Amplify-and-Forward Relay Selection with Outdated Channel Estimates

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***Abstract*—We study the effect of outdated channel state information on the outage and error rate performance of amplify-and-forward (AF) relay selection, where only one out of the set of available relays is activated. We consider two variations of AF relay selection, namely *best relay selection* and *partial relay selection*, when the selection is based upon outdated channel estimates. For both these variations, closed-form expressions for the outage probability are obtained, along with approximate expressions for the symbol error rate in the medium to high signal-to-noise-ratio (SNR) regime. The diversity gain and coding gain of the above schemes are also explicitly derived. Numerical results manifest that the outage performance of AF relay selection is highly dependent on the level of correlation between the actual channel conditions and their corresponding (outdated) estimates. This result has a significant impact on the deployment of relay selection in practical applications, implying that a high feedback rate may be required in practice in order to attain the full benefits of relay selection. It is further shown that it may be preferable, in terms of outage and symbol error probability, not to include links in the relay selection process that experience a sufficiently high maximum Doppler shift, since in those cases partial relay selection outperforms best relay selection.**

***Index Terms*—Amplify-and-forward, outdated channel state information, relay selection.**

I. Introduction

VARIOUS cooperative relay schemes have been explored in the literature because their deployment in wireless networks has the potential of offering a number of significant performance benefits, including hotspot throughput improvements and cellular signal coverage enhancements [1]- [4]. As a result, future mobile broadband communication networks such as 3GPP LTE-Advanced (Release 11), IEEE 802.16j, and IEEE 802.16m are expected to support relay based communication. Relays in wireless networks can be classified as decode-and-forward (DF) relays, which decode and possibly re-encode the information before forwarding it, and amplify-and-forward (AF) relays, which forward the signal without hard decoding.

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The advantages of cooperative diversity come at the expense of a loss in spectral efficiency since the source and all the relays must transmit in orthogonal channels [5]. The inefficient utilization of the channel resources can be mitigated by relay selection. One such scheme is the *best relay selection* protocol [6]. In this scheme, a single best relay is selected for retransmission to the destination. Hence, only two orthogonal channels (regardless of the number of relays) are required in this case. It was shown in [7] that by selecting the relay with the best end-to-end path between the source and the destination, a diversity gain on the order of the number of relays in the network can be realized. However, in some applications such as resource-constrained ad hoc and sensor networks, monitoring the connectivity of all links can limit the network lifetime. Such challenges have motivated the development of *partial relay selection* schemes, which require channel state information (CSI) of only the source-relay links, or alternatively, only the relay-destination links [9].

There have been several works in the literature dealing with the concept of relay selection (see, e.g., [6]- [26] and the references therein). Most of these works deal with the case where perfect CSI is available for relay selection, with the exception of the recent works in [22]- [27], where the issue of imperfect CSI was tackled. For example, in [22], the outage probability and the asymptotic behavior of DF best relay selection with outdated channel estimates was studied, and in [23], the capacity of AF relaying with symmetric level of imperfect CSI on the source-relay and relay-destination channels was investigated. However, the effect of outdated CSI on the diversity order and the outage performance of AF relay selection for the asymmetric case, where the outdated channel estimates in the source-relay and relay-destination links are not necessarily symmetrically correlated with the actual channel values, has not been investigated in the literature. Finally, an analysis of the diversity behavior of relay selection in cases where CSI imperfection does not necessarily stem from the time-varying nature of fading channels, but stems from noisy channel estimation as well, is provided in [27].

It is worth pointing out that the time varying nature of fading channels renders the case of outdated CSI highly relevant since the available CSI is rarely perfect in practice, because of feedback delays. To this end, in this paper, we provide closed-form expressions for the outage probability and the symbol error rate (SER) for the AF best relay selection and the partial relay selection schemes, described above, along with an explicit analysis for the achievable diversity order and coding gain. Such analysis leads to interesting numerical results, which demonstrate that the performance of relay selection is highly dependent on the correlation between the actual CSI of the participating links and the corresponding estimates. It is also shown that AF best relay selection is more sensitive to outdated CSI than DF best relay selection, indicating that the relative implementation simplicity of AF relaying, as compared to DF relaying, is compensated by its more stringent requirement for accurate channel estimation, when applied in best relay selection setups.

In addition, this work demonstrates that partial relay selection may outperform best relay selection in cases where the maximum Doppler frequency in either the source-relay or the relay-destination links is higher than a certain threshold. We note that this is in sharp contrast to the case where relay selection is based upon perfect CSI knowledge, a case extensively analyzed in the literature, where there exists a huge performance gap between best relay selection and partial relay selection, especially when a large number of relays are employed. This is because for perfect CSI the diversity order of best relay selection grows linearly with the number of relays, whereas that of partial relay selection is always unity.

The remainder of this paper is organized as follows. The system model as well as the modelling of the outdated CSI are presented in the ensuing section. In Section III, closed-form expressions for the outage probability of best relay selection and partial relay selection with outdated channel estimates are derived, while the corresponding SER expressions for both selection schemes in the high signal-to-noise-ratio (SNR) regime are given in Section IV. Numerical performance results for the systems under consideration and related discussions are provided in Section V. Finally, Section VI concludes the paper.

II. System Model

We consider the cooperative relaying setup illustrated in Fig. 1, which consists of a single source terminal, S, N relays denoted by R_i, $i=1,\dots,N$, and a single destination terminal, D. All relays operate in the half-duplex AF mode; they also employ the so-called CSI-assisted variable gain, where the relaying gain depends on the instantaneous channel amplitude of the corresponding S-R_i link. The fading in all S-R_i and R_i-D paths is assumed to be independent and identically distributed according to the Rayleigh distribution. In addition, no direct S-D link is assumed. Hence, S can communicate with D only via the relay terminals. We denote the circularly symmetric complex Gaussian channel gain between nodes A and B as h_{AB}; we use the notation γ_{AB} to refer to the instantaneous SNR of link A-B, so that $\gamma_{AB} = |h_{AB}|^2/N_0$, where we have assumed for simplicity that all nodes transmit with unit power and N_0 represents the additive white Gaussian noise (AWGN) power. Moreover, we use the notations $f_X(\cdot)$ and $F_X(\cdot)$ to refer to the probability density function (pdf) and the cumulative density function (cdf) of random variable (RV) X , respectively.

Among the N available relays only the relay with the highest instantaneous SNR is supposed to forward the signal transmitted by the source, S, to the destination, D. Selecting the "best" relay from the set of available relays results in a somewhat opportunistic usage of the relays, as pointed out in [7]. The selection is assumed to be made in a central unit (CU), which collects all information regarding the instantaneous channel states and feeds the result of the selection process back to the relays.

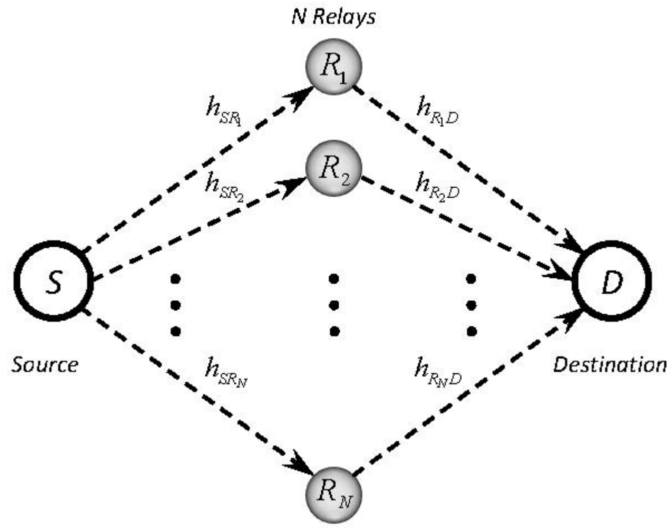

Fig. 1. System model.

A. CSI Model

In contrast to the majority of works on CSI-assisted AF relays, in this paper, we assume that the CU has *outdated CSI*. Thus, the selection of the "best" relay is not based on the current time instant because of e.g. a feedback delay. Therefore, denoting the outdated channel gains of the S-R_i and R_i-D links at the time of selection by $\hat{h}_{SR_i}$ and $\hat{h}_{R_iD}$, respectively, we model the imperfect CSI that the CU has for the ith relay as [22], [28]

$$\hat{h}_{SR_i} = \rho_1 h_{SR_i} + \left(\sqrt{1-\rho_1^2}\right) w_{SR_i} \tag{1}$$

and

$$\hat{h}_{R_iD} = \rho_2 h_{R_iD} + \left(\sqrt{1-\rho_2^2}\right) w_{R_iD} \tag{2}$$

where w_{SR_i} and w_{R_iD} are circularly symmetric complex Gaussian RVs having the same variance as RVs h_{SR_i} and h_{R_iD}, respectively; ρ_1 (ρ_2) is the correlation coefficient between h_{SR_i} and $\hat{h}_{SR_i}$ (h_{R_iD} and $\hat{h}_{R_iD}$), which, using the Jakes' autocorrelation model [29], is given by

$$\rho_1 = J_0(2\pi f_{d,SR} T_d) \tag{3}$$
$$\rho_2 = J_0(2\pi f_{d,RD} T_d). \tag{4}$$

Here, $J_0(\cdot)$ denotes the zeroth order Bessel function of the first kind [30, Eq. (8.411)]; $f_{d,AB}$ is the maximum Doppler frequency on the A-B link, and T_d is the time difference between the actual channel value and its estimate. Due to the CSI-assisted AF mode of operation, the end-to-end SNR for the ith relay, γ_i, $i=1,...,N$, can be expressed as [5]

$$\gamma_i = \frac{\gamma_{SR_i}\gamma_{R_iD}}{\gamma_{SR_i}+\gamma_{R_iD}+1}. \tag{5}$$

It is noted that although the CSI at the CU used for relay selection is outdated, the CSI of the selected path at the relay and the destination is assumed perfect, so as not to cause additional detection errors. The reasoning behind this assumption lies in the fact that the time-repetition rates between the above two processes are generally different. It is further assumed that there exists symmetry among the S-R_i and R_i-D links, respectively, in the sense that the fading on the S-R_i links is independent and identically distributed (i.i.d.), so that the average SNR in all the S-R_i links is given by $\bar{\gamma}_{SR}$; the R_i-D links are also assumed to be i.i.d., with average SNR $\bar{\gamma}_{RD}$. Hence, due to the symmetry, the pdfs of γ_{SR_i} and γ_{R_iD}, $i = 1, ..., N$, are denoted by $f_{\gamma_{SR}}(\cdot)$ and $f_{\gamma_{RD}}(\cdot)$, respectively; the cdfs of γ_{SR_i} and γ_{R_iD} are likewise denoted by $F_{\gamma_{SR}}(\cdot)$ and $F_{\gamma_{RD}}(\cdot)$, respectively.

B. Best Relay Selection

In this paper, we use the term "best relay selection" to refer to the case where the decision on the selected relay is determined by both the S-R_i and R_i-D links. In particular, we assume that the relay with the strongest "bottleneck" link is selected. Hence, the end-to-end SNR that the CU uses for selection based on outdated estimates is given by [7], [8]

$$\hat{\gamma}_i = \min\left(\hat{\gamma}_{SR_i}, \hat{\gamma}_{R_iD}\right) \tag{6}$$

where $\hat{\gamma}_{AB} = \left|\hat{h}_{AB}\right|^2 / N_0$ denotes the estimate of γ_{AB}. The CU thus activates the relay that satisfies the following condition

$$k = \arg\max_i \left(\hat{\gamma}_i\right). \tag{7}$$

It is noted that (6) represents an upper bound of the estimate of γ_i given in (5), resulting in approximately the same performance as if the best relay is selected based on the exact end-to-end SNR, shown in (5).

C. Partial Relay Selection

In cases where the CU has information only of either the S-R_i or the R_i-D path, the selection procedure is modified accordingly so that only one of the two links is taken into account. In the sequel, partial relay selection refers to the case where only the S-R_i CSI is available; the selected relay is thus determined by [9]

$$k = \arg\max_i \left(\hat{\gamma}_{SR_i}\right). \tag{8}$$

Likewise, the analysis can be easily extended to the case where the selection is based upon the channel conditions of the R_i-D links only. That is, a single relay is selected according to

$$k = \arg\max_i \left(\hat{\gamma}_{R_iD}\right). \tag{9}$$

III. Outage Probability

The outage probability is defined as the probability that the system cannot support a certain target data rate, which is equivalent to the probability that the overall SNR is lower than a threshold, γ_T; this threshold is related to the target rate r via $\gamma_T = 2^{2r} - 1$.

A. Best Relay Selection

Proposition 1: The outage probability of AF best relay selection with outdated channel estimates is given as shown in (10) at the top of the next page, where $\mathcal{S}(x, y, n, \rho) = x\left[1 + n\left(1-\rho^2\right)\right] + y(n+1)\left(1-\rho^2\right)$.

Proof: The proof is given in Appendix A. ■

One may note that, despite its length, (10) can be readily evaluated since it involves only simple algebraic operations, while the first order modified Bessel function of the second kind [30, Eq. (8.432.1)], $K_1(\cdot)$, is embedded in most mathematical software packages. Moreover, we note that the outage expression is simplified for the case where the fading in all the links involved is i.i.d., and ρ_1 and ρ_2 are equal to each other, so that by assuming $\bar{\gamma}_{SR} = \bar{\gamma}_{RD} = \bar{\gamma}$ and $\rho_1 = \rho_2 = \rho$, (10) reduces to (11) shown at the top of page 5. We note that the i.i.d. fading assumption implies that the relays are symmetrically located in a cluster, between the source and the destination.

1) Asymptotic Outage Behavior: In order to obtain useful insights into the diversity order and the coding gain of best relay selection in outdated CSI, an asymptotic outage analysis is presented. For this purpose, let us consider the symmetric scenario of i.i.d fading, i.e., $\bar{\gamma}_{SR} = \bar{\gamma}_{RD} = \bar{\gamma}$. Using the McLaurin series representation for the exponential and the $K_1(\cdot)$ function in (10), and considering only the first order terms, after some algebraic manipulations we obtain

$$F_{\gamma_k}(\gamma_T) \approx \begin{cases} 2^N\left(\frac{\gamma_T}{\bar{\gamma}}\right)^N, & \text{if } \rho_1 = \rho_2 = 1 \\ \mathcal{A}_1\frac{\gamma_T}{\bar{\gamma}} + \mathcal{A}_2\frac{\gamma_T}{\bar{\gamma}}, & \text{if } \rho_1 < 1 \text{ or } \rho_2 < 1 \end{cases} \tag{12}$$

where $\mathcal{A}_1$ and $\mathcal{A}_2$ are SNR-independent constants defined as

$$\mathcal{A}_1 = N\sum_{n=0}^{N-1}\frac{(-1)^n\binom{N-1}{n}}{2n+1}\left(1 + \frac{2n}{\mathcal{S}(1,1,n,\rho_2)}\right) \tag{13}$$

$$\mathcal{A}_2 = N^2\sum_{n=0}^{N-1}\sum_{m=0}^{N-1}\frac{(-1)^{n+m}\binom{N-1}{n}\binom{N-1}{m}\left(2-\rho_1^2\right)}{(m+1)\,\mathcal{S}(1,1,n,\rho_1)}. \tag{14}$$

Two interesting Corollaries follow from (12)-(14), as summarized below.

Corollary 1: The diversity order of AF best relay selection with outdated CSI is given as

$$d = \begin{cases} N, & \text{if } \rho_1 = \rho_2 = 1 \\ 1, & \text{if } \rho_1 < 1 \text{ or } \rho_2 < 1 \end{cases}. \tag{15}$$

Proof: The proof is provided in Appendix B. ■

The main result obtained from *Corollary 1* is that AF best relay selection achieves full diversity order only if the channel estimates of both the S-R_i and R_i-D links are perfectly updated. Any deviation from the ideal case of perfectly updated CSI in either the S-R_i or the R_i-D links results in unitary diversity order, i.e., loss of the diversity potential, regardless of the number of available relays. We note, however, that this corresponds to the strict mathematical definition of the diversity order, which refers to infinitely high SNRs. Further discussions on the practical aspects of the diversity loss caused by outdated CSI are provided in Section V-B.

In fact, *Corollary* 1 is in agreement with the intuition that outdated channel estimates lead to a reduced diversity order,

$$
\begin{aligned}
F_{\gamma_k}(\gamma_T) &= N\sum_{n=0}^{N-1}\frac{\left(\exp\left(-\frac{(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})(n+1)\gamma_T}{\bar{\gamma}_{RD}\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},n,\rho_2)}\right)-1\right)n\bar{\gamma}_{SR}+e^{-\frac{\gamma_T}{\bar{\gamma}_{RD}}}\left(1-e^{\frac{\gamma_T}{\bar{\gamma}_{RD}}}\right)(n+1)\bar{\gamma}_{RD}}{\left[(-1)^{n+1}\binom{N-1}{n}\right]^{-1}(n+1)\left(\bar{\gamma}_{RD}+n\bar{\gamma}_{SR}+n\bar{\gamma}_{RD}\right)}\\
&+N^2\sum_{n=0}^{N-1}\sum_{m=0}^{N-1}\frac{(-1)^{n+m}\binom{N-1}{m}\binom{N-1}{n}}{\left(\bar{\gamma}_{RD}+m\bar{\gamma}_{SR}+m\bar{\gamma}_{RD}\right)\left(\bar{\gamma}_{SR}+n\bar{\gamma}_{SR}+n\bar{\gamma}_{RD}\right)}\Bigg\{e^{-\frac{\gamma_T(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\bar{\gamma}_{SR}\\
&\times\left[e^{\frac{\gamma_T}{\bar{\gamma}_{SR}}}\bar{\gamma}_{RD}-2\sqrt{\frac{\bar{\gamma}_{RD}\gamma_T(\gamma_T+1)}{\bar{\gamma}_{SR}}}K_1\left(2\sqrt{\frac{\gamma_T(\gamma_T+1)}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)\right]+\frac{n\bar{\gamma}_{RD}e^{-\frac{\gamma_T}{\bar{\gamma}_{RD}}}}{(n+1)}\Bigg[\bar{\gamma}_{RD}-2e^{-\frac{(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})(n+1)\gamma_T}{\bar{\gamma}_{SR}\mathcal{S}(\bar{\gamma}_{RD},\bar{\gamma}_{SR},n,\rho_1)}}\\
&\times\sqrt{\frac{\bar{\gamma}_{RD}(\gamma_T+1)(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})(n+1)\gamma_T}{\bar{\gamma}\mathcal{S}(\bar{\gamma}_{RD},\bar{\gamma}_{SR},n,\rho_1)}}K_1\left(\sqrt{\frac{4\bar{\gamma}_{RD}(\gamma_T+1)(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})(n+1)\gamma_T}{\bar{\gamma}_{SR}\mathcal{S}(\bar{\gamma}_{RD},\bar{\gamma}_{SR},n,\rho_1)}}\right)\Bigg]\\
&+m\bar{\gamma}_{SR}\exp\left(-\frac{\gamma_T\left[\bar{\gamma}_{SR}^2(m+1)+\bar{\gamma}_{SR}\bar{\gamma}_{RD}\left(2+m\left(2-\rho_2^2\right)\right)+\bar{\gamma}_{RD}^2(m+1)\left(1-\rho_2^2\right)\right]}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},m,\rho_2)}\right)\\
&\times\left[\bar{\gamma}_{SR}e^{\frac{\gamma_T}{\bar{\gamma}_{SR}}}-\frac{2\sqrt{\gamma_T(\gamma_T+1)\bar{\gamma}_{SR}(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})(m+1)}K_1\left(2\sqrt{\frac{(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})(m+1)\gamma_T(\gamma_T+1)}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},m,\rho_2)}}\right)}{\sqrt{\bar{\gamma}_{RD}\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},m,\rho_2)}}\right]\\
&+\frac{mn\bar{\gamma}_{SR}(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})}{n+1}\Bigg[\frac{\bar{\gamma}_{RD}\exp\left(-\frac{(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})(m+1)\gamma_T}{\bar{\gamma}_{RD}\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},m,\rho_2)}\right)}{(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})(m+1)}\\
&+\frac{\exp\left(-\frac{(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})\gamma_T\left[\bar{\gamma}_{SR}^2(m+1)(n+1)\left(1-\rho_1^2\right)+\bar{\gamma}_{RD}^2(m+1)(n+1)\left(1-\rho_2^2\right)+\bar{\gamma}_{SR}\bar{\gamma}_{RD}\left(2+n\left(2-\rho_1^2\right)+m\left(2-\rho_2^2+n\left(2-\rho_1^2-\rho_2^2\right)\right)\right)\right]}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}\mathcal{S}(\bar{\gamma}_{RD},\bar{\gamma}_{SR},n,\rho_1)\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},m,\rho_2)}\right)}{\sqrt{\bar{\gamma}_{SR}(m+1)}}\\
&\times 2\sqrt{\frac{\bar{\gamma}_{RD}\gamma_T(\gamma_T+1)(n+1)\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},m,\rho_2)}{\mathcal{S}(\bar{\gamma}_{RD},\bar{\gamma}_{SR},n,\rho_1)}}K_1\left(\sqrt{\frac{4(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})^2(m+1)(n+1)\gamma_T(\gamma_T+1)}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}\mathcal{S}(\bar{\gamma}_{RD},\bar{\gamma}_{SR},n,\rho_1)\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},m,\rho_2)}}\right)\Bigg]\Bigg\} \qquad (10)
\end{aligned}
$$

since, loosely speaking, the notion of diversity lies in taking full advantage of the best actual available path between source and destination. As also pointed out in [22], outdated channel estimates imply that there a exists a non-negligible probability that, instead of the actual best relay, the n-th best relay is selected, resulting in a diversity order of $N-(n-1)$. In the infinitely high SNR region, the probability that the worst (i.e., the N-th best) relay is selected becomes non-negligible, leading to a diversity order of one.

Corollary 2: The coding gain of AF best relay selection with outdated CSI, $\mathcal{C}_B$, is given by

$$
\mathcal{C}_B=\begin{cases}1/2, & \text{if } \rho_1=\rho_2=1\\ \left[N^2\sum\limits_{n=0}^{N-1}\sum\limits_{m=0}^{N-1}\frac{(-1)^{n+m}\binom{N-1}{n}\binom{N-1}{m}\left(2-\rho_1^2\right)}{(m+1)\left[(2n+1)\left(1-\rho_1^2\right)+1\right]}\right. & \\ \quad +N\sum\limits_{n=0}^{N-1}\frac{(-1)^n\binom{N-1}{n}}{2n+1} & \text{if } \rho_1<1 \text{ or } \rho_2<1\\ \quad \left.\times\left(1+\frac{2n}{(2n+1)\left(1-\rho_2^2\right)+1}\right)\right]^{-1}, & \end{cases} \qquad (16)
$$

Proof: For the case of $\rho_1<1$ or $\rho_2<1$, the proof is derived straightforwardly from (12), (13) and (14), by expressing the outage probability in the form $F_{\gamma_k}(\gamma_T)\approx(\mathcal{C}_B\bar{\gamma}/\gamma_T)^{-d}$. For the case of $\rho_1=\rho_2=1$, the proof is provided in Appendix B[1]. ■

B. Partial Relay Selection

Proposition 2: The outage probability of AF partial relay selection with outdated channel estimates is given by

$$
\begin{aligned}
F_{\gamma_k}(\gamma_T) &= 1-2N\sum_{m=0}^{N-1}\Bigg[(-1)^m\binom{N-1}{m}\\
&\times\sqrt{\frac{\gamma_T(\gamma_T+1)}{\bar{\gamma}_{RD}\bar{\gamma}_{SR}(m+1)\left[1+m\left(1-\rho_1^2\right)\right]}}\\
&\times\exp\left(-\frac{\left[\bar{\gamma}_{RD}+\bar{\gamma}_{SR}+m\left(\bar{\gamma}_{RD}+\bar{\gamma}_{SR}-\bar{\gamma}_{SR}\rho_1^2\right)\right]\gamma_T}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}\left[1+m\left(1-\rho_1^2\right)\right]}\right)\\
&\times K_1\left(2\sqrt{\frac{(m+1)\gamma_T(\gamma_T+1)}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}\left[1+m\left(1-\rho_1^2\right)\right]}}\right)\Bigg]. \qquad (17)
\end{aligned}
$$

Proof: The proof is provided in Appendix C. ■

1) Asymptotic Outage Behavior: Using the McLaurin series representation in (17) and using a similar approach as in Section III-A1, we obtain the asymptotic outage probability of partial relay selection for the symmetric case of $\bar{\gamma}_{SR}=$

[1]The case of $\rho_1=\rho_2=1$ corresponds to best relay selection with perfect CSI, the asymptotic analysis of which is well-studied in the literature. For completeness of the analysis, *Corollary* 2 is rigorously proved based on the analysis of Section III, in Appendix B.

$$
\begin{aligned}
F_{\gamma_k}(\gamma_T) = {} & \sum_{n=0}^{N-1} \frac{N\left(n+1+ne^{-\frac{(2n+1)\gamma_T\rho^2}{\bar{\gamma}\mathcal{S}(1,1,n,\rho)}}-(2n+1)\,e^{\frac{\gamma_T}{\bar{\gamma}}}\right)}{e^{\frac{\gamma_T}{\bar{\gamma}}}\left[(-1)^{n+1}\binom{N-1}{n}\right]^{-1}(n+1)(2n+1)} + \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} \frac{N^2(-1)^{n+m+1}(n+1)^{-1}\binom{N-1}{n}\binom{N-1}{m}}{\bar{\gamma}(2m+1)(2n+1)S(1,1,m,\rho)} \\
& \times \left[\frac{2n+1}{n+1}\left(\bar{\gamma}e^{-\frac{\gamma_T}{\bar{\gamma}}}+\frac{m\bar{\gamma}e^{-\frac{2(m+1)\gamma_T}{\bar{\gamma}\mathcal{S}(1,1,m,\rho)}}}{m+1}\right)-2e^{-\frac{2\gamma_T}{\bar{\gamma}}}\sqrt{\gamma_T+\gamma_T^2}K_1\left(\frac{2\sqrt{\gamma_T+\gamma_T^2}}{\bar{\gamma}}\right)\right. \\
& -2\left[m\frac{e^{-\frac{\gamma_T[\mathcal{S}(1,1,m,\rho)+2m+2]}{\bar{\gamma}\mathcal{S}(1,1,m,\rho)}}K_1\left(2\sqrt{\frac{2(m+1)\left(\gamma_T+\gamma_T^2\right)}{\bar{\gamma}^2\mathcal{S}(1,1,m,\rho)}}\right)}{\sqrt{(m+1)\mathcal{S}(1,1,m,\rho)}\left[2\left(\gamma_T+\gamma_T^2\right)\right]^{-\frac{1}{2}}}+n\frac{e^{-\frac{\gamma_T[\mathcal{S}(1,1,n,\rho)+2n+2]}{\bar{\gamma}\mathcal{S}(1,1,n,\rho)}}K_1\left(2\sqrt{\frac{2(n+1)\left(\gamma_T+\gamma_T^2\right)}{\bar{\gamma}^2\mathcal{S}(1,1,n,\rho)}}\right)}{\sqrt{(n+1)\mathcal{S}(1,1,n,\rho)}\left[2\left(\gamma_T+\gamma_T^2\right)\right]^{-\frac{1}{2}}}\right] \\
& \left.+4mn\sqrt{\frac{e^{-\frac{4\gamma_T\left[(n+2)-(3n+2)\left(1-\rho^2\right)+m\left((4n+3)\left(1-\rho^2\right)+1\right)\right]}{\bar{\gamma}\mathcal{S}(1,1,m,\rho)\mathcal{S}(1,1,n,\rho)}}(n+1)\mathcal{S}(1,1,m,\rho)\,K_1^2\left(4\sqrt{\frac{(m+1)(n+1)\left(\gamma_T+\gamma_T^2\right)}{\bar{\gamma}^2\mathcal{S}(1,1,m,\rho)\mathcal{S}(1,1,n,\rho)}}\right)}{(m+1)\mathcal{S}(1,1,n,\rho)\left(\gamma_T+\gamma_T^2\right)^{-1}}}\right]. \qquad (11)
\end{aligned}
$$

$\bar{\gamma}_{RD} = \bar{\gamma}$ as follows

$$
\begin{aligned}
F_{\gamma_k}(\gamma_T) \approx {} & N\sum_{m=0}^{N-1}\frac{(-1)^m}{m+1}\binom{N-1}{m} \\
& \times\left(\frac{m+1}{m(1-\rho_1^2)+1}+1\right)\left(\frac{\gamma_T}{\bar{\gamma}}\right). \qquad (18)
\end{aligned}
$$

The tightness of the approximation given in (18) is discussed later, in Section V.

We note that, due to the fact that only the S-R or the R-D channels are used for relay selection, the diversity order of partial relay selection is one, regardless of the value of ρ_1. The coding gain, $\mathcal{C}_P$, is summarized in the ensuing Corollary.

Corollary 3: The coding gain of AF partial relay selection is given by

$$
\begin{aligned}
\mathcal{C}_P = {} & \left\{N\sum_{m=0}^{N-1}\frac{(-1)^m}{m+1}\binom{N-1}{m}\right. \\
\times \quad & \left.\left(\frac{m+1}{m(1-\rho_1^2)+1}+1\right)\right\}^{-1}, \ 0 \le \rho_1 \le 1 \qquad (19)
\end{aligned}
$$

Proof: By expressing the outage probability in the form $F_{\gamma_k}(\gamma_T) \approx (\mathcal{C}_P\bar{\gamma}/\gamma_T)^{-d}$, the (19) follows directly from (18). ■

IV. Error Rate Analysis

In this section, we derive closed-form approximate expressions for the SER of best relay selection and partial relay selection in the medium-to-high SNR regime. Having found an expression for the outage probability of single relay selection with outdated CSI, we may also derive an expression for the SER as follows.

For BPSK and M-PAM, as well M-PSK, M-QAM, and M-FSK modulations and sufficiently high values of SNR, the SER can be expressed as [21]

$$
P_e = \mathcal{E}_{\gamma_k}\left\langle BQ\left(\beta\sqrt{\gamma_k}\right)\right\rangle \qquad (20)
$$

where $Q(\cdot)$ is the Gaussian Q-function, $\mathcal{E}\langle\cdot\rangle$ is the expectation operator, and B and β are constants depending on the type of modulation, e.g., for BPSK $B=1$; $\beta=2$. Let us assume a RV X that follows the standard Gaussian distribution, i.e., $X \sim \mathcal{N}(0,1)$. Using the definition of the Q-function, (20) can be expressed as

$$
\begin{aligned}
P_e &= \mathcal{E}_{\gamma_k}\left\langle B\Pr\left\{X > \sqrt{\beta\gamma_k}\right\}\right\rangle = \mathcal{E}_{\gamma_k}\left\langle \frac{B}{2}\Pr\left\{\gamma_k < \frac{X^2}{\beta}\right\}\right\rangle \\
&= B\int_0^\infty F_{\gamma_k}\left(\frac{x^2}{\beta}\right)f_X(x)\,dx \qquad (21)
\end{aligned}
$$

where $f_X(x)$ is the pdf of RV X.

A. Best Relay Selection

The SER of best relay selection with outdated channel estimates is obtained by substituting (10) into (21). Nonetheless, the involvement of special functions in (10) renders the derivation of an exact closed-form SER expression a cumbersome, if not impossible, task. Instead, we focus on deriving a medium-to-high SNR approximation for the SER of best relay selection, as follows. First, we obtain an approximate expression for the outage probability, by using $K_1(z) \approx 1/z$, $0 < z \ll \sqrt{2}$ [31, Eq. (9.6.9)] in (10), yielding (22) shown at the top of page. Then, by inserting (22) into (21) and after some elementary integrations we obtain an expression for the SER as shown in (23) at the top of page.

B. Partial Relay Selection

Similarly as for the case of best relay selection, deriving an exact closed-form expression for the SER of partial relay selection is cumbersome. For sufficiently high SNR values, however, an approximate expression for the outage probability of partial relay selection is obtained in a similar way as in

$$
\begin{aligned}
F_{\gamma_k}(\gamma_T) \approx\; & N \sum_{n=0}^{N-1} \frac{\left(1-\exp\left(-\frac{(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})(n+1)\gamma_T}{\bar{\gamma}_{RD}\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},n,\rho_2)}\right)\right)\bar{\gamma}_{SR}n + \left(1-e^{-\frac{\gamma_T}{\bar{\gamma}_{RD}}}\right)\bar{\gamma}_{RD}(n+1)}{\left[(-1)^n\binom{N-1}{n}\right]^{-1}(n+1)\left(\bar{\gamma}_{RD}+n\bar{\gamma}_{SR}+n\bar{\gamma}_{RD}\right)} \\
& + N^2 \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} \frac{(-1)^{n+m+1}\binom{N-1}{m}\binom{N-1}{n}}{\left(\bar{\gamma}_{RD}+m\bar{\gamma}_{SR}+m\bar{\gamma}_{RD}\right)\left(\bar{\gamma}_{SR}+n\bar{\gamma}_{SR}+n\bar{\gamma}_{RD}\right)} \Bigg\{ \bar{\gamma}_{SR}\bar{\gamma}_{RD}\exp\left(-\frac{\gamma_T}{\bar{\gamma}}\right) \\
& + \frac{\bar{\gamma}_{SR}^2 m \exp\left(-\frac{\gamma_T\left[\bar{\gamma}_{SR}^2(m+1)+\bar{\gamma}_{SR}\bar{\gamma}_{RD}\left(2+m\left(2-\rho_2^2\right)\right)+\bar{\gamma}_{RD}^2(m+1)\left(1-\rho_2^2\right)\right]}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},m,\rho_2)}\right)}{m+1} \\
& + \frac{\bar{\gamma}_{RD}^2 n \exp\left(-\gamma_T\left(\frac{1}{\bar{\gamma}_{RD}}+\frac{(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})(n+1)}{\bar{\gamma}_{SR}\mathcal{S}(\bar{\gamma}_{RD},\bar{\gamma}_{SR},n,\rho_1)}\right)\right)}{n+1} \\
& + \frac{\bar{\gamma}_{SR}\bar{\gamma}_{RD} mn \exp\left(-\frac{\gamma_T\left[\bar{\gamma}_{SR}^2(m+1)(n+1)\left(1-\rho_1^2\right)+\bar{\gamma}_{RD}^2(m+1)(n+1)\left(1-\rho_2^2\right)+\bar{\gamma}_{SR}\bar{\gamma}_{RD}\left(2+n\left(2-\rho_1^2\right)+m\left(2-\rho_2^2+n\left(2-\rho_1^2-\rho_2^2\right)\right)\right)\right]}{\bar{\gamma}\mathcal{S}(\bar{\gamma}_{RD},\bar{\gamma}_{SR},n,\rho_1)\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},m,\rho_2)}\right)}{(n+1)(m+1)} \\
& - \frac{\left(\bar{\gamma}_{SR}+n\bar{\gamma}_{SR}+n\bar{\gamma}_{RD}\right)}{n+1}\left[\bar{\gamma}_{RD}\exp\left(-\frac{\gamma_T}{\bar{\gamma}_{RD}}\right)+\frac{m\bar{\gamma}_{SR}\exp\left(-\frac{(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})(m+1)\gamma_T}{\bar{\gamma}_{RD}\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},m,\rho_2)}\right)}{(m+1)}\right]\Bigg\}. \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
P_e \approx\; & \frac{NB}{2}\sum_{n=0}^{N-1} \frac{\bar{\gamma}_{SR}n\left(1-\sqrt{\frac{\beta\bar{\gamma}_{RD}\left(\bar{\gamma}_{SR}\left(1+n\left(1-\rho_2^2\right)\right)+\bar{\gamma}_{RD}(1+n)\left(1-\rho_2^2\right)\right)}{\beta\bar{\gamma}_{RD}\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},n,\rho_2)+2(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})(n+1)}}\right)+\bar{\gamma}_{RD}(n+1)\left(1-\sqrt{\frac{\beta\bar{\gamma}_{RD}}{\beta\bar{\gamma}_{RD}+2}}\right)}{\left[(-1)^n\binom{N-1}{n}\right]^{-1}(n+1)\left(\bar{\gamma}_{RD}+n\bar{\gamma}_{SR}+n\bar{\gamma}_{RD}\right)} \\
& + \frac{N^2 B}{2}\sum_{n=0}^{N-1}\sum_{m=0}^{N-1} \frac{(-1)^{n+m+1}\binom{N-1}{m}\binom{N-1}{n}}{(m+1)(n+1)\left(\bar{\gamma}_{RD}+m\bar{\gamma}_{SR}+m\bar{\gamma}_{RD}\right)\left(\bar{\gamma}_{SR}+n\bar{\gamma}_{SR}+n\bar{\gamma}_{RD}\right)} \\
& \times\Bigg\{\bar{\gamma}_{SR}\bar{\gamma}_{RD}\sqrt{\frac{\beta\bar{\gamma}}{\beta\bar{\gamma}+2}} + \frac{\bar{\gamma}_{SR}^2 m(n+1)}{\sqrt{1+\frac{2\bar{\gamma}_{SR}^2(m+1)+2\bar{\gamma}_{SR}\bar{\gamma}_{RD}\left(2+m\left(2-\rho_2^2\right)\right)+2\bar{\gamma}_{RD}^2(m+1)\left(1-\rho_2^2\right)}{\beta\bar{\gamma}_{SR}\bar{\gamma}_{RD}\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},m,\rho_2)}}} \\
& + \frac{\bar{\gamma}_{RD}^2 n(m+1)}{\sqrt{1+\frac{2\bar{\gamma}_{SR}\mathcal{S}(\bar{\gamma}_{RD},\bar{\gamma}_{SR},n,\rho_1)+2\bar{\gamma}_{RD}(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})(n+1)}{\beta\bar{\gamma}_{SR}\mathcal{S}(\bar{\gamma}_{RD},\bar{\gamma}_{SR},n,\rho_1)}}} \\
& + \frac{\bar{\gamma}_{SR}\bar{\gamma}_{RD}mn}{\sqrt{1+\frac{2\left[\bar{\gamma}_{SR}^2(m+1)(n+1)\left(1-\rho_1^2\right)+\bar{\gamma}_{RD}^2(m+1)(n+1)\left(1-\rho_2^2\right)+\bar{\gamma}_{SR}\bar{\gamma}_{RD}\left(2+n\left(2-\rho_1^2\right)+2m\left(2-\rho_2^2+n\left(2-\rho_1^2-\rho_2^2\right)\right)\right)\right]}{\beta\bar{\gamma}\mathcal{S}(\bar{\gamma}_{RD},\bar{\gamma}_{SR},n,\rho_1)\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},m,\rho_2)}}} \\
& - \frac{\left(\bar{\gamma}_{SR}+n\bar{\gamma}_{SR}+n\bar{\gamma}_{RD}\right)}{(m+1)^{-1}}\left[\frac{\bar{\gamma}_{RD}\sqrt{\beta\bar{\gamma}_{RD}}}{\sqrt{\beta\bar{\gamma}_{RD}+2}}+\frac{m\bar{\gamma}_{SR}}{(m+1)\sqrt{1+\frac{2(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})(m+1)}{\beta\bar{\gamma}_{RD}\mathcal{S}(\bar{\gamma}_{SR},\bar{\gamma}_{RD},m,\rho_2)}}}\right]\Bigg\}. \qquad (23)
\end{aligned}
$$

pt

Section IV-A, yielding

$$
F_{\gamma_k}(\gamma_T) \approx 1 - N\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{m+1} \times e^{-\left(\frac{m+1}{(m(1-\rho_1^2)+1)\bar{\gamma}_{SR}}+\frac{1}{\bar{\gamma}_{RD}}\right)\gamma_T}. \qquad (24)
$$

Then, an approximation for the SER of partial relay selection is obtained from (24) and (21) as

$$
P_e \approx \frac{1}{2} - \frac{N}{2}\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{m+1} \times \sqrt{\frac{\Psi}{\Psi+2\bar{\gamma}_{RD}(m+1)+\bar{\gamma}_{SR}\left(1+m\left(1-\rho_1^2\right)\right)}} \qquad (25)
$$

where we set $\Psi = \beta\bar{\gamma}_{SR}\bar{\gamma}_{RD}\left(1+m\left(1-\rho_1^2\right)\right)$.

V. Numerical Results and Discussion

In this section, we illustrate the effects of outdated CSI on the outage performance of the best relay selection and partial relay selection schemes. It is noted that all curves provided in Figs. 2-9 were confirmed by simulations (not shown here).

A. Performance Results

In Fig. 2, the outage probability of best relay selection for different correlation coefficients ρ_1 and ρ_2 is depicted versus the normalized average SNR of the S-R and R-D links; the normalization is with respect to the outage threshold SNR, γ_T, which in our examples equals unity. The number of available relays is $N = 3$, while all participating links are assumed to be symmetrical, so that $\bar{\gamma}_{SR_i} = \bar{\gamma}_{R_iD}$, $i = 1, 2, 3$. The main result extracted from Fig. 2 is that outdated CSI at the CU significantly affects the outage performance of best relay selection, since the corresponding outage curves strongly depend on the values of ρ_1, ρ_2. As expected from Corollary 1, when either ρ_1 or ρ_2 deviate from unity the diversity order of best relay selection drops to one, whereas the diversity order equals the number of available relays in the perfect CSI case (i.e., $\rho_1 = \rho_2 = 1$). One may note that this deviation from the perfect CSI case in terms of diversity order is evident even when these correlation coefficients take relatively high values, e.g., $\rho_1 = \rho_2 = 0.95$. Moreover, it is seen that the approximate expression given in (22) provides an outage approximation which becomes a tight lower bound for SNR values equal to or greater than 16 dB. For clarity of exposition, only the approximate expressions for the cases of $(\rho_1, \rho_2) = (1, 1)$ and $(\rho_1, \rho_2) = (0.95, 0.95)$ are shown in Fig. 2, yet the behavior of the remaining cases is very similar. We further note that, due to symmetry, the effect of outdated estimates for the S-R_i links on the overall outage performance is identical to that of the R_i-D links, so that the outage curves of the cases where $(\rho_1, \rho_2) = (0.5, 1)$ and $(\rho_1, \rho_2) = (1, 0.707)$ coincide with the curves for $(\rho_1, \rho_2) = (1, 0.5)$ and $(\rho_1, \rho_2) = (0.707, 1)$, respectively, which are shown in Fig. 2.

Similar conclusions regarding the outage performance of partial relay selection under various levels of imperfect CSI can be extracted from Fig. 3. Nevertheless, one may observe that the diversity order of partial relay selection is equal to unity, regardless of the value of ρ_1. This is due to the intrinsic drawback of partial relay selection in terms of diversity order, as compared to the best relay selection scheme, where the lack of CSI knowledge of either the S-R or the R-D link renders partial relay selection incapable of taking full advantage of the inherent diversity potential. To this end, it is pointed out that although in the best relay selection case any slight deviation from perfect CSI knowledge results in a significant outage performance degradation, the partial relay selection outage probability is affected to a much lesser extend when $\rho_1 \neq 1$, since the potential of partial relay selection to achieve a diversity gain is already degraded due to the lack of CSI knowledge of the R_i-D links. In addition, we note that the approximate expression given in (24) results in tight lower bounds for the outage probability, in the medium and high SNR regimes. As a cross check, one may observe from Figs. 2 and 3 that, as expected, the outage performance of best relay selection with $\rho_1 = \rho_2 = 0$ coincides with that of partial relay selection with $\rho_1 = 0$.

Interesting results regarding the relative outage performances of best relay selection and partial relay selection are derived from Fig. 4. In Fig. 4, the outage probabilities of both schemes are plotted versus ρ_2, which represents the correlation coefficient between the actual channel gain of the R_i-D links and the corresponding channel estimate, assuming $\gamma_T = 1$ and $N = 3$. Moreover, in Fig. 4, we have set $\rho_1 = 1$, implying that the S-R_i channel estimates are perfect, which may correspond to the downlink of a practical relaying system with fixed (infrastructure-based) relays. It is also noted that similar results are obtained for $\rho_1 < 1$; however, those results are omitted here for brevity. In fact, we observe from Fig. 4 that there exists a crossing point between the outage performances of the best and the partial relay selection cases. This implies that *depending on the level of CSI imperfection of the R_i-D links* (or equivalently of the S-R_i links if partial relay selection takes only the R_i-D links into account)*, partial relay selection may outperform best relay selection in terms of outage probability, and vice versa*.

A comparison between the outage performance of AF best relay selection and DF best relay selection is presented in Fig. 5 for several values of $\rho = \rho_1 = \rho_2$. The results for DF best relay selection were generated based on [22, Eq. (2)], which assumed $\rho = \rho_1 = \rho_2$. It is generally noted that the performance of AF best relay selection with outdated CSI is inferior to that of its DF counterpart. In particular, we notice that the same diversity gain is achieved by both AF and DF schemes for both perfect and imperfect CSI, respectively. However, there exists a gap between the coding gains achieved by the above schemes. Interestingly, we note that this coding-gain gap increases as ρ approaches unity, yet for the special case of $\rho = 1$ this gap diminishes so that in the high SNR regime the performances of AF and DF best relay selection with perfect CSI are almost identical. As a general result, it is inferred from Fig. 5 that AF best relay selection is more sensitive to CSI imperfection than DF best relay selection. Hence, one may conclude that the higher complexity of DF relaying, as compared to AF relaying, is compensated by its lower demand for accurate channel estimations, when applied in best relay selection applications.

Figs. 6 and 7 illustrate the SER of best relay selection and partial relay selection, respectively, versus the average channel conditions in the S-R_i and R_i-D links, $i = 1, 2, 3$, for various values for ρ_1 and ρ_2. The modulation format assumed in all cases was BPSK. The curves represented by solid lines correspond to the exact SER, as obtained by substituting (10) into (21) and then performing numerical integration, while the dotted curves were obtained by evaluating (23). As a general comment, Figs. 6 and 7 demonstrate that the SER performance of best relay selection and partial relay selection is affected by outdated channel estimates in the same way as the outage probability. Particularly for the best relay selection scheme, it is emphasized that even a small deviation from the perfect CSI case ($\rho_1 = \rho_2 = 1$) results in a significant deterioration in terms of error performance. Additionally, the expressions in (23) and (25) provide a lower bound on the exact SER, which becomes tight for SNR values greater than 18 dB.

The effect of the number of available relays on the SER performance of best relay selection with outdated CSI is illustrated in Fig. 8. Assuming outdated CSI in both the S-R and R-D links such that $\rho_1 = \rho_2 =$0.5, we notice that, as expected, an increase in N results in a performance improvement in terms of the coding gain, yet the diversity gain remains unaffected. The case of $N = 1$, which corresponds to no relay selection since is only a single relay is available, is also depicted for comparison purposes. One may notice that for small values of N, increasing the number of available relays leads to a considerable coding gain improvement. However, as N grows large, the coding gain improvement due to larger

N is smaller.

Finally, Fig. 9 depicts a comparison of the SER performance of best and partial relay selection versus ρ_2, assuming that the estimates of the S-R_i channels are perfectly updated, while the corresponding R_i-D estimates are outdated. One may observe that there exists a crossing point between the best and partial relay selection performances, depending on the corresponding channel strengths. The importance of the existence of such a crossing point is discussed in the ensuing subsection.

B. Discussion

Here, we provide some discussions regarding how outdated CSI affects the implementation of best and partial relay selection in practical applications. Using Jakes' model for the autocorrelation of a Rayleigh faded channel, shown in (3) and (4), we infer that the case of $\rho_1 = 0.95$ (or $\rho_2 = 0.95$) corresponds to updating the channel estimates with a rate $1/T_d$ which is greater than or equal to $14 f_{d,SR}$ ($= 14 f_{d,RD}$). Let us consider the typical case of a mobile transceiver operating at the frequency of 1.8 GHz, which is assumed to be moving at a speed of 50 km/h. Then, maintaining the channel autocorrelation at a value of 0.95 requires that the time interval between consecutive estimations is not greater than about $700\mu\,\mathrm{sec}$. This indicates that even when the channel estimates are relatively frequently updated, the maximum performance of relay selection still cannot be achieved, since $\rho = 0.95$ corresponds to a severe performance degradation as shown in Figs. 2 and 6.

In light of the above, one may conclude that, in terms of the achievable diversity order, *the (indubitable) benefits of best relay selection in cooperative communications come at the cost of high complexity in time-varying fading channels*, due its high sensitivity to outdated channel estimates. This result confirms the intuition that the performance of relay selection is strongly dependent on the available CSI. It is worth mentioning, however, that this diversity loss when $\rho < 1$ refers to the asymptotically high SNR regime, in the sense that the outage curve maintains its slope for medium to high SNRs, and it gradually converges to unitary slope as the SNR grows infinitely large. In this respect, one may conclude that *when ρ approaches one the system retains its diversity properties throughout the practical SNR range*, i.e., for $\bar{\gamma} < 40$ dB. This observation sheds some light onto the diversity impact of outdated CSI in relay selection over time-varying channels, manifesting that, although it is impossible to achieve full diversity from the theoretical point of view, the diversity properties are preserved for an SNR interval which increases as ρ approaches one.

As illustrated in Figs. 4 and 9, best relay selection and partial relay selection may outperform one another, depending on how close the actual channel values are to their estimates. To be more precise, since ρ_2 is defined as in (4), for the particular scenario where the value of ρ_1 is fixed the relative performances of best and partial relay selection are determined by the maximum Doppler frequency of the R_i-D links. Therefore, from a system designer's perspective, given a maximum allowable value of channel estimation repetition rate, it is preferable to include the R_i-D SNR in the relay selection process only if the maximum Doppler frequency of the R_i-D links is lower than a given threshold. This threshold is determined by the values of $\bar{\gamma}_{SR}$ and $\bar{\gamma}_{RD}$, through (4) and Figs. 4 and 9.

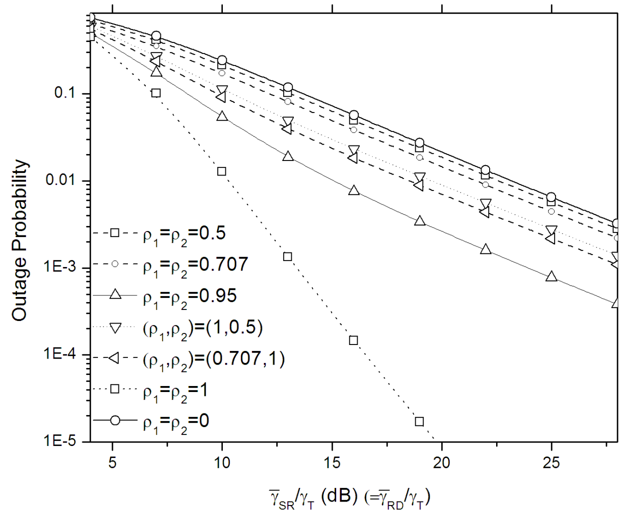

Fig. 2. Outage probability of the best relay selection scheme versus the normalized average SNR of the S-R_i and R_i-D links, for $N = 3$ and $\bar{\gamma}_{SR} = \bar{\gamma}_{RD}$.

Finally, we note that although the analysis conducted in this paper has been presented for i.i.d. S-R and R-D links, the asymptotic performances of the considered schemes retain the same characteristics for the case where the S-R (or the R-D) links are not necessarily i.i.d. The reason is that the asymptotic analysis considers infinitely high average SNRs, such that any multiplication of the individual average SNRs by a finite number (e.g., for the case of two available relays, $\bar{\gamma}_{SR1} = z\bar{\gamma}_{SR2}$, where z is a positive finite number) would result in scaling the corresponding SER (or outage) curves, yet without affecting their slope.

VI. Conclusions

We studied the effect of outdated channel estimates on amplify-and-forward best relay selection and partial relay selection. Closed-form expressions for the outage and the symbol error probability were derived in terms of the average channel strengths and the level of imperfection of the channel estimates. Interestingly, numerical results showed that even a small deviation of the channel estimates from the actual values leads to severe performance degradation in best relay selection schemes, both in terms of outage and symbol error probability. This implies that best relay selection requires relatively frequent channel estimations and feedback to the central unit in order to attain full diversity order. In addition, an outage comparison between AF and DF best relay selection revealed that DF relaying is less sensitive to outdated CSI than its AF counterpart, when applied in best relay selection setups, compensating thus its higher complexity. Furthermore, it is noticed that partial relay selection may outperform best relay selection in cases where the channel estimates in either

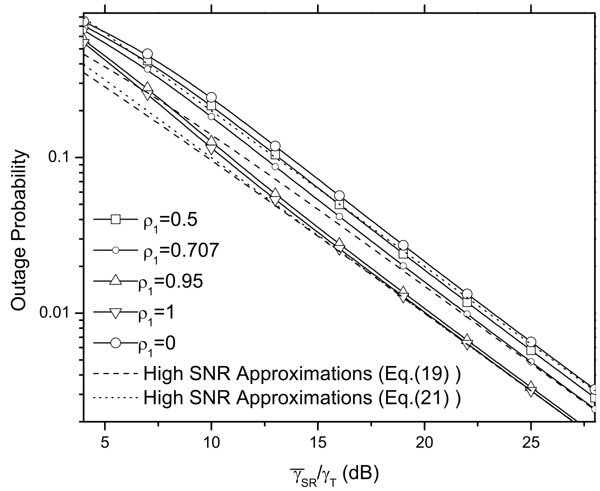

Fig. 3. Outage probability of the partial relay selection scheme versus the normalized average SNR of the S-R_i and R_i-D links, for $N = 3$ and $\bar{\gamma}_{SR} = \bar{\gamma}_{RD}$.

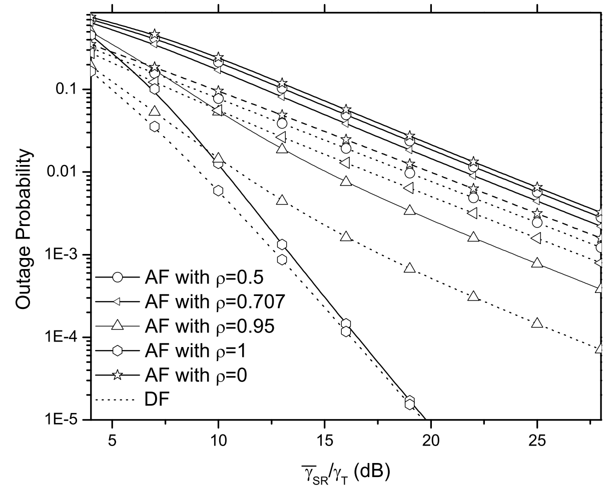

Fig. 5. Comparison of the outage probabilities of AF and DF best relay selection versus the normalized average SNR of the S-R_i and R_i-D links, for $N = 3$, $\bar{\gamma}_{SR} = \bar{\gamma}_{RD}$ and $\rho = \rho_1 = \rho_2$.

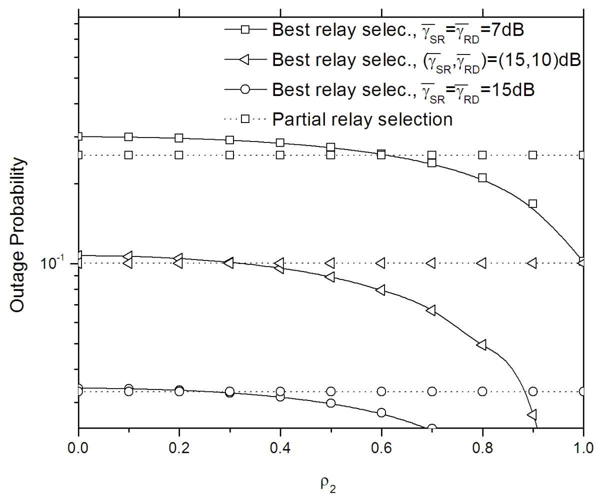

Fig. 4. Outage probability of best and partial relay selection versus ρ_2, for $\rho_1 = 1$ and $N = 3$.

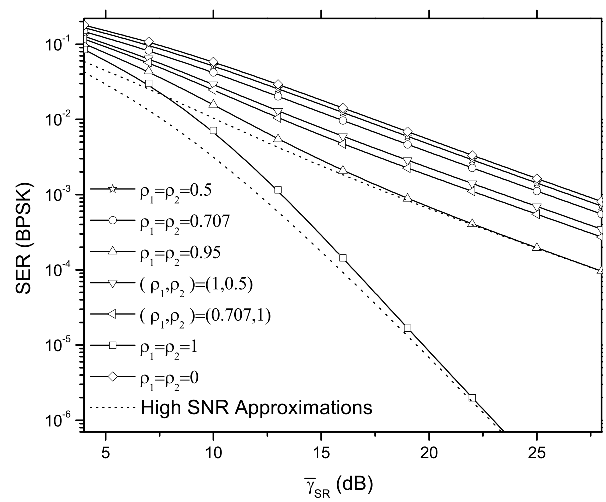

Fig. 6. SER of the best relay selection scheme versus the average SNR of the S-R_i and R_i-D links, for BPSK modulation, $N = 3$ and $\bar{\gamma}_{SR} = \bar{\gamma}_{RD}$.

the source-relay or the relay-destination links are not updated frequently enough.

Appendix

A. Proof of Proposition 1

Proof: The outage probability coincides with the cdf $F_{\gamma_k}(\gamma)$, which is obtained as

$$F_{\gamma_k}(\gamma_T) = \Pr\left(\frac{\gamma_{SR_k}\gamma_{R_kD}}{\gamma_{SR_k} + \gamma_{R_kD} + 1} < \gamma_T\right)$$
$$= \int_0^\infty \Pr\left(\frac{\gamma_{SR_k}y}{\gamma_{SR_k} + y + 1} < \gamma_T\right) f_{\gamma_{R_kD}}(y)dy. \quad (26)$$

Splitting the integration interval in (26) into $[0, \gamma_T)$ and $[\gamma_T, \infty)$ and after applying some algebraic manipulations, $F_{\gamma_k}(\gamma_T)$ can be expressed as

$$F_{\gamma_k}(\gamma_T) = \int_0^{\gamma_T} f_{\gamma_{R_kD}}(y)dy + \int_{\gamma_T}^\infty F_{\gamma_{SR_k}}\left(\frac{\gamma_T y + \gamma_T}{y - \gamma_T}\right) f_{\gamma_{R_kD}}(y)dy. \quad (27)$$

Hence, in order to derive the outage probability we need expressions for the functions $F_{\gamma_{SR_k}}(\cdot)$ and $f_{\gamma_{R_kD}}(\cdot)$, which are obtained as shown below.

Derivation of $F_{\gamma_{SR_k}}(\cdot)$ and $f_{\gamma_{R_kD}}(\cdot)$: Due to the fact that the distributions of γ_{SR_i} and $\hat{\gamma}_{SR_i}$ are correlated exponential distributions, their joint pdf is given by [22]

$$f_{\gamma_{SR_i},\hat{\gamma}_{SR_i}}(x, y) = \frac{e^{-\frac{x+y}{(1-\rho_1^2)\bar{\gamma}_{SR}}}}{(1-\rho_1^2)\bar{\gamma}_{SR}^2} I_0\left(\frac{2\sqrt{\rho_1^2 xy}}{(1-\rho_1^2)\bar{\gamma}_{SR}}\right) \quad (28)$$

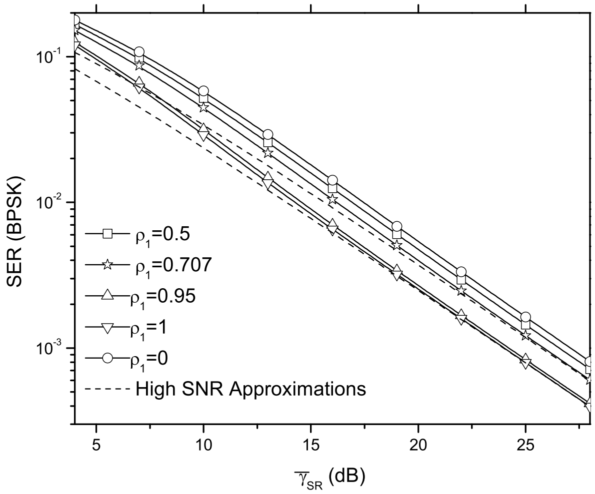

Fig. 7. SER of the partial relay selection scheme versus the average SNR of the S-R_i and R_i-D links, for BPSK modulation, $N = 3$ and $\bar{\gamma}_{SR} = \bar{\gamma}_{RD}$.

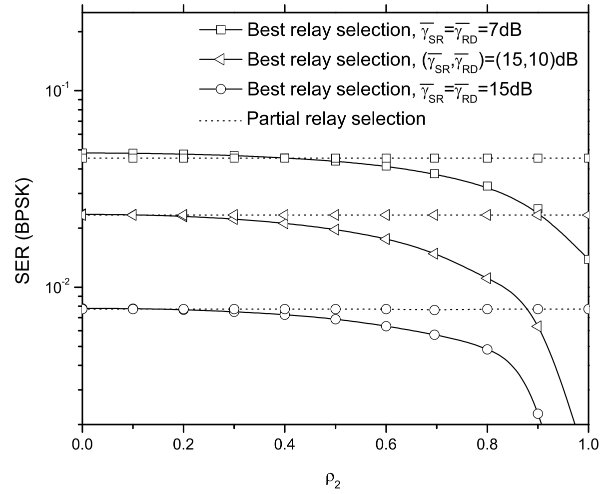

Fig. 9. SER of best and partial relay selection versus ρ_2, for BPSK modulation, $\rho_1 = 1$ and $N = 3$.

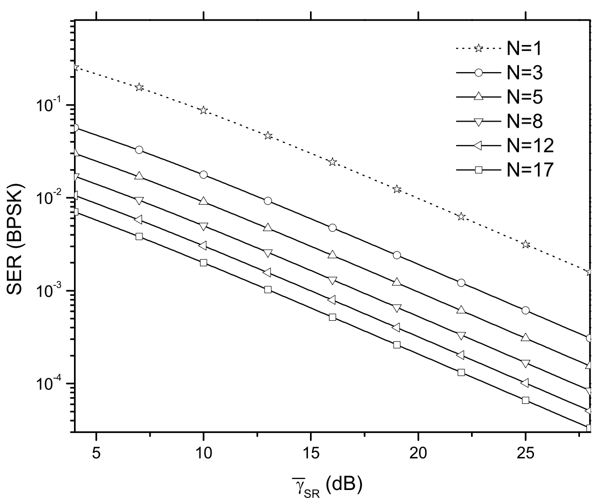

Fig. 8. SER of best relay selection versus the average SNR of the S-R_i and R_i-D links, for BPSK modulation, $\rho_1 = \rho_2 = 0.5$, and $\bar{\gamma}_{SR} = \bar{\gamma}_{RD}$.

where $I_0(\cdot)$ denotes the zeroth order modified Bessel function of the first kind [30, Eq. (8.447.1)]. The pdf of the SNR of the source-relay part of the selected path, γ_{SR_k}, is given by

$$f_{\gamma_{SR_k}}(x) = \int_0^\infty f_{\gamma_{SR_k}|\hat{\gamma}_{SR_k}}(x|y) f_{\hat{\gamma}_{SR_k}}(y) dy. \tag{29}$$

Then, since γ_{SR_i}, as well as $\hat{\gamma}_{SR_i}$, $i = 1, ..., N$, are i.i.d. RVs, the conditional pdfs $f_{\gamma_{SR_i}|\hat{\gamma}_{SR_i}}(\cdot|\cdot)$ are identical for all $i = 1, ..., N$, and expressed as $f_{\gamma_{SR}|\hat{\gamma}_{SR}}(\cdot|\cdot)$. Moreover, it follows from the theory of the concomitants of order statistics that also the conditional pdfs $f_{\gamma_{SR_k}|\hat{\gamma}_{SR_k}}(\cdot|\cdot)$ are identical to $f_{\gamma_{SR}|\hat{\gamma}_{SR}}(\cdot|\cdot)$ [32], yielding

$$f_{\gamma_{SR_k}|\hat{\gamma}_{SR_k}}(x|y) = \frac{f_{\gamma_{SR},\hat{\gamma}_{SR}}(x, y)}{f_{\hat{\gamma}_{SR}}(y)} \tag{30}$$

where $f_{\hat{\gamma}_{SR}}(y) = \frac{1}{\bar{\gamma}_{SR}} e^{-\frac{y}{\bar{\gamma}_{SR}}}$, since the pdf of $\hat{\gamma}_{SR_i}$ is the same as that of γ_{SR_i}, because the RVs h_{SR_i} and w_{SR_i} in (1) have the same pdf. Therefore, substituting (28) into (30) we obtain

$$f_{\gamma_{SR_k}|\hat{\gamma}_{SR_k}}(x|y) = \frac{e^{-\frac{x+\rho_1^2 y}{(1-\rho_1^2)\bar{\gamma}_{SR}}}}{(1-\rho_1^2)\bar{\gamma}_{SR}} I_0\left(\frac{2\sqrt{\rho_1^2 xy}}{(1-\rho_1^2)\bar{\gamma}_{SR}}\right). \tag{31}$$

The cdf of $\hat{\gamma}_{SR_k}$, $F_{\hat{\gamma}_{SR_k}}(\cdot)$, is expressed as $F_{\hat{\gamma}_{SR_k}}(x) = \sum_{i=1}^N \Pr\{\hat{\gamma}_{SR_i} \le x \cap k = i\}$. Due to symmetry among the N end-to-end paths, $F_{\hat{\gamma}_{SR_k}}(\cdot)$ takes the form

$$\begin{aligned} F_{\hat{\gamma}_{SR_k}}(x) &= N \Pr\{\hat{\gamma}_{SR_i} \le x \cap k = i\} \\ &= N \int_0^x f_{\hat{\gamma}_{SR_i}}(y) \Pr\{k = i \,|\hat{\gamma}_{SR_i} = y\} dy. \end{aligned} \tag{32}$$

Considering that the "best" relay is selected according to (6) and (7), it is the weakest of the S-R_i, R_i-D links, as estimated at the CU, which determines $\hat{\gamma}_i$, $i = 1, \ldots, N$. Therefore, we can express the probability $\Pr\{k = i \,|\hat{\gamma}_{SR_i} = y\}$ in (32) as the summation of two mutually exclusive events, corresponding to the cases where $\hat{\gamma}_{SR_i} < \hat{\gamma}_{R_iD}$ and $\hat{\gamma}_{SR_i} \ge \hat{\gamma}_{R_iD}$, yielding

$$\begin{aligned} F_{\hat{\gamma}_{SR_k}}(x) = N\Bigg[&\int_0^x \int_0^y f_{\hat{\gamma}_{SR_i}}(y) f_{\hat{\gamma}_{R_iD}}(\omega) [F_{\hat{\gamma}_i}(\omega)]^{N-1} d\omega dy \\ &+ \int_0^x f_{\hat{\gamma}_{SR_i}}(y) \left(\int_y^\infty f_{\hat{\gamma}_{R_iD}}(\omega) d\omega\right) [F_{\hat{\gamma}_i}(y)]^{N-1} dy\Bigg] \end{aligned} \tag{33}$$

where, because of symmetry, the index i may take any value of the set $\{1, \ldots, N\}$. Given that $\hat{\gamma}_{SR}$ and $\hat{\gamma}_{RD}$ follow the same distribution as γ_{SR} and γ_{RD}, respectively, the cdf of $\hat{\gamma}_i$ is derived as $F_{\hat{\gamma}_i}(x) = 1 - \exp(-x/\bar{\gamma}_e)$, where $\bar{\gamma}_e = \bar{\gamma}_{SR}\bar{\gamma}_{RD}/(\bar{\gamma}_{SR} + \bar{\gamma}_{RD})$ [20]. Consequently, using the binomial expansion we obtain

$$[F_{\hat{\gamma}_i}(x)]^{N-1} = \sum_{n=0}^{N-1} (-1)^n \binom{N-1}{n} \exp\left(-\frac{nx}{\bar{\gamma}}\right). \tag{34}$$

The inner integral in the first line of (33) is evaluated as

$$\int_0^y f_{\hat{\gamma}_{R_iD}}(\omega)[F_{\hat{\gamma}_i}(\omega)]^{N-1}d\omega = \bar{\gamma}_e \sum_{n=0}^{N-1} \frac{(-1)^n \binom{N-1}{n}}{n\bar{\gamma}_{RD}+\bar{\gamma}_e}\left(1-e^{-\left(\frac{n}{\bar{\gamma}_e}+\frac{1}{\bar{\gamma}_{RD}}\right)y}\right). \quad (35)$$

Likewise, we may express the first line of (33) as

$$\int_0^x f_{\hat{\gamma}_{SR}}(y)\left(\int_0^y f_{\hat{\gamma}_{R_iD}}(\omega)[F_{\hat{\gamma}_i}(\omega)]^{N-1}d\omega\right)dy = \bar{\gamma}_e \sum_{n=0}^{N-1} \frac{(-1)^n}{n\bar{\gamma}_{RD}+\bar{\gamma}_e}\binom{N-1}{n}\left(1-e^{-\frac{x}{\bar{\gamma}_{SR}}}\right) - \frac{\bar{\gamma}_e}{\bar{\gamma}_{SR}}\sum_{n=0}^{N-1}\frac{(-1)^n\binom{N-1}{n}}{n\bar{\gamma}_{RD}+\bar{\gamma}_e}\frac{\left(1-e^{-\left(\frac{n}{\bar{\gamma}_e}+\frac{1}{\bar{\gamma}_{RD}}+\frac{1}{\bar{\gamma}_{SR}}\right)x}\right)}{\left(\frac{n}{\bar{\gamma}_e}+\frac{1}{\bar{\gamma}_{RD}}+\frac{1}{\bar{\gamma}_{SR}}\right)} \quad (36)$$

and the second line of (33) as

$$\int_0^x f_{\hat{\gamma}_{SR}}(y)\left(\int_y^\infty f_{\hat{\gamma}_{R_iD}}(\omega)d\omega\right)[F_{\hat{\gamma}_i}(y)]^{N-1}dy = \frac{1}{\bar{\gamma}_{SR}}\sum_{n=0}^{N-1}\frac{\binom{N-1}{n}\left(1-e^{-\left(\frac{1}{\bar{\gamma}_{SR}}+\frac{1}{\bar{\gamma}_{RD}}+\frac{n}{\bar{\gamma}_e}\right)x}\right)}{(-1)^n\left(\frac{1}{\bar{\gamma}_{SR}}+\frac{1}{\bar{\gamma}_{RD}}+\frac{n}{\bar{\gamma}_e}\right)}. \quad (37)$$

Hence, combining (33)-(37) we get the cdf of $\hat{\gamma}_{SR_k}$ which, after differentiation, yields

$$f_{\hat{\gamma}_{SR_k}}(x) = N\sum_{n=0}^{N-1}\binom{N-1}{n} \times\left[\frac{\bar{\gamma}_e\left[(-1)^n e^{-\frac{x}{\bar{\gamma}_{SR}}}+(-1)^{n+1}e^{-\left(\frac{1}{\bar{\gamma}_{SR}}+\frac{1}{\bar{\gamma}_{RD}}+\frac{n}{\bar{\gamma}_e}\right)x}\right]}{\bar{\gamma}_{SR}\left(\bar{\gamma}_e+n\bar{\gamma}_{RD}\right)} + \frac{(-1)^n e^{-\left(\frac{1}{\bar{\gamma}_{SR}}+\frac{1}{\bar{\gamma}_{RD}}+\frac{n}{\bar{\gamma}_e}\right)x}}{\bar{\gamma}_{SR}}\right]. \quad (38)$$

As a result, we may evaluate the pdf of γ_{SR_k} by substituting (31) and (38) into (29), yielding

$$f_{\gamma_{SR_k}}(x) = N\sum_{n=0}^{N-1}(-1)^n\binom{N-1}{n}\left[\frac{\exp\left(-\frac{x}{\bar{\gamma}_{SR}}\right)}{\bar{\gamma}_{SR}+n\left(\bar{\gamma}_{SR}+\bar{\gamma}_{RD}\right)} - \frac{\bar{\gamma}_{RD}}{\left[\bar{\gamma}_{SR}+n\left(\bar{\gamma}_{SR}+\bar{\gamma}_{RD}\right)\right]} \times\frac{\exp\left(-\frac{(n+1)\left(\bar{\gamma}_{SR}+\bar{\gamma}_{RD}\right)x}{\bar{\gamma}_{SR}(n+1)(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})-\bar{\gamma}_{SR}\rho_1^2[\bar{\gamma}_{SR}+n(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})]}\right)}{(n+1)\left(\bar{\gamma}_{SR}+\bar{\gamma}_{RD}\right)+\rho_1^2\left(\bar{\gamma}_{SR}+n\left(\bar{\gamma}_{SR}+\bar{\gamma}_{RD}\right)\right)} + \frac{\bar{\gamma}_e\bar{\gamma}_{RD}\exp\left(-\frac{\left(\bar{\gamma}_{SR}\bar{\gamma}_{RD}+n\bar{\gamma}_{SR}\bar{\gamma}_{RD}\right)x}{\bar{\gamma}_{SR}\left[n\bar{\gamma}_{SR}\bar{\gamma}_{RD}\left(1-\rho_1^2\right)+\bar{\gamma}_e\left(\bar{\gamma}_{SR}\bar{\gamma}_{RD}-\bar{\gamma}_{SR}\rho_1^2\right)\right]}\right)}{\bar{\gamma}_{SR}\left[n\left(\bar{\gamma}_{SR}+\bar{\gamma}_{RD}\right)\left(1-\rho_1^2\right)+\bar{\gamma}_e\left(\bar{\gamma}_{SR}\bar{\gamma}_{RD}-\bar{\gamma}_{SR}\rho_1^2\right)\right]}\right]. \quad (39)$$

The cdf of γ_{SR_k} can be derived directly from (39) as

$$F_{\gamma_{SR_k}}(z) = N\sum_{n=0}^{N-1}\frac{(-1)^n\binom{N-1}{n}n\bar{\gamma}_{RD}}{(n+1)\left[\bar{\gamma}_{SR}+n\left(\bar{\gamma}_{SR}+\bar{\gamma}_{RD}\right)\right]} \times\left[\frac{\bar{\gamma}_{SR}(n+1)}{n\bar{\gamma}_{RD}}\left(1-\exp\left(-\frac{x}{\bar{\gamma}_{SR}}\right)\right) + 1-e^{-\frac{(n+1)(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})x}{\bar{\gamma}_{SR}(n+1)(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})-\bar{\gamma}_{SR}\rho_1^2[\bar{\gamma}_{SR}+n(\bar{\gamma}_{SR}+\bar{\gamma}_{RD})]}}\right]. \quad (40)$$

Furthermore, the pdf of γ_{R_kD} is derived from (39), due to symmetry, by mutually interchanging $\bar{\gamma}_{SR}$ and $\bar{\gamma}_{RD}$, while substituting ρ_1 with ρ_2. Hence, the outage probability of best relay selection is derived by substituting (40) and $f_{\gamma_{R_kD}}(\cdot)$, as taken from (39), into (27). Then, applying the change of variables, $\omega = y - \gamma_T$, and making use of the integral [30, Eq. (3.324.1)], we obtain (10); the proof is thus complete. ■

B. Proof of Corollary 1; Proof of Corollary 2 for $\rho_1 = \rho_2 = 1$

Proof: It follows from (13) that $\mathcal{A}_1$ in (12) is a non-zero constant for each $\rho_2 \neq 1$; likewise, $\mathcal{A}_2 \neq 0$ for $\rho_1 \neq 1$. Then, using the definition of the diversity order, $d = -\lim_{\bar{\gamma}_{SR}\to\infty}\log\left(F_{\gamma_k}\left(\gamma_T\right)\right)/\log\left(\bar{\gamma}\right)$, it follows from (12) that the diversity order of best relay selection equals unity, unless $\rho_1 = \rho_2 = 1$. The latter case is studied separately in the ensuing paragraph.

The case of $\rho_1 = \rho_2 = 1$ corresponds to best relay selection with perfectly updated channel estimates; the diversity order of this scheme, where the direct S-D link is considered negligible, has been proved in [7] to be given by $d = N$, which is in accordance with (15). For completeness, we corroborate this result here via the expression in (10), assuming $\bar{\gamma}_{SR} = \bar{\gamma}_{RD} = \bar{\gamma}$. Using the Taylor expansion in (10), and the binomial expansion $\left[1-\exp\left(-2\gamma_T/\bar{\gamma}\right)\right]^N = \sum_{n=0}^{N}(-1)^n\binom{N}{n}\exp\left(-n2\gamma_T/\bar{\gamma}\right)$, we are able to group appropriately the resulting terms, such that by using [31, Eq. (6.6.2)] and [31, Eq. (6.6.4)] and some algebraic manipulations, the outage probability for $\rho_1 = \rho_2 = 1$ and high SNR takes the expression

$$F_{\gamma_k}\left(\gamma_T\right) \approx 2\left[1-\exp\left(-2\frac{\gamma_T}{\bar{\gamma}}\right)\right]^N + N\exp\left(-\frac{\gamma_T}{\bar{\gamma}}\right)\left[\mathcal{B}_{\exp\left(-2\frac{\gamma_T}{\bar{\gamma}}\right)}\left(\frac{1}{2},N\right)-\mathcal{B}_1\left(\frac{1}{2},N\right)\right] = 2\left[1-\exp\left(-2\frac{\gamma_T}{\bar{\gamma}}\right)\right]^N - N\exp\left(-\frac{\gamma_T}{\bar{\gamma}}\right)\mathcal{B}_{1-\exp\left(-2\frac{\gamma_T}{\bar{\gamma}}\right)}\left(N,\frac{1}{2}\right) \quad (41)$$

where $\mathcal{B}_z(p,q)$ denotes the incomplete Beta function defined in [30, Eq. (8.391)]. Additionally, noting that $\mathcal{B}_z(a,b)$ is proportional to $\frac{z^a}{a}$ in the high SNR regime, (41) reduces to

$$F_{\gamma_k}\left(\gamma_T\right) \approx \left[1-\exp\left(-2\frac{\gamma_T}{\bar{\gamma}}\right)\right]^N \approx 2^N\left(\frac{\gamma_T}{\bar{\gamma}}\right)^N. \quad (42)$$

The diversity order and coding gain are then obtained accordingly. ■

C. Proof of Proposition 2

Proof: For the partial relay selection case considered here, recall that the relay selection is implemented according to (8). Hence, the pdf of $\hat{\gamma}_{SR_k}$ is given by

$$f_{\hat{\gamma}_{SR_k}}(x) = N[F_{\hat{\gamma}_{SR}}(x)]^{N-1} f_{\hat{\gamma}_{SR}}(x). \tag{43}$$

Moreover, the pdf of $\hat{\gamma}_{R_kD}$ is the exponential distribution with average value $\bar{\gamma}_{RD}$, since the R_i-D, $i = 1, ..., N$, links are not taken into account for relay selection. Consequently, $f_{\gamma_{SR_k}}(\cdot)$ is obtained by substituting (31) and (43) into (29).Thus, $F_{\gamma_{SR_k}}(\cdot)$ is derived as

$$F_{\gamma_{SR_k}}(x) = N \sum_{m=0}^{N-1} \frac{(-1)^m \binom{N-1}{m} \left(1 - e^{-\frac{(m+1)}{(m(1-\rho_1^2)+1)\bar{\gamma}_{SR}}x}\right)}{m+1}. \tag{44}$$

Therefore, (17) is obtained by combining (27) and (44), completing the proof. ■

As a cross check, we notice that if the CU has perfect CSI knowledge of the S-R_i links (i.e., $\rho_1 = 1$), (17) reduces to

$$F_{\gamma_k}(\gamma_T) = 1 - 2N \sum_{m=0}^{N-1} \left[\frac{(-1)^m \binom{N-1}{m} \sqrt{\gamma_T(\gamma_T+1)}}{\sqrt{\bar{\gamma}_{RD}\bar{\gamma}_{SR}(m+1)}} \times e^{-\frac{\bar{\gamma}_{SR}+(m+1)\bar{\gamma}_{RD}}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}\gamma_T} K_1\left(2\sqrt{\frac{(m+1)\gamma_T(\gamma_T+1)}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)\right] \tag{45}$$

which coincides with [10, Eq. (4)].

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