Generalised approach for evaluation of outage performance in micro- and pico-cellular networks

G.K. Karagiannidis, S.A. Kotsopoulos and P.T. Mathiopoulos

Abstract: Outage analysis is one of the primary objectives in the design and operation of the current generations of cellular mobile communications systems, in order to increase spectrum efficiency and meet the quality of service and the grade of service demands. A general and unified semi-analytical approach is presented for the direct evaluation of the outage probability in the presence of L mutually independent co-channel interferers for micro- and pico-cellular mobile radio environments that follow Rice or Nakagami models. Outage probability is evaluated in a nested mode via the Laguerre numerical integration technique, avoiding the calculation of complex functions. The proposed formulation can be efficiently applied to practical wireless applications with arbitrary statistical characteristics for the modelling parameters, in both the case of an interference-limited environment (no minimum power constraint) and the existence of a minimum power constraint. Comments, comparison with other existing techniques and useful curves are also presented.

1 Introduction

Radio-frequency interference is one of the most important technical challenges that need to be considered in the design, operation and maintenance of current and future mobile cellular networks. It is recognised that the most important of all the interferences that need to be considered by the system designers in cellular planning is co-channel interference (CCI). In this case, an important parameter as far as the system's performance is concerned is the outage probability (OUP). It is well known that OUP is related to the failure of achieving adequate reception of the transmitting signal due to CCI [1, 2].

The evaluation of the OUP in mobile networks depends on the various statistical models, such as lognormal, Rayleigh, Rice and Nakagami, which have been used to describe the mobile radio channel [1].

Rice distribution contains Rayleigh distribution as a special case and provides the optimum fits to collected data in indoor mobile radio environments [3–5]. In this paper, the Rayleigh model is not studied as a separate situation, but as a special case of the Rice one. Effective techniques have been developed to determine the OUP for the Rician fading environment. Yao and Sheikh [6, 7] have presented a closed form for the probability distribution function (pdf) of the signal-to-interference ratio (SIR), but it is limited to the case of a Rician desired signal among L Rayleigh CCIs. A solution to the same problem has been presented by Wijk *et al.* [3]. This is a useful approach since each of the CCIs introduces its own local mean power (LMP) to the extracted expression. Muammar [8] has presented an expression for the OUP in the Rice environment, but this expression contains infinite series. The most general approach was presented by Tjhung *et al.* [9, 10]. According to this, a closed form was found for different values of the Rice factor K applying Turin's [11] and Bello's [12] results for the characteristic function of N complex Gaussian variables and the inverse Fourier transform of the pdf of the sum of L Rice CCIs.

Nakagami fading (m-distribution) [13] describes multipath scattering with relatively large delay-time spreads, with different clusters of reflected waves. Sometimes, the Nakagami model is used to approximate the Rician distribution [14]. Although this may be true for the main body of the pdf, it becomes highly inaccurate for the tails. As bit errors or outage mainly occur during deep fades, the tail of the pdf determines these performance measures. F-distribution can be used [15] to evaluate the OUP in a case with a single interferer, and a closed form is extracted for the OUP in the case of multiple Nakagami CCIs [16] with a restriction of integer fading parameters. Zhang presented a more general approach in [17, 18]; it was the first time that the outage problem was solved for arbitrary parameters, with a formulation that contains only one integral. An alternative formulation of the outage probability for arbitrary parameters [19] and a comparison [18] have also been presented.

Since the situation of a Rice desired signal among L Rayleigh CCIs (Rayleigh/Rice) can be taken as a special case of the Rice/Rice one, it is very interesting to examine the outage performance in a Nakagami/Rice environment. Lin *et al.* [20] presented an approach for the calculation of OUP in a Nakagami/Rice environment. However, only integer values for the m Nakagami parameters are assumed. Several methods have been introduced for the calculation of OUP in situations with shadowing (lognormal) phenomena, which affect the desired signal or the CCIs [9, 19, 21–25].

In this paper, a generalised and unified semi-analytical formulation for the direct evaluation of the OUP (DEOUP) is presented, assuming that all the involved CCIs are statistically independent. This proposed approach can be used for any involved statistical characteristics of both the desired signal and the CCIs, by considering arbitrary values

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for the mobile radio environment parameters. Moreover, it can be also used for shadowing (lognormal) phenomena, which affect the desired signal or the interferers.

2 Generalised direct evaluation of outage probability (DEOUP)

Let us assume a mobile cellular environment with a desired signal among L CCIs with signal's powers x_i (LMP or instantaneous) which follow an exponential-type pdf, $f_i(x_i)$, given in the form of

$$f_i(x_i) = H_i(x_i)\exp[-G_i(x_i)] \tag{1}$$

Furthermore, it is assumed that the desired signal's power x_0 follows a pdf, which has a cumulative distribution function (cdf), $F_0(x)$. The OUP in an interference-limited environment (only co-channel interference is dominated), denoted as P^I_{OUP}, is then given by

$$P^I_{OUP} = \frac{I}{\beta^L}\sum_{i=1}^{v} w_i \sum_{j=1}^{v} w_j \cdot\cdot \sum_{n=1}^{v} w_n \prod_{t=i,j...,n} H_i\left(G_i^{-1}(x_t)\right) F_0\left[\sum_{t=i,j,...,n} \beta G_i^{-1}(x_t)\right] \prod_{t=i,j,...,n} \beta \frac{d\left(G_i^{-1}(x_t)\right)}{dx_t} \tag{2}$$

w_t, x_t, v are the weight factors, the abscissas and the order of the Laguerre numerical integration method [4], respectively; β is the protection ratio, defined as the ratio of the power of the desired signal to the sum of the powers of the CCIs; and $G_i^{-1}(x)$ is the inverse function of $G_i(x)$. The proof of (2) is given in the Appendix.

In the case of the existence of a minimum signal power constraint ψ, the OUP denoted as P^{II}_{OUP} is given by

$$P^{II}_{OUP} = F_0(\psi) + P^I_{OUP} - F_0(\psi)P^I_{OUP} \tag{3}$$

P^I_{OUP} is the OUP with nominimum power constraint given by (2). The proof of (3) is also given in the Appendix.

3 Outage probability analysis in presence of *L* Rician interferers

The Rice pdf for the fast varying instantaneous power p is described by [26, 27]

$$f_{RICE}(p) = \frac{1}{\sigma^2} I_0\left(\frac{2\sqrt{Kp}}{\sigma}\right) \exp(-K)\exp\left(-\frac{p}{\sigma^2}\right) \tag{4}$$

where p is the signal's instantaneous power; I_0 is the zero-order modified Bessel function of the first kind; K is the Rice factor; and $2\sigma^2$ is the scattered power. When K is zero, the channel statistic becomes Rayleigh, whereas if K is infinite, the channel is Gaussian. Values of K in indoor pico-cellular systems usually range from 0 to 7 [3, 5]. The Rice cdf $F_{RICE}(x)$ can be easily derived from (4) and has the form [27]

$$F_{RICE}(x) = 1 - Q\left(\sqrt{2K} \cdot \frac{\sqrt{2x}}{\sigma}\right) \tag{5}$$

where $Q(a,b)$ is the Markum Q function [26].

Using (2) with

$$H_i(x_i) = \exp(-K_i)\frac{1}{\sigma_i^2} I_0\left(\frac{2\sqrt{K_i x_i}}{\sigma_i}\right), G_i(x_i) = \frac{x_i}{\sigma_i^2}$$

and, after some manipulations, $P^I_{OUP\_RICE}$ is found to be

$$P^I_{OUP\_RICE} = \exp\left(\sum_{i=1}^{L} -K_i\right) \sum_{i=1}^{\nu} w_i \sum_{j=1}^{\nu} w_j \cdot\cdot \sum_{n=l}^{\nu} w_n F_0\left(\sum_{i=1}^{L} \sigma_i^2 \beta x_i\right) \left(\prod_{i=1}^{L} I_0\left(2\sqrt{K_i x_i}\right)\right) \tag{6}$$

$F_0(x)$ is the Rician cdf of the desired signal given by (5); and $2\sigma_i^2$ and K_i are the scattered power and the Rice factor of the ith interferer, respectively.

3.1 Numerical results and discussion

Equation (6) is used to evaluate the OUP for several values of K, σ and β common in the cellular indoor radio systems. *SIR* is defined here as

$$SIR\ (\text{dB}) = 10\log_{10} \frac{\sigma_0^2(1+K_0)}{\sum_{i=1}^{L} \sigma_i^2(1+K_i)} \tag{7}$$

where $2\sigma_0^2$ and K_0 are the scattered power and the Rice factor of the desired signal, respectively.

First, an indoor mobile radio environment is considered with a Rician desired signal ($K_0 = 5.8$) among three Rayleigh interferers with distinct LMPs, $P_{01} = 0.25$, $P_{02} = 0.1$ and $P_{03} = 0.3$. The protection ratio β is selected to be 15 dB. Table 1 shows the results for the OUP by using Wijk *et al.*'s [3] and DEOUP techniques. It can be seen that the numerical results differ by less than 0.01 for small values of *SIR* and by less than 0.001 for higher values of *SIR*. In Table 1 the DEOUP and Tjhungs *et al.*'s techniques are also compared [9] for the general case of a Rice desired signal among L Rice CCIs. Moreover, in Table 1 a comparison is made between the DEOUP and Yao and Sheikh's techniques [6]. The results confirmed that Yao and Sheikh's, Tjhung *et al.*'s and the DEOUP techniques yield accurate results with a difference of less than 0.01. Note that Tjhung *et al.*'s technique cannot be used when all the

Table 1: Outage probabilities $P^I_{OUT\_RICE}$ evaluated by several proposed techniques

SIR (dB)	$P_{01} = 0.25$, $P_{02} = 0.1$, $P_{03} = 0.3$, $K_0 = 5.8$, $\beta = 15$ dB		$L = 3$, $K_1 = 4.8$, $K_2 = 5$, $K_3 = 5.6$, $K_0 = 5.8$, $\beta = 15$ dB		$L = 3$, $K_1 = K_2 = K_3 = 0$, $K_0 = 5.8$, $\beta = 18$ dB	
	DEOUP	[3]	DEOUP	[9]	DEOUP	[6], [9]
0	0.999045	0.999680	0.995457	1.000000	0.999045	0.999757
5	0.993809	0.992566	0.995328	0.999944	0.993884	0.994044
10	0.896937	0.897941	0.967613	0.979657	0.886953	0.909122
15	0.478077	0.478665	0.520590	0.530641	0.469239	0.487206
20	0.087178	0.087303	0.072741	0.071077	0.089081	0.085005
25	0.008759	0.008772	0.007104	0.006951	0.009137	0.008430
30	0.001189	0.001190	0.001069	0.001061	0.001216	0.00116619

interferers have distinct LMPs and all of them are Rayleigh (i.e. $K_i = 0$ for every i). In Fig. 1 the OUP is calculated and presented using the DEOUP method, with and without a constraint in minimum signal power for several values of the K_0. The solid lines in Fig. 1 denote the case with no minimum constraint, and the dotted lines denote the case with a constraint $\psi = -15\,\text{dB}\ P_d$; P_d is the total power (scattered plus LOS) of the desired signal given as $P_d = 2\sigma_0^2(1 + K_0)$. The number of interferers is three, with the Rice factor taking values measured in experiments in a multistorey building [3]. These values are $K_0 = [5.8, 3.5, 1.5]$, $K_1 = 0$, $K_2 = 1.2$, $K_3 = 2.4$, $P_{01} = 0.25$, $P_{02} = 0.02$, $P_{03} = 1.22$, and the protection ratio $\beta = 15\,\text{dB}$. The Rice factor of the desired signal is shown to have a large effect on the probability of outage. This is the same result as in a previous study [21] of the problem of Rician/Rayleigh with lognormal shadowing.

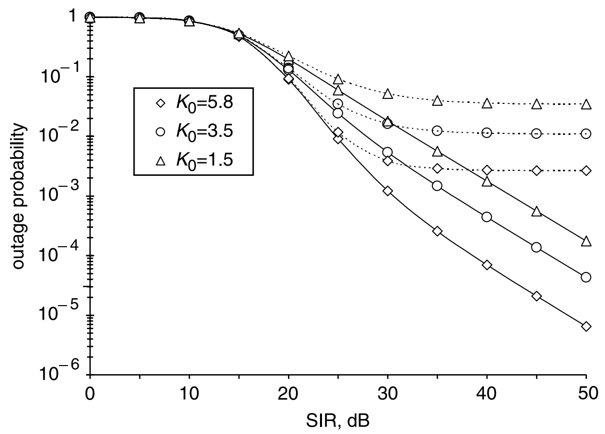

Fig. 1 *Outage probabilities $P^{I}_{OUT\_RICE}$ and $P^{II}_{OUT\_RICE}$ evaluated using DEOUP technique for several values of K_0*

4 Outage probability due to Nakagami signals

The instantaneous power ξ of a Nakagami variable is Gamma distributed with pdf given by [13]

$$f_{NAK}(\xi) = \left(\frac{m}{\Omega}\right)^m \frac{\xi^{m-1}}{\Gamma(m)} \exp\left(-\frac{m}{\Omega}\xi\right) \tag{8}$$

where $\Gamma(x)$ is the Gamma function; Ω represents the average signal power; and m is a severity parameter, which can take values from 0.5 through to infinity.

The Nakagami cdf can be easily found to be [27]

$$F_{NAK}(x) = P\left(m, \frac{m}{\Omega}x\right) \tag{9}$$

$P(x)$ is the well known incomplete Gamma function [28].

Applying (2) with

$$H_i(x_i) = \left(\frac{m_i}{\Omega_i}\right)^{m_i} \frac{x^{m_i-1}}{\Gamma(m_i)},\ G_i(x_i) = \frac{m_i}{\Omega_i}x_i$$

and, after some manipulations, the $P^{I}_{OUP\_NAK}$ is found to be [22]

$$P^{I}_{OUP\_NAK} = \frac{1}{\prod_{i=1}^{L} \Phi(m_i)} \sum_{i=1}^{\nu} w_i \sum_{j=1}^{\nu} w_j \cdots \sum_{n=1}^{\nu} w_n \left(\prod_{t=i,j,\ldots,n} x_t^{m_t-1}\right) F_0\left(\sum_{t=i,j,\ldots,n} \frac{\beta\Omega_t}{m_t} x_t\right) \tag{10}$$

$F_0(x)$ is the Nakagami cdf of the desired signal given by (9); and m_i, Ω_i are the Nakagami parameters of the ith CCI.

4.1 Results and discussion

The key feature of Zhang's method is the numerical calculation of the integrand in ([18], eqn. 7). However, such formulas, when evaluated by numerical integration (the selected technique here is the piece-wisely Gaussian quadrature), have the form of 0.5 plus or minus a sum [29]. When the tails of the distribution are sought, that sum is close to ± 0.5, and many steps of numerical integration of the oscillatory integrand of ([18], eqn. 25) are needed to determine the sum accurately enough.

To compare the method proposed in this paper to Zhang's, as far as the consumption of time and the accuracy are concerned, several OUP numerical results are shown in Table 2 for six and three Nakagami interferers. The observed calculation time T is also depicted. The calculations were performed on a Pentium II (333 MHz) PC with the use of Mathcad 2000 software. As we can see in Table 2, the calculation time using the proposed technique is higher than the mean time of Zhang's method for six interferers, and it is quicker in the case of three interferers. Moreover,

Table 2: Outage probabilities $P^{I}_{OUT\_NAKAGAMI}$ in the presence of six and three Nakagami interferers, using Zhang [18] and DEOUP methods

m_0	$L = 6$				$L = 3$			
	Zhang [18]		DEOUP		Zhang [18]		DEOUP	
	T	P^{I}_{OUT}	T	P^{I}_{OUT}	T	P^{I}_{OUT}	T	P^{I}_{OUT}
SIR/β = 25 dB								
1	2	0.003156	18	0.003080	2	0.003156	$\ll 1$	0.003098
2	2	0.000021	18	0.00002	2	2.5×10^{-4}	$\ll 1$	0.000024
3	3	1.81×10^{-7}	18	0.0000001	1	2.72×10^{-8}	$\ll 1$	2.67×10^{-7}
4	2	9.6×10^{-10}	18	1.56×10^{-9}	1	8.95×10^{-10}	$\ll 1$	3.56×10^{-9}
SIR/β = 15 dB								
1	3	0.031092	18	0.030338	2	0.031002	$\ll 1$	0.030430
2	3	0.00205	18	0.002002	3	0.00237	$\ll 1$	0.002334
3	2	1.61×10^{-4}	18	0.000158	2	2.42×10^{-4}	$\ll 1$	0.000239
4	2	1.41×10^{-5}	18	0.000013	2	3×10^{-5}	$\ll 1$	0.000029

$m = [0.8, 1.2, 1.8, 2.2, 2.5, 4.9)$, $\Omega = [1.3, 1.8, 2.6, 3, 3.2, 6]$, $\beta = 18\,\text{dB}$

the time consumed using Zhang's technique is about the same for small and large number of interferers. Using the proposed method, the calculation time for four interferers was observed to be 0.8 s and the corresponding value for five interferers is 2.5 s. Hence, it is obvious that the proposed method offers an advantage as far as the calculation speed is concerned, especially for small numbers of interferers (less than four). Taking into consideration the accuracy of the computation, the two methods give slightly different results, due to the alternative ways in which each of them approximates numerically the OUP.

Fig. 2 depicts the $P^I_{OUP\_NAK}$ for two Nakagami interferers with parameters $m=[1.44, 0.85]$ and $\Omega=[5.5, 3.2]$ against the protection ratio β (dB) for several values of m_0. *SIR* is defined here as

$$SIR\ (\mathrm{dB}) = 10\log_{10}\frac{\Omega_0}{\sum_{i=1}^{L}\Omega_i} \tag{11}$$

As we can see from Fig. 2, the influence of the protection ratio to the outage performance is particularly important and depends on m_0 for low values of β. On the contrary, for high values of β, OUP increases and tends to be independent of m_0. This happens because the high demands in quality of service (QoS) (high values for β) dominate the improvement offered by the low fading (high values of m_0).

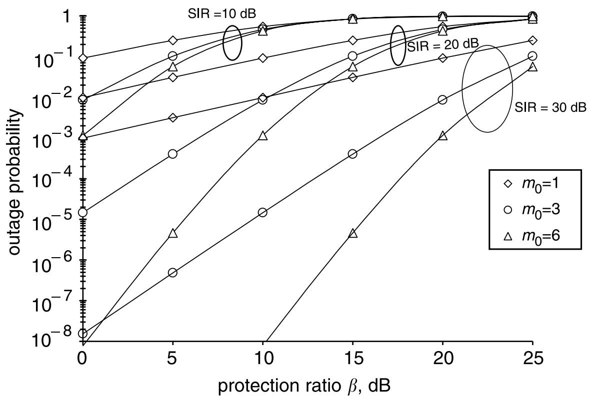

Fig. 2 *Outage probabilities $P^I_{OUT\_NAKAGAMI}$ against β evaluated using DEOUP technique with $m=[1.44, 0.85]$, $\Omega=[5.5, 3.2]$ and several values of SIR*

5 Outage probability in case of different statistical characteristics between desired signal and interferers

All the above-described radio environmental scenarios have a common assumption [6] that all the receiving signals, desired and CCIs, have the same statistical characteristics. However, for micro- and pico-cellular systems this is not true. For example in a micro-cellular environment a Rayleigh or a Nakagami pdf may model the distant co-channel interferers, since the appropriate modelling of the desired signal should be Rician. Therefore, in such a situation, different fading statistics characterise the desired and undesired signals. In a pico-cellular indoor environment, a Rician desired signal among L Nakagami interferers with arbitrary parameters seems to be the most realistic scenario.

5.1 Outage probability of Rician signal among L Nakagami interferers

Using (2) the expression for the P^I_{OUP} in a Nakagami/Rice environment with arbitrary parameters, both for the desired signal and the interferers, is given by [30]

$$P^I_{OUP\_NAK\_RICE} = \frac{1}{\prod_{i=1}^{L}\Gamma(m_i)}\sum_{i=1}^{\nu} w_i \sum_{j=1}^{\nu} w_j \cdots \sum_{n=1}^{\nu} w_n \left(\prod_{t=i,j,\ldots,n} x_t^{m_t-1}\right) F_0\left(\sum_{t=i,j,\ldots,n}\frac{\beta\Omega_t}{m_t}x_t\right) \tag{12}$$

$F_0(x)$ is the Rician cdf of the desired signal given by (5), and *SIR* is given here as

$$SIR\ (\mathrm{dB}) = 10\log_{10}\frac{2\sigma_0^2(1+K_0)}{\sum_{i=1}^{L}\Omega_i} \tag{13}$$

OUP is depicted in Fig. 3 as a function of *SIR* for several values of K_0. There are three Nakagami interferers with $m=[1.2, 2.7, 3.8]$, $\Omega=[5.4, 5.6, 6.3]$ and the protection ratio $\beta=15$ dB. The solid lines in Fig. 3 denote the case with no minimum constraint, and the dotted lines denote the case with a constraint $\psi=-15$ dB P_d. Note that the OUP decreases for high values of the Rice factor K_0. A slight change in K_0 leads to a significant change in outage performance, especially for large *SIR*. This happens because an increase in the Rice factor means that the desired signal contains a large LOS component and a small diffuse scattered component. Hence, the desired signal does not suffer from severe fading, which degrades the outage performance.

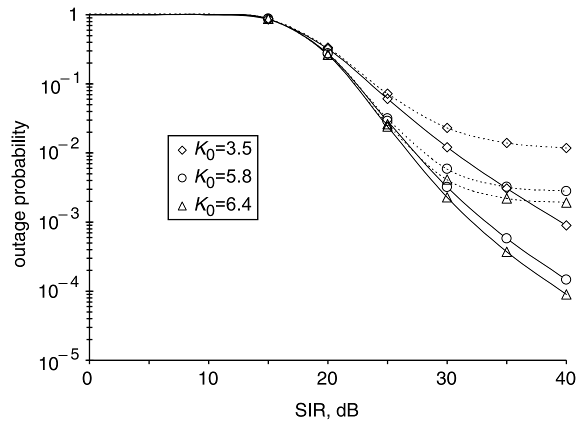

Fig. 3 *Outage probabilities $P^I_{OUT\_NAK\_RICE}$ and $P^{II}_{OUT\_NAK\_RICE}$ against SIR evaluated using the DEOUP technique with $m=[1.2, 2.7, 3.8]$, $\Omega=[5.4, 5.6, 6.3]$, $\beta=15$ dB and several values of K_0*

In Fig. 4, outage probabilities are depicted as a function of K_0 for several values of *SIR*. There are three interferers, with $m=[1.2, 2.7, 3.8]$, $\Omega=[5.4, 5.6, 6.3]$ and $\beta=15$ dB. We observe here that an increase of K_0 leads to an improvement of the outage performance, but this improvement is not important especially for short values in *SIR*. In a real indoor pico-cellular environment [3], the Rice factor belongs to a range from 1 to 7. In this range (as shown in Fig. 5), a small increase in *SIR* (5 dB) leads to a significant improvement of the outage performance (about one order), whereas an increase in K_0 does not improve this kind of performance equally. In Fig. 5, outage probability is shown in relation to the protection ratio β for three Nakagami

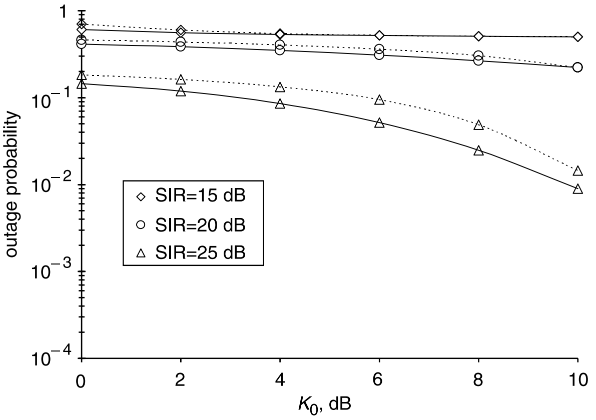

Fig. 4 *Outage probabilities $P^{I}_{OUT\_NAK\_RICE}$ and $P^{II}_{OUT\_NAK\_RICE}$ against K_0 evaluated using the DEOUP technique with $m = [1.2, 2.7, 3.8]$, $\Omega = [5.4, 5.6, 6.3]$, $\beta = 15\,dB$ and several values of SIR*

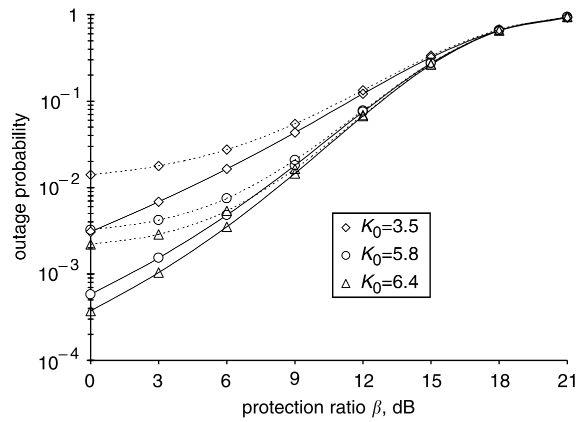

Fig. 5 *Outage probabilities $P^{I}_{OUT\_NAK\_RICE}$ and $P^{II}_{OUT\_NAK\_RICE}$ against β evaluated using DEOUP technique with $m = [1.2, 2.7, 3.8]$, $\Omega = [5.4, 5.6, 6.3]$, $SIR = 20\,dB$ and several values of K_0*

interferers with the parameters of Fig. 4 and several values of the Rice factor K_0 (3.5, 5.8, 6.4). As can be seen the influence of the protection ratio to the outage performance is particularly important and depends on K_0 for low values of β. On the contrary, for high values of β OUP increases and tends to be independent of K_0. This happens because the high demands in QoS (high values for β) dominate the improvement offered by the LOS communication (high values of K_0). In Figs. 4 and 5 the dotted denote the case with a constraint $\psi = -15\,dB\ P_d$.

6 Conclusions

We have presented a general and unified approach for the evaluation of the outage probability in micro- and pico-cellular mobile radio systems that involve Nakagami and Rice fading. The obtained formulation provides accuracy and speed, and can be used for both the case of an interference-limited environment (no minimum power constraint) and the existence of a minimum power constraint. Moreover, the proposed formulation can be efficiently applied to practical wireless applications with arbitrary values for the modelling parameters, and for situations with the same or different statistics between the desired signal and CCIs. Finally, it can also be used for pure shadowing (lognormal) phenomena, but analysis and the corresponding results of this case have not been presented here because of constraints on the length of the paper.

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8 Appendix

In this Appendix, the necessary mathematical analysis for the proof of (2) and (3) is presented. Let $\sum_{i=1}^{L} x_i$ be the sum of the powers of the L mutually independent co-channel interferers, which follow the exponential-type pdf of (1), and we define $w = x_0 - \beta \sum_{i=1}^{L} x_i$. The OUP in an interference-limited environment can then be expressed as

$$P_{OUP}^{I} = Probability(w < 0) \tag{14}$$

The pdf of the product $\beta\ x_i$ is given as [31]

$$f_{\beta xi}(x_i) = \frac{1}{\beta} f_i\left(\frac{x_i}{\beta}\right) = \frac{1}{\beta} H_i\left(\frac{x_i}{\beta}\right) \exp\left[-G_i\left(\frac{x_i}{\beta}\right)\right] \tag{15}$$

Let $\Phi_w(r)$, $\Phi_0(r)$, $\Phi_{\beta xi}(r)$ be the characteristic functions of the variables w, x_0 and $\beta\ x_i$, respectively. The $\Phi_w(r)$ can be expressed as

$$\Phi_w(r) = \Phi_0(r) \prod_{i=1}^{L} \int_0^{\infty} \exp(-jrx_i) f_{\beta x_i}(x_i) dx_i \tag{16}$$

Making the transformation $G_i(x_i/\beta) = r_i$, which gives $x_i = \beta G_i^{-1}(r_i)$, (16) can be written as

$$\Phi_w(r) = \frac{1}{\beta^L} \Phi_0(r)$$

$$\cdot \int_0^{\infty} \cdot\cdot \int_0^{\infty} \exp\left[-jr\left(\sum_{i=1}^{L} \beta G_i^{-1}(r_i)\right)\right] \left[\prod_{i=1}^{L} H_i\left[G_i^{-1}(r_i)\right]\right]$$

$$\times \exp\left(-\sum_{i=1}^{L} r_i\right) \prod_{i=1}^{L} \beta \frac{d[G_i^{-1}(r_i)]}{dr_i} dr_l \cdot \cdot dr_L \tag{17}$$

Using (14), we have

$$P_{OUP}^{I} = \int_{-\infty}^{0} f_w(\tau) d\tau$$

$$= \frac{1}{2\pi} \int_{-\infty}^{0} \int_{-\infty}^{\infty} \Phi_w(r) \exp(-jr\tau) dr d\tau \tag{18}$$

Now, using (17) and (18) and, taking into account the fact that

$$\int_{-\infty}^{\infty} \Phi_0(r) \exp\left[-jr\left(\tau + \sum_{i=1}^{L} \beta G_i^{-1}(r_i)\right)\right] dr$$

$$= 2\pi f_0\left(\tau + \sum_{i=1}^{L} \beta G_i^{-1}(r_i)\right) \tag{19}$$

P_{OUP}^{I} can be written after straightforward procedure as

$$P_{OUP}^{I} = \frac{1}{\beta^L} \int_0^{\infty} \cdots \int_0^{\infty} \left[\prod_{i=1}^{L} H_i\left[G_i^{-1}(r_i)\right]\right]$$

$$\times \exp\left(-\sum_{i=1}^{L} r_i\right) \prod_{i=1}^{L} \beta \frac{d\left\lfloor G_i^{-1}(r_i)\right\rfloor}{dr_i} F_0$$

$$\times \left[\sum_{i=1}^{L} \beta G_i^{-1}(r_i)\right] dr_1 \cdots dr_L \tag{20}$$

Equation (20) involves L integrals for L CCIs, and its second part can be calculated numerically with high desired accuracy using the Laguerre numerical integration technique [32]. Applying this integration method, (2) is extracted.

In the case of the existence of a minimum signal power constraint ψ, OUP is given by

$$P_{OUP}^{II} = 1 - Probability\left[\left(x_0 - \beta \sum_{i=1}^{L} x_i > 0\right) \cap (x_0 > \psi)\right] \tag{21}$$

Taking into account the fact that the two events in the brackets of (21) are statistically independent, P_{OUP}^{II} assumes the form

$$P_{OUP}^{II} = 1 - \left(1 - P_{OUP}^{I}\right)[1 - Probability(x_0 < \psi)] \tag{22}$$

After some simplifications, this finally results in (3).