetric cooperative diversity networks ($\Omega_X = \Omega_Y = \Omega_Z = \Omega$), where the product $\Omega\Gamma_0$ is denoted by Γ. The three nodes (S, R, and D) are assumed to introduce the same maximum Doppler frequency, $f_{mS} = f_{mR} = f_{mD} = f_m$.

Fig. 1 illustrates the behavior of the AORs and the AODs of such a symmetric cooperative diversity network, employing either DF or SR relaying with mobile nodes operating over fading channels with $\Omega = 1$. The normalized AORs, N_I/f_m, and AODs, $T_I \cdot f_m$, are plotted against the transmit SNR, Γ_0. The spectral efficiency is set to $R_0 = 0.5$ bps/Hz. The analytical results are validated by computer simulations.

TABLE I
ASYMPTOTIC EXPRESSIONS FOR AOR AND AOD FOR HIGH SNR.

Protocol	AOR	AOD
Direct	$\frac{2\sqrt{\pi} f_m \sqrt{2^{R_0} - 1}}{\sqrt{\Gamma}}$	$\frac{\sqrt{2^{R_0} - 1}}{2\sqrt{\pi} f_m \sqrt{\Gamma}}$
DF	$\frac{2\sqrt{\pi} f_m \sqrt{2^{2R_0} - 1}}{\sqrt{\Gamma}}$	$\frac{\sqrt{2^{2R_0} - 1}}{2\sqrt{\pi} f_m \sqrt{\Gamma}}$
SR	$\frac{(\sqrt{2}+3)\sqrt{\pi} f_m (2^{2R_0} - 1)^{3/2}}{\Gamma^{3/2}}$	$\frac{\sqrt{2^{2R_0} - 1}}{(\sqrt{2}+3)\sqrt{\pi} f_m \sqrt{\Gamma}}$

Fig. 1a reveals that the AOR of the SR protocol has a steeper slope at high SNR, as compared to the AORs of the DF protocol and direct transmission, which is consistent with (31). This behavior resembles the behavior of the OP which decays as $1/SNR^{d_k}$ [2]. So, at high SNR, the AOR decays at a slope equal to the respective cooperative diversity amplitudes reduced by 1/2.

Fig. 1b shows that the AODs for the DF and SR protocols are parallel to the AOD for direct transmission at high SNR, which is consistent with (32). Comparing the asymptotic expressions (presented in Table I) for the AODs of the SR and the DF protocols to that for direct transmission, we notice that the SR protocol incurs 3 dB performance gain over direct transmission and DF relaying suffers from 3.8 dB performance loss. The poorer performance of the DF protocol is also manifested in its higher OP [2]. For lower SNRs, direct transmission exhibits better performance than the SR protocol in terms of the AOD.

VI. Conclusion

In this paper, we have analyzed the AOR and AOD of two repetition-based DF cooperative diversity protocols in time-varying fading channels. The derived closed-form expressions for these two parameters quantify the temporal correlation of the capacity outage events in slow fading channels. Unlike the AOR of the DF protocol, the AOR of the SR protocol exhibits a diversity gain, which is consistent with their respective outage probability behaviors. Compared to direct transmission, the AODs of both cooperative protocols either increase or decrease depending on the network parameters such as the target spectral efficiency, the average received SNRs, and the Doppler rates. The presented analysis of AORs and AODs can provide important insight into the benefits of cooperative diversity.

Appendix A

In order to determine the LCR of random process $U(t)$, we find the time derivative of both sides of (13), yielding

$$\dot{U} = \frac{X}{U}\dot{X} + \frac{Z}{U}\dot{Z} . \quad (A.1)$$

Conditioned on $Z = z$, joint PDF $f_{U\dot{U}}(u, \dot{u})$ is obtained as

$$f_{U\dot{U}}(u, \dot{u}) = \int_0^\infty f_{U\dot{U}|Z}(u, \dot{u}\,|z)\, f_Z(z)\, dz , \quad (A.2)$$

where $f_Z(z)$ is the PDF of channel amplitude Z in the second hop of the relayed path. The conditional joint PDF $f_{U\dot{U}|Z}(u,\dot{u}|z)$ is expressed as

$$f_{U\dot{U}|Z}\left(u,\dot{u}\left|z\right.\right) = f_{\dot{U}|UZ}\left(\dot{u}\left|u,z\right.\right) f_{U|Z}\left(u\left|z\right.\right), \quad \text{(A.3)}$$

where $f_{U|Z}(u|z)$ denotes the conditional PDF of U for some given $Z=z$, which, after applying a simple transformation of random variables (RVs), is given by

$$f_{U|Z}\left(u\left|z\right.\right) = \frac{2u}{\Omega_X}\exp\left(-\frac{u^2-z^2}{\Omega_X}\right), \quad 0 \le z \le u\ . \quad \text{(A.4)}$$

In (A.3), $f_{\dot{U}|UZ}(\dot{u}|u,z)$ is the conditional PDF of $\dot{U}$, given $U=u$ and $Z=z$. In this case, $\dot{U}$ is a linear combination of the two independent zero-mean Gaussian RVs, $\dot{X}$ and $\dot{Z}$, with variances given by (2). Thus, under such conditions, $\dot{U}$ is a Gaussian RV with zero mean and variance

$$\sigma^2_{\dot{U}|UZ} = \frac{u^2-z^2}{u^2}\sigma^2_{\dot{X}} + \frac{z^2}{u^2}\sigma^2_{\dot{Z}}\,. \quad \text{(A.5)}$$

Applying (A.2) and (A.3) in definition (7), and changing the orders of integration, we obtain

$$\begin{aligned} N_U\left(G_0\right) &= \int_0^{G_0}\left(\int_0^{\infty}\dot{u} f_{\dot{U}|UZ}\left(\dot{u}\left|G_0,z\right.\right)d\dot{u}\right) \\ &\times\ f_{U|Z}\left(G_0\left|z\right.\right) f_Z\left(z\right)dz\,, \end{aligned} \quad \text{(A.6)}$$

where the innermost integral is obtained as

$$\int_0^{\infty}\dot{u} f_{\dot{U}|UZ}\left(\dot{u}\left|G_0,z\right.\right)d\dot{u} = \sqrt{\frac{\sigma^2_{\dot{U}|UZ}}{2\pi}}\,. \quad \text{(A.7)}$$

We obtain (21), (22), and (23) by inserting (1), (A.4), (A.5), and (A.7) into (A.6), and then integrating (A.6) over variable z. Specifically, (21) is obtained by introducing the change of variables $t = 1 + z^2(\sigma^2_{\dot{Z}} - \sigma^2_{\dot{X}})/(G_0^2\sigma^2_{\dot{X}})$ into (A.6) and then using the definition of the incomplete Gamma function, $\Gamma(\cdot,\cdot)$ [16, Eq. (6.5.3)]. For $\Omega_X = \Omega_Z$, (A.6) reduces to an elementary integral of the form $\int[\sigma^2_{\dot{X}}(1-z) + \sigma^2_{\dot{Z}} z]^{1/2}dz$, directly yielding (22).

Appendix B

Eq. (29) is derived as

$$\begin{aligned} \Pr\left\{\sqrt{2}X > G_0 \cap U < G_0\right\} &= \Pr\left\{\frac{G_0}{\sqrt{2}} < X < \sqrt{G_0^2 - Z^2}\right\} \\ &= \iint\limits_{G_0/\sqrt{2}<x<\sqrt{G_0^2-z^2}} f_X\left(x\right) f_Z\left(z\right)dxdz \\ &= \int_{G_0/\sqrt{2}}^{G_0} f_X\left(x\right)dx \int_0^{\sqrt{G_0^2-x^2}} f_Z\left(z\right)dz \\ &= \int_{G_0/\sqrt{2}}^{G_0} f_X\left(x\right) F_Z\left(\sqrt{G_0^2-x^2}\right)dx. \end{aligned} \quad \text{(B.1)}$$

Eq. (30) is derived as

$$\begin{aligned} \Pr\left\{\sqrt{2}X < G_0 \cap U > G_0\right\} &= \Pr\left\{\sqrt{G_0^2 - Z^2} < X < \frac{G_0}{\sqrt{2}}\right\} \\ &= \iint\limits_{\sqrt{G_0^2-z^2}<x<G_0/\sqrt{2}} f_X\left(x\right) f_Z\left(z\right)dxdz \\ &= \int_0^{G_0/\sqrt{2}} f_X\left(x\right)dx \int_{\sqrt{G_0^2-x^2}}^{\infty} f_Z\left(z\right)dz \\ &= \int_0^{G_0/\sqrt{2}} f_X\left(x\right)\left[1 - F_Z\left(\sqrt{G_0^2-x^2}\right)\right]dx, \end{aligned} \quad \text{(B.2)}$$

where $f_X(\cdot)$ and $F_Z(\cdot)$ stem from (1).

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