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Effect of Feedback Delay on Amplify-and-Forward Relay Networks With Beamforming

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***Abstract*—In this paper, the decremental effect of beamforming with feedback delay on the performance of a two-hop amplify-and-forward (AF) relay network over Rayleigh-fading channels is investigated. An antenna configuration in which the source and the destination are equipped with multiple antennas, whereas the relay is equipped with a single antenna, is assumed. We derive new expressions for the outage probability and the average bit error rate (BER), which are useful for a large number of modulation schemes. To gain further insights, simple outage probability and average BER approximations at high signal-to-noise ratio (SNR) are also presented. It is shown that, whenever a feedback delay exists, the network is not capable of offering diversity gains. Furthermore, source and relay power allocation results show significantly different behavior with feedback delay. Numerical results supported by simulations are provided to show that feedback delay can severely degrade the performance of the considered AF relay system.**

***Index Terms*—Amplify-and-forward (AF), average bit error rate (BER), beamforming, feedback delay, relays.**

I. Introduction

Wireless communication systems can benefit from relay deployment since the technology promises extended signal coverage, improved throughputs, and spatial diversity [1], [2]. One of the relaying protocols described in the literature is amplify and forward (AF). The performance of single-antenna AF relay networks has now been well investigated [3]–[5].

Work such as [6] has also demonstrated that significant benefits can be gained if multiple antennas are deployed in relaying networks. A practical transmission scheme for AF relaying systems employing multiple antennas is beamforming [7]–[13]. The performance of beamforming against relay selection, considering both unlimited and limited feedback, has been studied in [7]. In [8], the performance of a two-hop relay network with transmit beamforming at the source and maximal ratio combining (MRC) at the destination has been analyzed. The performance of the same system, by considering antenna correlation effects at the source and the destination, is reported in [9]. In [10], the outage performance with beamforming, considering only limited feedback, has been studied. In [11], the performance of a two-hop fixed gain network over Nakagami-m fading channels has been analyzed. In [12], assuming the absence/presence of the source–destination direct link, optimal beamforming codebook designs for an AF relay system with limited feedback was presented. In [13], a practical scenario in which a multiantenna-equipped source is communicating to a single-antenna-equipped destination via a relay has been considered. Despite the significant practical interest, the authors in [13] have limited their analysis to a situation of *perfect* channel state information (CSI) at the source.

In beamforming systems, the received signal-to-noise ratio (SNR) maximization is achieved by providing CSI to the transmitter. In frequency-division-duplex systems, such knowledge is provided by the feedback of CSI from the receiver to the transmitter. Feedback involves delay, and as a result, in practice, the available CSI at the transmitter and the actual channel may be different. The use of outdated CSI for beamforming degrades the system performance. Although this performance degradation for point-to-point systems is now well understood (see, e.g., [14]) and for partial relay selection [15], so far, in the existing literature, the effect of feedback delay on the performance of AF relaying with beamforming has not been investigated.

In this paper, the effect of feedback delay on the end-to-end performance of a two-hop AF relay network where multiple antennas at the source are used for beamforming is investigated. We derive closed-form expressions for the system's outage probability and the average bit error rate (BER) applicable for a range of modulation schemes. To gain valuable insights, in the high-SNR regime, we also present asymptotic outage probability and average BER expressions. The impact of different antenna configurations, feedback delay, and SNR imbalance on the performance is illustrated through some analytical results.

II. System Model

Consider a wireless network where a source S equipped with N_t antennas communicates with a destination D equipped with N_r antennas through a single antenna relay R [8], [9], [11]. In this network, we assume that S does not have a direct link to D. The communication from S to D via relay R takes place in two time slots. In the first time slot, S beamforms its signal to R. The received signal at R can be written as

$$y_R(t) = \sqrt{P_1}\mathbf{w}_t^{\dagger}\mathbf{h}_{sr}(t)x(t) + n_1(t) \quad (1)$$

where $x(t)$ is the data symbol, P_1 is the transmit power, $\mathbf{h}_{sr}(t) = [h_{sr}^1(t), \ldots, h_{sr}^{N_t}(t)]^T$ is the channel vector from S to R with Rayleigh fading entries, and $n_1(t)$ is the additive white Gaussian noise (AWGN) at R with one-sided power spectrum density σ_1^2. The transpose and the conjugate transpose are denoted by $(\cdot)^T$ and $(\cdot)^{\dagger}$, respectively. According to the principles of maximal ratio transmission, we choose

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$\mathbf{w}_t = (\mathbf{h}_{sr}(t-T_d)/\|\mathbf{h}_{sr}(t-T_d)\|_F)$, where $\|\cdot\|_F$ is the Frobenius norm. Note that $\mathbf{w}_t$ is calculated from the *outdated channel* $\mathbf{h}_{sr}(t-T_d)$, instead of the channel $\mathbf{h}_{sr}(t)$, with T_d being the feedback delay. The received scalar signal at R is then multiplied by a gain G/ and transmitted to the destination. The received signal at D is given by

$$\mathbf{y}_D(t) = \sqrt{P_2}\mathbf{h}_{rd}(t)G\left(\sqrt{P_1}\mathbf{w}_t^\dagger \mathbf{h}_{sr}(t)x + n_1(t)\right) + \mathbf{n}_2(t) \quad (2)$$

where P_2 is the relay transmit power, $\mathbf{h}_{rd}(t) = [h_{rd}^1(t), \ldots, h_{rd}^{N_r}(t)]$ is the channel vector from R to D with Rayleigh fading entries, and $\mathbf{n}_2(t)$ is the AWGN vector at D with one-sided power spectral density $E[\mathbf{n}_2(t)\mathbf{n}_2^\dagger(t)] = \sigma_2^2 \mathbf{I}_{N_r}$. The expectation operator is $E[\cdot]$, and $\mathbf{I}_{N_r}$ denotes the identity matrix of size N_r. According to the principles of MRC, we then multiply the received signal $\mathbf{y}_D(t)$ by a vector $\mathbf{w}_r = (\mathbf{h}_{rd}(t)/\|\mathbf{h}_{rd}(t)\|_F)$ and write

$$r_D(t) = \sqrt{P_1 P_2}\mathbf{w}_r^\dagger \mathbf{h}_{rd}(t) G \mathbf{w}_t^\dagger \mathbf{h}_{sr}(t)x + \sqrt{P_2}\mathbf{w}_r^\dagger \mathbf{h}_{rd}(t) G n_1(t) + \mathbf{w}_r^\dagger \mathbf{n}_2(t). \quad (3)$$

Using the CSI-assisted AF relay gain [5], [8][1]

$$G^2 = \frac{1}{P_1 \left\|\mathbf{w}_t^\dagger \mathbf{h}_{sr}\right\|_F^2 + \sigma_1^2} \quad (4)$$

and after some manipulations, the end-to-end SNR can be written as

$$\gamma_{\rm eq1} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1} \quad (5)$$

where $\gamma_1 = \|\mathbf{w}_t^\dagger \mathbf{h}_{sr}(t)\|_F^2 \bar{\gamma}_1$, $\gamma_2 = \|\mathbf{h}_{rd}(t)\|_F^2 \bar{\gamma}_2$, and $\bar{\gamma}_i = (P_i/\sigma_i^2)$ for $i = 1, 2$.

III. Performance Analysis

In this section, we derive important performance metrics, e.g., the outage probability and the average BER for the two-hop network under investigation.

A. Outage Probability

The outage probability P_o is an important quality-of-service measure defined as the probability that $\gamma_{\rm eq1}$ drops below an acceptable SNR threshold $\gamma_{\rm th}$. To study the outage probability of (5), it is necessary to obtain the cumulative distribution function (cdf) $F_Z(\gamma_{\rm th})$ of the random variable, which is defined as

$$Z = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + c} \quad (6)$$

where c is a positive constant. After applying some algebraic manipulations along similar lines as in [8], $F_Z(\gamma_{\rm th})$ can be expressed as

$$F_Z(\gamma_{\rm th}) = 1 - \int_0^\infty f_{\gamma_1}(x+\gamma_{\rm th})\left(1 - F_{\gamma_2}\left(\gamma_{\rm th} + \frac{\gamma_{\rm th}^2 + c\gamma_{\rm th}}{x}\right)\right)dx \quad (7)$$

where $f_{\gamma_1}(\cdot)$ and $F_{\gamma_2}(\cdot)$ denote the probability density function (pdf) of γ_1 and the cdf of γ_2, respectively.

[1]In the presence of a feedback delay, the ability of the relay to perfectly track the current CSI is subjective to the pilot placement arrangement employed for channel estimation at the relay. Analyzing the impact of any specific pilot placement on the relay gain and system performance is beyond the scope of the current work.

To obtain $F_Z(\gamma_{\rm th})$, expressions for the pdf of γ_1 and the cdf of γ_2 are needed. To model the relationship between $\mathbf{h}_{sr}(t)$ and $\mathbf{h}_{sr}(t-T_d)$, we employ the following widely adopted time-varying channel model:

$$\mathbf{h}_{sr}(t) = \rho_d \mathbf{h}_{sr}(t-T_d) + \sqrt{1-|\rho_d|^2}\mathbf{e}(t) \quad (8)$$

where ρ_d is the normalized correlation coefficient between $h_{sr}^j(t)$ and $h_{sr}^j(t-T_d)$, with $j = 1, \ldots, N_t$. The error vector $\mathbf{e}(t) \sim {\rm CN}(\mathbf{0}, \mathbf{I}_{N_t})$, where ${\rm CN}(\boldsymbol{\varphi}, \boldsymbol{\Xi})$ denotes the complex Gaussian distribution with mean $\boldsymbol{\varphi}$ and covariance $\boldsymbol{\Xi}$, is uncorrelated with $\mathbf{h}_{sr}(t)$. For Jake's fading spectrum, $\rho_d = J_0(2\pi f_d T_d)$, where f_d is the Doppler frequency, and $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind [16, Sec. (9.1)].

The pdf of γ_1 using [14, (15)] can be written as[2]

$$f_{\gamma_1}(x) = \frac{1}{\bar{\gamma}_1^{N_t}} \sum_{i=0}^{N_t-1} \frac{\binom{N_t-1}{i}\left(|\rho_d|^2\right)^{N_t-i-1}\left(\bar{\gamma}_1\left(1-|\rho_d|^2\right)\right)^i}{(N_t-i-1)!} \times x^{N_t-i-1} e^{-\frac{x}{\bar{\gamma}_1}} \quad (9)$$

and the cdf of γ_2 is given by

$$F_{\gamma_2}(x) = 1 - e^{-\frac{x}{\bar{\gamma}_2}} \sum_{j=0}^{N_r-1} \frac{\left(\frac{x}{\bar{\gamma}_2}\right)^j}{j!}. \quad (10)$$

Substituting (9) and (10) into (7), we obtain

$$F_Z(\gamma_{\rm th}) = 1 - \frac{e^{-\left(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2}\right)\gamma_{\rm th}}}{\bar{\gamma}_1^{N_t}} \times \sum_{i=0}^{N_t-1} \frac{\binom{N_t-1}{i}\left(|\rho_d|^2\right)^{N_t-i-1}\left(\bar{\gamma}_1\left(1-|\rho_d|^2\right)\right)^i}{(N_t-i-1)!} \times \sum_{j=0}^{N_r-1} \frac{\left(\frac{\gamma_{\rm th}}{\bar{\gamma}_2}\right)^j}{j!}\mathcal{I}_1 \quad (11)$$

where

$$\mathcal{I}_1 = \int_0^\infty (x+\gamma_{\rm th})^{N_t-i-1}(x+\gamma_{\rm th}+c)^j x^{-j} e^{-(x/\bar{\gamma}_1)-(\gamma_{\rm th}^2 + c\gamma_{\rm th}/x\bar{\gamma}_2)}dx.$$

Using the binomial theorem and with the help of the identity [17, eq. (3.471.9)], we obtain

$$\mathcal{I}_1 = \sum_{p=0}^{N_t-i-1}\binom{N_t-i-1}{p}\gamma_{\rm th}^{N_t-i-p-1}\sum_{q=0}^{j}\binom{j}{q}(\gamma_{\rm th}+c)^{j-q} \times 2\left(\frac{\bar{\gamma}_1\left(\gamma_{\rm th}^2+c\gamma_{\rm th}\right)}{\bar{\gamma}_2}\right)^{\frac{p+q-j+1}{2}} K_{p+q-j+1}\left(2\sqrt{\frac{\gamma_{\rm th}^2+c\gamma_{\rm th}}{\bar{\gamma}_1\bar{\gamma}_2}}\right) \quad (12)$$

where $K_\nu(\cdot)$ is the νth-order modified Bessel function of the second kind [16, Sec. (9.6)]. Now, the outage probability follows by substituting $c = 1$ into (12) and then using (11).

Note that the derived outage probability (valid for arbitrary ρ_d) can be further simplified in many special cases. For example, when D has a single antenna and the feedback delay goes to infinity, the outage

[2]It is noted that, [14, eq. (15)] includes a small typo, which we have corrected in (9). The power of the term $1/\bar{\gamma}_1$ outside the $\sum$ must be N_t and not $N_t - 1$.

probability can be obtained from (11) and (12) by substituting $N_r = 1$, $i = N_t - 1$, and $|\rho_d|^2 = 0$ and is given by

$$F_{\gamma_{\text{eq1}}}(\gamma_{\text{th}}) = 1 - 2e^{-\left(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2}\right)\gamma_{\text{th}}} \sqrt{\frac{\gamma_{\text{th}}^2 + \gamma_{\text{th}}}{\bar{\gamma}_1\bar{\gamma}_2}} \times K_1\left(2\sqrt{\frac{\gamma_{\text{th}}^2 + \gamma_{\text{th}}}{\bar{\gamma}_1\bar{\gamma}_2}}\right). \tag{13}$$

B. Outage Probability at High SNR

Although (11) is exact and valid for any given SNR, it is difficult to gain insights such as the effect of parameters N_t and ρ_d on the outage probability. Therefore, as $\bar{\gamma}_1$ and $\bar{\gamma}_2$ tend to infinity with fixed ratio μ, we can analyze the system's asymptotic outage probability with the help of *Theorem 1*.

Theorem 1: The outage probability at high SNR for $\rho_d = 1$ is given by[3]

$$P_o^\infty = \begin{cases} \frac{1}{N_t!}\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^{N_t} + o\left(\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^{N_t+1}\right), & N_t < N_r \\ \frac{1}{N!}\left(1+\frac{1}{\mu^N}\right)\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^{N} + o\left(\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^{N+1}\right), & N_t = N_r = N \\ \frac{1}{\mu^{N_r} N_r!}\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^{N_r} + o\left(\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^{N_r+1}\right), & N_t > N_r. \end{cases} \tag{14}$$

The outage probability for $\rho_d < 1$ is given by

$$P_o^\infty = \begin{cases} \left(\left(1-|\rho_d|^2\right)^{N_t-1} + \frac{1}{\mu}\right)\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right) + o\left(\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^2\right), & N_r = 1 \\ \left(\left(1-|\rho_d|^2\right)^{N_t-1}\right)\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right) + o\left(\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^2\right), & N_r > 1 \end{cases} \tag{15}$$

where $\mu = (\bar{\gamma}_2/\bar{\gamma}_1)$.

Proof: See the Appendix. ■

We observe that a longer delay (low ρ_d) degrades the system performance by increasing the outage probability. More importantly, in the presence of feedback delay, no diversity gains can be achieved. In this two-hop system, the worst link dominates the performance at high SNR. According to (8), coefficients of the noise vector $\mathbf{e}(t)$, although undesirable, contributes in determining the beamforming weights of the $S-R$ channel $\mathbf{h}_{sr}(t)$. As such, although signals from N_t different paths are received at R, we do not expect an improvement in the signal quality since $\mathbf{e}(t)$ is uncorrelated with $\mathbf{h}_{sr}(t)$.

C. Average BER

For many modulation formats used in wireless applications, the average BER expressed as $E[a\, Q(\sqrt{b\gamma_{\text{eq1}}})]$ is given by

$$P_b = \frac{a}{\sqrt{2\pi}} \int_0^\infty F_{\gamma_{\text{eq1}}}\left(\frac{t^2}{b}\right) e^{-\frac{t^2}{2}}\, dt \tag{16}$$

where $a,\ b > 0$, and $Q(x) = (1/\sqrt{2\pi})\int_x^\infty e^{-(y^2/2)}dy$ is the Gaussian Q-function. Moreover, our BER derivations can also be extended to square/rectangular M-QAM since P_b can be written as a finite-weighted sum of $a\, Q(\sqrt{b\gamma_{\text{eq1}}})$ terms.

To the best of our knowledge, (16) does not have a closed-form solution. To overcome this, we substitute $c = 0$ into (11) and tightly lower bound the average BER as

$$P_b \geq \frac{a}{2} - \sqrt{\frac{2}{\pi}} \sum_{i=0}^{N_t-1} \frac{\binom{N_t-1}{i}\left(|\rho_d|^2\right)^{N_t-i-1}\left(b\bar{\gamma}_1\left(1-|\rho_d|^2\right)\right)^i}{(b\bar{\gamma}_1)^{N_t}(N_t-i-1)!} \times \sum_{j=0}^{N_r-1} \frac{(b\bar{\gamma}_2)^{-j}}{j!} \sum_{p=0}^{N_t-i-1} \binom{N_t-i-1}{p} \times \sum_{q=0}^{j} \binom{j}{q} \left(\frac{\bar{\gamma}_1}{\bar{\gamma}_2}\right)^{\frac{p+q-j+1}{2}} \mathcal{I}_2. \tag{17}$$

In (17), $\mathcal{I}_2 = \int_0^\infty t^{2(N_t-i+j)} e^{-((1/b\bar{\gamma}_1)+(1/b\bar{\gamma}_2)+(1/2))t^2} K_{p+q-j+1}((2t^2/b\sqrt{\bar{\gamma}_1\bar{\gamma}_2}))dt$. Applying a simple variable transformation and using [17, eq. (6.621.3)], $\mathcal{I}_2$ has a closed-form solution given by

$$\mathcal{I}_2 = \frac{\sqrt{\pi}(2\alpha_2)^{\eta_2}}{2(\alpha_1+\alpha_2)^{\eta_1+\eta_2}} \frac{\Gamma(\eta_1+\eta_2)\Gamma(\eta_1-\eta_2)}{\Gamma\left(\eta_1+\frac{1}{2}\right)} \times {}_2F_1\left(\eta_1+\eta_2, \eta_2+\frac{1}{2}; \eta_1+\frac{1}{2}; \frac{\alpha_1-\alpha_2}{\alpha_1+\alpha_2}\right) \tag{18}$$

where $\eta_1 = N_t - i + j + (1/2)$, $\eta_2 = p + q - j + 1$, $\alpha_1 = (1/b\bar{\gamma}_1) + (1/b\bar{\gamma}_2) + (1/2)$, $\alpha_2 = (2/b\sqrt{\bar{\gamma}_1\bar{\gamma}_2})$, $\Gamma(z)$ is the gamma function, and ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gauss hypergeometric function [16, eq. (15.1.1)].

D. Average BER at High SNR

Asymptotic analysis on relaying networks are important since such studies provide valuable insights that are useful to design engineers. It is shown in [18] that, at high SNRs, the error performance can be calculated based on the behavior of the pdf of the instantaneous channel power gain around zero. First, consider the case of $\rho_d = 1$. By following [18, Prop. (1)] and using (14), the average BER in the high-SNR regime can be expressed as[4]

$$P_b^\infty = \frac{2^q a \Psi\, \Gamma\left(q+\frac{3}{2}\right)}{\sqrt{\pi}} (b\bar{\gamma}_1)^{-(q+1)} + o\left(\bar{\gamma}_1^{-(q+2)}\right) \tag{19}$$

where $q = \min(N_t, N_r) - 1$, and

$$\Psi = \begin{cases} \frac{1}{N_t!}, & N_t < N_r \\ \frac{1}{N!}\left(1+\frac{1}{\mu^N}\right), & N_t = N_r = N \\ \frac{1}{\mu^{N_r} N_r!}, & N_t > N_r. \end{cases} \tag{20}$$

Equation (19) implies that the array gain G_a and the diversity gain G_d can be written as

$$G_a = b\left(\frac{2^q a\Psi\, \Gamma\left(q+\frac{3}{2}\right)}{\sqrt{\pi}}\right)^{-\frac{1}{q+1}} \tag{21}$$

$$G_d = \min(N_t, N_r). \tag{22}$$

[3]We have defined $f(x) = o(g(x)), x \to x_0$, if $\lim_{x\to x_0}(f(x)/g(x)) = 0$.

[4]A high-SNR average-symbol-error-rate approximation has also been presented in [8, eq. (18)]. However, our expression is much simpler and clearly shows how network parameters affect the system's error performance at high SNR.

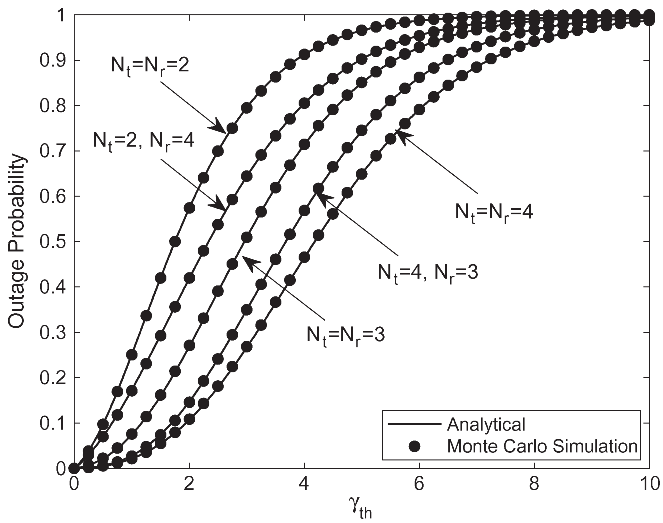

Fig. 1. Outage probability for various antenna configurations. $f_dT_d = 0.1$, $\bar{\gamma}_1 = 3$ dB, and $\bar{\gamma}_2 = 7$ dB.

From (22), we can infer that beamforming with no feedback delay can achieve the maximum possible diversity order of this system [8].

Now consider the case of $\rho_d < 1$. The average BER in the high-SNR regime is given by

$$P_b^{\infty} = \frac{a\psi}{2b\bar{\gamma}_1} + o\left(\bar{\gamma}_1^{-2}\right) \tag{23}$$

where

$$\psi = \begin{cases} \left((1-|\rho_d|^2)^{N_t-1} + \frac{1}{\mu}\right), & N_r = 1 \\ \left((1-|\rho_d|^2)^{N_t-1}\right), & N_r > 1. \end{cases} \tag{24}$$

Equation (23) implies that the G_a and G_d of the system with feedback delay can be written as $G_a = (2b/a\psi)$ and $G_d = 1$, respectively.

IV. Numerical and Simulation Results

In this section, we confirm the derived analytical results through comparison with Monte Carlo simulations and discuss the impact of feedback delay on system's performance. We assume Jake's fading spectrum, and in the examples, $\rho_d = 0.903, 0.642,$ and 0.472 correspond to $f_dT_d = 0.1, 0.2,$ and 0.3, respectively.

Fig. 1 shows the outage probability for various antenna configurations. We see that the analytical results obtained using (11) exactly match the simulations. As expected, the outage probability is significantly improved as the number of antenna increases. Thus, deployment of multiple antennas improve the performance of this network.

Fig. 2 shows the outage probability for various feedback delays. We see that a large feedback delay can significantly degrade system performance. When the feedback delay is large, the system's outage performance is mostly determined by low SNR thresholds. The outage curves shift to high SNR thresholds when the feedback delay tends to zero, indicating that the system performance improves for small feedback delays.

Fig. 3 shows the outage probability using source and relay power allocations β using our analytical results in (11). In particular, we set $\bar{\gamma}_1 + \bar{\gamma}_2 = \bar{\gamma}$ and have $\bar{\gamma}_1 = \beta\bar{\gamma}$ and $\bar{\gamma}_2 = (1-\beta)\bar{\gamma}$. As expected, with no feedback delay ($\rho_d = 1$), equal power allocation ($\beta = 0.5$) is

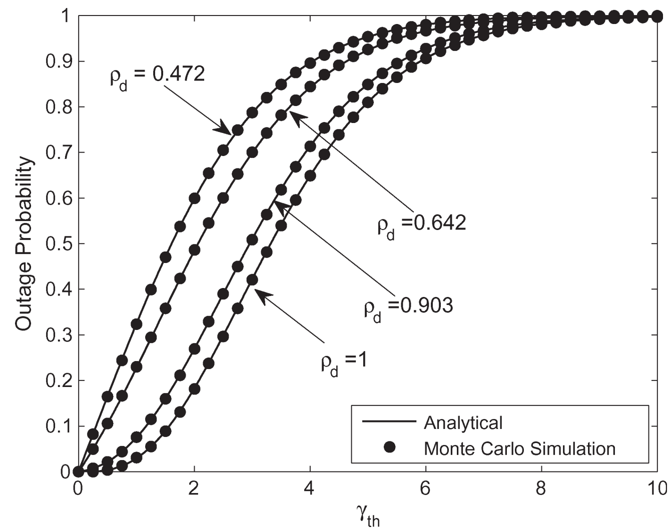

Fig. 2. Outage probability for different ρ_d's. $N_t = N_r = 3$, $\bar{\gamma}_1 = 3$ dB, and $\bar{\gamma}_2 = 7$ dB.

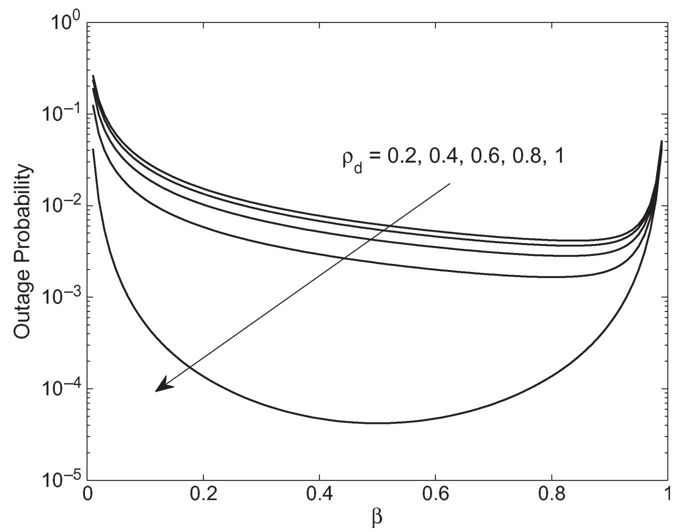

Fig. 3. Outage probability for different β's with $N_t = N_r = 2$, $\gamma_{\mathrm{th}} = -5$ dB, and $\bar{\gamma} = 20$ dB.

optimal, i.e., yields the lowest outage probability. This observation is expected since, for the considered antenna configuration (equal number of antennas at the source and the destination), the network is symmetrical with respect to $S-R$ and $R-D$ link SNRs. However, interestingly, when there is a feedback delay, $\beta = 0.5$ yields a suboptimal result. The optimal β is close to 0.9. The optimal β slightly changes for different ρ_d's. In the presence of feedback delays, the $S-R$ and $R-D$ links are not symmetrical. The $S-R$ link is weaker, compared with the $R-D$ link, and becomes the bottleneck for the network's performance. Therefore, to improve the performance, more power must be allocated to the $S-R$ link. For large feedback delays ($\rho_d = 0.2-0.6$), the outage probability is barely affected by the variations of β. For example, when $\rho_d = 0.2$, the outage probabilities at $\beta = 0.1$ and 0.9 are approximately 3×10^{-2} and 4×10^{-3}, respectively. However, when the feedback delay is decreased ($\rho_d = 0.8 \rightarrow 1$), the situation is significantly improved. When a large feedback delay exists, the $S-R$ link is severely weakened, and although allocating more power to the $S-R$ link improves the system's outage probability, it does so marginally. Fig. 3 also shows that, for the given β's in the range of 0.1–0.9, when a large feedback delay exists ($\rho_d = 0.2-0.6$), only

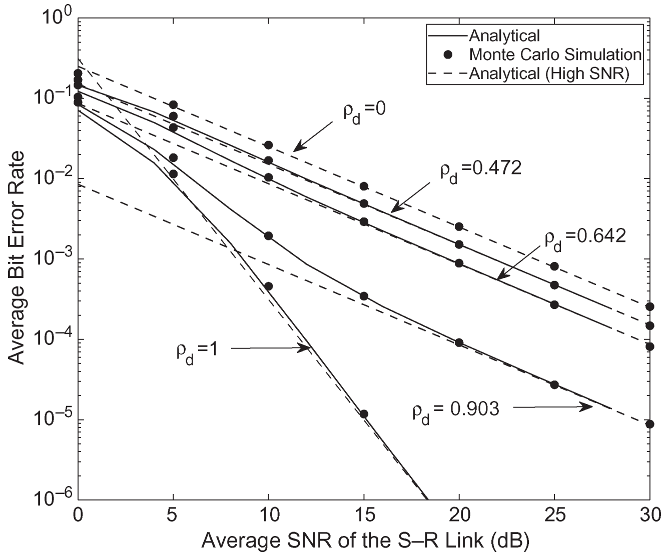

Fig. 4. Average BER using BPSK modulation with $N_t = N_r = 3$.

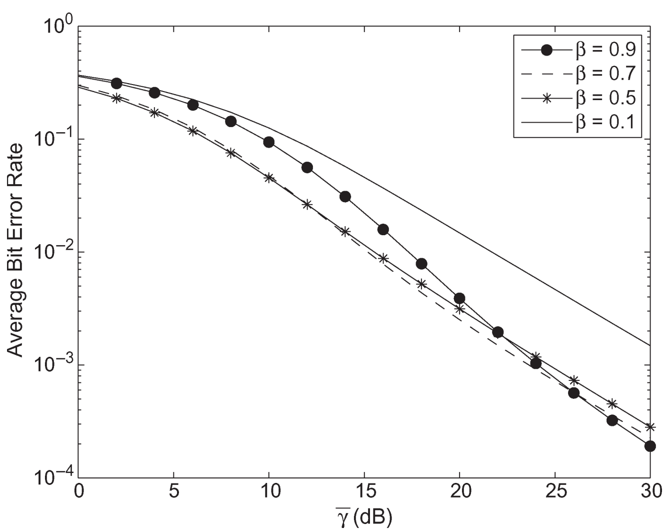

Fig. 5. Average BER using BPSK modulation with $N_t = N_r = 2$. $\rho_d = 0.642 (f_d T_d = 0.2)$.

a marginal improvement in the outage probability can be observed. As such, efficient feedback schemes to counteract the effect of feedback delay must be developed.

Fig. 4 shows the average BER of binary phase-shift keying (BPSK) modulation for various feedback delays. We have also plotted the simple approximations presented in (19) and (23) to illustrate their utility to compare the system's performance at high SNR. We see that the analytical results obtained from (17) complement Monte Carlo simulations in most of the SNR regions. Analytical results are accurate for all but very low SNRs, i.e., $\bar{\gamma}_1 > 5$ dB. Fig. 4 shows that feedback delays introduce a significant amount of BER loss. This can be seen by comparing the ideal case without feedback delay against feedback delay scenarios. In the ideal case, the system enjoys a third-order diversity gain, whereas with feedback delay, no diversity gains can be achieved. For example, a feedback delay of $f_d T_d = 0.1$ causes an SNR loss of approximately 15 dB at a BER of 10^{-5}. Higher feedback delays further degrade the performance, as can be seen from cases $f_d T_d = 0.1$ and 0.2.

Finally, in Fig. 5, we show the impact of power allocation on the average BER performance. Again, for the considered antenna configuration, the results are different from the case of no feedback delay in two ways: First, we see that equal power allocation ($\beta = 0.5$) does not provide the minimum average BER. Second, depending on $\bar{\gamma}$, different β values give the lowest average BER. Results not provided for the case of no feedback delay showed that $\beta = 0.5$ gives the lowest BER for all $\bar{\gamma}$. Therefore, the impact of feedback delay must be accounted for in the system design phase of AF relay networks. Since feedback delay is unavoidable in some practical cases, it must be kept at a very low level so that the performance of AF relay networks with beamforming is not degraded.

V. Conclusion

Beamforming performance of AF relay networks under perfect conditions has been well studied in the available literature. In contrast, we have investigated the effect of outdated CSI due to feedback delay on the performance of a two-hop AF relay network with beamforming. We have derived new expressions for the system's outage probability and the average BER. Accurate outage probability and average BER approximations parameterized by the number of transmit antennas, the number of receive antennas, and the normalized correlation coefficient ρ_d were also derived to quantitate the diversity and array gain at the high SNR region. It was shown that feedback delay is highly decremental to the system's performance since no diversity advantage can be achieved. Investigations into source and relay power allocation revealed that the optimal solution that minimizes the outage probability with feedback delay does not follow the equal power allocation strategy, as in the case with no feedback delay.

Appendix
Outage Probability at High Signal-to-Noise Ratio

In this Appendix, we obtain a first-order expansion expression for $F_{\gamma_{\mathrm{eq1}}}(\gamma_{\mathrm{th}})$. To do so, the following lemma is useful.

Lemma 1: The cdf of $\gamma_{\mathrm{eq2}} = \min(\gamma_1, \gamma_2)$ has the same first nonzero Taylor series coefficients around zero as the cdf of γ_{eq1}.

Proof: The proof for the case of $c = 0$ is given in [5, Appendix]. The proof for $c = 1$ follows similar steps and is omitted. ■

To proceed, we note that the cdf of γ_{eq2} can be written as

$$F_{\gamma_{\mathrm{eq2}}}(\gamma_{\mathrm{th}}) = 1 - \left(1 - F_{\gamma_1}(\gamma_{\mathrm{th}})\right)\left(1 - F_{\gamma_2}(\gamma_{\mathrm{th}})\right). \tag{25}$$

The cdf of γ_1 can be derived using (9) as

$$F_{\gamma_1}(x) = 1 - \left(|\rho_d|^2\right)^{N_t - 1} e^{-\frac{x}{\bar{\gamma}_1}} \times \sum_{i=0}^{N_t-1} \binom{N_t - 1}{i} \left(\frac{1 - |\rho_d|^2}{|\rho_d|^2}\right)^i \sum_{j=0}^{N_t-i-1} \frac{\left(\frac{x}{\bar{\gamma}_1}\right)^j}{j!}. \tag{26}$$

Therefore, $F_{\gamma_{\mathrm{eq2}}}(\gamma_{\mathrm{th}})$ can be written as

$$F_{\gamma_{\mathrm{eq2}}}(\gamma_{\mathrm{th}}) = 1 - \left(\left(|\rho_d|^2\right)^{N_t - 1} e^{-\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_1}} \sum_{i=0}^{N_t-1} \binom{N_t - 1}{i} \left(\frac{1 - |\rho_d|^2}{|\rho_d|^2}\right)^i \times \sum_{j=0}^{N_t-i-1} \frac{\left(\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_1}\right)^j}{j!} \right) \left(e^{-\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_2}} \sum_{k=0}^{N_r-1} \frac{\left(\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_2}\right)^k}{k!} \right). \tag{27}$$

Now, we consider several cases of interest: 1) no feedback delay ($\rho_d = 1$). $F_{\gamma_{\text{eq2}}}(\gamma_{\text{th}})$ can be simplified as

$$F_{\gamma_{\text{eq2}}}(\gamma_{\text{th}}) = 1 - \left(e^{-\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}} \sum_{j=0}^{N_t-1} \frac{\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^j}{j!}\right) \times \left(e^{-\frac{\gamma_{\text{th}}}{\bar{\gamma}_2}} \sum_{k=0}^{N_r-1} \frac{\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_2}\right)^k}{k!}\right). \quad (28)$$

Substituting $\bar{\gamma}_2 = \mu\bar{\gamma}_1$ and $x = (\gamma_{\text{th}}/\bar{\gamma}_1)$, we write

$$F_{\gamma_{\text{eq2}}}(\gamma_{\text{th}}) = F_X(x) = 1 - e^{-\left(1+\frac{1}{\mu}\right)x} \left(\sum_{j=0}^{N_t-1} \frac{x^j}{j!}\right) \times \left(\sum_{k=0}^{N_r-1} \frac{\left(\frac{x}{\mu}\right)^k}{k!}\right). \quad (29)$$

We see that the behavior of $F_{\gamma_{\text{eq2}}}(\gamma_{\text{th}})$ for large $\bar{\gamma}_1$ and $\bar{\gamma}_2$ is equivalent to the behavior of $F_X(x)$ around $x = 0$. Using the McLaurin series expansion for the exponential function in (29) yields

$$\begin{aligned} F_X(x) &= 1 - \left(\sum_{\ell=0}^{\infty} \frac{(-x)^\ell}{\ell!} \sum_{j=0}^{N_t-1} \frac{x^j}{j!}\right) \times \left(\sum_{m=0}^{\infty} \frac{\left(-\frac{x}{\mu}\right)^m}{m!} \sum_{k=0}^{N_r-1} \frac{\left(\frac{x}{\mu}\right)^k}{k!}\right) \\ &= 1 - \left(1 - \frac{x^{N_t}}{N_t!} + o(x^{N_t+1})\right) \times \left(1 - \frac{x^{N_r}}{\mu^{N_r} N_r!} + o(x^{N_r+1})\right). \end{aligned} \quad (30)$$

If $N_t < N_r$, we have

$$F_X(x) = \frac{1}{N_t!} x^{N_t} + o(x^{N_t+1}). \quad (31)$$

Now, if $N_t > N_r$, we have

$$F_X(x) = \frac{1}{\mu^{N_r} N_r!} x^{N_r} + o(x^{N_r+1}) \quad (32)$$

and if $N_t = N_r = N$, we have

$$F_X(x) = \frac{\left(1 + \frac{1}{\mu^N}\right)}{N!} x^N + o(x^{N+1}). \quad (33)$$

Now, consider the case of arbitrary feedback delay, i.e., $\rho_d < 1$; we can express $F_X(x)$ as

$$F_X(x) = 1 - \left(\left(|\rho_d|^2\right)^{N_t-1} e^{-x} \sum_{i=0}^{N_t-1} \binom{N_t-1}{i} \left(\frac{1-|\rho_d|^2}{|\rho_d|^2}\right)^i \times \sum_{j=0}^{N_t-i-1} \frac{x^j}{j!}\right) \left(e^{-\frac{x}{\mu}} \sum_{k=0}^{N_r-1} \frac{\left(\frac{x}{\mu}\right)^k}{k!}\right). \quad (34)$$

Collecting only the first-order terms yields

$$\begin{aligned} F_X(x) = 1 &- \left(|\rho_d|^2\right)^{N_t-1} \left(1 - x + o(x^2)\right) \times \left(\sum_{i=0}^{N_t-1} \binom{N_t-1}{i} \left(\frac{1-|\rho_d|^2}{|\rho_d|^2}\right)^i + \sum_{i=0}^{N_t-2} \binom{N_t-1}{i} \left(\frac{1-|\rho_d|^2}{|\rho_d|^2}\right)^i x + o(x^2)\right) \\ &\times \left(1 - \frac{x^{N_r}}{\mu^{N_r} N_r!} + o(x^{N_r+1})\right). \end{aligned} \quad (35)$$

Simplifying (35), $F_X(x)$ can be expressed as

$$F_X(x) = 1 - \left(1 - \left(1 - |\rho_d|^2\right)^{N_t-1} x + o(x^2)\right) \times \left(1 - \frac{x^{N_r}}{\mu^{N_r} N_r!} + o(x^{N_r+1})\right). \quad (36)$$

Now, for $N_r = 1$, we get

$$F_Y(x) = \left(\left(1 - |\rho_d|^2\right)^{N_t-1} + \frac{1}{\mu}\right) x + o(x^2) \quad (37)$$

and for $N_r > 1$, $F_X(x)$ can be simplified as

$$F_X(x) = \left(\left(1 - |\rho_d|^2\right)^{N_t-1}\right) x + o(x^2). \quad (38)$$

Now, by substituting $x = (\gamma_{\text{th}}/\bar{\gamma}_1)$ into (31)–(33), (37), and (38) and using Lemma 1, the desired results can be obtained.

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Spatial Fading Correlation for Local Scattering: A Condition of Angular Distribution

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***Abstract*—Local scattering in the vicinity of the receiver or the transmitter leads to the formation of a large number of multipath components along different spatial angles. A condition of angular distribution, which is valid for only a uniform linear array, is proposed in this paper to justify whether the spatial fading correlation (SFC) remains simple as a Bessel function. If an angular distribution satisfies the condition, a class of angular distributions is revealed and results in simplifying the analysis of the SFC. To demonstrate its practical use, we apply the condition to several angular distributions that are considered in previous works. It is found that cosine and von Mises distributions follow the condition, whereas uniform, Gaussian, and Laplacian distributions do not satisfy the condition, and then, one needs to calculate the sinusoidal coefficients in the SFC computation.**

***Index Terms*—Antenna array, local scattering, spatial fading correlation (SFC).**

I. Introduction

In wireless communications, local scattering around the transmitter or the receiver leads to the formation of a large number of multipath components. In multiantenna communications systems, the receiver features the correlation among the impulse responses of different pairs of antenna elements, namely, spatial fading correlation (SFC), as an impact on link quality.

The SFC plays an important role in the performance analysis of a wireless communications system because most of the performance metrics, e.g., bit error probability [1]–[3] and channel capacity [4]–[6], depend on it. Therefore, several works pay attention to the SFC of antenna arrays [7]–[15]. In [16] and [17], the SFC of a circular array is derived for uniform, cosine, and Gaussian angular distributions, respectively, which are chosen as the candidates for fitting several measurement results. However, the direct computation of the SFC requires extensive integrations, which are complicated and possibly infeasible for a complex angular distribution.

In this paper, a condition of the angular distributions is proposed to justify whether the SFC from a linear antenna array remains simple as a Bessel function [18, Ch. 1]. For a class of the angular distributions, the test of the condition requires only differentiations, which are simpler than the integrations that are required in the direct method. This can help facilitate the analysis of the fading correlation in the wireless channels. It is discovered that the proposed condition defines a group of the angular distributions (see various families of the distributions in [19, Ch. 7] and [20, Ch. 5]). It means that, if an angular distribution satisfies the condition, the SFC remains only a simple form for the calculation. To demonstrate its usage in practice, we apply the condition to several angular distributions that are considered in previous works. It is found that the cosine and von Mises distributions follow the condition, whereas the uniform, Gaussian, and Laplacian distributions do not satisfy the condition, and then, one needs to calculate the sinusoidal coefficients in the SFC computation.

The gap between this paper and the previous works can be seen as follows: The SFC in various environments is studied in [7]–[15]. The effects of the SFC on the system performance are investigated in [1]–[6]. In [21], [22, Sec. 2.2.2], and [23], tedious calculation is avoided by approximating the SFC for a small angular spread. The contributions of this paper are summarized here.

1) A condition is proposed to test the angular distribution of local scattering in the vicinity of the transmitter or the receiver, which can happen in the multipath channels that take into account the spatial angle observed by a uniform linear array.
2) The test requires only differentiations instead of integrations. For a class of the angular distributions, the solution of the SFC is known *a priori* as a simple form of the zeroth-order Bessel function. The proposed condition can be applied to any angular distribution, provided that the uniform linear array is taken into account.

For another class of the angular distributions that do not satisfy the proposed condition, we also provide the derivation of the sinusoidal coefficients for $\phi \in (-\pi, \pi]$. Although we consider in this paper the azimuth plane $\phi \in (-\pi, \pi]$ [18], the scattering over the half-circle $\phi \in (-(1/2)\pi, (1/2)\pi]$ [24], as well as the 3-D scattering, can be extended in a straightforward treatment based on our derivation idea.

Some mathematical notations are involved as follows: $E_\phi\{\cdot\}$ is the expectation with respect to ϕ, whose probability density function (pdf) is $p_\phi(\phi)$. $J_0(\cdot)$ and $J_k(\cdot)$ are the zeroth- and kth-order Bessel functions of the first kind. $I_0(\cdot)$ and $I_k(\cdot)$ are the zeroth- and kth-order modified Bessel functions of the first kind. The error function $\mathrm{erf}(z)$ is defined as $\mathrm{erf}(z) = (2/\sqrt{\pi}) \int_0^z e^{-u^2} du$. $(\cdot)^*$ is the conjugate of a complex argument $\cdot$. $\Re(\cdot)$ and $\Im(\cdot)$ are the real and imaginary components, respectively. $\lfloor \cdot \rfloor$ denotes the integer part of a variable $\cdot$.

II. Spatial Correlation

For the uniform linear array, the time delay at the nth antenna element is given by

$$\psi_n = \frac{1}{c} d(n-1) \sin(\phi) \tag{1}$$

where c is the wave propagation speed, which is herein equivalent to the speed of light; d is the distance between adjacent antenna elements; and ϕ is the direction of the emitting or incoming ray, which is measured from the perpendicular axis of the array. The received signal

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