Capacity Bounds for AF Dual-hop Relaying in $\mathcal{G}$ Fading Channels

Caijun Zhong, *Member, IEEE*, Michail Matthaiou, *Member, IEEE*, George K. Karagiannidis, *Senior Member, IEEE*, Aiping Huang, *Senior Member, IEEE*, and Zhaoyang Zhang, *Member, IEEE*

***Abstract*—We investigate the ergodic capacity of amplify-and-forward (AF) dual-hop relaying systems in composite Nakagami-m/inverse-Gaussian fading channels. This type of fading, which is known in the literature as $\mathcal{G}$ fading, has recently attracted increasing research interest due to its ability to better approximate the Nakagami-m/lognormal model, compared with the Nakagami-m/gamma model. We study both fixed- and variable-gain relaying systems and present analytical upper and lower bounds for the ergodic capacity of dual-hop relaying systems with not necessarily identical hops; these bounds provide an efficient means to evaluate the ergodic capacity of AF dual-hop relaying systems over $\mathcal{G}$ fading channels. We also establish sufficient conditions for the existence of the bounds, depending on the values of the fading parameters. In both cases, our simulation results demonstrate that the proposed upper and lower bounds remain relatively tight for different fading conditions.**

***Index Terms*—Amplify-and-forward (AF) relaying, dual-hop transmission, ergodic capacity, Nakagami-m/inverse-Gaussian (IG) fading.**

I. Introduction

DUAL-HOP relaying, where an intermediate relay node helps forward the signal from the source node to the destination node, is an efficient means to extend the coverage and improve the throughput of communication systems. Among various relaying protocols proposed in the literature, the amplify-and-forward (AF) relaying scheme is of particular interest in both academia and industry due to its low implementation cost [1]. In this case, relays simply amplify the source signal and forward it to the destination while performing no decoding. Depending on the availability of instantaneous channel state information (CSI) at the relay node, AF relaying schemes generally fall into two categories: 1) fixed-gain relaying (or blind relaying) [2] and 2) variable-gain relaying (or CSI-assisted relaying) [3]. The former configurations do not require knowledge of instantaneous CSI but need to know the average fading power of the previous hop, whereas variable-gain relaying systems amplify the received signal using the instantaneous CSI of the previous hop.

Understanding the ergodic capacity performance of AF dual-hop relaying systems in various propagation conditions has been an active area of research. The ergodic capacity of AF dual-hop relaying systems in Rayleigh fading channels was investigated in [4]–[7], whereas the cases of Nakagami-m and Weibull fading channels were studied in [8]–[10], respectively. The seminal work of [11], which established a generic moment-generating-function-based framework for the performance analysis of relaying systems, is also worth mentioning. Finally, [12] proposed a general framework for analyzing the ergodic capacity of variable-gain multihop relaying systems, although the presented results either apply for identically distributed fading distributions (e.g., Nakagami-m) or rely on the classical moment-based approach of [11].

The common characteristic of [4]–[10] is that they consider only small-scale fading and neglect large-scale fading (or shadowing). This can be attributed to the difficulty in averaging the end-to-end signal-to-noise ratio (SNR) over the shadowing distribution. In the analysis of composite fading channels, the prevalent model is the Nakagami-m/lognormal model, which has been extensively used to approximate the fading fluctuations in radar and radio-frequency communication systems [13]. Note that the Nakagami-m distribution has been shown to yield a good fit with empirical data in various propagation scenarios [14], [15]. This can, for example, occur for multipath scattering with relatively large delay-time spreads or when two strong paths of comparable power are dominating the other multipath components. However, the main disadvantage of the Nakagami-m/lognormal model is that the composite probability density function (p.d.f.) cannot be expressed in closed form. Consequently, the most important performance measures of interest, such as the ergodic capacity and symbol error rate, cannot be analytically evaluated. For this reason, some alternative models were recently proposed, with the most tractable being the Nakagami-m/gamma model, which is often referred to as the generalized-K model [16]–[19]. The performance of relaying systems in generalized-K fading channels has been investigated in [12], [20]–[22], and the references therein.

Manuscript received October 2, 2011; revised December 14, 2011 and January 21, 2012; accepted February 1, 2012. Date of publication February 15, 2012; date of current version May 9, 2012. This work was supported in part by the National Key Basic Research Program of China under Grant 2012CB316104 and in part by the Nature Science Foundation of China under Grant 60972058, the Fundamental Research Funds for the Central Universities under Grant 2012QNA5011, and the Zhejiang Provincial Natural Science Foundation of China No. Y1110368. The work of M. Matthaiou was supported in part by the Swedish Governmental Agency for Innovation Systems (VINNOVA) within the VINN Excellence Center Chase. The review of this paper was coordinated by Prof. S. Muhaidat.

C. Zhong, A. Huang, and Z. Zhang are with the Institute of Information and Communication Engineering, Zhejiang University, Hangzhou 310027, China (e-mail: caijunzhong@zju.edu.cn; aiping.huang@zju.edu.cn; ning_ming@zju.edu.cn).

M. Matthaiou is with the Department of Signals and Systems, Chalmers University of Technology, SE-412 96 Gothenburg, Sweden (e-mail: michail.matthaiou@chalmers.se).

G. K. Karagiannidis is with the Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, 54 124 Thessaloniki, Greece (e-mail: geokarag@auth.gr).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TVT.2012.2188110

On the other hand, it was pointed out in [23] that the gamma distribution does not yield a good approximation of the lognormal distribution with large variance. In addition, the accuracy of this approximation significantly deteriorates in the tails of the distribution (i.e., low outage region). Motivated by this limitation, Karmeshu and Agrawal [23] suggested to replace the gamma model with the inverse-Gaussian (IG) model; it was further demonstrated that the composite Rayleigh/IG model gives better characterization of fading channels, in comparison with the Rayleigh/gamma distribution. Later, in [24], Laourine *et al.* introduced the more general Nakagami-m/IG model (hereafter referred to as the $\mathcal{G}$ distribution) and studied the performance of such fading channels. Despite the importance of this composite fading model, to the best of our knowledge, very little is known about the performance of relaying systems in such fading channels.

On this basis, in this paper, we study the ergodic capacity of AF dual-hop relaying systems in composite $\mathcal{G}$ fading channels. In particular, we provide new analytical upper and lower capacity bounds for both fixed-gain and variable-gain AF dual-hop relaying systems. The derived bounds involve only standard mathematical functions and therefore can be easily and efficiently evaluated. More importantly, they apply to the case where the first and second hops experience nonidentically distributed fading. Our simulation results indicate that these bounds remain relatively tight across the entire SNR range and under different fading conditions. Thus, they can provide an efficient means to evaluate the capacity of AF dual-hop relaying systems in composite $\mathcal{G}$ fading channels. It is also shown that the ergodic capacity of AF dual-hop relaying systems increases when the fading of either hop is less severe.

The remainder of the paper is organized as follows: In Section II, the AF dual-hop relaying system model is presented, while Section III introduces some new and fundamental results on the statistics of IG variates. In Section IV, we provide new analytical upper and lower bounds on the ergodic capacity of fixed-gain relaying systems, while Section V elaborates on the case of variable-gain relaying systems. Finally, Section VI concludes the paper.

II. System Model

Let us consider a dual-hop relaying system with one source, relay, and destination node, as shown in Fig. 1. In this case, the end-to-end input-output relationship can be succinctly expressed as [2], [3]

$$y = h_2 G(h_1 x + n_1) + n_2 \tag{1}$$

where x denotes the source symbol with $\mathtt{E}\{xx^*\} = P_1$, and $(\cdot)^*$ and $\mathtt{E}\{\cdot\}$ denote complex conjugate and expectation, respectively. The term h_1 represents the random fading coefficient of the source–relay link, whereas h_2 is the random fading coefficient of the relay–destination link. In addition, G is the power scaling factor, which depends on the relay operation mode, and will be explicitly defined in the corresponding sections. In addition, n_1 and n_2 are the additive Gaussian noises at the relay and destination nodes with power N_1 and N_2, respectively.

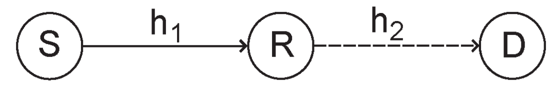


Fig. 1. System model. S, R, and D stand for source, relay, and destination nodes, respectively, whereas h_1 and h_2 denote the channel fading coefficients for the source–relay and relay–destination links, respectively.

We assume that $|h_1|$ and $|h_2|$ are independent and follow the $\mathcal{G}$ composite distribution, where $|x|$ denotes the amplitude of a complex number x. This implies that the small-scale fading follows a Nakagami-m distribution, which is characterized by the Nakagami-m parameter, whereas the large-scale fading follows an IG distribution with parameters λ_i and θ_i. In the following, we are particularly interested in the statistics of the power of each hop, i.e., $|h_i|^2$, $i = 1, 2$. For the case under consideration, this can be expressed as a product of two independent random variables [24]

$$|h_i|^2 = u_i v_i$$

where u_i follows the gamma distribution with pdf, i.e.,

$$f_{u_i}(x) = \frac{m_i^{m_i}}{\Gamma(m_i)} x^{m_i - 1} e^{-m_i x}, \qquad x > 0, m_i \geq 0.5$$

whereas v_i follows the inverse Gaussian distribution with parameters λ_i, and θ_i with pdf

$$p_{v_i}(x) = \sqrt{\frac{\lambda_i}{2\pi x^3}} \exp\left(-\frac{\lambda_i (x - \theta_i)^2}{2\theta_i^2 x}\right), \qquad x, \theta_i, \lambda_i \geq 0.$$

Following the methodology of [24], it can be shown that the pdf of $|h_i|^2$ reads as

$$f_{|h_i|^2}(x) = \frac{\alpha_i x^{m_i - 1}}{\left(\sqrt{x + \gamma_i}\right)^{m_i + \frac{1}{2}}} K_{m_i + \frac{1}{2}}\left(\beta_i \sqrt{x + \gamma_i}\right) \tag{2}$$

where $K_\nu(\cdot)$ denotes the νth-order modified Bessel function of the second kind [25, eq. (8.407.1)], whereas $\alpha_i \triangleq (m_i^{m_i}/\Gamma(m_i))(\lambda_i/\theta_i^2 \beta_i)^{m_i + 1/2} \sqrt{2\lambda_i/\pi} \exp(\lambda_i/\theta_i)$, $\beta_i \triangleq \sqrt{2\lambda_i m_i/\theta_i^2}$, and $\gamma_i \triangleq \lambda_i/(2m_i)$.

From (1), it is easy to see that the end-to-end SNR is equal to

$$\rho = \frac{G^2 |h_1|^2 |h_2|^2 P_1}{G^2 |h_2|^2 N_1 + N_2}.$$

In the following analysis, the hop SNRs defined as $\rho_1 \triangleq P_1 |h_1|^2 / N_1$ and $\rho_2 \triangleq P_2 |h_2|^2 / N_2$ will be extensively used. Unfortunately, their statistical characterization is not always straightforward, particularly when shadowing is taken into account. In this light, we henceforth present some novel results on the statistical properties of ρ_i for the case of composite $\mathcal{G}$ fading.

III. Statistical Properties of Functions Involving the Hop Signal-to-Noise Ratios ρ_i

In this section, we provide a set of new statistical results for functions involving $\rho_i \triangleq P_i/N_i |h_i|^2$ and $i = 1, 2$. Note that the following results will be handy when studying the ergodic capacity of AF dual-hop systems. Having this in mind, we start with the first positive moment of ρ_i.

Lemma 1: The first positive moment of ρ_i is given by

$$\mathtt{E}\{\rho_i\} = \frac{P_i}{N_i}\theta_i. \tag{3}$$

Proof: Due to the independence of u_i and v_i, we have that

$$\mathtt{E}\{\rho_i\} = \frac{P_i}{N_i}\mathtt{E}\{u_i\}\mathtt{E}\{v_i\}.$$

Utilizing the fact that $\mathtt{E}\{v_i\} = \theta_i$ [26, eq. (2.6)] yields the desired result. ∎

Lemma 2: The first negative moment of ρ_i is given by

$$\mathtt{E}\left\{\rho_i^{-1}\right\} = \frac{N_i}{P_i}\frac{m_i}{m_i - 1}\left(\frac{1}{\lambda_i} + \frac{1}{\theta_i}\right). \tag{4}$$

Proof: See Appendix I-A. ∎

Lemma 3: The expectation of the logarithm of ρ_i is given by

$$\mathtt{E}\{\ln\rho_i\} = \ln\frac{P_i\theta_i}{N_i m_i} + \psi(m_i) + \exp\left(\frac{2\lambda_i}{\theta_i}\right)\mathtt{Ei}\left(-\frac{2\lambda_i}{\theta_i}\right) \tag{5}$$

where $\psi(x)$ is the digamma function [25, eq. (8.360.1)], and $\mathtt{Ei}(x) = \int_{-\infty}^{x}(e^t/t)dt, x < 0$ is the exponential integral function [25, eq. (8.211.1)].

Proof: See Appendix I-B. ∎

Lemma 4: The first moment of $\rho_i/(a + \rho_i)$ can be expressed as

$$\mathtt{E}\left\{\frac{\rho_i}{a + \rho_i}\right\} = \alpha_i\left((-b)^{m_i}R_{m_i}(b, \gamma_i - b, \beta_i) + \sum_{k=1}^{m_i}\binom{m_i}{k}(-b)^{m_i-k}I_{m_i,k}(b, 1, \gamma_i - b, \beta_i)\right) \tag{6}$$

where $R_n(u, v, w)$ is defined as

$$R_n(u, v, w) \triangleq \int_1^\infty \frac{K_{n+\frac{1}{2}}\left(w\sqrt{ux + v}\right)}{x\left(\sqrt{ux + v}\right)^{n+\frac{1}{2}}}dx \tag{7}$$

whereas $I_{p,q}(u, v, w, z)$ reads as

$$I_{p,q}(u, v, w, z) \triangleq (q - 1)! \times \sum_{s=1}^{q}\frac{2^s u^{q-s}}{(vz)^s(q - s)!}\frac{K_{p-s+\frac{1}{2}}\left(z\sqrt{uv + w}\right)}{\left(\sqrt{uv + w}\right)^{p-s+\frac{1}{2}}}. \tag{8}$$

Proof: See Appendix I-C. ∎

IV. Fixed-Gain AF Dual-Hop Systems

In this section, we elaborate on the ergodic capacity of fixed-gain AF dual-hop relaying systems. As was previously mentioned, fixed-gain relaying schemes do not require instantaneous CSI at the relay node, and as such, they are more attractive from a practical point of view. For fixed-gain relaying systems, the relaying gain is given by [2]

$$G_f = \sqrt{\frac{P_2}{\mathtt{E}\left\{|h_1|^2\right\}P_1 + N_1}}$$

where P_2 is the power of the transmitted signal at the output of the relay. Hence, the end-to-end SNR for fixed-gain AF dual-hop relaying systems is equal to

$$\rho_f = \frac{\rho_1\rho_2}{c + \rho_2}$$

where $c \triangleq P_2/(G_f^2 N_1)$. With this definition in hand, we can obtain the ergodic capacity of the system under consideration as follows:

$$C_{\mathrm{f}} = \frac{1}{2}\mathtt{E}\left\{\log_2\left(1 + \frac{\rho_1\rho_2}{c + \rho_2}\right)\right\} \tag{9}$$

where the factor 1/2 accounts for the fact that the entire communication takes place in two time slots. Unfortunately, an exact evaluation of the ergodic capacity in (9) is, in general, intractable due to the presence of the nonlinear log function. Motivated by this, we hereafter seek to deduce upper and lower bounds on C_{f}. We start with the following Jensen's upper bound.

Theorem 1: The ergodic capacity of fixed-gain AF dual-hop relaying systems in $\mathcal{G}$ fading channels is upper bounded by (10), shown at the bottom of the next page, where $d_2 \triangleq cN_2/P_2$, and $R_n(u, v, w)$ and $I_{p,q}(u, v, w, z)$ are defined in (7) and (8), respectively.

Proof: Exploiting the concavity of the log function and applying Jensen's inequality, the ergodic capacity of fixed-gain AF dual-hop systems can be upper bounded by

$$C_{\mathrm{f}}^{u1} = \frac{1}{2}\log_2\left(1 + \mathtt{E}\{\rho_1\}\mathtt{E}\left\{\frac{\rho_2}{c + \rho_2}\right\}\right).$$

The proof concludes after invoking Lemma 1, Lemma 4, and factorization.

Corollary 1: When $m_2 = 1$, the capacity upper bound (10) reduces to

$$C_{\mathrm{f}}^{u1} = \frac{1}{2}\log_2\left(1 - \frac{P_1\theta_1\alpha_2}{N_1}\left(\frac{\sqrt{2\pi}e^{-\beta_2\sqrt{\gamma_2}}}{\beta_2^{3/2}\sqrt{\gamma_2}} - d_2 A_1\right)\right) \tag{11}$$

where $A_1 \triangleq R_1(d_2, \gamma_2 - d_2, \beta_2)$.

Proof: When $m_2 = 1$, we notice that $I_{1,1}(u, v, w, z) = 2K_{1/2}(z\sqrt{uv + w})/(vz\sqrt{uv + w}^{1/2})$. The desired result can then be obtained after some mathematical simplifications. ∎

The main limitation with the Jensen's upper bound C_{f}^{u1} is that it is not sufficiently tight in the high-SNR regime. Motivated by this, we propose the following upper bound, which is asymptotically exact in the high-SNR regime.

Theorem 2: The ergodic capacity of fixed-gain AF dual-hop systems in $\mathcal{G}$ fading channels is upper bounded by $C_{\rm f}^{u2}$ as in (12), shown at the bottom of the page.

Proof: We know from [21] that the ergodic capacity of fixed-gain AF dual-hop systems is upper bounded by

$$C_{\rm f}^{u2} = \frac{1}{2}\log_2\left(1 + c\mathtt{E}\left\{{\rho_1}^{-1}\right\}\mathtt{E}\left\{{\rho_2}^{-1}\right\} + \mathtt{E}\left\{{\rho_1}^{-1}\right\}\right) + \frac{1}{2}\mathtt{E}\left\{\log_2\rho_1 + \log_2\rho_2 - \log_2(c+\rho_2)\right\}. \quad (13)$$

To this end, the desired result can be obtained by combining Lemma 2, Lemma 3, and a result from [24, eq. (21)]. ■

Note that, taking into account the condition for the existence of the first negative moment in (13) (see Lemma 2), the upper bound $C_{\rm f}^{u2}$ exists only for $m_i > 1$. This implies, that the second upper bound does not exist if small-scale fading in one of the hops is Rayleigh distributed. Now, we turn our attention to the ergodic capacity lower bound and present the following new result:

Theorem 3: The ergodic capacity of fixed-gain AF dual-hop systems in $\mathcal{G}$ fading channels is lower bounded by $C_{\rm f}^{l}$ given in (14), shown at the bottom of the page.

Proof: From [21], the ergodic capacity of fixed-gain dual-hop relaying systems is lower bounded by

$$C_{\rm f}^{l} = \frac{1}{2}\log_2\left(1 + \exp\left(\mathtt{E}\left\{\ln\rho_1\right\} + \mathtt{E}\left\{\ln\rho_2\right\} - \mathtt{E}\left\{\ln(c+\rho_2)\right\}\right)\right).$$

As a next step, we invoke Lemma 3 and a result from [24, eq. (21)], and the desired result can be obtained after some basic algebraic manipulations. ■

We note that all bounds require m_2 to be a positive integer for $C_{\rm f}^{u2}$, i.e., $m_2 = 2, 3, \ldots$. When both the first- and second-hop channels are subjected to Rayleigh/IG fading, we have the following simplified lower bound.

Corollary 2: When $m_1 = m_2 = 1$, the ergodic capacity lower bound in (14) reduces to $C_{\rm f}^{l}$ in (15), shown at the bottom of the next page, where $\gamma_0 = 0.57721$ is the Euler–Mascheroni constant.

Proof: When $m_i = 1$, we have

$$\mathtt{E}\left\{\log_2(c+\rho_2)\right\} = \ln c + \frac{2\alpha_2}{\beta_2}R_0(b_2, \gamma_2 - b_2, \beta_2) \quad (16)$$

and the desired result immediately follows. ■

$$C_{\rm f}^{u1} = \frac{1}{2}\log_2\left(1 + \frac{P_1\theta_1\alpha_2(-d_2)^{m_2}}{N_1}\left(R_{m_2}(d_2, \gamma_2 - d_2, \beta_2) + \sum_{k=1}^{m_2}\binom{m_2}{k}(-d_2)^{-k}I_{m_2,k}(d_2, 1, \gamma_2 - d_2, \beta_2)\right)\right) \quad (10)$$

$$\begin{aligned} C_{\rm f}^{u2} =& \frac{1}{2}\log_2\left(1 + \frac{N_1 m_1}{P_1(m_1-1)}\left(\frac{1}{\lambda_1} + \frac{1}{\theta_1}\right)\left(\frac{cN_2 m_2}{P_2(m_2-1)}\left(\frac{1}{\lambda_2} + \frac{1}{\theta_2}\right) + 1\right)\right) \\ &+ \frac{\log_2 e}{2}\left(\ln\frac{P_1P_2\theta_1\theta_2}{N_1N_2m_1m_2} + \psi(m_1) + \psi(m_2) + \exp\left(\frac{2\lambda_1}{\theta_1}\right)\mathtt{Ei}\left(-\frac{2\lambda_1}{\theta_1}\right) + \exp\left(\frac{2\lambda_2}{\theta_2}\right)\mathtt{Ei}\left(-\frac{2\lambda_2}{\theta_2}\right)\right) \\ &- \frac{\log_2 c}{2} - \frac{\log_2 e}{2}\sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k(-d_2)^{m_2-k}}{\beta_2^k(m_2-k)!}R_{m_2-k}(d_2, \gamma_2 - d_2, \beta_2) \\ &- \frac{\log_2 e}{2}\sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}\sum_{j=1}^{m_2-k}\binom{m_2-k}{j}(-d_2)^{m_2-k-j}I_{m_2-k,j}(d_2, 1, \gamma_2 - d_2, \beta_2) \end{aligned} \quad (12)$$

$$\begin{aligned} C_{\rm f}^{l} =& \frac{1}{2}\log_2\Bigg(1 + \frac{P_1P_2\theta_1\theta_2}{N_1N_2m_1m_2}\exp\left(\psi(m_1) + \psi(m_2) + \exp\left(\frac{2\lambda_1}{\theta_1}\right)\mathtt{Ei}\left(-\frac{2\lambda_1}{\theta_1}\right)\right) \\ &\times\exp\Bigg(\exp\left(\frac{2\lambda_2}{\theta_2}\right)\mathtt{Ei}\left(-\frac{2\lambda_2}{\theta_2}\right) - \ln c - \sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}(-d_2)^{m_2-k}R_{m_2-k}(d_2, \gamma_2 - d_2, \beta_2) \\ &- \sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}\sum_{j=1}^{m_2-k}\binom{m_2-k}{j}(-d_2)^{m_2-k-j}I_{m_2-k,j}(d_2, 1, \gamma_2 - d_2, \beta_2)\Bigg)\Bigg) \end{aligned} \quad (14)$$

It is worth pointing out that the ergodic capacity upper and lower bounds presented in the preceding theorems only involve standard functions and, hence, can be very fast and efficiently evaluated in popular software packages such as Matlab or Mathematica. More importantly, C_{f}^{u2} and C_{f}^{l} become exact at high SNRs, as shown in the following corollary.

Corollary 3: At high SNRs, the upper bound C_{f}^{u2} and lower bound C_{f}^{l} become exact as

$$C_{\mathrm{f}}^{u2} = C_{\mathrm{f}}^{l} = C_{\mathrm{f}}^{e} \tag{17}$$

where C_{f}^{e} is defined as in (18), shown at the bottom of the page.

To illustrate the tightness of the proposed ergodic capacity upper and lower bounds, we compare the analytical results against Monte Carlo simulation results. For all simulations, the parameters of the IG distribution are obtained by matching the first and the second moment with those of the lognormal distribution as per [24]

$$\lambda = \frac{\exp \mu}{2\sinh(\sigma^2/2)} \quad \theta = \exp\left(\mu + \frac{\sigma^2}{2}\right)$$

where $\sinh(x)$ is the sine hyperbolic function, whereas μ and σ are the mean and the standard deviation of a lognormal distribution, respectively.

Fig. 2 shows the impact of small-scale fading on the ergodic capacity of fixed-gain AF dual-hop systems. In the simulations, we set $\mu_i = 0.115$ and $\sigma_i = 0.115$, $i = 1, 2$, which corresponds to an infrequent light shadowing scenario [27]. As can be readily seen, the second upper bound C_{f}^{u2} is much tighter than the first upper bound C_{f}^{u1} in the high-SNR regime; in fact, C_{f}^{u2} almost overlaps with C_{f}^{l} when $P_1/N_1 \geq 20$ dB, thereby demonstrating that C_{f}^{u2} and C_{f}^{l} are asymptotically exact at high SNRs. We also observe the intuitive result that the ergodic capacity of the system improves when the fading severity level is reduced, i.e., when m_i increases from 2 to 20. Moreover, the tightness of all three bounds improves when m_i becomes larger. To further examine the impact of m_i, in Fig. 3, we set $P_1/N_1 = 10$ dB, $P_2/N_2 = 2(P_1/N_1)$, $\mu_1 = \mu_2 = 0.115$, and $\sigma_1 = \sigma_2 =$

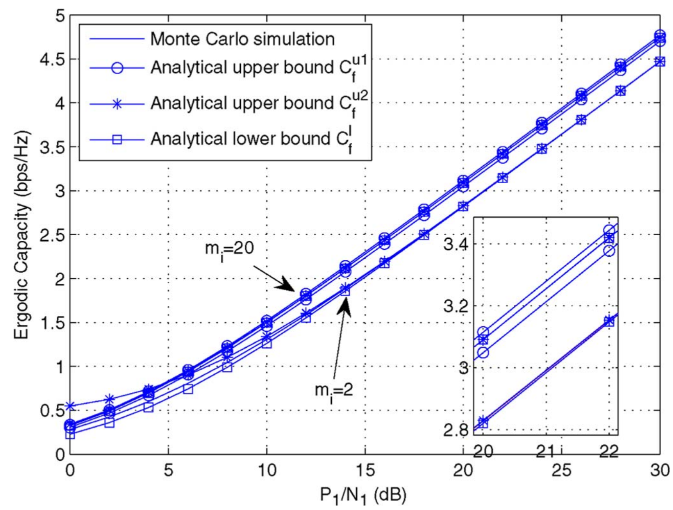


Fig. 2. Ergodic capacity of fixed-gain AF dual-hop systems in composite $\mathcal{G}$ fading channels: Simulation results versus analytical upper bounds $P_2/N_2 = 3(P_1/N_1)$.

0.115. As can be seen, when m_i increases, the ergodic capacity of the system improves; however, this improvement gradually diminishes when m_i becomes sufficiently large, i.e., $m_i > 10$.

Similarly, Fig. 4 shows the impact of shadowing on the ergodic capacity of fixed-gain AF dual-hop systems when $m_i = 20$. As we can observe, when the shadowing effect becomes more frequent and heavy, the ergodic capacity of the system is reduced.

V. Variable-Gain AF Dual-Hop Systems

In this section, we study the ergodic capacity of variable-gain AF dual-hop relaying systems. The variable-gain relaying scheme exploits the instantaneous CSI at the relay node, such that the corresponding relay amplification factor is given by [3]

$$G_v = \sqrt{\frac{P_2}{|h_1|^2 P_1 + N_1}}.$$

$$C_{\mathrm{f}}^{l} = \frac{1}{2}\log_2\left(1 + \frac{P_1P_2}{N_1N_2c}\exp\left(-2\gamma_0 + \exp\left(\frac{2\lambda_1}{\theta_1}\right)\mathtt{Ei}\left(-\frac{2\lambda_1}{\theta_1}\right) + \exp\left(\frac{2\lambda_2}{\theta_2}\right)\mathtt{Ei}\left(-\frac{2\lambda_2}{\theta_2}\right)\right)\exp\left(-\frac{2\alpha_2}{\beta_2}R_0(d_2, \gamma_2 - d_2, \beta_2)\right)\right) \tag{15}$$

$$\begin{aligned} C_{\mathrm{f}}^{e} = &\frac{\log_2 e}{2}\left(\ln\frac{P_1P_2}{N_1N_2m_1m_2} + \psi(m_1) + \psi(m_2) + \exp\left(\frac{2\lambda_1}{\theta_1}\right)\mathtt{Ei}\left(-\frac{2\lambda_1}{\theta_1}\right)\right) \\ &- \frac{\log_2 c}{2} + \frac{\log_2 e}{2}\left(\exp\left(\frac{2\lambda_2}{\theta_2}\right)\mathtt{Ei}\left(-\frac{2\lambda_2}{\theta_2}\right) - \sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}(-d_2)^{m_2-k}R_{m_2-k}(d_2, \gamma_2 - d_2, \beta_2)\right) \\ &- \frac{\log_2 e}{2}\sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}\sum_{j=1}^{m_2-k}\binom{m_2-k}{j}(-d_2)^{m_2-k-j}I_{m_2-k,j}(d_2, 1, \gamma_2 - d_2, \beta_2) \end{aligned} \tag{18}$$

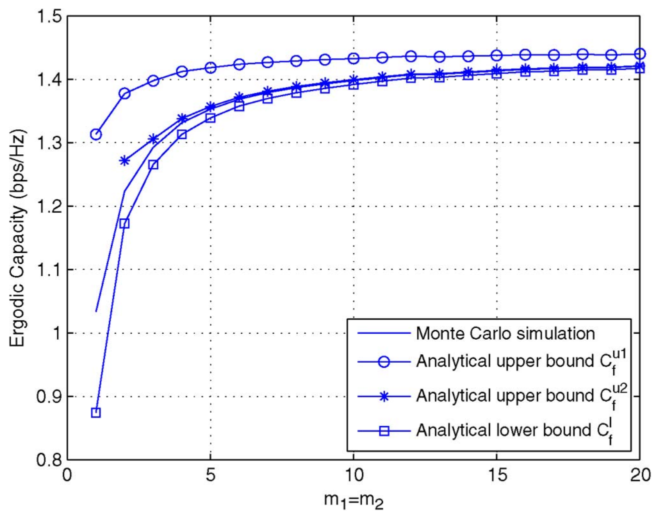


Fig. 3. Ergodic capacity of fixed-gain AF dual-hop systems in composite $\mathcal{G}$ fading channels with $P_2/N_2 = 2(P_1/N_1)$ and $P_1/N_1 = 10$ dB: Impact of m_i.

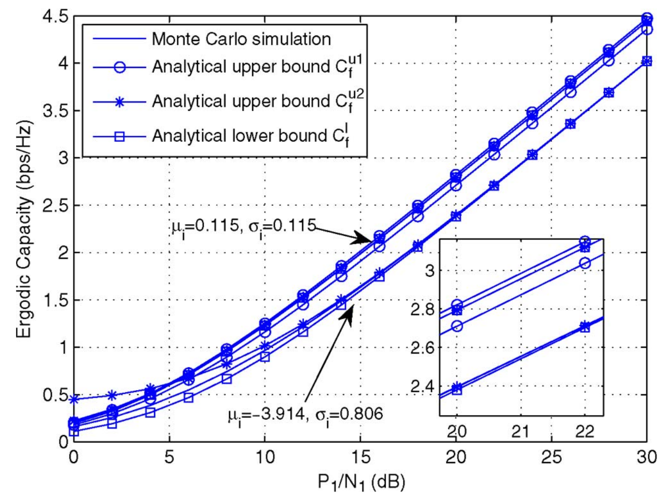


Fig. 4. Ergodic capacity of fixed-gain AF dual-hop systems in composite $\mathcal{G}$ fading channels: Simulation results versus analytical upper and lower bounds, i.e., $P_1/N_1 = P_2/N_2$.

Hence, the end-to-end SNR for variable-gain AF dual-hop relaying systems is equal to

$$\rho_v = \frac{\rho_1\rho_2}{\rho_1+\rho_2+1}. \tag{19}$$

From (19), the ergodic capacity of the system can be expressed as

$$C_{\rm v} = \frac{1}{2}\mathtt{E}\left\{\log_2\left(1+\frac{\rho_1\rho_2}{\rho_1+\rho_2+1}\right)\right\}.$$

Similar to the fixed-gain relaying case, an exact expression for $C_{\rm v}$ is, in general, very difficult to derive. Hence, we focus on deriving closed-form upper and lower bounds on $C_{\rm v}$.

Theorem 4: The ergodic capacity of variable-gain AF dual-hop relaying systems in $\mathcal{G}$ fading channels is upper bounded by $C_{\rm v}^u$ as in (20), shown at the bottom of the page, where $f_i \triangleq P_i/N_i$, $i = 1, 2$.

Proof: From [7], the ergodic capacity of variable-gain AF dual-hop systems can be alternatively expressed as

$$C_{\rm v} = \frac{1}{2}\mathtt{E}\left\{\log_2(1+\rho_1)\right\} + \frac{1}{2}\mathtt{E}\left\{\log_2(1+\rho_2)\right\} - \frac{1}{2}\mathtt{E}\left\{\log_2(1+\rho_1+\rho_2)\right\}. \tag{21}$$

Since the first two terms can be expressed in closed-form expressions, the key task is to bound the third term. It is easy to show that $f(x,y) = \log_2(1+e^x+e^y)$ is a convex function with respect to x and y; hence, we have

$$\mathtt{E}\left\{\log_2(1+\rho_1+\rho_2)\right\} \geq \log_2\left(1+e^{\mathrm{E}\{\ln\rho_1\}}+e^{\mathrm{E}\{\ln\rho_2\}}\right).$$

The desired result is finally obtained by using Lemma 3. ∎

When both the first- and second-hop channels are subjected to Rayleigh/IG fading, we have the following simple-capacity upper bound:

$$\begin{aligned}
C_{\rm v}^u =\ & \frac{\log_2 e}{2}\sum_{k=1}^{m_1}\frac{\alpha_1\Gamma(m_1)2^k}{\beta_1^k(m_1-k)!}\left(-\frac{1}{f_1}\right)^{m_1-k} R_{m_1-k}\left(\frac{1}{f_1},\gamma_1-\frac{1}{f_1},\beta_1\right)\\
&+\frac{\log_2 e}{2}\sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}\left(-\frac{1}{f_2}\right)^{m_2-k} R_{m_2-k}\left(\frac{1}{f_2},\gamma_2-\frac{1}{f_2},\beta_2\right)\\
&+\frac{\log_2 e}{2}\sum_{k=1}^{m_1}\frac{\alpha_1\Gamma(m_1)2^k}{\beta_1^k(m_1-k)!}\sum_{j=1}^{m_1-k}\binom{m_1-k}{j}\left(-\frac{1}{f_1}\right)^{m_1-k-j} I_{m_1-k,j}\left(\frac{1}{f_1},1,\gamma_1-\frac{1}{f_1},\beta_1\right)\\
&+\frac{\log_2 e}{2}\sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}\sum_{j=1}^{m_2-k}\binom{m_2-k}{j}\left(-\frac{1}{f_2}\right)^{m_2-k-j} I_{m_2-k,j}\left(\frac{1}{f_2},1,\gamma_2-\frac{1}{f_2},\beta_2\right)\\
&-\frac{1}{2}\log_2\left(1+\frac{f_1}{m_1}\exp\left(\psi(m_1)+\exp\left(\frac{2\lambda_1}{\theta_1}\right)\mathtt{Ei}\left(-\frac{2\lambda_1}{\theta_1}\right)\right)+\frac{f_2}{m_2}\exp\left(\psi(m_2)+\exp\left(\frac{2\lambda_2}{\theta_2}\right)\mathtt{Ei}\left(-\frac{2\lambda_2}{\theta_2}\right)\right)\right)
\end{aligned} \tag{20}$$

Corollary 4: When $m_1 = m_2 = 1$, the ergodic capacity upper bound in (20) reduces to

$$C_{\mathrm{v}}^u = \frac{\alpha_1 \log_2 e}{\beta_1} R_0\left(\frac{1}{f_1}, \gamma_1 - \frac{1}{f_1}, \beta_1\right) + \frac{\alpha_2 \log_2 e}{\beta_2} R_0\left(\frac{1}{f_2}, \gamma_2 - \frac{1}{f_2}, \beta_2\right) - \frac{1}{2}\log_2\Bigg(1 + f_1 \exp\left(-\gamma_0 + \exp\left(\frac{2\lambda_1}{\theta_1}\right)\mathtt{Ei}\left(-\frac{2\lambda_1}{\theta_1}\right)\right) + f_2 \exp\left(-\gamma_0 + \exp\left(\frac{2\lambda_2}{\theta_2}\right)\mathtt{Ei}\left(-\frac{2\lambda_2}{\theta_2}\right)\right)\Bigg). \quad (22)$$

Proof: The result immediately follows by invoking (16). ■

Now, we turn our attention to the ergodic capacity lower bound, and we have the following key result:

Theorem 5: The ergodic capacity of variable-gain AF dual-hop relaying systems in $\mathcal{G}$ fading channels is lower bounded by C_{v}^l, as in (23), shown at the bottom of the page.

Proof: Applying Jensen's inequality on the third term of (21), we have

$$\mathtt{E}\{\log_2(1 + \rho_1 + \rho_2)\} \le \log_2\left(1 + \mathtt{E}\{\rho_1\} + \mathtt{E}\{\rho_2\}\right).$$

Hence, the desired result can be obtained by invoking Lemma 1. ■

When both the first- and second-hop channels are subjected to Rayleigh/IG fading, we have the following simple capacity lower bound.

Corollary 5: When $m_1 = m_2 = 1$, the ergodic capacity lower bound in (23) reduces to

$$C_{\mathrm{v}}^l = \frac{\alpha_1 \log_2 e}{\beta_1} R_0\left(\frac{1}{f_1}, \gamma_1 - \frac{1}{f_1}, \beta_1\right) + \frac{\alpha_2 \log_2 e}{\beta_2} R_0\left(\frac{1}{f_2}, \gamma_2 - \frac{1}{f_2}, \beta_2\right) - \frac{1}{2}\log_2\left(1 + f_1\theta_1 + f_2\theta_2\right). \quad (24)$$

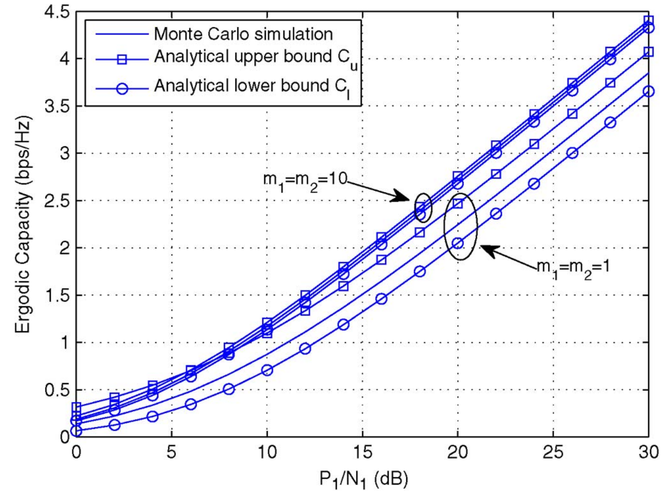


Fig. 5. Ergodic capacity of variable-gain AF dual-hop systems in composite $\mathcal{G}$ fading channels: Simulation results versus analytical upper and lower bounds, i.e., $P_1/N_1 = P_2/N_2$.

Proof: The result immediately follows by invoking (16). ■

We note that both the upper bound in (20) and the lower bound in (23) require that both m_1 and m_2 are positive integers. We can now examine the tightness of the aforemtentioned upper and lower capacity bounds. Fig. 5 shows the impact of small-scale fading on the tightness of the capacity bounds when $\mu_i = 0.115$, $\sigma_1 = 0.115$, and $i = 1, 2$. Generally speaking, the effect of m_i on the capacity becomes less pronounced as m_i gets larger (i.e., the relatively difference between the capacity curves gets smaller).

In addition, Fig. 6 shows how the large-scale fading affects the performance of the capacity bounds when $m_i = 10$, $i = 1, 2$. For all cases under consideration, it can be readily observed that the capacity bounds perform much better for the less severe fading scenarios.

Finally, Fig. 7 shows the impact of asymmetric fading channels on the system capacity. The curves indicate that the capacity bounds become tighter when the fading level of either hops improves.

$$C_{\mathrm{v}}^l = \frac{\log_2 e}{2}\sum_{k=1}^{m_1}\frac{\alpha_1\Gamma(m_1)2^k}{\beta_1^k(m_1-k)!}\left(-\frac{1}{f_1}\right)^{m_1-k} R_{m_1-k}\left(\frac{1}{f_1}, \gamma_1 - \frac{1}{f_1}, \beta_1\right) + \frac{\log_2 e}{2}\sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}\left(-\frac{1}{f_2}\right)^{m_2-k} R_{m_2-k}\left(\frac{1}{f_2}, \gamma_2 - \frac{1}{f_2}, \beta_2\right) + \frac{\log_2 e}{2}\sum_{k=1}^{m_1}\frac{\alpha_1\Gamma(m_1)2^k}{\beta_1^k(m_1-k)!}\sum_{j=1}^{m_1-k}\binom{m_1-k}{j}\left(-\frac{1}{f_1}\right)^{m_1-k-j} I_{m_1-k,j}\left(\frac{1}{f_1}, 1, \gamma_1 - \frac{1}{f_1}, \beta_1\right) + \frac{\log_2 e}{2}\sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}\sum_{j=1}^{m_2-k}\binom{m_2-k}{j}\left(-\frac{1}{f_2}\right)^{m_2-k-j} I_{m_2-k,j}\left(\frac{1}{f_2}, 1, \gamma_2 - \frac{1}{f_2}, \beta_2\right) - \frac{1}{2}\log_2\left(1 + f_1\theta_1 + f_2\theta_2\right) \quad (23)$$

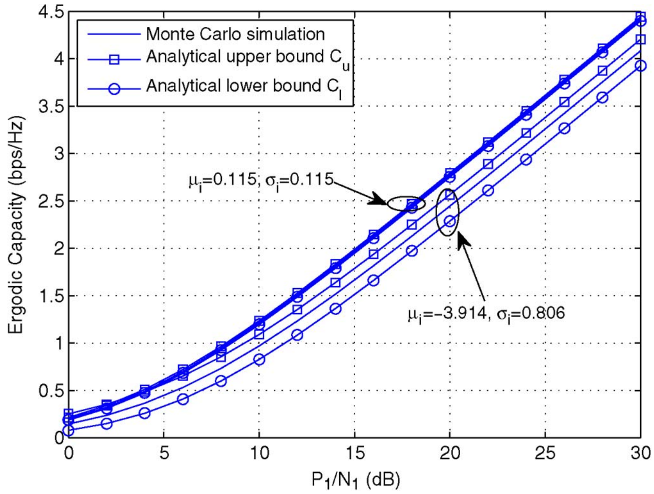


Fig. 6. Ergodic capacity of variable-gain AF dual-hop systems in composite $\mathcal{G}$ fading channels: Simulation results versus analytical upper and lower bounds, i.e., $P_1/N_1 = P_2/N_2$.

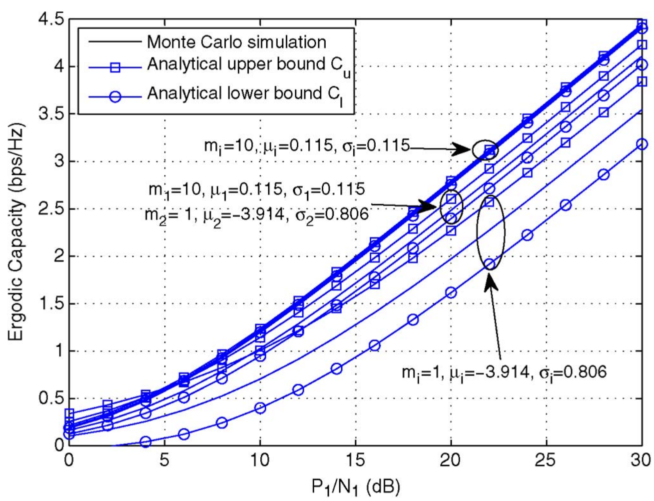


Fig. 7. Ergodic capacity of variable-gain AF dual-hop systems in composite $\mathcal{G}$ fading channels: Simulation results versus analytical upper and lower bounds, i.e., $P_1/N_1 = P_2/N_2$.

VI. Conclusion

The capacity characterization of AF dual-hop relaying systems has so far not properly addressed the effects of shadowing. On this basis, we have considered the generic $\mathcal{G}$ distribution, which is the combination of small-scale Nakagami-m fading and large-scale IG fading. We note that the $\mathcal{G}$ fading model can more effectively approximate the Nakagami-m/lognormal model than the widely used Nakagami-m/gamma model. On the other hand, the mathematical analysis becomes more challenging.

In this paper, we have investigated the ergodic capacity of AF dual-hop relaying systems in $\mathcal{G}$ fading channels. More specifically, some new analytical upper and lower bounds were derived for the ergodic capacity of both fixed- and variable-gain AF dual-hop relaying systems. It was demonstrated that the performance of the capacity bounds remains good across the entire SNR range and under different fading conditions; therefore, they can be used to efficiently assess the ergodic capacity performance of AF dual-hop systems over composite channels. Moreover, our numerical results suggested that the ergodic capacity of the system is degraded when the small-scale fading becomes more severe or when the shadowing becomes more frequent and heavy.

Appendix I
Main Proofs

A. Proof of Lemma 2

Due to the independence of u_i and v_i, the first negative moment of ρ_i simplifies to

$$\mathtt{E}\left\{\rho_i^{-1}\right\} = \frac{N_i}{P_i}\mathtt{E}\left\{u_i^{-1}\right\}\mathtt{E}\left\{v_i^{-1}\right\}.$$

The first negative moment of u_i can be computed as

$$\mathtt{E}\left\{u_i^{-1}\right\} = \frac{m_i^{m_i}}{\Gamma(m_i)}\int_0^\infty x^{m_i-2}e^{-m_i x}\,dx = \frac{m_i}{m_i-1}$$

where we have used [25, eq. (3.381.5)] to solve the corresponding integral. It is noteworthy that $\mathtt{E}\{u_i^{-1}\}$ requires $m_i > 1$ to exist.[1] Now, the first negative moment of v_i can be computed as

$$\mathtt{E}\left\{v_i^{-1}\right\} = \sqrt{\frac{\lambda_i}{2\pi}}\int_0^\infty x^{-\frac{5}{2}}\exp\left(-\frac{\lambda_i(x-\theta_i)^2}{2\theta_i^2 x}\right)dx$$

$$= \sqrt{\frac{\lambda_i}{2\pi}}\exp\left(\frac{\lambda_i}{\theta_i}\right)\int_0^\infty x^{-\frac{5}{2}}\exp\left(-\frac{\lambda_i x}{2\theta_i^2}-\frac{\lambda_i}{2x}\right)dx \tag{25}$$

$$= \sqrt{\frac{\lambda_i}{2\pi}}\exp\left(\lambda_i\theta_i\right)2\theta_i^{-\frac{3}{2}}K_{\frac{3}{2}}\left(\frac{\lambda_i}{\theta_i}\right) \tag{26}$$

$$= \frac{1}{\lambda_i}+\frac{1}{\theta_i} \tag{27}$$

where, from (25) to (26), we have used [25, eq. (3.471.12)], whereas from (26) to (27), we have used a property of Bessel functions of order equal to an integer plus one-half [25, eq. (8.468)]. ■

B. Proof of Lemma 3

The expectation of the logarithm of ρ_i can be trivially expressed as

$$\mathtt{E}\{\ln\rho_i\} = \ln\frac{P_i}{N_i} + \mathtt{E}\{\ln u_i\} + \mathtt{E}\{\ln v_i\}.$$

With the help of the integral identity [25, eq. (4.352.1)], the expectation of $\ln u_i$ becomes

$$\mathtt{E}\{\ln u_i\} = \psi(m_i) - \ln m_i.$$

[1] For a detailed discussion of this condition, see [21].

To compute the expectation of $\ln v_i$, we first work out the general moment of v_i as

$$\mathtt{E}\left\{v_i^n\right\} = \sqrt{\frac{2\lambda_i}{\pi}} \exp\left(\frac{\lambda_i}{\theta_i}\right) \theta_i^{n-\frac{1}{2}} K_{n-\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right).$$

To this end, the expectation of $\ln v_i$ can be derived as

$$\mathtt{E}\{\ln v_i\} = \left.\frac{d\mathtt{E}\left\{v_i^n\right\}}{dn}\right|_{n=0}$$

where we have used the following derivative property:

$$\frac{dx^n}{dn} = x^n \ln x.$$

Hence, we get

$$\mathtt{E}\{\ln v_i\} = \sqrt{\frac{2\lambda_i}{\pi\theta_i}} \ln\theta_i \exp\left(\frac{\lambda_i}{\theta_i}\right) K_{\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right) + \left.\sqrt{\frac{2\lambda_i}{\pi\theta_i}} \exp\left(\frac{\lambda_i}{\theta_i}\right) K_{n-\frac{1}{2}}^{\{1,0\}}\left(\frac{\lambda_i}{\theta_i}\right)\right|_{n=0}$$

where $K_\nu^{\{1,0\}}(x)$ denotes the derivative of the Bessel-K function with respect to the order ν. Using [28], $K_{n-1/2}^{\{1,0\}}(\lambda_i/\theta_i)|_{n=0}$ can be explicitly expressed as

$$\begin{aligned}\left.K_{n-\frac{1}{2}}^{\{1,0\}}\left(\frac{\lambda_i}{\theta_i}\right)\right|_{n=0} &= \frac{\pi}{2}\left(I_{\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right)+I_{-\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right)\right)\left(\mathsf{Chi}\left(\frac{2\lambda_i}{\theta_i}\right)-\mathsf{Shi}\left(\frac{2\lambda_i}{\theta_i}\right)\right)\\ &\quad - \frac{\theta_i}{\lambda_i}K_{\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right)+\sqrt{\frac{\pi\theta_i}{\lambda_i}}\left(I_{\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right)+I_{-\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right)\right)K_{\frac{1}{2}}\left(\frac{2\lambda_i}{\theta_i}\right)\end{aligned}$$

where $I_\nu(\cdot)$ denotes the νth-order modified Bessel function of the first kind [25, eq. (8.406.1)], whereas $\mathsf{Chi}(x)$ and $\mathsf{Shi}(x)$ are the hyperbolic cosine integral and hyperbolic sine integral, respectively. After some algebraic manipulations, we arrive at

$$\begin{aligned}\mathtt{E}\{\ln v_i\} &= \left(\ln\theta_i - \frac{\theta_i}{\lambda_i}\right) + \sqrt{\frac{\pi\lambda_i}{2\theta_i}}\exp\left(\frac{\lambda_i}{\theta_i}\right)\left(I_{\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right)+I_{-\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right)\right)\\ &\quad\times\left(\mathsf{Chi}\left(\frac{2\lambda_i}{\theta_i}\right)-\mathsf{Shi}\left(\frac{2\lambda_i}{\theta_i}\right)+\frac{\theta_i}{\lambda_i}\exp\left(-\frac{2\lambda_i}{\theta_i}\right)\right). \quad (28)\end{aligned}$$

We now invoke the following properties of Bessel functions [25, eq. (8.467)]

$$I_{\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi z}}\sinh(z)$$

$$I_{-\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi z}}\cosh(z)$$

and combine them with the relationship $\sinh(z) + \cosh(z) = \exp(z)$ to reformulate (28) as

$$\mathtt{E}\{\ln v_i\} = \ln\theta_i + \exp\left(\frac{2\lambda_i}{\theta_i}\right)\left(\mathsf{Chi}\left(\frac{2\lambda_i}{\theta_i}\right) - \mathsf{Shi}\left(\frac{2\lambda_i}{\theta_i}\right)\right).$$

We now recall the integral definitions of $\mathsf{Chi}(x)$ and $\mathsf{Shi}(x)$ [25, eq. (8.221)]

$$\mathsf{Chi}(x) = \gamma_0 + \ln x + \int_0^x \frac{\cosh(t)-1}{t}\,dt \quad (29)$$

$$\mathsf{Shi}(x) = \int_0^x \frac{\sinh(t)}{t}\,dt. \quad (30)$$

We can now combine (29) and (30) with the trigonometric relationship $\cosh(z) - \sinh(z) = \exp(-z)$ to get

$$\begin{aligned}\mathtt{E}\{\ln v_i\} &= \ln\theta_i + \exp\left(\frac{2\lambda_i}{\theta_i}\right)\\ &\quad\times\left(\gamma_0 + \ln\left(\frac{2\lambda_i}{\theta_i}\right) + \int_0^{\frac{2\lambda_i}{\theta_i}} \frac{e^{-t}-1}{t}\,dt\right)\\ &= \ln\theta_i + \exp\left(\frac{2\lambda_i}{\theta_i}\right)\mathtt{Ei}\left(-\frac{2\lambda_i}{\theta_i}\right).\end{aligned}$$

Note that, for the evaluation of the integral, we have used the integral identity [25, eq. (8.212.1)]. This concludes the proof. ■

C. Proof of Lemma 4

With the pdf given in (2), the considered first moment can be reexpressed as

$$\begin{aligned}\mathtt{E}\left\{\frac{\rho_i}{a+\rho_i}\right\} &= \mathtt{E}\left\{\frac{|h_i|^2}{\frac{aN_i}{P_i}+|h_i|^2}\right\}\\ &= \alpha_i\int_0^\infty \frac{x^{m_i}}{(x+b)\left(\sqrt{x+\gamma_i}\right)^{m_i+\frac{1}{2}}}\\ &\quad\times K_{m_i+\frac{1}{2}}\left(\beta_i\sqrt{x+\gamma_i}\right)dx\end{aligned}$$

where $b \triangleq aN_i/P_i$. We now make a change of variables $t = x + b$ and apply the binomial theorem to get

$$\begin{aligned}\mathtt{E}\left\{\frac{\rho_i}{a+\rho_i}\right\} &= \alpha_i\sum_{k=0}^{m_i}\binom{m_i}{k}(-b)^{m_i-k}\\ &\quad\times\int_b^\infty \frac{t^{k-1}}{\left(\sqrt{t-b+\gamma_i}\right)^{m_i+\frac{1}{2}}}K_{m_i+\frac{1}{2}}\left(\beta_i\sqrt{t-b+\gamma_i}\right)dt.\end{aligned} \quad (31)$$

For the first term $k = 0$, we get

$$\begin{aligned}\mathsf{term}_1 &= \alpha_i(-b)^{m_i}\int_1^\infty \frac{K_{m_i+\frac{1}{2}}\left(\beta_i\sqrt{bx-b+\gamma_i}\right)}{x\left(\sqrt{bx-b+\gamma_i}\right)^{m_i+\frac{1}{2}}}\,dx\\ &= \alpha_i(-b)^{m_i}R_{m_i}(b,\gamma_i-b,\beta_i).\end{aligned} \quad (32)$$

Note that an explicit expression for $R_n(u, v, w)$ is provided in [24, App. C]. With the help of [24, App. A], the rest of the terms $k \geq 1$ can be computed as

$$\mathsf{term}_2 = \alpha_i \sum_{k=1}^{m_i} \binom{m_i}{k} (-b)^{m_i-k} I_{m_i,k}(b, 1, \gamma_i - b, \beta_i). \quad (33)$$

Substituting (32) and (33) into (31) yields the desired result after appropriate factorization. ■

REFERENCES

[1] Y. Yang, H. Hu, J. Xu, and G. Mao, "Relay technologies for WiMAX and LTE-advanced mobile systems," *IEEE Commun. Mag.*, vol. 47, no. 10, pp. 100–105, Oct. 2009.

[2] M. O. Hasna and M.-S. Alouini, "A performance study of dual-hop transmissions with fixed gain relays," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1963–1968, Nov. 2004.

[3] M. O. Hasna and M.-S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 1126–1131, Nov. 2003.

[4] G. Farhadi and N. C. Beaulieu, "On the ergodic capacity of wireless relaying systems over Rayleigh fading channels," *IEEE Trans. Wireless Commun.*, vol. 7, no. 11, pp. 4462–4467, Nov. 2008.

[5] S. Ikki and M. Ahmed, "Exact error probability and channel capacity of best-relay cooperative diversity networks," *IEEE Signal Process. Lett.*, vol. 16, no. 12, pp. 1051–1054, Dec. 2009.

[6] O. Waqar, M. Ghogho, and D. McLernon, "Tight bounds for ergodic capacity of dual-hop fixed-gain relay networks under Rayleigh fading," *IEEE Commun. Lett.*, vol. 15, no. 4, pp. 413–415, Apr. 2011.

[7] L. Fan, X. Lei, and W. Li, "Exact closed-form expression for ergodic capacity of amplify-and-forward relaying in channel-noise-assisted cooperative networks with relay selection," *IEEE Commun. Lett.*, vol. 15, no. 3, pp. 332–333, Mar. 2011.

[8] D. B. Costa and S. Aïssa, "Capacity analysis of cooperative systems with relay selection in Nakagami-m fading," *IEEE Commun. Lett.*, vol. 13, no. 9, pp. 637–639, Sep. 2009.

[9] H. Q. Ngo, T. Q. S. Quek, and H. Shin, "Random coding error exponent for dual-hop Nakagami-m fading channels with amplify-and-forward relaying," *IEEE Commun. Lett.*, vol. 13, no. 11, pp. 823–825, Nov. 2009.

[10] S. Ikki and M. Ahmed, "Performance analysis of dual hop relaying over non-identical Weibull fading channels," in *Proc. IEEE VTC*, Barcelona, Spain, Apr. 2009.

[11] M. D. Renzo, F. Graziosi, and F. Santucci, "A unified framework for performance analysis of CSI-assisted cooperative communications over fading channels," *IEEE Trans. Commun.*, vol. 57, no. 9, pp. 2551–2557, Sep. 2009.

[12] O. Waqar, D. McLernon, and M. Ghogho, "Exact evaluation of ergodic capacity for multihop variable-gain relay networks: A unified framework for generalized fading channels," *IEEE Trans. Veh. Technol.*, vol. 59, no. 8, pp. 4181–4187, Oct. 2010.

[13] M. K. Simon and M.-S. Alouini, *Digital Communication Over Fading Channels*, 2nd ed. New York: Wiley, 2005.

[14] H. Suzuki, "A statistical model for urban radio propagation," *IEEE Trans. Commun.*, vol. COM-25, no. 7, pp. 673–680, Jul. 1977.

[15] A. U. Sheikh, M. Handforth, and M. Abdi, "Indoor mobile radio channel at 946 MHz: Measurements and modeling," in *Proc. IEEE VTC*, Secaucus, NJ, May 1993, pp. 73–76.

[16] P. M. Shankar, "Error rates in generalized shadowed fading channels," *Wireless Pers. Commun.*, vol. 28, no. 3, pp. 233–238, Feb. 2004.

[17] P. S. Bithas, N. C. Sagias, P. T. Mathiopoulos, G. K. Karagiannidis, and A. A. Rontogiannis, "On the performance analysis of digital communications over generalized-K fading channels," *IEEE Commun. Lett.*, vol. 10, no. 5, pp. 353–355, May 2006.

[18] I. M. Kostic, "Analytical approach to performance analysis for channel subject to shadowing and fading," *Inst. Elect. Eng. Proc. Commun.*, vol. 152, no. 6, pp. 821–827, Dec. 2005.

[19] A. Laourine, M.-S. Alouini, S. Affes, and A. Stéphenne, "On the capacity of generalized-K fading channels," *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2441–2445, Jul. 2008.

[20] L. Wu, K. Niu, Z. He, W. Xu, and J. Lin, "Ergodic capacity of dual-hop transmissions over composite multipath/shadowing channels," *IET Electron. Lett.*, vol. 45, no. 19, pp. 975–976, Sep. 2009.

[21] C. Zhong, M. Matthaiou, G. K. Karagiannidis, and T. Ratnarajah, "Generic ergodic capacity bounds for fixed-gain AF dual-hop relaying systems," *IEEE Trans. Veh. Technol.*, vol. 60, no. 8, pp. 3814–3824, Oct. 2011.

[22] L. Wu, J. Lin, K. Niu, and Z. He, "Performance of dual-hop transmissions with fixed gain relays over generalized-K fading channels," in *Proc. IEEE ICC*, Dresden, Germany, Jun. 2009.

[23] Karmeshu and R. Agrawal, "On efficacy of Rayleigh-inverse Gaussian distribution over K-distribution for wireless fading channels," *Wireless Commun. Mobile Comput.*, vol. 7, no. 1, pp. 1–7, Jan. 2007.

[24] A. Laourine, M.-S. Alouini, S. Affes, and A. Stéphenne, "On the performance analysis of composite multipath/shadowing channels using the $\mathcal{G}$-distribution," *IEEE Trans. Commun.*, vol. 57, no. 4, pp. 1162–1170, Apr. 2009.

[25] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 5th ed. Orlando, FL: Academic, 1994.

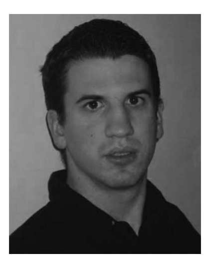
[26] R. S. Chhikara and J. L. Folks, *The Inverse Gaussian Distribution: Theory, Methodology, and Applications*. New York: Marcel Dekker, 1989.

[27] C. Loo, "A statistical model for a land mobile satellite link," *IEEE Trans. Veh. Technol.*, vol. VT-34, no. 3, pp. 122–127, Aug. 1985.

[28] *The Wolfram Functions Site*. [Online]. Available: http://functions.wolfram.com/Bessel-TypeFunctions/BesselK/20/01/01/0007/

Caijun Zhong (S'07–M'10) received the B.S. degree in information engineering from Xi'an Jiaotong University, Xi'an, China, in 2004 and the M.S. degree in information security and the Ph.D. degree in telecommunications from University College London, London, U.K., in 2006 and 2010, respectively.

From September 2009 to September 2011, he was a Research Fellow with the Institute for Electronics, Communications and Information Technologies, Queen's University Belfast, Belfast, U.K. Since September 2011, he has been with the Institute of Information and Communication Engineering, Zhejiang University, Hangzhou, China, where he is currently an Assistant Professor. His research interests include multivariate statistical theory, multiple-input–multiple-output communications systems, cooperative communications, and cognitive radio systems.



Michail Matthaiou (S'05–M'08) was born in Thessaloniki, Greece, in 1981. He received the Diploma degree in electrical and computer engineering from the Aristotle University of Thessaloniki, in 2004, the M.Sc. degree (with distinction) in Communications Systems and Signal Processing from the University of Bristol, Bristol, U.K., in 2005, and the Ph.D. degree from the University of Edinburgh, Edinburgh, U.K., in 2008.

From September 2008 to May 2010, he was with the Institute for Circuit Theory and Signal Processing, Munich University of Technology, Munich, Germany, working as a Postdoctoral Research Associate. In June 2010, he joined the Department of Signals and Systems, Chalmers University of Technology, Göteborg, Sweden, as an Assistant Professor, and in 2011, he received the Docent title. His research interests include signal processing for wireless communications, random matrix theory and multivariate statistics for multiple-input–multiple-output systems, and performance analysis of fading channels.

Dr. Matthaiou was an Exemplary Reviewer for the IEEE COMMUNICATIONS LETTERS for 2010. He has been a member of Technical Program Committees for several IEEE conferences, such as GLOBECOM and DSP. He currently serves as an Associate Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS and the IEEE COMMUNICATIONS LETTERS, as well as a Lead Guest Editor for the Special Issue on "Large-Scale Multiple Antenna Wireless Systems" of the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS. He is an Associate Member of the IEEE Signal Processing Society SPCOM Technical Committee. He received the 2011 IEEE ComSoc Young Researcher Award for the Europe, Middle East, and Africa Region and coreceived the 2006 IEEE Communications Chapter Project Prize for the best M.Sc. dissertation in the area of communications.

George K. Karagiannidis (SM'03) was born in Pithagorion, Greece. He received the University Diploma and the Ph.D. degree from the University of Patras, Patras, Greece, in 1987 and 1999, respectively, both in electrical and computer engineering.

From 2000 to 2004, he was a Senior Researcher with the Institute for Space Applications and Remote Sensing, National Observatory of Athens, Athens, Greece. In June 2004, he joined the faculty of Aristotle University of Thessaloniki, Thessaloniki, Greece, where he is currently an Associate Professor (four-level academic rank system) of digital communications systems with the Department of Electrical and Computer Engineering and Head of the Telecommunications Systems and Networks Laboratory. He is the author or coauthor of more than 160 technical papers published in scientific journals and presented at international conferences. He is also the author of the Greek edition of a book on "Telecommunications Systems" and a coauthor of the book *Advanced Wireless Communications Systems* (Cambridge, 2012). His research interests are digital communications systems, with emphasis on communications theory, energy-efficient multiple-input–multiple-output and cooperative communications, cognitive radio, wireless security, and optical wireless communications.

Dr. Karagiannidis has been a member of technical program committees for several IEEE conferences, such as the IEEE International Conference on Communications, GLOBECOM, and the IEEE Vehicular Technology Conference. In the past, he was the Editor for Fading Channels and Diversity of the IEEE TRANSACTIONS ON COMMUNICATIONS, the Senior Editor for the IEEE COMMUNICATIONS LETTERS, and an Editor for the *EURASIP Journal of Wireless Communications & Networks*. He was a Lead Guest Editor for the Special Issue on "Optical Wireless Communications" of the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS and currently serves as a Guest Editor for the Special Issue on "Large-Scale Multiple Antenna Wireless Systems" (to be published in February 2013). Since January 2012, he has been the Editor-in-Chief of the IEEE COMMUNICATIONS LETTERS. He was a corecipient of the Best Paper Award from the Wireless Communications Symposium at the ICC, Glasgow, U.K., in June 2007.

Aiping Huang (SM'08) received the degree from Nanjing Institute of Post and Telecommunications, Nanjing, China, in 1977, the M.S. degree in communication and electronic systems from Southeast University, Nanjing, in 1982, and the Licentiate of Technology degree from Helsinki University of Technology, Espoo, Finland, in 1997.

From 1977 to 1980, she was an Engineer with the Design and Research Institute, the Post and Telecommunication Ministry, China. From 1982 to 1994, she was with Zhejiang University, Hangzhou, China, as an Assistant Professor and then an Associate Professor with the Department of Scientific Instrumentation. She was a Research Scientist with Helsinki University of Technology from 1994 to 1998. She is currently a Full Professor and Director of the Institute of Information and Communication Engineering, Zhejiang University. She is also an Adjunct Professor with the Nanjing University of Posts and Telecommunications. She has published a book and more than 100 papers in refereed journals and conference proceedings on signal processing, communications, and networks. Her current research interests include cognitive wireless networks and planning/optimization of cellular mobile communication systems.

Prof. Huang currently serves as Vice Chair of the IEEE Communications Society Nanjing Chapter, Council Member of the China Institute of Communications, and member of the Standing Committee of the Zhejiang Provincial Institute of Communications.

Zhaoyang Zhang (M'02) received the B.Sc. degree in radio technology and the Ph.D. degree in communication and information systems from Zhejiang University, Hangzhou, China, in 1994 and 1998, respectively.

Since 1998, he has been with the Institute of Information and Communication Engineering, Zhejiang University, where he is currently a Full Professor. He has authored or coauthored about 150 papers and two books. His current research interests include information theory and coding theory, cooperative and mobile relays, cognitive radio and cognitive radio networks, and communications and network signal processing.

Dr. Zhang is currently serving as an Associate Editor for the *International Journal of Communication Systems*.