Gallager's Exponent Analysis of STBC MIMO Systems over η-μ and κ-μ Fading Channels

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***Abstract*—In this paper, we analytically investigate Gallager's exponent for space-time block codes over multiple-input multiple-output block-fading channels with Gaussian input distribution. As a suitable metric of the fundamental tradeoff between communication reliability and information rate, Gallager's exponent can be used to determine the required codeword length to achieve a prescribed error probability at a given rate below the channel capacity. We assume that the receiver has full channel state information (CSI), while the transmitter has no CSI and performs equal power allocation across all transmit antennas. In the following, novel exact expressions for Gallager's exponent are derived for two well-known channel fading models, namely η-μ and κ-μ fading models. More importantly, the implications of fading parameters and channel coherence time on Gallager's exponent are investigated. In addition, we present new expressions for the Shannon capacity, cutoff rate and expurgated exponent for the above mentioned fading models, while in the high signal-to-noise ratio regime, simplified closed-form expressions are also derived. Finally, we highlight the fact that the presented analysis encompasses all previously known results on Nakagami-m, Rician, Rayleigh and Hoyt fading channels, as special cases.**

***Index Terms*—Capacity, cutoff rate, expurgated exponent, MIMO fading channels, random coding exponent.**

I. INTRODUCTION

OVER the past few years, multiple-input multiple-output (MIMO) systems have been well investigated in terms of capacity over various types of fading channels (see e.g., [1]–[4] and references therein). In the related literature, ergodic Shannon capacity has been used as the typical metric to evaluate the performance of MIMO systems. However, this metric gives knowledge only of the maximum achievable information rate and may not be sufficient to reflect the fundamental limits of MIMO communication systems. More specifically, it was shown in [5] that the error probability tends to zero as the block length tends to infinity, when the rate is less than the channel capacity. On the other hand, for most practical scenarios, the block length cannot be too long because of the limitations on delay and encoding/decoding complexity. For this reason, this fundamental tradeoff needs to be thoroughly exploited. As an easily computable lower bound, an information-theoretic metric, namely Gallager's exponent (or reliability function), has been proposed in [6] to determine the probability of error, P_e, as a function of the codeword length, L, and the information rate R.

Since then, a number of works have investigated Gallager's exponent of various single-antenna communication systems over Rayleigh flat fading channels [7], [8]. Moreover, there have been some advances in understanding Gallager's exponent of multiple-antenna systems as well. In this context, [9] derived Gallager's exponent for MIMO Rayleigh block-fading channels with spatial fading correlation and subject to an average power constraint, assuming perfect channel state information (CSI) at the receiver. The relationship between probability of error, information rate, codeword length and signal-to-noise ratio (SNR) for fast Rayleigh fading MIMO-ARQ channels was examined in [10].

Yet, very few results on Gallager's exponent of multiple-antenna systems with space-time block codes (STBC) in non-Rayleigh fading conditions are available. Only recently, [11] and [12] investigated Gallager's exponent of STBC systems operating in Nakagami-m and generalized-K fading channels, respectively. While these prior works have significantly improved our knowledge on Gallager's exponent of STBC systems, a general analytic framework which will account for more realistic fading models seems to be missing from the open literature. In particular, the above fading distributions rely inherently on the assumption of a homogeneous scattering environment, which is often unrealistic since the surfaces are spatially correlated in most propagation environments [13]. To address such non-homogeneous environments, the η-μ and κ-μ distributions have been proposed in [14]–[18]. These fading models can provide better fit to experimental data than the

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Rayleigh, Rician and Nakagami-m fading models [18], [19]. Moreover, the η-μ distribution involves the Hoyt (Nakagami-q), Nakagami-m and Rayleigh distributions as special cases, while the κ-μ distribution includes the Rician, Nakagami-m, and Rayleigh distributions as special cases. Motivated by these important observations, we herein analytically investigate Gallager's exponent of STBC systems over η-μ and κ-μ fading channels.

The contributions of this paper can now be summarized as follows: We first provide novel, analytical expressions for Gallager's exponent of STBC systems over η-μ and κ-μ fading channels. Note that, although the considered models incur significant mathematical challenges, all the presented results can be easily evaluated and efficiently programmed in most standard software packages (e.g., MATLAB, MATHEMATICA). We further elaborate on the Shannon capacity and cutoff rate, which can be directly derived from Gallager's exponent. Additionally, new formulas for the expurgated exponent over these fading channels are presented to extend Gallager's exponent. In order to get additional insights into the impact of fading parameters, a high-SNR analysis is pursued to investigate the effects of coherence time and codeword length error probability. We note that the presented results extend and complement all previous results on Rayleigh, Rician and Nakagami-m fading channels [7], [8], [11]. For the sake of completeness, we provide the link to these previous results and also present the corresponding results for Hoyt fading channels which, to the best of our knowledge, have not been reported elsewhere.

The remainder of the paper is organized as: In Section II, the MIMO system model and Gallager's exponent used throughout the paper are introduced. In Section III, we provide new, analytical expressions for Gallager's exponent of STBC over η-μ and κ-μ MIMO fading channels along with a detailed high-SNR analysis. Some special cases of interest are also assessed. A set of numerical results is given in Section III, while Section IV concludes this paper.

Notation: We use upper and lower case boldface letters to denote matrices and vectors, respectively. The symbol $(\cdot)^\dagger$ represents the Hermitian transpose, the trace operator of a square matrix is denoted by $\mathrm{tr}(\cdot)$, $\mathrm{etr}(\cdot) = e^{\mathrm{tr}}(\cdot)$, while $\|\cdot\|_F$ denotes the matrix Frobenius norm. The expectation operator of a random variable is given by $\mathtt{E}\{\cdot\}$, the matrix determinant reads as $\det(\cdot)$, while $\lceil\cdot\rceil$ denotes the ceiling operation to the nearest integer.

II. System Model and Gallager's Exponent

A. System model

We consider a single-user MIMO system with N_t transmit antennas and N_r receive antennas whose complex input-output relationship can be expressed as

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N} \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ denotes the fading channel matrix with entries $h_i\,(i = 1, 2, \ldots, N_t N_r)$, while $\mathbf{X} \in \mathbb{C}^{N_t \times N_c}$ is the transmit matrix containing N_c symbols. Also, $\mathbf{Y} \in \mathbb{C}^{N_r \times N_c}$ represents the received signal matrix and $\mathbf{N} \in \mathbb{C}^{N_r \times N_c}$ is the complex zero-mean additive white Gaussian noise (AWGN) matrix with the variance of its elements being N_0. Considering N_b independent coherence intervals, the block codeword length of a reliable communication link is $N_b N_c$. Moreover, the input signal matrix is subject to an average power constraint of the form $\mathtt{E}\left\{\mathrm{tr}\left(\mathbf{X}\mathbf{X}^\dagger\right)\right\} = N_c \mathrm{tr}\left(\mathbf{Q}\right) \leq N_c P$, where $\mathbf{Q}$ is the $N_t \times N_t$ positive semidefinite input covariance matrix and P is the total transmit power. Assuming that the transmitter has no CSI, it is meaningful to assume that uniform power allocation is being performed across the transmit antennas, such that $\mathbf{Q} = \frac{P}{N_t}\mathbf{I}$.

For STBC, the MIMO channel can be represented as $N_t \times N_r$ parallel single-input single-output channels for each data symbol [20]. Thus, the effective output symbol SNR is given by [4, Eq. (4)], [12, Eq. (7)]

$$\gamma_o = \frac{\gamma}{N_t}\|\mathbf{H}\|_F^2 \tag{2}$$

where $\gamma = \frac{P}{R_c N_0}$ is the effective transmit SNR, and R_c is the information code rate. Hereafter, we assume, for the sake of clarity and without significant loss of generality, full-rate STBC such that $R_c = 1$.

B. Gallager's exponent

1) Random coding exponent: With maximum-likelihood decoding, an upper bound on the error probability of MIMO channels with continuous inputs and outputs is given by [6]

$$P_e \leq \left(\frac{2e^{r\delta}}{\xi}\right)^2 \exp\left(-N_b N_c E_r\left(p_{\mathbf{X}}\left(\mathbf{X}\right), R, N_c\right)\right). \tag{3}$$

The above bound is given in terms of several arbitrary parameters, namely $r \geq 0$, $\delta \geq 0$, which are defined as

$$\xi \approx \frac{\delta}{\sqrt{2\pi N_b \sigma_\xi^2}} \tag{4}$$

$$\sigma_\xi^2 = \int_{\mathbf{X}} \left[\mathrm{tr}\left(\mathbf{X}\mathbf{X}^\dagger\right) - N_c P\right]^2 p_{\mathbf{X}}\left(\mathbf{X}\right) d\mathbf{X}. \tag{5}$$

To achieve a desired error probability at an information rate R, the codeword length can be obtained by solving for N_b in (3) with $L = N_c \times \lceil N_b \rceil$. The random coding exponent is defined as [6]

$$E_r\left(p_{\mathbf{X}}\left(\mathbf{X}\right), R, N_c\right) \triangleq \max_{0\leq\rho\leq1}\left(\max_{r\geq0} E_0\left(p_{\mathbf{X}}\left(\mathbf{X}\right), \rho, r, N_c\right) - \rho R\right) \tag{6}$$

where $E_0\left(p_{\mathbf{X}}\left(\mathbf{X}\right), \rho, r, N_c\right)$ is given in (7) at the bottom of this page.

$$E_0\left(p_{\mathbf{X}}\left(\mathbf{X}\right), \rho, r, N_c\right) \triangleq -\frac{1}{N_c}\ln\left(\int_{\mathbf{H}} p_{\mathbf{H}}\left(\mathrm{H}\right) \int_{\mathbf{Y}} \left(\int_{\mathbf{X}} p_{\mathbf{X}}\left(\mathbf{X}\right) e^{r\left[\mathrm{tr}\left(XX^\dagger\right) - N_c P\right]} p(\mathbf{Y}|\,\mathbf{X}, \mathbf{H})^{\frac{1}{1+\rho}} d\mathbf{X}\right)^{1+\rho} d\mathbf{Y} d\mathbf{H}\right). \tag{7}$$

In general, it is very difficult to minimize the upper bound by optimizing the input probability density function (p.d.f.) $p_{\mathbf{X}}(\mathbf{X})$. Subject to the considered power constraint, we choose the capacity-achieving Gaussian distribution for $p_{\mathbf{X}}(\mathbf{X})$ as [6]

$$p_{\mathbf{X}}(\mathbf{X}) = \pi^{-N_t N_c} \det(\mathbf{Q})^{-N_c} \mathrm{etr}\left(-\mathbf{Q}^{-1}\mathbf{X}\mathbf{X}^{\dagger}\right). \quad (8)$$

This choice of the input distribution may facilitate the solution of (7), though it is optimal for the random coding exponent calculation only when the rate R approaches the channel capacity. Then, (7) becomes

$$\begin{aligned}\tilde{E}_0(\rho,\beta,N_c) &\triangleq E_0\left(\frac{P}{N_t}\mathbf{I}_{N_t},\rho,r,N_c\right)\Bigg|_{\beta=N_t-rP} \\ &= \underbrace{(1+\rho)(N_t-\beta)+N_t(1+\rho)\ln(\beta/N_t)}_{A(\rho,\beta)} \\ &\quad -\frac{1}{N_c}\ln\left(\mathrm{E}\left\{\det\left(\mathbf{I}_{N_r}+\frac{\gamma\mathbf{H}\mathbf{H}^{\dagger}}{\beta(1+\rho)}\right)^{-N_c\rho}\right\}\right). \quad (9)\end{aligned}$$

Then, the random coding exponent can be written as

$$E_r(R,N_c) = \max_{0\le\rho\le1}\left(\max_{0\le\beta\le N_t}\tilde{E}_0(\rho,\beta,N_c)-\rho R\right). \quad (10)$$

As in [21], the error probability is given by

$$P_e \le \frac{8\pi}{N_t}(N_t-\beta^*(\rho))^2 N_b N_c e^{(2-N_b N_c E_r(R,N_c))} \quad (11)$$

where $\beta^*(\rho)$ is the value of β that maximizes $\tilde{E}_0(\rho,\beta,N_c)$, defined in (9) for each ρ, and is in the range $0<\beta\le N_t$.

2) Shannon capacity: From [22], the information rate R can be expressed as

$$R = \frac{\partial\tilde{E}_0(\rho,\beta^*(\rho),N_c)}{\partial\rho}. \quad (12)$$

Note that R becomes identical to Shannon capacity, $\langle C\rangle$, for $\rho=0$ and $\beta^*(0)=N_t$, such that

$$\langle C\rangle \triangleq \frac{\partial\tilde{E}_0(\rho,\beta^*(\rho),N_c)}{\partial\rho}\Bigg|_{\rho=0,\beta^*(0)=N_t}. \quad (13)$$

3) Cutoff rate: As an important information-theoretic metric, the cutoff rate R_0 determines a lower bound to the Shannon capacity and the corresponding value of the zero-rate random coding exponent. By setting $\rho=1$ and $\beta^*(1)=N_t$ in (9), the cutoff rate becomes [21]

$$R_0 = -\frac{1}{N_c}\ln\left(\mathrm{E}\left\{\det\left(\mathbf{I}_{N_r}+\frac{\gamma\mathbf{H}\mathbf{H}^{\dagger}}{2N_t}\right)^{-N_c}\right\}\right). \quad (14)$$

4) Expurgated exponent: The random coding exponent is defined by selecting the unbiased codeword based on the input distribution. Therefore, the good and bad codewords have equal contribution to the average error probability. Another error metric, namely expurgated exponent, is proposed by expurgating the bad codewords and is given by [6]

$$\begin{aligned}&E_{ex}(p_{\mathbf{X}}(\mathbf{X}),R,N_c) \\ &\quad\triangleq \max_{\rho\ge1}\left(\max_{r\ge0}E_x(p_{\mathbf{X}}(\mathbf{X}),\rho,r,N_c)-\rho R\right) \quad (15)\end{aligned}$$

where $E_x(p_{\mathbf{X}}(\mathbf{X}),\rho,r,N_c)$ is expressed in (16) at the bottom of this page. The expurgated exponent with Gaussian input distribution and equal power allocation is given by [6]

$$E_{ex}(R,N_c) = \max_{\rho\ge1}\left(\max_{0\le\beta\le N_t}\tilde{E}_x(\rho,\beta,N_c)-\rho R\right) \quad (17)$$

where

$$\begin{aligned}\tilde{E}_x(\rho,\beta,N_c) &\triangleq \underbrace{2\rho(N_t-\beta)+2\rho N_t\ln(\beta/N_t)}_{A'(\rho,\beta)} \\ &\quad -\frac{1}{N_c}\ln\left(\mathrm{E}\left\{\det\left(\mathbf{I}_{N_r}+\frac{\gamma\mathbf{H}\mathbf{H}^{\dagger}}{2\rho\beta}\right)^{-N_c\rho}\right\}\right). \quad (18)\end{aligned}$$

III. Gallager's Exponent Analysis in η-μ and κ-μ Fading Channels

In this section, we present a detailed Gallager's exponent analysis for η-μ and κ-μ fading channels. Note that only independent and identically distributed (i.i.d.) fading is considered here; however, we can address the cases of non-identically distributed or correlated fading using the similar methodology as described below.

A. η-μ fading channels

The η-μ distribution models the small-scale variation of the fading signal in a non-homogeneous environment with the p.d.f. of the instantaneous SNR given by [18, Eq. (26)]

$$f_{\eta\text{-}\mu}(\omega) = \frac{2\sqrt{\pi}\mu^{\mu+\frac{1}{2}}h^{\mu}\omega^{\mu-\frac{1}{2}}}{\Gamma(\mu)H^{\mu-\frac{1}{2}}\Omega^{\mu+\frac{1}{2}}}e^{\left(-\frac{2\mu h\omega}{\Omega}\right)}I_{\mu-\frac{1}{2}}\left(\frac{2\mu H\omega}{\Omega}\right)$$

where $I_v(x)$ is the v-th order modified Bessel function of the first kind [23, Eq. (8.445)], $\Gamma(x)$ is the Gamma function [23, Eq. (8.310.1)], and $\Omega = \mathrm{E}\{\omega\}$ denotes the average power. The parameters h and H related to η are different in two formats. More specifically, according to format 1, $h = \left(2+\eta^{-1}+\eta\right)/4$ and $H = \left(\eta^{-1}-\eta\right)/4$, where $0<\eta<\infty$ is the scattered-wave power ratio between the in-phase and quadrature components. For format 2, $h = 1/\left(1-\eta^2\right)$ and $H = \eta/\left(1-\eta^2\right)$, where $-1<\eta<1$ represents the

$$\begin{aligned}E_x(p_{\mathbf{X}}(\mathbf{X}),\rho,r,N_c) \triangleq -\frac{1}{N_c}\ln\int_{\mathbf{H}}p_{\mathbf{H}}(\mathrm{H})\Bigg\{\int_{\mathbf{X}'}\int_{\mathbf{X}}&p_{\mathbf{X}}(\mathbf{X})p_{\mathbf{X}}(\mathbf{X}')e^{r\left[\mathrm{tr}\left(XX^{\dagger}\right)+\mathrm{tr}\left(X'X'^{\dagger}\right)-2N_cP\right]} \\ &\times\left[\int_{\mathbf{Y}}\sqrt{p(\mathbf{Y}|\mathbf{X},\mathbf{H})p(\mathbf{Y}|\mathbf{X}',\mathbf{H})}d\mathbf{Y}\right]^{1/\rho}d\mathbf{X}d\mathbf{X}'\Bigg\}^{\rho}d\mathbf{H}. \quad (16)\end{aligned}$$

correlation coefficient between the scattered-wave in-phase and quadrature components of each cluster of multipath. In both formats, the parameter μ denotes the half number of multipath clusters. Note that the η-μ distribution includes the Hoyt (Nakagami-q) distribution as special case for $\mu = 0.5$ and $\eta = q^2$. We also note that the Nakagami-m distribution can be obtained by setting $\mu = m$ and $\eta \to 0$ or $\eta \to \infty$ in format 1 of the η-μ distribution. Moreover, the Rayleigh distribution is obtained by setting $\mu = 0.5$ and $\eta = 1$ in format 1 or $\mu = 0.5$ and $\eta = 0$ in format 2. Alternatively, it can be attained by setting $\mu = m/2$ and $\eta \to 1$ in format 1 or $\eta \to 0$ in format 2 [18]. Without significant loss of generality, we only consider format 1 in the following analysis.

According to [18], [24], it is known that the sum of M i.i.d. squared η-μ RVs with parameters η, μ, and Ω is also an η-μ RV with parameters η, $M\mu$ and $M\Omega$. As such, we can easily obtain the p.d.f. of $z = \sum_{i=1}^{N_t N_r} |h_i|^2$ as follows,

$$p_{\eta-\mu}(z) = \frac{2\sqrt{\pi} h^{\mu N_t N_r}}{\Gamma(\mu N_t N_r)} \left(\frac{\mu}{\Omega}\right)^{\mu N_t N_r + \frac{1}{2}} \left(\frac{z}{H}\right)^{\mu N_t N_r - \frac{1}{2}} \times \exp\left(-\frac{2\mu h z}{\Omega}\right) I_{\mu N_t N_r - \frac{1}{2}}\left(\frac{2\mu H z}{\Omega}\right). \quad (19)$$

By using the representation of $I_v(x)$ in terms of a generalized hypergeometric function ${}_pF_q(\cdot)$ [23, Eq. (9.238.2)] and [25, Eq. (6.1.18)], we can obtain an alternative expression as

$$p_{\eta-\mu}(z) = \frac{z^{2\mu N_t N_r - 1} h^{\mu N_t N_r}}{\Gamma(2\mu N_t N_r)} \left(\frac{2\mu}{\Omega}\right)^{2\mu N_t N_r} e^{\left(-\frac{2\mu(h+H)}{\Omega} z\right)} \times {}_1F_1\left(\mu N_t N_r; 2\mu N_t N_r; \frac{4\mu H}{\Omega} z\right). \quad (20)$$

1) Random coding exponent analysis: Based on the theoretical analysis presented in Section II, we first obtain the exact random coding exponent as follows:

Proposition 1: The random coding exponent of STBC over η-μ MIMO fading channels can be expressed as in (21) at the bottom of this page, where $U(\cdot)$ is the Tricomi hypergeometric function [25, Eq. (13.1.3)].

Proof: We can directly substitute (19) into (9) and thereafter use the infinite series representation of $I_v(x)$ from [23, Eq. (8.445)]. The involved integral is evaluated with the help of the following identity

$$\int_0^\infty \frac{(1+ax)^{-v}}{x^{1-q} e^{px}} dx = \frac{p^{v-q}}{a^v} \Gamma(q)\, U\left(v; v-q+1; \frac{p}{a}\right) \quad (22)$$

which is a combination of Kummer's transformation [26, Eq. (07.33.17.0007.01)] and [3, Eq. (39)]. Then, the proof concludes after invoking [25, Eq. (6.1.18)] and appropriate simplifications. ■

In order to assess the convergence of the infinite series in (21), we assume that $T_0 - 1$ terms are used; hence, the associated truncation error Ξ_0 can be upper bounded as

$$\begin{aligned}
\Xi_0 &= \sum_{l=T_0}^{\infty} \frac{\Gamma(\mu N_t N_r + l)}{l!} \left(\frac{H}{h}\right)^{2l} \times U\left(N_c\rho; N_c\rho - 2\mu N_t N_r - 2l + 1; \frac{2\mu h\beta(1+\rho)}{\Omega\gamma}\right) \\
&< U\left(N_c\rho; N_c\rho - 2\mu N_t N_r - 2T_0 + 1; \frac{2\mu h\beta(1+\rho)}{\Omega\gamma}\right) \times \sum_{l=T_0}^{\infty} \frac{\Gamma(\mu N_t N_r + l)}{\Gamma(l+1)} \left(\frac{H}{h}\right)^{2l} \\
&= U\left(N_c\rho; N_c\rho - 2\mu N_t N_r - 2T_0 + 1; \frac{2\mu h\beta(1+\rho)}{\Omega\gamma}\right) \\
&\quad \times \left(\frac{\Gamma(\mu N_t N_r)}{\left(1 - \left(\frac{H}{h}\right)^2\right)^{-\mu N_t N_r}} - \sum_{l=0}^{T_0-1} \frac{\Gamma(\mu N_t N_r + l)}{\Gamma(l+1)} \left(\frac{H}{h}\right)^{2l}\right)
\end{aligned} \quad (23)$$

where we have used the fact that $U(a; b-n; z)$ is a monotonically decreasing function in n.

As was previously mentioned, our analysis elaborates also on the Shannon capacity, cutoff rate and expurgated component of STBC systems. We now present new results for these metrics over η-μ fading channels.

Corollary 1: The Shannon capacity of STBC over η-μ MIMO fading channels can be expressed as

$$\langle C \rangle = \frac{\exp\left(\frac{2\mu h N_t}{\Omega\gamma}\right)}{\Gamma(\mu N_t N_r) h^{\mu N_t N_r}} \sum_{l=0}^{\infty} \frac{\Gamma(\mu N_t N_r + l)}{l!} \left(\frac{H}{h}\right)^{2l} \times \sum_{n=1}^{2\mu N_t N_r + 2l} E_{2\mu N_t N_r + 2l + 1 - n}\left(\frac{2\mu h N_t}{\Omega\gamma}\right) \quad (24)$$

where $E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$, $n = 0, 1, 2, \ldots$ and $\mathrm{Re}(x) > 0$ denotes the exponential integral function of order n [25, Eq. (5.1.4)].

Proof: By plugging (19) into (13), we end up with the following integral expression

$$\langle C \rangle = \frac{2\sqrt{\pi} h^{\mu N_t N_r}}{\Gamma(\mu N_t N_r)} \sum_{l=0}^{\infty} \frac{H^{2l} (\mu/\Omega)^{2\mu N_t N_r + 2l}}{l!\,\Gamma(\mu N_t N_r + l + 1/2)} \times \int_0^\infty \ln\left(1 + \frac{\gamma z}{\beta(1+\rho)}\right) z^{2\mu N_t N_r + 2l - 1} e^{\left(-\frac{2\mu h z}{\Omega}\right)} dz. \quad (25)$$

With the aid of [3, Eq. (40)] and [9, Eq. (46)], we can evaluate the integral in (25) and arrive at the desired result in (24). ■

$$E_r(R, N_c, \eta, \mu) = \max_{0 \le \rho \le 1} \max_{0 \le \beta \le N_t} \left(A(\rho, \beta) - \frac{1}{N_c} \ln\left(\frac{h^{-\mu N_t N_r}}{\Gamma(\mu N_t N_r)} \left(\frac{2\mu h\beta(1+\rho)}{\Omega\gamma}\right)^{N_c\rho} \sum_{l=0}^{\infty} \frac{\Gamma(\mu N_t N_r + l)}{l!} \left(\frac{H}{h}\right)^{2l} \times U\left(N_c\rho; N_c\rho - 2\mu N_t N_r - 2l + 1; \frac{2\mu h\beta(1+\rho)}{\Omega\gamma}\right)\right)\right) - \rho R. \quad (21)$$

As for Proposition 1, we can prove the convergence of the infinite series in (24). By assuming $T_1 - 1$ terms are used, the associated truncation error Ξ_1 can be upper bounded as

$$\begin{aligned}\Xi_1 &= \sum_{l=T_1}^{\infty} \frac{\Gamma(\mu N_t N_r + l)}{l!}\left(\frac{H}{h}\right)^{2l} \\ &\times \sum_{n=1}^{2\mu N_t N_r+2l} E_{2\mu N_t N_r+2l+1-n}\left(\frac{2\mu h N_t}{\Omega\gamma}\right) \\ &< \nu E_\nu\left(\frac{2\mu h N_t}{\Omega\gamma}\right)\left(\Gamma(\mu N_t N_r)\left(1-\left(\frac{H}{h}\right)^2\right)^{\mu N_t N_r}\right. \\ &\left. - \sum_{l=0}^{T_1-1} \frac{\Gamma(\mu N_t N_r + l)}{\Gamma(l+1)}\left(\frac{H}{h}\right)^{2l}\right) \qquad (26)\end{aligned}$$

where $\nu = 2\mu N_t N_r + 2T_1 - 2$, and we have exploited the fact that $E_n(x)$ is a monotonically decreasing function in n.

Corollary 2: The cutoff rate R_0 of STBC over η-μ MIMO fading channels can be expressed as

$$\begin{aligned}R_0 = &-\frac{1}{N_c}\ln\left(\frac{h^{-\mu N_t N_r}}{\Gamma(\mu N_t N_r)}\left(\frac{4\mu h N_t}{\Omega\gamma}\right)^{N_c} \sum_{l=0}^{\infty} \frac{\Gamma(\mu N_t N_r + l)}{l!}\right. \\ &\left.\times \left(\frac{H}{h}\right)^{2l} U\left(N_c; N_c + 1 - 2\mu N_t N_r - 2l; \frac{4\mu h N_t}{\Omega\gamma}\right)\right). \qquad (27)\end{aligned}$$

Proof: The proof follows a similar line of reasoning as in Proposition 1, by plugging (19) into (14) and using (22). ■

Corollary 3: The expurgated exponent of STBC over η-μ MIMO fading channels can be expressed as in (28) at the bottom of this page.

Proof: Substituting (19) into (18) and also using the integral identity (22), the desired result can be obtained after some algebra. ■

2) High-SNR analysis: We now investigate Gallager's exponent in the high-SNR regime and present closed-form expressions for all related figures of merit.

Corollary 4: The random coding exponent of STBC over η-μ fading channels at high SNRs and for $N_c\rho < 2\mu N_t N_r$ can be expressed as in (29) at the bottom of this page.

Proof: By considering the initial expression (9) and keeping only the dominant term therein as $\gamma \to \infty$, we can obtain the desired result in (29) with the aid of (20) and the following integral identity [23, Eq. (7.522.9)]

$$\int_0^{\infty} x^{\sigma-1} e^{-ax} {}_1F_1(\alpha;\beta;\lambda x)\,dx = \frac{\Gamma(\sigma)}{a^{\sigma}} {}_2F_1\left(\alpha,\sigma;\beta;\frac{\lambda}{a}\right) \qquad (30)$$

where $\mathrm{Re}(\sigma) > 0,\ \mathrm{Re}(a) > \mathrm{Re}(\lambda)$. Note that the condition on the arguments of (30) is satisfied in our setting by taking $N_c\rho < 2\mu N_t N_r$. ■

Corollary 5: The Shannon capacity of STBC over η-μ MIMO fading channels at high SNRs can be expressed as

$$\begin{aligned}\langle C\rangle^{\infty} &= \sum_{l=0}^{\infty} \frac{\Gamma(\mu N_t N_r + l) H^{2l}}{l!\Gamma(\mu N_t N_r) h^{\mu N_t N_r + 2l}} \\ &\times \left(\psi(2\mu N_t N_r + 2l) - \ln\left(\frac{2\mu h N_t}{\Omega\gamma}\right)\right) \qquad (31)\end{aligned}$$

where $\psi(\cdot)$ is Euler's digamma function [23, Eq. (8.360.1)].

Proof: The proof follows by taking γ large in (25), then using the integral identity [23, Eq. (4.352.1)] and simplifying the resulting expression. ■

Corollary 6: The cutoff rate of STBC over η-μ MIMO fading channels at high SNRs and for $N_c < 2\mu N_t N_r$ can be expressed as

$$\begin{aligned}R_0^{\infty} = &-\ln\left(\frac{2\mu N_t\left(1+\eta^{-1}\right)}{\Omega\gamma}\right) \\ &- \frac{\ln\left(\frac{\Gamma(2\mu N_t N_r - N_c)}{\Gamma(2\mu N_t N_r)} {}_2F_1\left(\mu N_t N_r, N_c; 2\mu N_t N_r; 1-\frac{1}{\eta}\right)\right)}{N_c}. \qquad (32)\end{aligned}$$

Proof: The proof concludes by following a similar line of reasoning as in Corollary 4. ■

Corollary 7: The expurgated exponent of STBC over η-μ MIMO fading channels at high SNRs and for $N_c\rho < 2\mu N_t N_r$ can be expressed as in (33) at the top of next page.

Proof: By taking γ large in (18), the proof boils down to the computation of $\mathrm{E}\left\{\left(\frac{\gamma z}{2\rho\beta}\right)^{-N_c\rho}\right\}$. Combining (20) with the expectation operation, the proof concludes by invoking (30). We also set $N_c\rho < 2\mu N_t N_r$ to satisfy the condition on the arguments of (30). ■

$$\begin{aligned}E_{ex}(R, N_c, \eta, \mu) = \max_{\rho \ge 1}\Bigg(\max_{0\le\beta\le N_t}\Bigg(A'(\rho,\beta) - \frac{1}{N_c}\ln\Bigg(\frac{h^{-\mu N_t N_r}}{\Gamma(\mu N_t N_r)}\left(\frac{4\mu h\beta\rho}{\Omega\gamma}\right)^{N_c\rho} \\ \times \sum_{l=0}^{\infty}\frac{\Gamma(\mu N_t N_r + l)}{l!}\left(\frac{H}{h}\right)^{2l} U\left(N_c\rho; N_c\rho + 1 - 2\mu N_t N_r - 2l; \frac{4\mu h\beta\rho}{\Omega\gamma}\right)\Bigg)\Bigg) - \rho R\Bigg). \qquad (28)\end{aligned}$$

$$\begin{aligned}E_r^{\infty}(R, N_c, \eta, \mu) = \max_{0\le\rho\le 1}\Bigg(\max_{0\le\beta\le N_t}\Bigg(A(\rho,\beta) - \rho\ln\left(\frac{\mu\beta\left(1+\eta^{-1}\right)(1+\rho)}{\Omega\gamma}\right) \\ - \frac{1}{N_c}\ln\left(\frac{\Gamma(2\mu N_t N_r - N_c\rho)}{\Gamma(2\mu N_t N_r)} {}_2F_1\left(\mu N_t N_r, N_c\rho; 2\mu N_t N_r; 1-\eta^{-1}\right)\right)\Bigg) - \rho R\Bigg). \qquad (29)\end{aligned}$$

$$E_{ex}^{\infty}\left(R,N_c,\eta,\mu\right)=\max_{\rho\ge 1}\left(\max_{0\le\beta\le N_t}\left(A'\left(\rho,\beta\right)-\rho\ln\left(\frac{2\mu\rho\beta\left(1+\eta^{-1}\right)}{\Omega\gamma}\right)\right.\right.$$
$$\left.\left.-\frac{1}{N_c}\ln\left(\frac{\Gamma\left(2\mu N_tN_r-N_c\rho\right)}{\Gamma\left(2\mu N_tN_r\right)}{}_2F_1\left(\mu N_tN_r,N_c\rho;2\mu N_tN_r;1-\eta^{-1}\right)\right)\right)-\rho R\right). \quad (33)$$

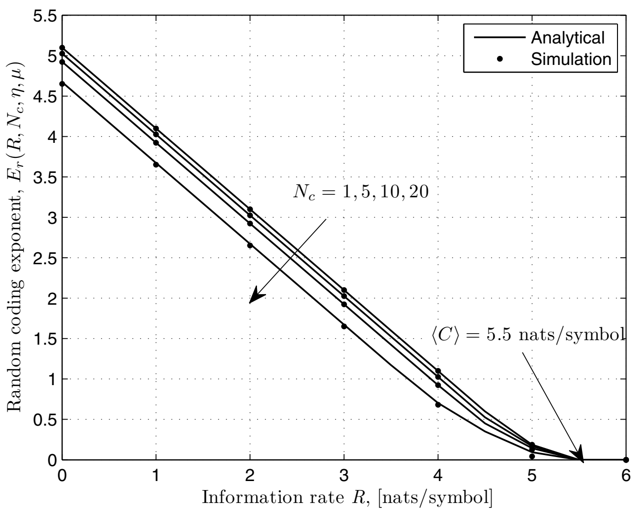

Fig. 1. Analytical and simulated random coding exponent against information rate for STBC systems over η-μ fading channels ($N_t = N_r = 4$, $\eta = 0.5$, $\mu = 1$, $\Omega = 2.5$, and $\gamma = 15$ dB).

3) Numerical results: In this subsection, the above theoretical analysis is validated through a set of Monte-Carlo simulations. We first generate 10^6 random realizations of the small-scale fading matrix $\mathbf{H}$ according to (19), and thereafter obtain the simulated random coding exponent, Shannon capacity, cutoff rate and expurgated exponent via the corresponding expressions (21), (24), (27), and (28), respectively.

In Fig. 1, the simulated random coding exponent is plotted along with the analytical expression (21). Note that the random coding exponent decreases monotonically with the parameter N_c, which means a less reliable communication can be achieved for a larger coherence time. Moreover, it is impossible to transmit any information at a positive rate with arbitrary small error probability when $N_c \to \infty$. As expected, the Shannon capacity is independent of N_c and represents the upper bound of R.

In order to get more insights into the effects of η and μ on coding requirements for STBC systems, the codeword length L required to achieve a fixed error probability and rate, i.e., $P_e \le 10^{-6}$, $R = 4$ bits/symbol, is tabulated in Table I. We observe from Table I that for each value of γ, the codeword lengths for channels with small values of η and μ are much longer than those for channels with large values of η and μ. This is due to the advantages of having more multipath clusters. For example, the required codeword length for the case of $\eta = 0.2$ and $\mu = 0.2$ is almost 5.4 times the codeword length for the case of $\eta = 0.5$ and $\mu = 1$ for $\gamma = 12.5$ dB.

Figure 2 illustrates the simulated, analytical (27) and high-

TABLE I
REQUIRED CODEWORD LENGTHS OF STBC SYSTEMS OVER η-μ FADING CHANNELS AT A RATE $R = 4$ BITS/SYMBOL (2.77 NATS/SYMBOL) WITH $P_e \le 10^{-6}$, $N_t = N_r = 2$, $\Omega = 2.5$, AND $N_c = 5$

SNR γ (dB)	$\mu = 0.2$ $\eta = 0.2$	$\mu = 0.5$ $\eta = 1$	$\mu = 1$ $\eta = 0.5$
10	460	110	70
12.5	160	45	30
15	85	30	20
17.5	55	20	15
20	35	15	10

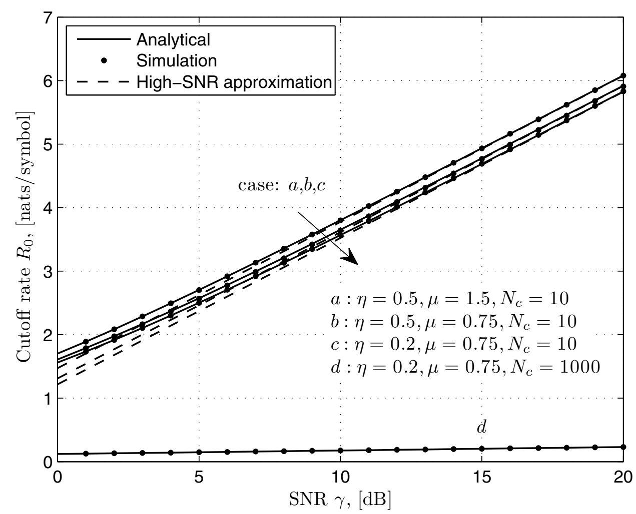

Fig. 2. Analytical, simulated and high-SNR approximation cutoff rate against the transmit SNR for STBC systems over η-μ fading channels ($N_t = N_r = 4$ and $\Omega = 2.5$).

SNR approximation (32) cutoff rate as a function of the transmit SNR for STBC over η-μ MIMO fading channels with different η, μ and N_c. It can be seen that the cutoff rate is a monotonically increasing function of γ, η and μ. Note that the effect of μ on the cutoff rate is more pronounced than that of η. We also note that as N_c increases, R_0 reduces to zero, while the Shannon capacity is independent of N_c. This difference reveals that the cutoff rate is more useful than the Shannon capacity in reflecting the reliability of block-fading channels, which is consistent with the results in [6], [21].

In Fig. 3, the random coding (21) and expurgated exponents (28) are plotted as a function of R for different values of η and μ. As expected, a performance improvement is observed as μ increases, which corresponds to more multipath clusters. Likewise, we also observe that Gallager's exponent increases when η increases from 0.2 to 0.5, which indicates that a shorter code is required to achieve the same level of reliable

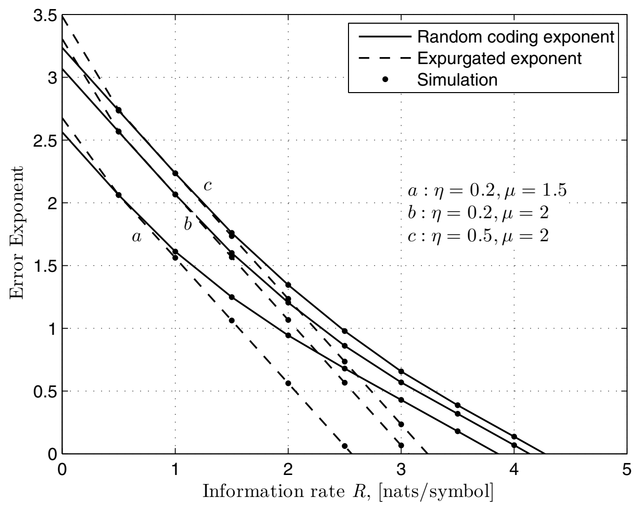

Fig. 3. Analytical and simulated results for both random coding and expurgated exponents against information rate, R, for STBC systems over η-μ fading channels ($N_t = N_r = 2$, $N_c = 20$, $\Omega = 2.5$, $\gamma = 15$ dB).

communications. However, the impact of μ on Gallager's exponent is more pronounced than that of η.

B. κ-μ fading channels

The κ-μ distribution models the small-scale variation of the fading signal in a non-homogeneous environment. In a physical κ-μ fading model, the phases of the scattered waves are random and have similar delay times, while the spreads of different clusters are relatively large within each cluster. It is assumed that each cluster has a dominant component with arbitrary power, and that the clusters of multipath waves have scattered waves with identical powers. The p.d.f. of the κ-μ SNR is given by [18, Eq. (10)]

$$p_{\kappa\text{-}\mu}(\omega) = \frac{\mu(1+\kappa)^{\frac{\mu+1}{2}}\omega^{\frac{\mu-1}{2}}}{\exp(\mu\kappa)\,\kappa^{\frac{\mu-1}{2}}\Omega^{\frac{\mu+1}{2}}}\exp\left(-\frac{\mu(1+\kappa)\omega}{\Omega}\right) \times I_{\mu-1}\left(2\mu\sqrt{\frac{\kappa(1+\kappa)\omega}{\Omega}}\right) \tag{34}$$

where κ denotes the ratio between the total power of the dominant components and the total power of the scattered waves, while μ is related to the number of the multipath clusters. Note that the κ-μ distribution includes the Rician and Nakagami-m distributions as special cases for $\mu = 1$ and $\kappa \to 0$, respectively.

We start our analysis by invoking [18], which showed that the sum of M i.i.d. squared κ-μ RVs with parameters κ, μ, and Ω is also a κ-μ RV with parameters κ, $M\mu$, and $M\Omega$. Then, we can obtain the p.d.f. of $z = \sum_{i=1}^{N_tN_r}|h_i|^2$ as

$$p_{\kappa\text{-}\mu}(z) = \sum_{l=0}^{\infty}\frac{(\mu\kappa N_tN_r)^l}{l!\Gamma(\mu N_tN_r+l)\,z}\left(\frac{\mu(1+\kappa)z}{\Omega}\right)^{\mu N_tN_r+l} \times \exp\left(-\mu\kappa N_tN_r - \frac{\mu(1+\kappa)z}{\Omega}\right) \tag{35}$$

where we have used [23, Eq. (8.445)]. Note that the infinite series expression in (35), although is not in closed-form, is more amenable to mathematical manipulations.

1) Random coding exponent analysis: We first present analytical results on Gallager's exponent, Shannon capacity and cutoff rate of STBC over MIMO κ-μ fading channels.

Proposition 2: The random coding exponent of STBC over κ-μ MIMO fading channels can be expressed as in (36) at the bottom of this page.

Proof: Following a similar line of reasoning as in Proposition 1, the proof concludes by substituting (35) into (9) and using (22). ■

Note that the convergence of the infinite series in (36), can be trivially demonstrated following the technique of (23).

Corollary 8: The Shannon capacity of STBC over MIMO κ-μ fading channels can be expressed as

$$\langle C\rangle = \exp\left(\frac{(1+\kappa)\mu N_t}{\Omega\gamma} - \mu\kappa N_tN_r\right)\sum_{l=0}^{\infty}\frac{(\mu\kappa N_tN_r)^l}{l!} \times \sum_{n=1}^{\mu N_tN_r+l}E_{\mu N_tN_r+l+1-n}\left(\frac{(1+\kappa)\mu N_t}{\Omega\gamma}\right). \tag{37}$$

Proof: Following a similar line of reasoning as in Corollary 1, we can conclude the proof after some basic algebraic manipulations. ■

Corollary 9: The cutoff rate of STBC over MIMO κ-μ fading channels can be expressed as

$$R_0 = -\frac{1}{N_c}\ln\left(\exp(-\mu\kappa N_tN_r)\left(\frac{2\mu N_t(1+\kappa)}{\Omega\gamma}\right)^{N_c} \times\sum_{l=0}^{\infty}\frac{(\mu\kappa N_tN_r)^l}{l!}U\left(N_c; N_c-\mu N_tN_r-l+1; \frac{2\mu N_t(1+\kappa)}{\Omega\gamma}\right)\right).$$

Proof: The proof is trivial and therefore omitted. ■

Corollary 10: The expurgated exponent of STBC over κ-μ MIMO fading channels can be obtained as in (38) at the bottom of next page.

Proof: The proof follows a similar line of reasoning as in Corollary 3. ■

$$E_r(R, N_c, \kappa, \mu) = \max_{0\le\rho\le 1}\left(\max_{0\le\beta\le N_t}\left(A(\rho,\beta) - \frac{1}{N_c}\ln\left(\exp(-\mu\kappa N_tN_r)\left(\frac{\mu(1+\kappa)\beta(1+\rho)}{\Omega\gamma}\right)^{N_c\rho} \times\sum_{l=0}^{\infty}\frac{(\mu\kappa N_tN_r)^l}{l!}U\left(N_c\rho, N_c\rho-\mu N_tN_r-l+1, \frac{\mu(1+\kappa)\beta(1+\rho)}{\Omega\gamma}\right)\right)\right) - \rho R\right). \tag{36}$$

2) High-SNR analysis: In order to obtain additional insights, we now elaborate on the high-SNR regime. We begin with the following result:

Corollary 11: The random coding exponent of STBC over MIMO κ-μ fading channels at high SNRs and for $N_c\rho < \mu N_t N_r$ can be expressed as in (39) at the bottom of this page.

Proof: We first take $\gamma \to \infty$ in (9) and keep only the dominant term. Then, the desired result in (39) can be obtained with the help of [23, Eq. (9.212.1)] and the following integral identity [27, Eq. (3.15.2.5)]

$$\int_0^\infty x^n e^{-px} I_v\left(a\sqrt{x}\right) dx = \frac{\Gamma\left(n+\frac{v}{2}+1\right)}{\Gamma\left(v+1\right)} \frac{\left(\frac{v}{2}\right)^v}{p^{n+\frac{v}{2}+1}} {}_1F_1\left(n+\frac{v}{2}+1; v+1; \frac{a^2}{4p}\right) \tag{40}$$

where $\mathrm{Re}\left(2n+v\right) > -2, \mathrm{Re}\left(p\right) > 0$. Note that the condition on the arguments of (40) requires $N_c\rho < \mu N_t N_r$. ■

Corollary 12: The Shannon capacity of STBC over κ-μ MIMO fading channels at high SNRs can be written as

$$\langle C\rangle^\infty = \exp\left(-\mu\kappa N_t N_r\right) \sum_{l=0}^{\infty} \frac{\left(\kappa\mu N_t N_r\right)^l}{l!} \times \left(\psi\left(\mu N_t N_r + l\right) - \ln\left(\frac{\mu\left(1+\kappa\right) N_t}{\Omega\gamma}\right)\right). \tag{41}$$

Proof: The proof follows by applying a similar methodology as in Corollary 5. ■

Corollary 13: The cutoff rate of STBC over κ-μ MIMO fading channels at high SNRs and for $N_c < \mu N_t N_r$ can be expressed as

$$R_0^\infty\left(R, N_c, \kappa, \mu\right) = -\ln\left(\frac{2\mu N_t\left(1+\kappa\right)}{\Omega\gamma}\right) - \frac{1}{N_c}\ln\left(\frac{\Gamma\left(\mu N_t N_r - N_c\right)}{\Gamma\left(\mu N_t N_r\right)} {}_1F_1\left(N_c; \mu N_t N_r; -\mu\kappa N_t N_r\right)\right). \tag{42}$$

Proof: The proof concludes by following a similar line of reasoning as in Corollary 11. ■

Corollary 14: The expurgated exponent of STBC over κ-μ MIMO fading channels at high SNRs and for $N_c\rho < \mu N_t N_r$ can be expressed as in (43) at the bottom of this page.

Proof: The proof follows by applying a similar methodology as for the random coding exponent results presented in Corollary 7. ■

3) Numerical results: Table II shows the required codeword length L for MIMO κ-μ fading channels with $N_t = N_r = 2$, $\Omega = 2.5$, and $N_c = 5$ at $P_e \leq 10^{-6}$ when γ varies from 10 to 20 dB. It is clear from Table II that there is a considerable reduction in the required codeword length when the values of κ and μ increase. This is expected since for large values of κ, the dominant components of signals have more power, thereby reducing the signal's envelope fluctuations.

TABLE II
REQUIRED CODEWORD LENGTHS OF STBC OVER κ-μ MIMO FADING CHANNELS AT A RATE $R = 4$ BITS/SYMBOL (2.77 NATS/SYMBOL) WITH $P_e \leq 10^{-6}$, $N_t = N_r = 2, \Omega = 2.5$ AND $N_c = 5$

SNR γ (dB)	$\kappa = 0$ $\mu = 1$	$\kappa = 5$ $\mu = 0.5$	$\kappa = 10$ $\mu = 2$
10	110	80	40
12.5	45	40	20
15	30	25	15
17.5	20	20	10
20	15	15	10

$$E_{ex}\left(R, N_c, \kappa, \mu\right) = \max_{\rho\geq 1}\left(\max_{0\leq\beta\leq N_t}\left(A'\left(\rho,\beta\right) - \frac{1}{N_c}\ln\left(\exp\left(-\mu\kappa N_t N_r\right)\left(\frac{2\rho\mu\beta\left(1+\kappa\right)}{\Omega\gamma}\right)^{N_c\rho} \times \sum_{l=0}^{\infty}\frac{\left(\mu\kappa N_t N_r\right)^l}{l!} U\left(N_c\rho; N_c\rho - \mu N_t N_r - l + 1; \frac{2\rho\mu\beta\left(1+\kappa\right)}{\Omega\gamma}\right)\right)\right) - \rho R\right). \tag{38}$$

$$E_r^\infty\left(R, N_c, \kappa, \mu\right) = \max_{0\leq\rho\leq 1}\left(\max_{0\leq\beta\leq N_t}\left(A\left(\rho,\beta\right) - \rho\ln\left(\frac{\mu\beta\left(1+\kappa\right)\left(1+\rho\right)}{\Omega\gamma}\right) - \frac{1}{N_c}\ln\left(\frac{\Gamma\left(\mu N_t N_r - N_c\rho\right)}{\Gamma\left(\mu N_t N_r\right)} {}_1F_1\left(N_c\rho; \mu N_t N_r; -\mu\kappa N_t N_r\right)\right)\right) - \rho R\right). \tag{39}$$

$$E_{ex}^\infty\left(R, N_c, \kappa, \mu\right) = \max_{\rho\geq 1}\left(\max_{0\leq\beta\leq N_t}\left(A'\left(\rho,\beta\right) - \rho\ln\left(\frac{2\rho\mu\beta\left(1+\kappa\right)}{\Omega\gamma}\right) - \frac{1}{N_c}\ln\left(\frac{\Gamma\left(\mu N_t N_r - N_c\rho\right)}{\Gamma\left(\mu N_t N_r\right)} {}_1F_1\left(N_c\rho; \mu N_t N_r; -\mu\kappa N_t N_r\right)\right)\right) - \rho R\right). \tag{43}$$

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

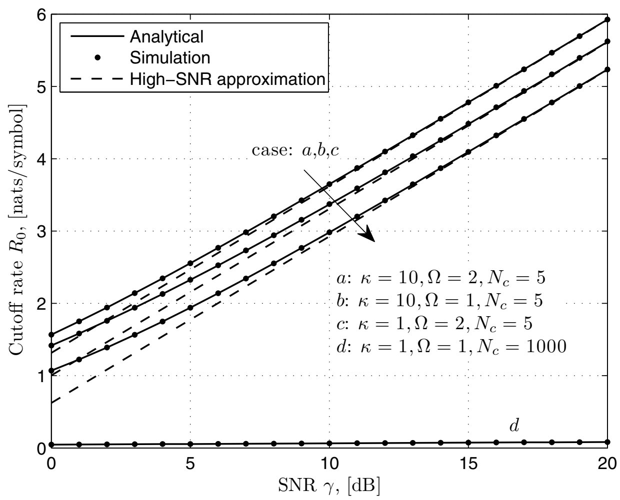

Fig. 4. Analytical, simulated and high-SNR approximation cutoff rate against the transmit SNR for STBC systems over κ-μ fading channels ($N_t = N_r = 4$ and $\mu = 0.5$).

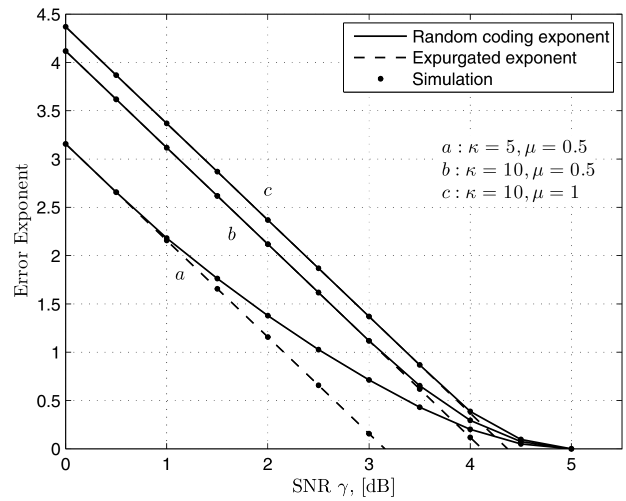

Fig. 6. Analytical and simulated results for both random coding and expurgated exponents against information rate, R, for STBC systems over κ-μ fading channels ($N_t = N_r = 2$, $N_c = 5$, $\Omega = 2.5$, $\gamma = 15$ dB).

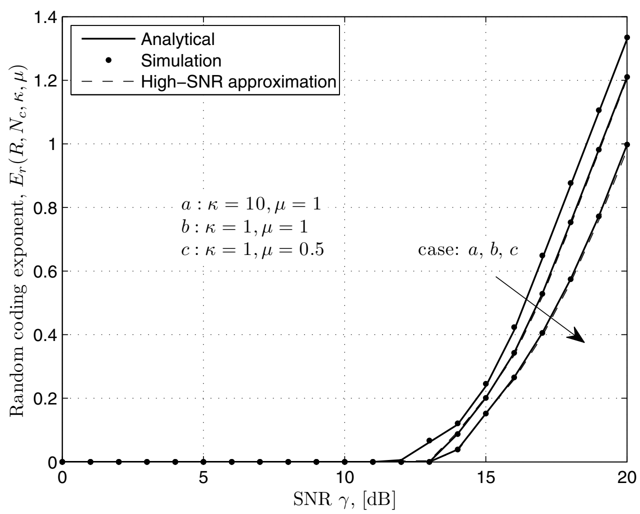

Fig. 5. Analytical, simulated and high-SNR approximation random coding exponent against the transmit SNR for STBC systems over κ-μ fading channels ($N_t = N_r = 4$, $N_c = 5$, $R = 4$ nats/symbol and $\Omega = 1$).

In Fig. 4, it can be seen that the analytical cutoff rate of STBC over κ-μ MIMO fading channels exhibit a good match with the simulation result. The high-SNR approximation (42) remains sufficiently tight across a wide SNR range. Note that an increase in κ and Ω leads to higher values of cutoff rate. As reflected in Fig. 2, the cutoff rate reduces to zero when N_c increases to infinity.

The accuracy of the high-SNR approximation (39) for the random coding exponent of STBC over κ-μ MIMO fading models is illustrated in Fig. 5. We have assumed an information rate $R = 4$ nats/symbol and that the minimum SNR for reliable communication should be more than 12 dB. Clearly, the high-SNR expressions become sufficiently tight even at moderate SNRs and can accurately predict the random coding exponent for most practical SNR values. Note that the values of random coding exponent increase when κ and μ increase, whilst the effect of μ is more pronounced than that of κ. This is line with the conclusions drawn from Fig. 3.

In Fig. 6, the random coding and expurgated exponent are depicted as a function of the SNR with different fading parameters. The outputs of a Monte-Carlo simulator are compared with the exact expressions of Proposition 2 and Corollary 10, respectively. Once more, the match between theory and simulation is excellent for all cases under consideration. Note that the expurgated exponent is the lower bound of random coding exponent in all cases considered here. Both exponents are monotonically increasing functions in κ and μ, which implies that the error probability of communication system is lower with larger values of fading parameters, which also coincides with the conclusions of Table II. Alternatively, increasing the power of the dominant component requires a smaller information rate to achieve the same value of error exponent.

C. Special cases

In this subsection, we provide simplified analytical expressions for a few of widely used fading channel models, namely Nakagami-m, Rician, Rayleigh, and Hoyt, respectively. Note that all subsequent results are presented with no proof since the mathematical manipulations involved are straightforward. The link to previously reported results, where available, is also provided.

1) Nakagami-m fading channels: By setting $\mu = m/2$ and $\eta = 1$ in the η-μ distribution, we can obtain the Nakagami-m distribution. Then, the random coding and expurgated exponents of Nakagami-m fading channels in (21) and (28) reduce to (44) and (45) at the bottom of next page, respectively. Note that (44) coincides with [11, Eq. (23)] after applying the transformation [28, Eq. (8.4.46.1)]. We can also derive the Shannon capacity and cutoff rate expressions of STBC over Nakagami-m fading channels as:

$$\langle C\rangle = \exp\left(\frac{mN_t}{\Omega\gamma}\right)\sum_{n=1}^{mN_tN_r} E_{mN_tN_r-n+1}\left(\frac{mN_t}{\Omega\gamma}\right) \tag{46}$$

$$R_0 = -\frac{2mN_t}{\Omega\gamma} - \frac{\ln\left(U\left(N_c; N_c - mN_tN_r + 1; \frac{2mN_t}{\Omega\gamma}\right)\right)}{N_c}. \tag{47}$$

Note that (47) coincides with [11, Eq. (30)] after applying the transformation [28, Eq. (8.4.46.1)]. Note that the above expressions can be alternatively derived from the κ-μ formulations by setting $\kappa \to 0$ and $\mu = m$.

2) Rician fading channels: The Rician distribution can be obtained by setting $\mu = 1$ in the κ-μ distribution. Then, the random coding and expurgated exponents of κ-μ fading channels in (36) and (38) reduce to (48) and (49) at the bottom of this page, respectively. Similarly, we can derive the expressions of the Shannon capacity and cutoff rate for STBC systems over Rician fading channels

$$\begin{aligned}\langle C\rangle &= \exp\left(\frac{(K+1)\,N_t}{\Omega\gamma} - KN_tN_r\right)\\ &\times \sum_{l=0}^{\infty}\frac{(KN_tN_r)^l}{l!}\sum_{n=1}^{N_tN_r+l} E_{N_tN_r+l-n+1}\left(\frac{(K+1)\,N_t}{\Omega\gamma}\right)\\ R_0 &= -\frac{1}{N_c}\ln\left(\exp\left(-KN_tN_r\right)\left(\frac{2N_t\,(1+K)}{\Omega\gamma}\right)^{N_c}\right.\\ &\left.\times\sum_{l=0}^{\infty}\frac{(KN_tN_r)^l}{l!}U\left(N_c; N_c-N_tN_r-l+1; \frac{2N_t\,(1+K)}{\Omega\gamma}\right)\right).\end{aligned}$$

3) Rayleigh fading channels: We now turn our attention to the classical Rayleigh fading model. When considering the Rayleigh distribution, we simply set $\mu = 0.5$ and $\eta = 1$ in the η-μ distribution.

Therefore, the random coding and expurgated exponents of η-μ fading channels in (21) and (28) reduce respectively to

$$\begin{aligned}E_r\,(R, N_c) &= \max_{0\le\rho\le 1}\max_{0\le\beta\le N_t}\left(A\,(\rho,\beta) - \frac{1}{N_c}\ln\left(\left(\frac{\beta\,(1+\rho)}{\Omega\gamma}\right)^{N_c\rho}\right.\right.\\ &\left.\left.\times\, U\left(N_c\rho; N_c\rho - N_tN_r + 1; \frac{\beta\,(1+\rho)}{\Omega\gamma}\right)\right)\right) - \rho R\\ E_{ex}\,(R, N_c) &= \max_{\rho\ge 1}\max_{0\le\beta\le N_t}\left(A'\,(\rho,\beta) - \frac{1}{N_c}\ln\left(\left(\frac{2\beta\rho}{\Omega\gamma}\right)^{N_c\rho}\right.\right.\\ &\left.\left.\times\, U\left(N_c\rho; N_c\rho - N_tN_r + 1; \frac{2\beta\rho}{\Omega\gamma}\right)\right)\right) - \rho R.\end{aligned}$$

Note that the Shannon capacity and cutoff rate expressions for STBC over Rayleigh fading channels can be simply written as

$$\langle C\rangle = \exp\left(\frac{N_t}{\Omega\gamma}\right)\sum_{n=1}^{N_tN_r} E_{N_tN_r-n+1}\left(\frac{N_t}{\Omega\gamma}\right) \tag{50}$$

$$R_0 = -\frac{1}{N_c}\ln\left(\left(\frac{2N_t}{\Omega\gamma}\right)^{N_c} U\left(N_c, N_c - N_tN_r + 1, \frac{2N_t}{\Omega\gamma}\right)\right).$$

Note that (50) coincides with [29, Eq. (20)] after some algebra.

4) Hoyt fading channels: By setting $\mu = 0.5$ and $\eta = q^2$ in the η-μ distribution, the random coding and expurgated exponents of Hoyt fading channels can be expressed as in (51) and (52) at the bottom of next page, respectively.

In the same way, the expressions for the Shannon capacity and cutoff rate of STBC systems over Hoyt fading channels

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$$E_r\,(R, N_c, m) = \max_{0\le\rho\le 1}\max_{0\le\beta\le N_t}\left(A\,(\rho,\beta) - \frac{1}{N_c}\ln\left(\left(\frac{m\beta\,(1+\rho)}{\Omega\gamma}\right)^{N_c\rho} U\left(N_c\rho; N_c\rho - mN_tN_r + 1; \frac{m\beta\,(1+\rho)}{\Omega\gamma}\right)\right)\right) - \rho R \tag{44}$$

$$E_{ex}\,(R, N_c, m) = \max_{\rho\ge 1}\max_{0\le\beta\le N_t}\left(A'\,(\rho,\beta) - \frac{1}{N_c}\ln\left(\left(\frac{2mh\beta\rho}{\Omega\gamma}\right)^{N_c\rho} U\left(N_c\rho; N_c\rho - mN_tN_r + 1; \frac{2mh\beta\rho}{\Omega\gamma}\right)\right)\right) - \rho R. \tag{45}$$

$$\begin{aligned}E_r\,(R, N_c, K) &= \max_{0\le\rho\le 1}\max_{0\le\beta\le N_t}\left(A\,(\rho,\beta) - \frac{1}{N_c}\ln\left(\exp\left(-\kappa N_tN_r\right)\left(\frac{\beta\,(1+\kappa)\,(1+\rho)}{\Omega\gamma}\right)^{N_c\rho}\right.\right.\\ &\left.\left.\times\sum_{l=0}^{\infty}\frac{(\kappa N_tN_r)^l}{l!}U\left(N_c\rho; N_c\rho - N_tN_r - l + 1; \frac{\beta\,(1+\kappa)\,(1+\rho)}{\Omega\gamma}\right)\right)\right) - \rho R \qquad (48)\\ E_{ex}\,(R, N_c, K) &= \max_{\rho\ge 1}\max_{0\le\beta\le N_t}\left(A'\,(\rho,\beta) - \frac{1}{N_c}\ln\left(\exp\left(-\kappa N_tN_r\right)\left(\frac{2\rho\beta\,(1+\kappa)}{\Omega\gamma}\right)^{N_c\rho}\right.\right.\\ &\left.\left.\times\sum_{l=0}^{\infty}\frac{(\kappa N_tN_r)^l}{l!}U\left(N_c\rho; N_c\rho - N_tN_r - l + 1; \frac{2\rho\beta\,(1+\kappa)}{\Omega\gamma}\right)\right)\right) - \rho R. \qquad (49)\end{aligned}$$

are given by

$$\langle C\rangle = \frac{\exp\left(\frac{(1+q^2)^2 N_t}{4\Omega\gamma q^2}\right)}{\Gamma\left(\frac{N_tN_r}{2}\right)\left(\frac{1+q^2}{2q}\right)^{N_tN_r}}\sum_{l=0}^{\infty}\frac{\Gamma\left(\frac{N_tN_r}{2}+l\right)}{l!}\left(\frac{1-q^2}{1+q^2}\right)^{2l}$$
$$\times\sum_{n=1}^{N_tN_r+2l}E_{N_tN_r+2l-n+1}\left(\frac{\left(1+q^2\right)^2N_t}{4\Omega\gamma q^2}\right)$$
$$R_0{=}-\frac{1}{N_c}\ln\frac{\left(\frac{1+q^2}{2q}\right)^{-N_tN_r}\left(\frac{(1+q^2)^2N_t}{2\Omega\gamma q^2}\right)^{N_c}}{\Gamma\left(\frac{N_tN_r}{2}\right)}\sum_{l=0}^{\infty}\frac{\Gamma\left(\frac{N_tN_r}{2}+l\right)}{l!}$$
$$\times\left(\frac{1-q^2}{1+q^2}\right)^{2l}U\left(N_c;N_c-N_tN_r-2l+1;\frac{(1+q^2)^2N_t}{2\Omega\gamma q^2}\right).$$

IV. Conclusion

In this paper, a detailed Gallager's exponent analysis of MIMO systems employing STBC was presented in order to investigate the fundamental tradeoff between communication reliability and information rate. In particular, we considered the η-μ and κ-μ fading models, which have been exhaustively used in the performance analysis of wireless communication systems. For the considered models, new analytical expressions for the exact random coding exponent were derived that extend and complement several previous results on Rayleigh and Nakagami-m fading channels. Note that all the special functions in our results can be efficiently evaluated in standard software computer packages, such as MATLAB and MATHEMATICA. Moreover, we elaborated on the expurgated exponent, Shannon capacity and cutoff rate for which new analytical formulas were deduced. By exploring Gallager's exponent, we were able to derive the required codeword length to achieve a certain level of error probability and draw significant insights into the reliability-rate tradeoff in MIMO systems. Finally, we presented simplified high-SNR closed-form expressions of the above performance metrics and obtained additional physical insights into the implications of several parameters (e.g., fading parameters, coherence time) on the required codeword lengths for a prescribed error probability. For example, we noticed that larger values of μ, η and κ tend to increase Gallager's exponent or communication reliability.

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$$E_r\left(R,N_c,q\right)=\max_{0\le\rho\le1}\max_{0\le\beta\le N_t}\left(A\left(\rho,\beta\right)-\frac{1}{N_c}\ln\left(\frac{\left(\frac{1+q^2}{2q}\right)^{-N_tN_r}}{\Gamma\left(0.5N_tN_r\right)}\left(\frac{\beta\left(1+\rho\right)\left(1+q^2\right)^2}{4\Omega\gamma q^2}\right)^{N_c\rho}\right.\right.$$
$$\left.\left.\times\sum_{l=0}^{\infty}\frac{\Gamma\left(0.5N_tN_r+l\right)}{l!}\left(\frac{1-q^2}{1+q^2}\right)^{2l}U\left(N_c\rho,N_c\rho-N_tN_r-2l+1,\frac{\beta\left(1+\rho\right)\left(1+q^2\right)^2}{4\Omega\gamma q^2}\right)\right)\right)-\rho R \quad (51)$$

$$E_{ex}\left(R,N_c,q\right)=\max_{\rho\ge1}\max_{0\le\beta\le N_t}\left(A'\left(\rho,\beta\right)-\frac{1}{N_c}\ln\left(\frac{\left(\frac{1+q^2}{2q}\right)^{-N_tN_r}}{\Gamma\left(0.5N_tN_r\right)}\left(\frac{\beta\rho\left(1+q^2\right)^2}{2\Omega\gamma q^2}\right)^{N_c\rho}\right.\right.$$
$$\left.\left.\times\sum_{l=0}^{\infty}\frac{\Gamma\left(0.5N_tN_r+l\right)}{l!}\left(\frac{1-q^2}{1+q^2}\right)^{2l}U\left(N_c\rho,N_c\rho-N_tN_r-2l+1,\frac{\beta\rho\left(1+q^2\right)^2}{2\Omega\gamma q^2}\right)\right)\right)-\rho R. \quad (52)$$

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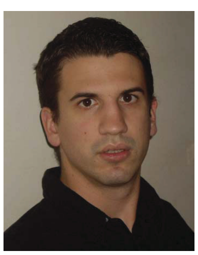

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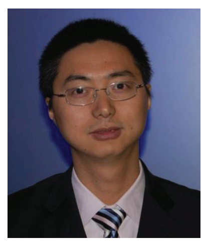

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