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Diversity Loss Due to Suboptimal Relay Selection

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***Abstract*—The performance of relay selection is degraded when the channel estimates used for relay selection are imperfect. Considering that the channel estimates are obtained via practical channel estimation techniques, we quantify the effects of imperfect channel state information (CSI) on the diversity order of relay selection. The analysis involves combining the effects of noise, time-varying channels, and feedback delays on the CSI used for relay selection into a unified model. Based on this model, the correlation between the actual and the estimated channel values, ρ, is expressed as a function of the signal-to-noise ratio (SNR), providing interesting insight into the behavior of the outage probability in high SNR. The resulting expression for the diversity order reveals that the behavior of the correlation in high SNR crucially affects the asymptotic performance of relay selection.**

In summary, based upon expressing ρ as a function of the SNR, we answer the following question: How fast should ρ tend to one, as the SNR tends to infinity, so that relay selection does not experience any diversity loss?

I. Introduction

The use of relay terminals in wireless communications has attracted the interest of both academia and industry, due to the plethora of advantages that it promises for future networks. Among the several variations of the wireless relaying concept, relay selection has gained particular interest from the research community, due to its ability to take advantage of multiple paths between the source and destination, yet with limited cost on bandwidth usage [1], [2].

As one may expect, given that relay selection involves activating the relay with the strongest channel, high performance is achieved only if the available knowledge about the instantaneous channel conditions of the links involved is sufficiently correlated to the corresponding actual channel conditions. To this end, there has been a recent trend in the literature to investigate the effect of imperfect channel state information (CSI) on relay selection schemes. Such works include [3]-[9], where it was assumed that the imperfection in the channel estimates was caused by feedback delay, which in time-varying fading environments renders the channel estimates different from their actual values. Moreover, in [10]-[11] the effects of channel estimation errors were considered, in addition to feedback delay, yet the estimation errors were assumed to be independent of the signal-to-noise ratio (SNR) thus causing zero diversity order in the high SNR regime.

Nonetheless, given that in practice imperfect estimation can also stem from noisy training symbols, it follows that a complete investigation of the diversity order of relay selection with imperfect CSI requires the consideration of the channel estimation error as a function of the SNR. This is our main rationale for conducting a thorough examination of the diversity potential of relay selection with imperfect channel estimates, generated by practical channel estimation techniques. Based upon a unified model, which reflects the channel estimation imperfection on the correlation between the actual and the estimated channel values, we provide insight into the diversity potential of relay selection for channel estimation in both static and time-varying fading scenarios. For the static channel scenario the CSI imperfection stems from the noise impairing the pilot symbols; for the time-varying channel scenario, CSI imperfection is caused by both noise and the time-varying nature of fading channels. It is shown that in Rayleigh fading the diversity order of relay selection can take any value between unity and the number of available relays, depending on the speed which the correlation coefficient between actual and estimated channel converges to unity, as the SNR approaches infinity.

II. Preliminaries

The scheme under consideration consists of a single source terminal, S, N half-duplex relays which are denoted by R_i, $i = 1, ..., N$, and are assumed to operate in the amplify-and-forward (AF) mode [1], and a single destination terminal, D.

Channel Model: Let h_{AB} denote the complex channel between the nodes A and B, where $A, B \in \{S, R_i, D : \; i = 1, ..., N\}$. Moreover, slow and Rayleigh distributed fading in each of the participating links is assumed, implying that h_{AB} is complex Gaussian random variable (RV). In addition, since in this work we focus our attention on the asymptotic properties of relay selection under imperfect CSI, we assume independent and identically distributed (i.i.d.) fading in each of the links involved.

Let γ_{AB} represent the instantaneous SNR of the link between any terminals A and B, i.e., $\gamma_{AB} = |h_{AB}|^2 / N_0$, where we have assumed that the source and the relays transmit with power one, and N_0 is the additive white Gaussian noise (AWGN) power. Due to the i.i.d. fading assumption, the average SNRs of all links involved are identical, and denoted by $\bar{\gamma}$. Moreover, we use the notation $f_X(\cdot)$ and $F_X(\cdot)$ to refer to the probability density function (pdf) and the cumulative distribution function (cdf) of the RV X, respectively.

All relays are assumed to employ the so-called CSI-assisted variable gain, where the relaying gain depends on the instantaneous channel amplitude of the corresponding S-R_i link. As a result, the end-to-end SNR associated with the S-R_i-D link, γ_i, is given by [2]

$$\gamma_i = \frac{\gamma_{SR_i} \gamma_{R_i D}}{\gamma_{SR_i} + \gamma_{R_i D} + 1}. \tag{1}$$

Relay Selection Process: Among the available relays, only a single relay, the relay associated with the strongest S-R_i-D channel, is activated in each transmission session. As addressed in several works in the literature (see, e.g., [12], [13]), an upper bound of γ_i is obtained as the minimum of γ_{SR_i} and $\gamma_{R_i D}$, i.e,

$$\gamma_i \leq \min(\gamma_{SR_i}, \gamma_{R_i D}). \tag{2}$$

Hence, (2) represents an alternative metric for identifying the relay with the strongest end-to-end channel [2], and is the relay selection metric adopted in this work. In particular, a central unit (CU) is assumed to collect the CSI for each of the links involved, and form estimates of the corresponding SNRs, which are denoted by $\hat{\gamma}_{AB}$. Then, the selected relay, denoted by R_κ, is picked based on the following rule

$$\kappa = \arg \max_{i=1,\ldots,N} \hat{\gamma}_i = \arg \max_{i=1,\ldots,N} \min \left(\hat{\gamma}_{SR_i}, \hat{\gamma}_{R_i D} \right). \quad (3)$$

A fundamental consideration throughout this paper is that the actual channel values of each of the links involved are not necessarily equal to their corresponding estimates, leading thus to suboptimal relay selection. That is, the relay which is considered as the "best" relay may not necessarily be the actual "best" relay, for a given time instance, resulting in performance degradation. In the same context, the CSI imperfection is assumed to affect the relay selection process, but not the symbol detection at the destination. This is because the number of pilot symbols used for detection is typically higher than that used for relay selection, and the channel estimates for detection can be updated with higher frequency. A thorough investigation of the considered imperfect CSI model follows.

III. Imperfect CSI Model

The physical causes of the considered CSI degradation are the time-varying nature of fading channels, as well as the finite pilot symbol power. In this work, both of these causes of imperfect CSI are integrated into one unified model, as shown below.

The level of CSI imperfection is quantified by the correlation coefficient between the actual squared channel envelope, $|h_{AB}|^2$, and its estimate, $|\hat{h}_{AB}|^2$, where A and B can be any terminals of the set $\{S, R_i, D: \ i = 1, ..., N\}$. This coefficient is defined as (in the sequel, all channel indices are dropped due to the i.i.d. fading assumption)

$$\rho = \frac{E\left\langle \left(|h|^2 - \Omega_h \right) \left(|\hat{h}|^2 - \Omega_{\hat{h}} \right) \right\rangle}{\sigma_{|h|^2} \sigma_{|\hat{h}|^2}} \quad (4)$$

where $\Omega_h = E\left\langle |h|^2 \right\rangle$, $\Omega_{\hat{h}} = E\left\langle |\hat{h}|^2 \right\rangle$ with $E\langle \cdot \rangle$ denoting expectation, and σ_X denotes the standard deviation of RV X.

Note that ρ reflects the effect of imperfect CSI on the SNR of the selected relay and hence on the overall performance of suboptimal relay selection. For this reason, the subsequent analysis focuses on expressing the performance degradation as a function of ρ, so that all physical phenomena that cause CSI degradation are, in fact, incorporated into ρ.

A. Versatile Imperfect CSI Case

In accordance with the intuition that imperfect channel estimation occurs in noisy environments, let us assume the versatile scenario where ρ is a general function of SNR. That is,

$$1 - \rho = g(\bar{\gamma}) \quad (5)$$

where $g(\cdot)$ is generally a non-increasing function of its argument, with $0 \leq g(\bar{\gamma}) \leq 1$. In order to obtain insight into the asymptotic dependence of the CSI error on the SNR, we expand $g(\bar{\gamma})$ into a Puiseux series [14], so that for high SNR we have

$$g(\bar{\gamma}) = b\bar{\gamma}^{-a} + o\left(\bar{\gamma}^{-a}\right) \quad (6)$$

where a, b are positive constants with $0 < b \leq 1$, and $o\left(\bar{\gamma}^{-a}\right)$ is defined such that $\lim_{\bar{\gamma}\to\infty} o\left(\gamma^{-a}\right) / \left(\bar{\gamma}^{-a}\right) = 0$.

It is emphasized that the versatile imperfect CSI model considered in (5) and (6) is general enough so as to accommodate the cases of imperfect CSI due to noise impairment and the time-varying nature of the underlying channels, respectively. Next, we study the scenarios of CSI imperfection in static channels and time-varying channels separately.

B. Static Channels, Noise-Impaired CSI

Let us consider the case of static channels, where channel estimation is implemented via averaging over L noisy pilot symbols. As a result, $g(\bar{\gamma})$ in (5) is a decreasing function of $\bar{\gamma}$, for which $\lim_{\bar{\gamma}\to\infty} g(\bar{\gamma}) = 0$ holds. Moreover, let us consider the scenario where the power allocated to pilot symbols, $\mathcal{E}_p$, is not necessarily equal to the power allocated to data transmission, $\mathcal{E}_d$. In order to account for the general case, we allow the ratio of $\mathcal{E}_p$ over $\mathcal{E}_d$ to be SNR-dependent, such that

$$\mathcal{E}_p = \beta\bar{\gamma}^{\alpha}\mathcal{E}_d \quad (7)$$

where β is a positive constant and α is a constant, the sign of which determines whether $\mathcal{E}_p$ increases or decreases with SNR. The estimated channel values are expressed as $\hat{h} = h + n_p$, where h and n_p denote the true channel component and the remaining noise component, respectively. Given that the channel estimates are derived by averaging over L pilot symbols, the noise variance of the estimation process equals $\sigma^2_{n_p} = N_0 / \left(\mathcal{E}_p L\right)$.

Lemma 1 (Correlation between $|h|^2$, $|\hat{h}|^2$): The correlation coefficient, ρ, between $|h|^2$ and $|\hat{h}|^2$, is given by

$$\rho = \frac{\Omega_h}{\Omega_{\hat{h}}} = \frac{\Omega_h}{\Omega_h + \sigma^2_{n_p}} = \frac{L\beta\bar{\gamma}^{\alpha+1}}{L\beta\bar{\gamma}^{\alpha+1} + 1}. \quad (8)$$

Proof: Since $\hat{h}$ equals a linear combination of complex Gaussian RVs, $|\hat{h}|^2$ is exponentially distributed. Hence, it follows from the theory of the moments of exponential RVs that $E\left\langle |h|^4 \right\rangle = 2\Omega_h^2$; $E\left\langle |\hat{h}|^4 \right\rangle = 2\Omega_{\hat{h}}^2$. Using this result, the proof follows from (4) after algebraic manipulations, in conjunction with (7) and the fact that $\bar{\gamma} = \mathcal{E}_d \Omega_h / N_0$. ■

Expanding (8) in a Taylor series for $\bar{\gamma} \to \infty$ and using (5), we obtain for $\alpha > -1$[1]

$$g(\bar{\gamma}) = 1 - \rho \approx \frac{1}{\beta L}\bar{\gamma}^{-(\alpha+1)} + o\left(\bar{\gamma}^{-(\alpha+1)}\right). \quad (9)$$

Therefore, using (6), from (9) we have $a = \alpha + 1$; $b = 1/(\beta L)$.

C. Time-Varying Channels

Next, the case of time-varying fading is studied, where the maximum Doppler frequency of each of the participating links is assumed identical, and denoted by f_d. Moreover, the autocorrelation function of the complex channel h is denoted by $\rho_h(T_d)$; based on the Jakes' model [15], $\rho_h(T_d)$ is given as $\rho_h(T_d) = \Omega_h J_0(2\pi f_d T_d)$, where $J_0(\cdot)$ denotes the zeroth order Bessel function of the first kind [16, Eq. (8.411)].

1) Finite Impulse Response (FIR) Channel Prediction: In time-varying environments, the channel estimation can be improved by utilizing the CSI available from previous time instances, so that the channel estimates are derived through a *channel prediction* process [17]. Let us consider an FIR channel prediction filter of length L, and denote the time interval between consecutive CSI acquisitions by T_d. In such case, following the

[1]The case of $\alpha < -1$ yields $\rho = 0$ for $\bar{\gamma} \to \infty$, and is out of the scope of this paper.

analysis in [17], the prediction parameters can be optimized so as to yield the minimum squared error between the actual and the predicted channel values, σ_e^2, resulting in

$$\sigma_e^2 = \Omega_h - \underline{\mathrm{u}}_h^H \underline{\mathrm{R}}^{-1} \underline{\mathrm{u}}_h. \quad (10)$$

In (10), $\underline{\mathrm{u}}_h$ denotes the L-dimensional autocorrelation vector, i.e., $\underline{\mathrm{u}}_h = [\rho_h(-T_d), ..., \rho_h(-LT_d)]^T$, and $\underline{\mathrm{R}}$ denotes an $L \times L$ symmetric Toeplitz matrix, the first row of which is given by $[\rho_h(0) + N_0/\mathcal{E}_p, \rho_h(-T_d), ..., \rho_h(-LT_d + T_d)]$. Here, $(\cdot)^T$ and $(\cdot)^H$ denote the transposition and Hermitian operators, respectively. Hence, ρ is derived by combining (4) and (10), as

$$\rho = \underline{\mathrm{u}}_h^H \underline{\mathrm{R}}^{-1} \underline{\mathrm{u}}_h / \Omega_h. \quad (11)$$

It follows then from (5) that the CSI error can be expressed as a function of the SNR as $g(\bar{\gamma}) = 1 - \rho = 1 - \underline{\mathrm{u}}_h^H \underline{\mathrm{R}}^{-1} \underline{\mathrm{u}}_h / \Omega_h$. Interestingly, it is noted that in the high-SNR regime and for $f_d > 0$, $g(\bar{\gamma})$ converges to a finite non-zero constant, i.e.,

$$\lim_{\bar{\gamma}\to\infty} g(\bar{\gamma}) = 1 - \underline{\mathrm{u}}_h^H \left. \underline{\mathrm{R}}^{-1} \right|_{N_0=0} \underline{\mathrm{u}}_h / \Omega_h = b > 0 \quad (12)$$

implying that the CSI error is independent of the SNR. Hence, considering (6), it follows that for the case where the channel estimates are obtained through an FIR channel prediction method, $a = 0$ holds.

Ideal but Outdated CSI: This special case of channel estimation was considered in [3]-[9], and in fact corresponds to noiseless FIR channel prediction with a one-tap predictor ($L = 1$), and is dubbed as "outdated CSI" here. It implies that the CSI based on which the "best" relay is selected is noise-free, yet the selection of the "best" relay is not based on the current time instant but on a previous one, because of, e.g., a feedback delay. Based on (11), it can be shown that the correlation coefficient, ρ, for the outdated CSI case equals $\rho = \rho_h^2(T_d)/\Omega_h^2$, a result which is in accordance with [3], [4]. Moreover, $g(\bar{\gamma})$ is a constant function in this case, and thus (6) yields $a = 0$; $b = 1 - \rho_h^2(T_d)/\Omega_h^2$.

2) Infinite Impulse Response (IIR) Channel Prediction: Let us now extend the channel prediction case to the scenario where the number of pilot symbols participating in the prediction process are infinitely large. As shown in Appendix A, this scenario leads to a correlation coefficient of

$$\begin{aligned}\rho \;=\; & 1 - \exp\left(T_d \int_{-f_d}^{f_d} \ln\left[\mathcal{S}_{hh}\left(e^{j2\pi f T_d}\right) + \left(\beta\bar{\gamma}^{\alpha+1}\right)^{-1}\right] df\right) \\ & \times \left(\beta\bar{\gamma}^{\alpha+1}\right)^{-(1-2f_dT_d)} + \left(\beta\bar{\gamma}^{\alpha+1}\right)^{-1}\end{aligned} \quad (13)$$

where $\mathcal{S}_{hh}(\cdot)$ represents the Fourier transform of $\rho_h(\cdot)$. Hence, combining (5) and (13) we obtain for high SNR and $f_d > 0$

$$\begin{aligned} g(\bar{\gamma}) \;=\; & \underbrace{\exp\left(T_d \int_{-f_d}^{f_d} \ln\left[\mathcal{S}_{hh}\left(e^{j2\pi f T_d}\right)\right] df\right) \beta^{-(1-2f_dT_d)}}_{b} \\ & \times \bar{\gamma}^{-\overbrace{(\alpha+1)(1-2f_dT_d)}^{a}}. \end{aligned} \quad (14)$$

Consequently, it is concluded that the parameters a and b of the asymptotic dependence of the CSI error on the SNR are given by $a = (\alpha+1)(1-2f_dT_d)$ and $b = \exp\left(T_d \int_{-f_d}^{f_d} \ln\left[\mathcal{S}_{hh}\left(e^{j2\pi f T_d}\right)\right] df\right) \beta^{-(1-2f_dT_d)}$.

TABLE I
PARAMETERS a AND b FOR THE CONSIDERED IMPERFECT CSI SCENARIOS.

Case	a	b
Outdated CSI	0	$1 - J_0^2(2\pi f_d T_d)$
Noisy CSI	$\alpha + 1$	$1/(\beta L)$
FIR Ch. Pred.	0	$1 - \underline{\mathrm{u}}_h^H \left. \underline{\mathrm{R}}^{-1} \right\|_{N_0=0} \underline{\mathrm{u}}_h / \Omega_h$
IIR Ch. Pred.	$(\alpha+1)(1-2f_dT_d)$	$\exp\left(\int_{-f_d}^{f_d} \ln\left[S_{hh}\left(e^{j2\pi f T_d}\right)\right] \times T_d df\right) \beta^{-(1-2f_dT_d)}$

The reader is referred to Table I for an overview of the parameters a and b for the practical channel estimation scenarios considered in this paper. An asymptotic performance analysis of suboptimal relay selection follows.

IV. Asymptotic Analysis

A. Outage Probability of Suboptimal Relay Selection

The outage probability is defined as the probability that the overall SNR lies below a given threshold, denoted here by T, i.e., $P_{out} = \Pr\{\gamma_\kappa < T\}$, where κ denotes the index of the selected relay and γ_κ is the corresponding end-to-end SNR. Observing that for all channel estimation scenarios considered in Section III, $\hat{h}$ is obtained as a linear combination of complex Gaussian RVs, it follows that $\hat{h}$ is also a complex Gaussian RV. Hence, $\hat{\gamma}$ is exponentially distributed. Consequently, the conditional pdf of the actual SNR, γ, conditioned on its estimate, $\hat{\gamma}$, is obtained from [18, Eq. (2.11)] as

$$f_{\gamma|\hat{\gamma}}(x, y) = \frac{\exp\left(-\frac{x}{\bar{\gamma}(1-\rho)} - \frac{y\rho}{\bar{\hat{\gamma}}(1-\rho)}\right)}{\bar{\gamma}(1-\rho)} I_0\left(2\frac{\sqrt{\rho x y}}{(1-\rho)\sqrt{\bar{\gamma}\bar{\hat{\gamma}}}}\right) \quad (15)$$

where $\bar{\hat{\gamma}}$ denotes the average estimated SNR and $I_0(\cdot)$ denotes the zeroth order modified Bessel function of the first kind [16, Eq. (8.447.1)]. It is emphasized that since the parameters $\bar{\gamma}$ and $\bar{\hat{\gamma}}$ are, as opposed to the outdated CSI case treated in [4], not necessarily equal to each other, the diversity investigation of suboptimal relay selection under the general imperfect CSI assumption requires that we conduct a new outage analysis of our scheme. Such outage analysis is similar to [4], yet [4, Eq. (14)] is substituted by (15).

In particular, the outage probability is derived as

$$P_{out} = \int_0^T f_{\gamma_{R_\kappa D}}(x) dx + \int_T^\infty F_{\gamma_{SR_\kappa}}\left(\frac{xT + T}{x - T}\right) f_{\gamma_{R_\kappa D}}(x) dx \quad (16)$$

where, due to the i.i.d. fading assumption, $f_{\gamma_{R_\kappa D}}(\cdot)$ is obtained as

$$f_{\gamma_{R_\kappa D}}(x) = \int_0^\infty f_{\gamma|\hat{\gamma}}(x, y) f_{\hat{\gamma}_{R_\kappa D}}(y) dy \quad (17)$$

and $F_{\gamma_{SR_\kappa}}(\cdot)$ follows straightforwardly from (17). The pdf of $\hat{\gamma}_{R_\kappa D}$, $f_{\hat{\gamma}_{R_\kappa D}}(\cdot)$, is derived from [4, Eq. (22)] by making all distribution parameters equal, yielding

$$f_{\hat{\gamma}_{R_\kappa D}}(y) = N \sum_{n=0}^{N-1} \frac{\binom{N-1}{n}}{(-1)^n} \frac{\exp\left(-\frac{y}{\bar{\hat{\gamma}}}\right) + 2n \exp\left(-\frac{2(1+n)y}{\bar{\hat{\gamma}}}\right)}{\bar{\hat{\gamma}}(1+2n)}. \quad (18)$$

Interestingly, we observe that by combining (18) and (17) and [16, Eq. (6.614.3)] we obtain an expression for $f_{\gamma_{R_\kappa D}}(\cdot)$ which is independent of $\bar{\hat{\gamma}}$; this expression coincides with its counterpart for outdated CSI, given in [4, Eq. (23)]. The fact that P_{out} is independent of $\bar{\gamma}$ can be explained by noting that it is the relative values of $\hat{\gamma}_i$ that determine relay selection, not their absolute values. Hence scaling all the estimated SNRs by the same factor does not affect the relay selection process. As a result, we conclude that, under i.i.d. Rayleigh fading and assuming correlation coefficient ρ between the actual and the estimated SNR in each intermediate link, *the outage probability of suboptimal relay selection, expressed as a function of* ρ, *is given by the same formula, irrespective of the channel estimation technique used*. Therefore, the outage probability of suboptimal AF relay selection is obtained as the i.i.d. fading version of [4, Eq. (25)]; an approximation is also available in [8, Eq. (16)]. It is emphasized, however, that *different channel estimation techniques lead to different dependences of* ρ *on the SNR, resulting ultimately in a different diversity behavior.* The diversity order of suboptimal relay selection is investigated next.

B. Diversity Analysis

Let us recall the high SNR expansion of ρ presented in Section III, i.e., $1-\rho \approx b\bar{\gamma}^{-a}$. The ensuing theorem provide insight into the asymptotic behavior of the scheme under consideration.

Theorem 1: The diversity order of suboptimal relay selection for practical channel estimation is given by

$$d = \begin{cases} (N-1)\,a+1 & \text{if } a<1 \\ N & \text{if } a \geq 1 \end{cases} \quad (19)$$

Proof: The proof is provided in Appendix B. ■

V. On The Diversity Potential of Relay Selection

Based upon the high SNR analysis of the previous section, interesting results regarding the diversity order of relay selection can be obtained. These results are presented below, for different types of channels and channel estimation techniques.

A. Channel Estimation over Static Channels

Let us first focus on the scenario where the channel estimates are obtained via training in static channels. In this case, as can be seen from Table I the exponent a can take any positive value, depending on how fast the training power increases with SNR, with respect to the data transmission power. In particular, it follows from Theorem 1 that full diversity is achieved by using a training power which increases with SNR at least as fast as the data transmission power. A slower increase with SNR results in a decreased diversity order. Further details regarding the latter argument are provided in Section VI, via numerical examples.

B. FIR Channel Prediction in Time-Varying Channels

Here, we concentrate our attention on the case of FIR channel prediction; note that the special case of outdated channel estimates is also included in this scenario, by setting the number of predictor taps equal to one. As shown in Table I, this case results in $a=0$. Interestingly, it follows from (19) that the diversity order equals one regardless of the number of available relays. In other words, *when the estimates of time-varying channels are obtained via an FIR predictor, the diversity order of relay selection reduces to that of the scheme where only a single relay is available*.

From another viewpoint, *when relay selection is performed over time-varying channels, its full diversity potential is completely lost, unless a predictor with an infinitely large length is employed*. A study of the latter case follows.

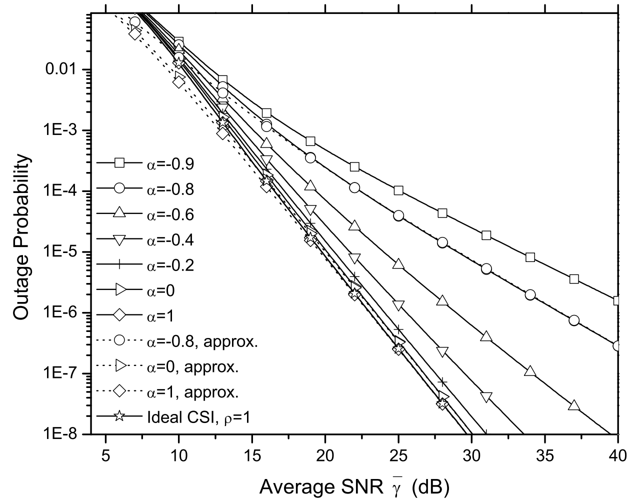

Fig. 1. Case of static channels: Outage probability of relay selection with noise-impaired channel estimates versus the average SNR, assuming $N=3$, outage threshold $T=1$, $\beta=1$, $L=1$, and several values of α.

C. IIR Channel Prediction in Time-Varying Channels

As implied by Table I and Theorem 1, the diversity loss of relay selection incurred by the time-varying nature of the underlying channels can be recovered via IIR channel prediction. Nevertheless, given that the full diversity order is recovered for $a \geq 1$, it follows that if the power of the channel estimation pilot symbols grows with SNR as fast as the data transmission power, i.e., $\alpha=0$, the resulting diversity order is still lower than the maximum value. As a result, it is concluded that *in order to achieve full diversity in relay selection over time-varying channels, IIR channel prediction is required, in conjunction with a training power which increases faster than the data transmission power, i.e.,* $\alpha>0$. The amount of training power required to achieve full diversity is determined by the Doppler spread of the estimated channel, in conjunction with the time difference between the consecutive noisy channel observations.

VI. Numerical Results

In this section, we provide some numerical results based on the analysis conducted in Section IV. All numerical results were also confirmed by simulations.

Fig. 1 considers the case of static channels, and illustrates the effect of noisy channel estimates on the outage probability of suboptimal relay selection. Specifically, in Fig. 1, $N=3$ available relays are assumed. We observe that the outage probability substantially depends on the parameter α, assuming $\beta=1$ and $L=1$. Recall from (7) that the parameters α and β reflect the relation between the power allocated to pilot symbols and the power used for data transmission. It is observed that full diversity is achieved for any $\alpha \geq 0$ (corresponding to $a \geq 1$), yet there exists a power gain loss compared to the perfect CSI case; this power gain loss is recovered for higher values of α, i.e., for $\alpha \geq 1$.

In Fig. 2, we consider the case of ideal yet outdated CSI, where the channel estimation is assumed noise-free, yet it suffers from feedback delay. Specifically, we assume the typical scenario of a

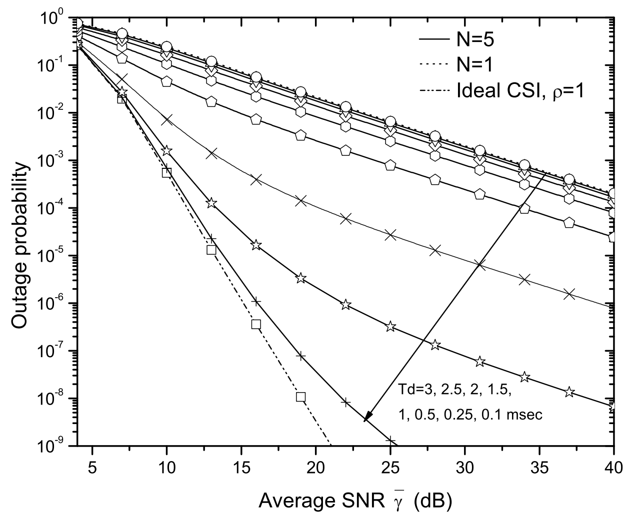

Fig. 2. Case of outdated CSI: Outage probability of relay selection versus the average SNR, assuming $N = 5$, outage threshold $T = 1$, relative speed of 50 km/h, carrier frequency of 2.4 GHz, and several values of T_d.

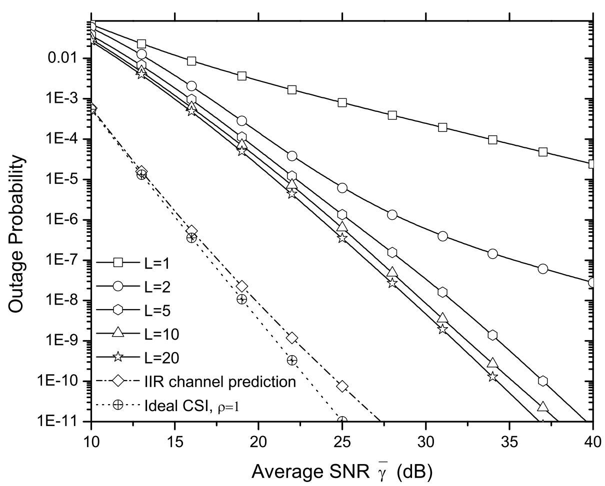

Fig. 3. Time-varying channels with FIR channel prediction: Outage probability of relay selection versus the average SNR, assuming $N = 5$, outage threshold $T = 1$, $T_d = 1$ msec, $\alpha = 0$, $\beta = 1$, relative speed of 50 km/h, carrier frequency of 2.4 GHz, and several values of the predictor length, L.

vehicle moving at 50 km/h and receiving at a frequency of 2.4 GHz via selecting one out of $N = 5$ available relays. This results in a maximum Doppler frequency of approximately $f_d = 100$ Hz. Under this assumption, we illustrate the dependence of the corresponding outage probability on the time interval between estimation updates, T_d.

We notice from Fig. 2 that the rate of estimation update significantly affects the outage performance of relay selection, in the sense that low update rates result in severe performance loss. This is in agreement with (19) where, given that for outdated channel estimates $a = 0$ holds, the diversity order equals unity regardless of T_d. Nonetheless, it should be pointed out that for low values of T_d the slope of the outage curves retains its full diversity characteristics in the practical SNR range, and exhibits its complete diversity loss only for very high SNRs that approach infinity. This observation sheds light on the diversity potential of relay selection with outdated channel estimates since, *although it is impossible to achieve full diversity from a theoretical perspective (i.e., when $\bar{\gamma} \to \infty$), it is still possible to achieve full diversity for finite SNR values, by decreasing T_d*. Fig. 2 also demonstrates that for relatively low channel estimation update rates (that is, for $T_d = 3$ msec), relay selection cannot take advantage of the large number of available relays, since the cases of $N = 5$ yields approximately the same performance as the case where only a single relay is available, i.e., $N = 1$. Furthermore, it is worth mentioning that the outage probability for the case where the mobile terminals are moving at the walking speed of 5 km/h can be also extracted from Fig. 2, by tenfolding the corresponding values of T_d (i.e., $T_d = 30, 25, ..., 1$ msec).

With reference to the moving-vehicle scenario considered in Fig. 2, the case of channel prediction in time-varying channels is investigated in Fig. 3. In particular, Fig. 3 depicts the outage probability of suboptimal relay selection for $T_d = 1$ msec and under several assumptions on the channel predictor length, L, including the case of $L \to \infty$ which corresponds to IIR channel prediction and serves here as benchmark. Fig. 3 reveals that by increasing the number of channel observations for FIR channel prediction, the outage probability is improved, yet even for $L = 20$ there exists a big gap to the ideal performance of relay selection, which corresponds to perfect CSI acquisition. On the contrary, we observe a dramatic improvement of the outage performance via IIR channel prediction, closely approaching the ideal outage performance.

APPENDIX A
IIR CHANNEL PREDICTION

Let us consider the process $y_l = h_l + n_l/\sqrt{\mathcal{E}_p}$, where h_l and n_l denote the channel value and the corresponding noise component at time instance l, respectively, with $l = 1, ..., \infty$. The variance of the prediction error is derived as [19]

$$\sigma_e^2 = \exp\left(T_d \int_{-1/(2T_d)}^{1/(2T_d)} \ln\left[\mathcal{S}_{hh}\left(e^{j2\pi f T_d}\right) + \frac{N_0}{\mathcal{E}_p}\right] df\right) - \frac{N_0}{\mathcal{E}_p} \tag{20}$$

where $N_0/\mathcal{E}_p$ is the variance of the noise component and $\mathcal{S}_{hh}(f)$ denotes the Fourier transform of $\rho_h(\tau)$. Therefore, considering that the spectrum of $\mathcal{S}_{hh}(f)$ is band-limited by f_d, (20) yields

$$\begin{aligned} \sigma_e^2 &= \exp\left(T_d \int_{-f_d}^{f_d} \ln\left[\mathcal{S}_{hh}\left(e^{j2\pi f T_d}\right) + \frac{N_0}{\mathcal{E}_p}\right] df\right) \\ &\quad \times \left(\frac{N_0}{\mathcal{E}_p}\right)^{1-2f_dT_d} - \frac{N_0}{\mathcal{E}_p}. \end{aligned} \tag{21}$$

Assuming $f_d > 0$, (13) is obtained from (21), (4), and (7).

APPENDIX B
PROOF OF THEOREM 1

Because of the slow fading assumption, we focus on deriving the diversity order of suboptimal relay selection via the asymptotic behavior of the outage probability. A high SNR expression for the outage probability of the scheme under consideration is obtained by using the approximation $K_1(z) \approx 1/z,\ 0 < z \ll \sqrt{2}$

[20, eq. (9.6.9)] in [4, Eq. (25)], in conjunction with (5) and (6), yielding

$$
\begin{aligned}
P_{out} = N \sum_{n=0}^{N-1} & \frac{\frac{n}{n+1}\left(1-e^{\frac{-2(n+1)T}{\bar{\gamma}\left[1+(2n+1)b\bar{\gamma}^{-a}\right]}}\right)+1-e^{-\frac{T}{\bar{\gamma}}}}{\left[(-1)^n\binom{N-1}{n}\right]^{-1}(2n+1)} \\
&+N^2 \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} \frac{(-1)^{n+m+1}\binom{N-1}{m}\binom{N-1}{n}}{(2m+1)(2n+1)}\left\{e^{-\frac{T}{\bar{\gamma}}}\right. \\
&+\frac{me^{-\frac{T}{\bar{\gamma}}\left(1+\frac{2(m+1)}{1+(2m-1)b\bar{\gamma}^{-a}}\right)}}{m+1}+\frac{ne^{-\frac{T}{\bar{\gamma}}\left(1+\frac{2(n+1)}{1+(2n-1)b\bar{\gamma}^{-a}}\right)}}{n+1} \\
&+\frac{mne^{-\frac{T\left[2(m+1)(n+1)b\bar{\gamma}^{-a}+2+m+n+(m+n)b\bar{\gamma}^{-a}+mn\left(b\bar{\gamma}^{-a}\right)^2\right]}{\bar{\gamma}\left[1+(2m-1)b\bar{\gamma}^{-a}\right]\left[1+(2n-1)b\bar{\gamma}^{-a}\right]}}}{(n+1)(m+1)} \\
&\left.-\frac{2n+1}{n+1}\left[e^{-\frac{T}{\bar{\gamma}}}+\frac{me^{-\frac{2(m+1)T}{\bar{\gamma}\left[1+(2m-1)b\bar{\gamma}^{-a}\right]}}}{m+1}\right]\right\}. \qquad (22)
\end{aligned}
$$

We notice from (22) that the asymptotic outage probability involves the term $\bar{\gamma}^{1-a}$, which tends to either infinity or zero depending on the value of a.

Case of $a<1$: In this case, $\bar{\gamma}^{1-a}$ tends to infinity as $\bar{\gamma}\to\infty$. Then, using a McLaurin series representation of (22) for $\bar{\gamma}\to\infty$ and keeping the first order term, followed by the use of the expansions $1/(1+x)\approx\sum_{k=0}^{\infty}x^k$ and $(1+x)/(1+\lambda x)\approx 1+\sum_{k=1}^{\infty}(\lambda-1)\lambda^{k-1}x^k$, for $x\to 0$, the asymptotic outage probability is expressed as

$$
\begin{aligned}
P_{out} \approx & \frac{N}{\bar{\gamma}}\sum_{n=0}^{N-1}\frac{(-1)^n\binom{N-1}{n}}{2n+1}\sum_{k=0}^{\infty}(-1)^k 2n\left[(2n+1)b\right]^k\bar{\gamma}^{-ak} \\
&+\frac{N^2}{\bar{\gamma}}\sum_{n=0}^{N-1}\sum_{m=0}^{N-1}\frac{\binom{N-1}{n}\binom{N-1}{m}2n}{(-1)^{n+m}(m+1)}\sum_{k=1}^{\infty}(2n+1)^{k-1}b^j\bar{\gamma}^{-ak}. \qquad (23)
\end{aligned}
$$

It follows then that for sufficiently high SNR, (23) equals zero for $\bar{\gamma}\to\infty$ and for any $k\leq N-2$, since the corresponding terms in the infinite summations in (23) cross each other out. Consequently, it is concluded that for high SNR the dominant term in (23) is proportional to $\bar{\gamma}^{-a(N-1)-1}$, yielding (19).

Case of $a>1$: In this case, the term $\bar{\gamma}^{1-a}$ approaches zero as $\bar{\gamma}\to\infty$. Then, using a McLaurin expansion of (22) and keeping only the high-SNR dominant term yields

$$
\begin{aligned}
P_{out} \approx N \sum_{n=0}^{N-1} & \frac{\frac{n}{n+1}\left(1-e^{\frac{-2(n+1)T}{\bar{\gamma}}}\right)+1-e^{-\frac{T}{\bar{\gamma}}}}{\left[(-1)^n\binom{N-1}{n}\right]^{-1}(2n+1)} \\
&+N^2 \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} \frac{(-1)^{n+m+1}\binom{N-1}{m}\binom{N-1}{n}}{(2m+1)(2n+1)}\left\{e^{-\frac{T}{\bar{\gamma}}}\right. \\
&+\frac{me^{-\frac{T}{\bar{\gamma}}(2m+3)}}{m+1}+\frac{ne^{-\frac{T}{\bar{\gamma}}(2n+3)}}{n+1}+\frac{mne^{-\frac{T(2+m+n)}{\bar{\gamma}}}}{(n+1)(m+1)} \\
&\left.-\frac{2n+1}{n+1}\left[e^{-\frac{T}{\bar{\gamma}}}+\frac{me^{-\frac{2(m+1)T}{\bar{\gamma}}}}{m+1}\right]\right\}. \qquad (24)
\end{aligned}
$$

Note that (24) is obtained from (22) by considering $b\bar{\gamma}^{-a}\approx 1-\rho=0$. Expressing (24) as an infinite Taylor series and noting that, similarly to (23), the terms proportional to $\bar{\gamma}^{-k}$, $k=1,...,N-1$ cross each other out, we conclude that P_{out} decays proportionally to $\bar{\gamma}^{-N}$, yielding (19).

References

[1] M. Dohler and Y. Li, "*Cooperative communications: Hardware, channel & PHY*," Wiley & Sons, 2010.

[2] M. Uysal (Ed.), "*Cooperative communications for improved wireless network transmission: Frameworks for virtual antenna array applications*". IGI-Global, 2009.

[3] J. L. Vicario, A. Bel, J. A. Lopez-Salcedo, and G. Seco, "Opportunistic relay selection with outdated CSI: Outage probability and diversity analysis," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 2872-2876, June 2009.

[4] D. S. Michalopoulos, H. A. Suraweera, G. K. Karagiannidis, and R. Schober, "Relay selection with outdated channel estimates," *IEEE Global Communications Conference (Globecom) 2010* Miami, FL, USA.

[5] H. A. Suraweera, M. Soysa, C. Tellambura, and H. K. Garg, "Performance analysis of partial relay selection with feedback delay", *IEEE Signal Processing Letters*, vol.17, pp.531-534, Jun 2010.

[6] D. S. Michalopoulos, N. D. Chatzidiamantis, R. Schober and G. K. Karagiannidis, "Relay selection with outdated channel estimates in Nakagami-m fading", to be presented at *IEEE International Conference on Communications (ICC)*, 2011. Available at http://ece.ubc.ca/~dio/ICCNak.pdf

[7] D. S. Michalopoulos, H. A. Suraweera, G. K. Karagiannidis, and R. Schober, "Amplify-and-forward relay selection with outdated channel estimates", submitted to *IEEE Trans. on Commun.*Available at http://ece.ubc.ca/~dio/OutdatedJ.pdf.

[8] M. Torabi and D. Haccoun, "Capacity analysis of opportunistic relaying in cooperative systems with outdated channel information", *IEEE Commun. Letters*, vol. 14, pp. 1137-1139, Dec 2010.

[9] G. Amarasuriya, C. Tellambura and M. Ardakani, "Feedback delay effect on dual-hop MIMO AF relaying with antenna selection,"*IEEE Global Telecommunications Conference (GLOBECOM)*, 2010.

[10] M. Seyfi, S. Muhaidat and J. Liang, "Performance analysis of relay selection with feedback delay and channel estimation errors," *IEEE Signal Proc. Letters*, vol. 18, Jan 2011.

[11] M. J. Taghiyar, S. Muhaidat and J. Liang, "On the performance of pilot symbol assisted modulation for cooperative systems with imperfect channel estimation,"*IEEE Wireless Communications and Networking Conference (WCNC)*, 2010.

[12] G. Amarasuriya, M. Ardakani and C. Tellambura, "Output-Threshold Multiple-Relay-Selection Scheme for Cooperative Wireless Networks," *IEEE Trans. Veh. Technol.*, vol.59, pp.3091-3097, Jul 2010.

[13] B. Medepally, and N. B. Mehta, "Voluntary energy harvesting relays and selection in cooperative wireless networks," *IEEE Trans. on Wireless Commun.*, vol.9, pp.3543-3553, Nov 2010.

[14] G. Cherlin, *Model Theoretic Algebra Selected Topics*. Lecture notes in mathematics 521. Springer-Verlag, 1976.

[15] W. C. Jakes, "*Microwave mobile communication"*, J. Wiley&Sons, NY, 1974.

[16] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series, and products,* New York, Academic Press, 7th edition, 2007.

[17] J. Makhoul, "Linear prediction: A tutorial overview", *IEEE Proceedings,* Vol. 63, No.4, Apr 1975.

[18] F. Downton, "Bivariate exponential distributions in reliability theory". J*ournal of the Royal Statistics Society*, Series B, vol 32, No. 3 (1970), pp. 408-417.

[19] B. Picinbono and J.-M. Kerilis, "Some properties of prediction and interpolation errors", *IEEE Trans. on Acoustics, Speech and Signal Processing,* vol.36, pp.525-531, Apr 1988.

[20] M. Abramowitz and I. A. Stegun, *Handbook of mathematical functions with formulas, graphs, and mathematical tables, 9th ed.* New York: Dover, 1970.