Performance Analysis of Space-Time Block Codes over Generalized-K Fading MIMO Channels

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***Abstract*—This paper elaborates on the performance of orthogonal space-time block codes (STBCs) for multiple-input multiple-output (MIMO) systems operating in generalized-*K* fading conditions. The considered fading model is generic since it intimately encompasses both small-scale fading (modeled via the Nakagami-*m* distribution) and large-scale fading (modeled via the gamma distribution). In the following, new exact analytical expressions are derived for the Shannon capacity along with asymptotic expressions in the high and low Signal-to-Noise ratio (SNR) regimes. We also present exact tractable expressions followed by first-order expansions for the marginal outage probability and symbol error rate (SER); further, we quantify the performance of STBCs in terms of diversity order and coding gain. The derived analytical expressions are validated via a set of Monte-Carlo simulations which also demonstrate the implications of the model parameters on the overall performance.**

I. Introduction

Orthogonal STBCs represent a simple and reliable transmission scheme which can afford the same diversity order as the classical maximal-ratio combining. Since their original establishment by Alamouti in [1] and the generalization by Tarokh *et al.* in [2], they have been extensively used in the design and performance analysis of MIMO communication systems. The main advantage of STBCs is that maximum-likelihood (ML) decoding can be performed with linear processing at the receiver, while the MIMO channel can be transformed into an equivalent scalar Gaussian channel with a response equal to the Frobenius norm of the channel matrix [3], [4].

Conceptually, the majority of related studies documented in the literature adopts the common assumption of Rayleigh or Ricean fading statistics (see for instance [5], [6] and references therein among others). Some theoretical investigations and measurement campaigns [7], [8] though, have demonstrated that the Nakagami-m distribution [9] yields a better fit with real-time data for various measured channels and, more importantly, encompasses both Rayleigh/Ricean distributions as special cases. This reveals that assessing the performance of STBC over MIMO Nakagami-m channels is a highly interesting topic.

Apart from small-scale fading, each MIMO link is most likely to experience path-loss and shadowing (large-scale fading) with the latter manifestation being rather critical when assessing MIMO performance since it can significantly diminish the benefits of MIMO technology. For this reason, it is of paramount importance to consider the effects of shadowing into the analysis of STBC MIMO systems. As such, we note the work in [10] which modeled the shadowing via the lognormal distribution and provided an integral representation of the outage capacity. In order to circumvent this, the author approximated the outage capacity numerically via Gauss-Hermite polynomials with this technique, however, being time-consuming (especially at low SNRs) and not amenable to further manipulations. Over the past years, lognormal shadowing has been successfully approximated by the analytically friendlier Gamma shadowing with the resulting fading model being usually referred to as the generalized K-distribution [11], [12], which will serve as our reference model henceforth.

In this paper, we provide a systematic statistical characterization of STBCs over generalized-K fading MIMO channels. The main paper contributions can be summarized as follows:

- After presenting exact analytical expressions for the probability density function (PDF) and cumulative density function (CDF) of the instantaneous STBC SNR, we derive a new, analytical formula for the exact Shannon capacity. In addition, we consider the asymptotically high and low-SNR regimes for which insightful expressions are also presented; in the latter case, the notions of *minimum energy per bit to reliably convey any positive rate* and *wideband slope* are introduced.
- In the second part of our analysis, we focus on the SER and outage probability measures for which exact analytical expressions along with first-order expansions are deduced. Moreover, we introduce a common parametrization in the area of wireless communications to quantify the STBC performance in terms of *diversity order* and *coding (or array) gain*.

Notation: We use upper and lower case boldface to denote matrices and vectors. The expectation is given by $\mathcal{E}[\bullet]$, while $(\bullet)^\dagger$ represents the Hermitian transpose and $\mathrm{tr}(\bullet)$, $\|\bullet\|$ yield the trace and Frobenius norm of a matrix, respectively. The real part is expressed as $\mathrm{Re}(\bullet)$.

II. MIMO STBC System Model and Statistics of the Instantaneous SNR

We consider a typical point-to-point MIMO system with N_r receive antennas and N_t transmit antennas. For the case of composite fading (both small and large-scale fading), we can model the $N_r \times N_t$ channel matrix as $\mathbf{Z} = \mathbf{H}\mathbf{\Xi}^{1/2}$. The entries of the small-scale channel matrix $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ are assumed to be independent and identically distributed (i.i.d.) random variables (RVs) with uniformly distributed phase in $[0, 2\pi)$, while their amplitude $x = |h_{i,j}|$ follows a Nakagami-m distribution

$$f_x(x) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m x^{2m-1} e^{-(m/\Omega)x^2}, \; x \geq 0, m \geq \frac{1}{2} \tag{1}$$

where $\Omega = \mathcal{E}[x^2]$ is the average power. The entries of the diagonal matrix $\mathbf{\Xi} \in \mathbb{R}^{N_t \times N_t}$ represent essentially the large-scale effects, and hence $\mathbf{\Xi} = \left(\frac{\xi}{D^v}\right) \mathbf{I}_{N_t}$, where D denotes the normalized with respect to 1 Km Tx-Rx distance, while v is the path-loss exponent with typical values ranging from $2-6$. The large-scale fading coefficient ξ is modeled as a gamma RV, $\xi \sim$ Gamma(k, θ), or

$$f_\xi(\xi) = \frac{\xi^{k-1}}{\Gamma(k)\theta^k} \exp\left(-\frac{\xi}{\theta}\right), \quad \xi, \theta, k \geq 0 \tag{2}$$

where k, $\theta = \mathcal{E}[\xi]/k$, are the so-called shape and scale parameters of the gamma distribution respectively while $\Gamma(\cdot)$ is the well-known gamma function. Omitting explicit details, the instantaneous SNR per symbol after space-time block decoding is given by [3]

$$\gamma_{\text{STBC}} \triangleq \frac{\overline{\gamma}}{R_c N_t} \|\mathbf{Z}\|_F^2 = \frac{\overline{\gamma}}{R_c N_t} \text{tr}\left(\mathbf{\Xi}^{1/2}\mathbf{H}^\dagger\mathbf{H}\mathbf{\Xi}^{1/2}\right) = \frac{\xi\overline{\gamma}}{D^v R_c N_t} \text{tr}\left(\mathbf{H}^\dagger\mathbf{H}\right) \tag{3}$$

where R_c is the STBC rate and $\overline{\gamma}$ is the average SNR per receive antenna. If we define

$$z = \text{tr}\left(\mathbf{H}^\dagger\mathbf{H}\right)$$

it can be easily seen that z is the sum of $N_r N_t$ statistically independent gamma RVs with common scale and shape parameters $(\Omega/m) > 0$ and m, respectively. Then, from [11], [12] we can infer that $z \sim$ Gamma$(N_t N_r m, \Omega/m)$, or

$$f_z(z) = \frac{z^{N_t N_r m - 1}}{\Gamma(N_t N_r m) \left(\frac{\Omega}{m}\right)^{N_t N_r m}} \exp\left(-\frac{mz}{\Omega}\right). \tag{4}$$

For the sake of simplicity, we rewrite (3) according to

$$\gamma_{\text{STBC}} = \frac{\xi\overline{\gamma}}{D^v R_c N_t} z = y \cdot z \tag{5}$$

and also define $\alpha = (D^v R_c N_t m)/(\overline{\gamma}\theta\Omega)$. Hence, the PDF of γ_{STBC} can be derived after some algebra and with the aid of [4, Eq. (31)], as

$$f_{\gamma_{\text{STBC}}}(\gamma) = \frac{2}{\Gamma(N_t N_r m)\Gamma(k)} \alpha^{\frac{k+N_t N_r m}{2}} \times \gamma^{\frac{k+N_t N_r m}{2}-1} K_{k-N_t N_r m}\left(2\sqrt{\alpha\gamma}\right). \tag{6}$$

where $K_v(\cdot)$ is the v-th order modified Bessel function of the second kind [13, Eq. (8.407.1)]. Using (6), we can express the CDF of γ_{STBC} as

$$F_{\gamma_{\text{STBC}}}(\gamma_{\text{th}}) \triangleq \int_0^{\gamma_{\text{th}}} f_{\gamma_{\text{STBC}}}(x)\, dx \tag{7}$$

which can be analytically evaluated by expressing its integrands in terms of Meijer's G-functions. In particular, we have via [13, Eq. (9.34.3)]

$$K_v(x) = \frac{1}{2} G_{0,2}^{2,0}\left[\frac{x^2}{4} \middle| \begin{matrix} - \\ \frac{v}{2}, -\frac{v}{2} \end{matrix}\right] \tag{8}$$

where $G_{p,q}^{m,n}\left[x, \middle| \begin{matrix} \alpha_1,\ldots,\alpha_p \\ \beta_1,\ldots,\beta_q \end{matrix}\right]$ corresponds to the Meijer's G-function [13, Eq. (9.301)]. Then, (7) can be rewritten as

$$F_{\gamma_{\text{STBC}}}(\gamma_{\text{th}}) = \frac{1}{\Gamma(N_t N_r m)\Gamma(k)} (\alpha\gamma_{\text{th}})^{\frac{k+N_t N_r m}{2}} \times G_{1,3}^{2,1}\left[\alpha\gamma_{\text{th}} \middle| \begin{matrix} 1 - \frac{k+N_t N_r m}{2} \\ \frac{k-N_t N_r m}{2}, -\frac{k-N_t N_r m}{2}, -\frac{k+N_t N_r m}{2} \end{matrix}\right] \tag{9}$$

$$= \frac{1}{\Gamma(N_t N_r m)\Gamma(k)} G_{1,3}^{2,1}\left[\alpha\gamma_{\text{th}} \middle| \begin{matrix} 1 \\ k, N_t N_r m, 0 \end{matrix}\right] \tag{10}$$

where we have initially used [13, Eq. (7.813.1)] and thereafter [13, Eq. (9.31.5)] to get from (9) to (10). We now present two alternative simplified CDF expressions depending on the values of the shape parameter of shadowing:

Corollary 1: For $k \in \mathbb{Z}$, the CDF of the STBC SNR in (3) becomes

$$F_{\gamma_{\text{STBC}}}(\gamma_{\text{th}}) = 1 - \frac{2}{\Gamma(N_t N_r m)} \sum_{\ell=0}^{k-1} \frac{1}{\ell!} (\alpha\gamma_{\text{th}})^{\frac{N_t N_r m + \ell}{2}} \times K_{N_t N_r m - \ell}\left(2\sqrt{\alpha\gamma_{\text{th}}}\right) \tag{11}$$

while for $(k - mN_t N_r) \notin \mathbb{Z}$, we get

$$\begin{aligned} F_{\gamma_{\text{STBC}}}(\gamma_{\text{th}}) &= \pi \csc\left(\pi\left(mN_r N_t - k\right)\right) \\ &\times \left(\frac{(\alpha\gamma_{\text{th}})^k {}_1F_2\left(k; 1+k, 1+k-N_r N_t m; \alpha\gamma_{\text{th}}\right)}{\Gamma(1+k)\Gamma(mN_r N_t)\Gamma(1+k-mN_r N_t)}\right. \\ &- \frac{(\alpha\gamma_{\text{th}})^{mN_r N_t}}{\Gamma(1+mN_r N_t)\Gamma(k)\Gamma(1+mN_r N_t - k)} \\ &\times \left. {}_1F_2\left(mN_r N_t; 1+mN_r N_t, 1+mN_r N_t - k; \alpha\gamma_{\text{th}}\right)\right) \end{aligned} \tag{12}$$

where ${}_pF_q(\cdot)$ denotes the generalized hypergeometric function, with p, q being non-negative integers [13, Eq. (9.14.1)].

Proof: From (5), the CDF of γ_{STBC} can be expressed as

$$F_{\gamma_{\text{STBC}}}(\gamma_{\text{th}}) = \int_0^\infty F_y\left(\frac{\gamma_{\text{th}}}{z}\right) f_z(z) dz \tag{13}$$

where $F_y(\cdot)$ denotes the CDF of y. The PDF of y is equal to

$$f_y(\xi) = \frac{\xi^{k-1}}{\Gamma(k)(a\theta)^k} \exp\left(-\frac{\xi}{a\theta}\right) \tag{14}$$

where $a = \frac{\bar{\gamma}}{D^v R_c N_t}$. Therefore, $F_y(\xi)$ is given by

$$F_y(\xi) = \frac{\gamma\left(k, \frac{\xi}{a\theta}\right)}{\Gamma(k)} = 1 - e^{-\frac{\xi}{a\theta}} \sum_{\ell=0}^{k-1} \frac{\left(\frac{\xi}{a\theta}\right)^\ell}{\ell!} \tag{15}$$

where $\gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt$ is the lower incomplete gamma function [13, Eq. (8.350.1)] while the second equality follows from [13, Eq. (8.352.1)]. We can then easily work out the final result by substituting (4), (15) into (13) and thereafter using [13, Eq. (3.471.9)]. The expression in (12) can be derived from [13, Eq. (9.303)] and using Euler's reflection formula $\pi/\sin(\pi x) = \Gamma(x)\Gamma(1-x)$. ■

We note that all these expressions will be particularly useful when analyzing the STBC performance in terms of capacity and SER. It is also worth emphasizing the similarity of the presented analytical expressions for the SNR statistics with previous results that have been reported in the context of keyhole Nakagami-m channels (e.g. see [4], [14], [15]).

III. Shannon Capacity

A. Exact analysis

By definition, the Shannon capacity of an orthogonal STBC is given (in bits/s/Hz) by

$$C \triangleq \frac{R_c}{\ln 2}\mathcal{E}\left[\ln\left(1+\frac{\bar{\gamma}}{R_c N_t}\|\mathbf{Z}\|_F^2\right)\right] \tag{16}$$

where the scaling factor $R_c/\ln 2$ accounts for the additional SNR gain [6]. The following theorem returns an analytical expression for the capacity of STBC over generalized-K fading MIMO channels and constitutes the key contribution of the paper:

Theorem 1: The capacity of STBC over generalized-K fading MIMO channels is given by

$$C = \frac{R_c}{\Gamma(N_t N_r m)\Gamma(k)\ln 2} G_{2,4}^{4,1}\left[\alpha \left| \begin{matrix} 0,1 \\ k, N_t N_r m, 0, 0 \end{matrix}\right.\right]. \tag{17}$$

Proof: We start by expressing the expectation in (16) in integral form according to

$$\begin{aligned} C &= \frac{R_c}{\ln 2}\int_0^\infty \ln(1+\gamma) f_{\gamma_{\text{STBC}}}(\gamma)d\gamma \\ &= \frac{R_c}{\ln 2}\int_0^\infty G_{2,2}^{1,2}\left[\gamma, \left|\begin{smallmatrix}1,1\\1,0\end{smallmatrix}\right.\right] f_{\gamma_{\text{STBC}}}(\gamma)d\gamma \end{aligned} \tag{18}$$

where (18) follows by expressing the logarithmic function via [16, Eq. (8.4.6.5)]. Substituting (6) into (18), expressing the involved Bessel function as in (8) and thereafter applying [13, Eq. (7.813.1)], we can obtain (17) after some algebra. ■

Note that the proposed capacity expression in Theorem 1 is expressed in terms of Meijer's G-function which can be easily evaluated and efficiently programmed in most standard software packages (e.g. MAPLE, MATHEMATICA).

B. High-SNR analysis

The capacity expression in (17), though exact, does not provide any insights into the implications of the fading parameters on the STBC performance. For this reason, we now consider the high-SNR regime and provide the following result:

Corollary 2: At high SNRs, the capacity of STBC over generalized-K fading MIMO channels is given by

$$\begin{aligned} C^\infty = R_c\Bigg(&\log_2\left(\frac{\bar{\gamma}}{R_c N_t}\right)+\frac{\psi(k)}{\ln 2}+\log_2(\theta)-\upsilon\log_2(D) \\ &+\frac{\psi(N_t N_r m)}{\ln 2}+\log_2\left(\frac{\Omega}{m}\right)\Bigg) \end{aligned} \tag{19}$$

where $\psi(x)$ is the well-known Euler's digamma function [13, Eq. (8.360.1)].

Proof: The proof follows trivially by taking $\bar{\gamma}$ large in (16) and successively using the integral identity [13, Eq. (4.352.1)]. ■

The above result reveals that at high SNRs the effects of small and large-scale fading are decoupled, while higher values of both k, θ increase the high-SNR capacity. On the other hand, a larger Tx-Rx distance effectively reduces capacity, due to the increased path-loss attenuation.

C. Low-SNR analysis

The low-SNR performance of K MIMO channels can be investigated by taking a first-order expansion of (16) around $\bar{\gamma} = 0^+$. Recent theoretic studies have demonstrated though that this approach can not adequately reflect the impact of the channel and can lead to misleading results in the low-SNR (or wideband) regime [18]. To this end, it is more meaningful to explore the low-SNR capacity in terms of the normalized transmit energy per information bit E_b/N_0 rather than per-symbol SNR. This representation reads as [18]

$$C\left(\frac{E_b}{N_0}\right) \approx \mathcal{S}_0 \log_2\left(\frac{\frac{E_b}{N_0}}{\frac{E_b}{N_0}_{\min}}\right) \tag{20}$$

where $\frac{E_b}{N_0}_{\min}$ and $\mathcal{S}_0$ are the two key parameters determining the low-SNR behavior, corresponding to the "minimum normalized energy per information bit required to convey any positive rate reliably" and the wideband slope, or

$$\frac{E_b}{N_0}_{\min} = \frac{1}{\dot{C}(0)}, \quad \mathcal{S}_0 = -2\ln 2\frac{\left(\dot{C}(0)\right)^2}{\ddot{C}(0)} \tag{21}$$

where $\dot{C}(\cdot)$, $\ddot{C}(\cdot)$ denote the first and second-order derivatives of the ergodic capacity (16) over the SNR $\bar{\gamma}$, respectively. For the case under consideration, we have that:

Corollary 3: For generalized-K fading MIMO channels with STBC, the minimum energy per bit to reliably convey any positive rate and the wideband slope are respectively

$$\frac{E_b}{N_0}_{\min} = \frac{\ln 2 D^\upsilon}{N_r k\theta\Omega} \tag{22}$$

$$\mathcal{S}_0 = \frac{2R_c N_t N_r}{(N_t N_r m+1)}\frac{km}{k+1}. \tag{23}$$

Proof: The proof follows by combining (16) with (21) and successively applying [13, Eq. (3.381.4)]. ■

Note that the wideband slope is independent of the scale parameter of shadowing and of the terminal distance while a higher number of receive antennas can significantly improve the STBC performance due to the additional power captured by the extra antennas. This observation is in agreement with previous results on Rayleigh-fading MIMO channels [18]. In addition, shadowing always reduces $\mathcal{S}_0$ especially for severe fading conditions (i.e. small k).

IV. SER AND OUTAGE PROBABILITY

A. SER analysis

We now analyze the SER performance of STBC introduced in Section II. We first invoke that for many modulation formats (e.g. BPSK, M-ary PSK, M-ary PAM) the average SER can be represented by the following generic formula

$$\text{SER} = \mathcal{E}_{\gamma_{\text{STBC}}}\left[AQ\left(\sqrt{2B\gamma_{\text{STBC}}}\right)\right] \tag{24}$$

where $Q(\cdot)$ is the Gaussian Q-function while A, B are modulation-specific constants. The following theorem returns an analytical expression for the exact SER:

Theorem 2: The exact SER of STBC over generalized-K fading MIMO channels is

$$\text{SER} = \frac{A}{2\sqrt{\pi}\Gamma(N_tN_rm)\Gamma(k)} G_{2,3}^{2,2}\left[\frac{\alpha}{B}\,\middle|\, \begin{matrix} 1, \frac{1}{2} \\ k, N_tN_rm, 0 \end{matrix}\right]. \tag{25}$$

Proof: The proof starts by re-expressing (24) in terms of the complementary error function $\text{erfc}(\cdot)$, as follows

$$\text{SER} = \frac{A}{2}\int_0^\infty \text{erfc}\left(\sqrt{B\gamma_{\text{STBC}}}\right) f_{\gamma_{\text{STBC}}}\left(\gamma_{\text{STBC}}\right) d\gamma_{\text{STBC}} \tag{26}$$

where we have used the fundamental property $Q(x) = 0.5\,\text{erfc}\left(x/\sqrt{2}\right)$. Substituting (6) into (26), and thereafter express the $\text{erfc}(\cdot)$ integrand with the aid of a Meijer-G function [16, Eq. (8.4.14.2)]

$$\text{erfc}\left(\sqrt{x}\right) = \frac{1}{\sqrt{\pi}} G_{1,2}^{2,0}\left[x\,\middle|\, \begin{matrix} 1 \\ 0, \frac{1}{2} \end{matrix}\right] \tag{27}$$

and likewise the Bessel function integrand according to (8), we can get

$$\text{SER} = \frac{A\alpha^{\frac{k+N_tN_rm}{2}}}{2\sqrt{\pi}\Gamma(N_tN_rm)\Gamma(k)}\int_0^\infty x^{\frac{k+N_tN_rm}{2}-1} \times G_{1,2}^{2,0}\left[Bx\,\middle|\, \begin{matrix} 1 \\ 0, \frac{1}{2} \end{matrix}\right] G_{0,2}^{2,0}\left[\alpha x\,\middle|\, \begin{matrix} - \\ \frac{k-N_tN_rm}{2}, -\frac{k-N_tN_rm}{2} \end{matrix}\right] dx.$$

The final result is obtained by solving the above integral via [16, Eq. (2.24.1.1)]. ■

To gain more insights, it is meaningful to explore the SER at high SNRs. Under these circumstances, a common parametrization in the area of wireless communications can be introduced via the two critical measures describing the system performance, these are the diversity order and coding gain.

Theorem 3: The SER of STBC over generalized-K fading MIMO channels at high SNRs and for $(k - mN_tN_r) \notin \mathbb{Z}$ is

$$\text{SER}_m^\infty = (G_a\overline{\gamma})^{-G_d} + o\left(\overline{\gamma}^{-G_d}\right) \tag{28}$$

where $p \triangleq \min(N_tN_rm, k)$ and $q \triangleq \max(N_tN_rm, k)$ while the diversity order and coding gain are respectively

$$G_d(m) = p \tag{29}$$

$$G_a(m) = \frac{\theta\Omega}{D^vR_cN_tm}\left(\frac{A}{2\sqrt{\pi}B^p}\frac{\Gamma\left(q-p\right)\Gamma\left(p+\frac{1}{2}\right)}{\Gamma\left(q\right)\Gamma\left(p+1\right)}\right)^{-\frac{1}{p}}. \tag{30}$$

Proof: A detailed proof is given in [19]. ■

Interestingly, the above theorem reveals that the diversity order depends only on the system dimensions, Nakagami-m factor and the shape parameter of gamma shadowing. This demonstrates the effect of shadowing on the diversity order since with only Nakagami-m fading, the diversity order is equal to N_tN_rm. As such, for severe shadowing conditions (i.e. small k), the diversity order is limited to k and therefore an increase in the number of antennas will not yield any additional diversity gains. Note that for $(k - mN_tN_r) \in \mathbb{Z}$ a similar parametrization is not straightforward due to the asymptotic performance of the Bessel function in (6). A detailed discussion can be found in [14].

B. Outage Probability analysis

For the case of non-ergodic channels (e.g. quasi-static or block-fading), it is more appropriate to resort to the notion of outage probability to characterize the performance of STBC operating in generalized-K fading channels. In general, the outage probability is a critical measure of system performance and is defined as the probability that the instantaneous received SNR, γ_{STBC}, falls below a predefined threshold γ_{th}. Clearly, the outage probability can be directly obtained via the SNR CDF expression in (10) and (11), or

$$P_{\text{out}} \triangleq \Pr\left(\gamma_{\text{STBC}} \leq \gamma_{\text{th}}\right) = F_{\gamma_{\text{STBC}}}\left(\gamma_{\text{th}}\right). \tag{31}$$

Moreover, since we are typically interested in small outage (e.g. 0.01, 0.001), we can exploit the outage performance for small values of γ_{th}. In light of this fact, the following theorem returns the outage probability in the high-SNR regime:

Theorem 4: The outage probability of STBC over generalized-K fading MIMO channels at high SNRs and for $(k - mN_tN_r) \notin \mathbb{Z}$ is

$$P_{\text{out}}^\infty = \frac{\Gamma(q-p)}{\Gamma(q)\Gamma(1+p)}\left(\alpha\gamma_{\text{th}}\right)^p + o(\gamma_{\text{th}})^p. \tag{32}$$

Proof: The proof follows trivially by keeping only the dominant term in (12) and thereafter use the Euler's reflection formula to simplify. ■

V. NUMERICAL AND SIMULATION RESULTS

In this section, the theoretical analysis presented in Sections III and IV is validated through a set of Monte-Carlo simulations. To this end, we first generate 20,000 random realizations of the small and large-scale fading matrices $\mathbf{H}$ and Ξ and thereafter obtain the instantaneous STBC SNR via (3). For the sake of simplicity, we hereafter consider the classical Alamouti scheme which is a one-rate code (i.e. $R_c = 1$ bits/s/Hz) employing two transmit antennas [1].

In Fig. 1, the Shannon capacity is plotted against the average SNR, $\overline{\gamma}$ for different MIMO configurations by keeping $N_t = 2$ and increasing only N_r. The exact analytical expression and the high-SNR approximation have been respectively generated via Theorem 1 and Corollary 2. It is easily observed that the match between theory and simulation is excellent in all cases under consideration, thereby validating the correctness of the proposed analytical expressions. As anticipated, a higher N_r increases capacity due to the enhanced receive spatial diversity that offers more degrees of freedom for the decoding of the incoming signals. Further, the high-SNR expressions become sufficiently tight at $\overline{\gamma} \geq 15$ dB and can explicitly predict the exact capacity for most practical SNR values.

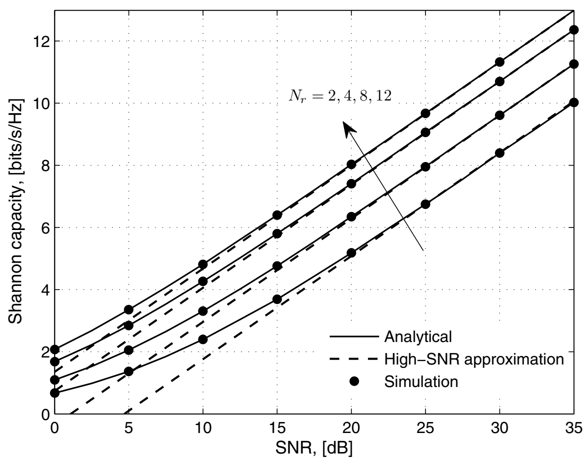

Fig. 1. Simulated capacity, analytical expression and high-SNR approximation against the SNR ($N_t = 2$, $m = 0.5$, $\upsilon = 4$, $k = 1$, $\theta = 2$, $D = 1.5$ Km).

In Fig. 2, the exact and high-SNR SER curves based on Theorem 2 and (28) respectively, are overlaid with the output of a Monte-Carlo simulator. We are considering QPSK modulation with $A = 2$, $B = 0.5$. Clearly, a higher Nakagami-m factor yields smaller SER due to reduced signal's envelope fluctuations, which is in agreement with [9], [12], [14].

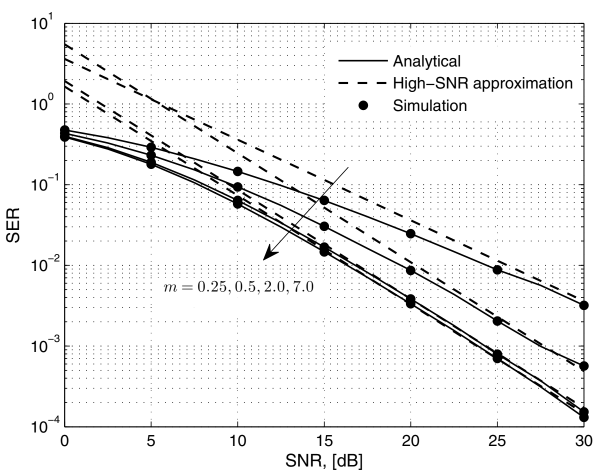

Fig. 2. Simulated SER, analytical expression and high-SNR approximation against the SNR ($N_t = 2$, $N_r = 2$, $\upsilon = 4$, $k = 1.35$, $\theta = 2$, $D = 1.5$ Km).

Finally, in Fig. 3, we consider the STBC subchannel outage probability against the SNR, $\overline{\gamma}$, for a fixed threshold value $\gamma_{\text{th}} = 0.5$ and increasing values of the shape parameter k. The analytical and high-SNR approximation curves have been respectively generated via (31) and (32). It is easily seen that a high k diminishes the effects of shadowing, thereby delivering smaller outage. However, the gap between the corresponding curves decreases as k increases which implies that its effect becomes less pronounced; note that consistent conclusions were also drawn in [12].

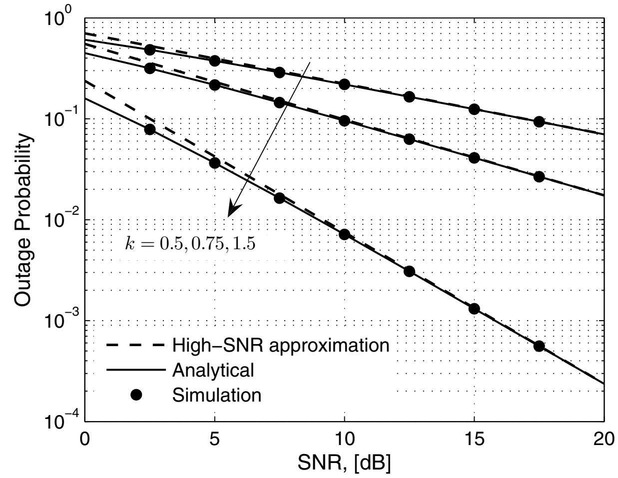

Fig. 3. Simulated outage probability, analytical expression and high-SNR approximation against the SNR ($N_t = 2$, $N_r = 4$, $m = 0.5$, $\upsilon = 4$, $\theta = 2$, $\gamma_{\text{th}} = 0.5$, $D = 1.5$ Km).

VI. Conclusion

In this paper, a detailed statistical characterization of STBC over generalized-K fading MIMO channels was presented. Our analysis extends and complements some recent results on Rayleigh/Ricean fading MIMO channels to account for the effects of path-loss and shadowing. The latter manifestation was modeled via the gamma distribution which is analytically friendlier than the classical lognormal model. New analytical expressions for the Shannon capacity were derived that apply for any arbitrary SNR. For large SNR, a simplified approximation was presented and, likewise, at low SNRs we considered the minimum energy per bit to reliably convey any positive rate and the wideband slope. In addition, we presented exact analytical formulas along with first-order expansions for the SER and outage probability. These expansions were particularly useful to quantify the STBC performance in terms of diversity order and coding gain.

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