On the Multivariate Nakagami-m Distribution With Exponential Correlation

George K. Karagiannidis, *Member, IEEE*, Dimitris A. Zogas, *Student Member, IEEE*, and Stavros A. Kotsopoulos

***Abstract*—In this letter, capitalizing on the proof of a theorem presented by Blumenson and Miller many years ago, a useful closed formula for the exponentially correlated n-variate Nakagami-m probability density function is proposed. Moreover, an infinite series approach for the corresponding cumulative distribution function is presented. Bounds on the error resulting from the truncation of the infinite series are also derived. Finally, in order to check the accuracy of the proposed formulation, numerical results are presented.**

***Index Terms*—Correlated fading, diversity, exponential correlation model, Nakagami-m fading channels, Rayleigh fading channels.**

I. Introduction

THE effect of correlated fading on the performance of wireless communications systems has received a great deal of research interest recently, due to the possible use of space diversity receivers in handheld phones or compact base stations [1], [2]. Several spatial correlation models have been proposed and used for the performance analysis of various wireless systems, corresponding to specific modulation, detection, and diversity schemes. One of them, frequently used in multichannel reception, is the exponential correlation model, which was treated in [3] by Aalo, who studied the performance of maximal-ratio diversity in correlated Nakagami-m fading. The correlation matrix of this model is described by $\sum_{i,j} \equiv \rho^{|i-j|}$ and corresponds to the scenario of multichannel reception from equispaced diversity antennas, since the correlation between the pairs of combined signals decays as the spacing between the antennas increases [1]. The exponential model was used later by several researchers, who dealt with the performance analysis of space diversity techniques [4]–[6]. This model was also proposed to model the single Rayleigh fading process [7]. However, care should be taken in the use of this model as a temporal fading correlation model, since it gives zero average fade duration for the Rayleigh fading envelope, which is not consistent with real-world measured data [8]. From the literature review, multivariate Rayleigh densities have been reported in many papers. A summary of these works can be found in [9]. Simon and Alouini in [10] presented an approach to the bivariate Rayleigh cumulative distribution function (cdf) in the form of a single integral with finite limits and an integrand composed of elementary functions. As far as Nakagami-m multivariate analysis is concerned, Nakagami in [11] defined the bivariate Nakagami-m probability density function (pdf) and Tan and Beaulieu in [12] presented an infinite series representation for the bivariate Nakagami-m cdf.

The main contribution of this letter is the derivation of useful, closed-form expressions for the multivariate, exponentially correlated Nakagami-m pdf and cdf, which can be used as a general theoretical tool in the performance analysis of wireless communications systems with multichannel reception or in the Markov modeling of the Nakagami-m fading channel. More specifically, in this letter, capitalizing on the proof for a theorem presented many years ago by Blumenson and Miller in [13], a useful closed formula for the n-variate exponentially correlated Nakagami-m pdf is proposed. Moreover, an infinite series approach for the corresponding cdf is presented and bounds on the error resulting from the truncation of these series are derived. This approach is an extension to the n-variate case of the formulation proposed by Tan and Beaulieu in [12] for the bivariate Nakagami-m cdf.

II. The Exponentially Correlated Nakagami-m Distribution

The Nakagami-m model describes multipath scattering with relatively large delay-time spreads, with different clusters of reflected waves [11]. It includes, as a special case, the Rayleigh model, the one-sided Gaussian model, and it can also approximate the classical Rician fading distribution. However, while this approximation may be true for the main body of the pdf, it becomes highly inaccurate for the tails. As bit errors or outage mainly occur during deep fades, the tail of the pdf mainly determines these performance measures [14].

If r is a Nakagami-m variable, then its corresponding pdf is described by[1]

$$f(r) = \frac{2r^{2m-1}}{\Gamma(m)\Omega^m} \exp\left(-\frac{r^2}{\Omega}\right), \qquad r \geq 0 \tag{1}$$

whereas $\Gamma(\cdot)$ is the Gamma function, $\Omega = \overline{r^2}/m$, with $\overline{r^2}$ being the average signal power and m representing the inverse normalized variance r^2, which must satisfy $m \geq 1/2$, describing the fading severity. Moreover, it is well known that r can be considered as the square root of the sum of squares of m independent Rayleigh or $2m$ independent Gaussian variates [2], [11]. Let $\mathbf{Y}_1, \mathbf{Y}_2, \ldots, \mathbf{Y}_{2m}$ be n-dimensional column vectors, which are independent and normally distributed with mean zero and exponential correlation matrix $\sum_{i,j} \equiv \rho^{|i-j|}$, $1 \leq i, j \leq n$ with

$$\rho = \frac{\operatorname{cov}\left(r_i^2, r_j^2\right)}{\sqrt{\operatorname{var}\left(r_i^2\right)\operatorname{var}\left(r_j^2\right)}}, \qquad 0 \leq \rho < 1. \tag{2}$$

[1]Equation (1) is another representation of the classical formula for the single Nakagami-m pdf [11] using $\Omega = \overline{r^2}/m$.

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G. K. Karagiannidis is with the Institute for Space Applications and Remote Sensing, National Observatory of Athens, 15236 Athens, Greece (e-mail: gkarag@space.noa.gr).

D. A. Zogas and S. A. Kotsopoulos are with the Electrical and Computer Engineering Department, University of Patras, 26110 Patras, Greece (e-mail: zogas@space.noa.gr; kotsop@ee.upatras.gr).

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If $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_{2m}$ are $2m$ dimensional column vectors, with $\mathbf{X}_k$ composed of the kth components of the $\mathbf{Y}_i$ and $r_1 = |\mathbf{X}_1|, r_2 = |\mathbf{X}_2|, \ldots, r_n = |\mathbf{X}_\mathrm{n}|$, where $|\mathbf{X}_1|$ means the norm of $\mathbf{X}_i$, then $r_1, r_2, \ldots, r_n$ are Nakagami-m variates and their joint pdf is given by

$$f(r_1, r_2, \ldots, r_n) = \frac{r_1^{m-1} r_n^m}{2^{m-1}\Gamma(m)(1-\rho^2)^{m(n-1)}} e^{-\frac{r_1^2+r_n^2}{2(1-\rho^2)}-g_1} \times \prod_{k=1}^{n-1} r_k \left(\frac{\rho}{1-\rho^2}\right)^{-(m-1)} \times I_{m-1}\left[\left(\frac{\rho}{1-\rho^2}\right) r_k r_{k+1}\right] \quad (3)$$

with $g_1 = \begin{cases} 0, & \text{for } n=2 \\ (\rho^2+1)/2(1-\rho^2)\sum_{k=2}^{n-1} r_k^2, & \text{for } n>2 \end{cases}$ and $I_v(\cdot)$ is the first kind and νth-order modified Bessel function. Equation (3) is derived using [13, (2.1)] with the appropriate substitution of the parameters.

The pdf in (3) is well defined for $\rho = 0$ due to the following result [15, (9.6.7)]:[2]

$$\lim_{\rho\to 0}\left[\left(\frac{\rho}{1-\rho^2}\right)^{-(m-1)} I_{m-1}\left(\left(\frac{\rho}{1-\rho^2}\right) r_k r_{k+1}\right)\right] = \frac{(r_k r_{k+1})^{m-1}}{2^{m-1}\Gamma(m)}. \quad (4)$$

Moreover, for $n = 2$ the bivariate Nakagami-m pdf, defined in [8], is derived. Although the parameter m in (3) seems to be restricted to a positive half integer or integer, it can be any positive number not less than 0.5, since the same argument for $n = 2$ is given in [11, p. 31].

The n-variate Nakagami-m cdf is by definition

$$F(R_1, R_2, \cdots, R_n) = \int_0^{R_1}\int_0^{R_2}\cdots\int_0^{R_n} f(r_1, r_2, \cdots, r_n) \times dr_1 dr_2 \cdots dr_n \quad (5)$$

or

$$F(R_1, R_2, \cdots, R_n) = \frac{(1-\rho^2)^m}{\Gamma(m)} \sum_{i_1, i_2, \ldots, i_{n-1}=0}^{\infty} \times \frac{g_2 \rho^{2(i_1+i_2+\ldots+i_{n-1})}}{\prod_{j=1}^{n-1}[i_j!\Gamma(i_j+m)]} \times \gamma\left(i_1+m, \frac{R_1^2}{2(1-\rho^2)}\right) \times \gamma\left(i_1+i_2+m, \frac{R_2^2}{2}\left(\frac{\rho^2+1}{1-\rho^2}\right)\right) \times \ldots \gamma\left(i_{n-2}+i_{n-1} + m, \frac{R_{n-1}^2}{2}\left(\frac{\rho^2+1}{1-\rho^2}\right)\right) \times \gamma\left(i_{n-1}+m, \frac{R_n^2}{2(1-\rho^2)}\right) \quad (6)$$

$$g_2 = \begin{cases} 1, & \text{for } n=2 \\ (\rho^2+1)^{-[i_1+2i_2+\ldots+2i_{n-2}+i_{n-1}+(n-2)m]}, & \text{for } n>2 \end{cases}$$

and $\gamma(\cdot)$ is the incomplete Gamma function [15, (6.5.2)]. For the derivation of (6), see Appendix A.

The error $E_{I,n}$, resulting after the truncation of the nested infinite series in (6), is given by

$$E_{I,n} = \sum_{i_1=I_1}^{\infty}\sum_{i_2=0}^{I_2-1}\cdots\sum_{i_{n-2}=0}^{I_{n-2}-1}\sum_{i_{n-1}=0}^{I_{n-1}-1} G_n + \sum_{i_1=0}^{\infty}\sum_{i_2=I_2}^{\infty}\cdots\sum_{i_{n-2}=0}^{I_{n-2}-1}\sum_{i_{n-1}=0}^{I_{n-1}-1} G_n + \cdots \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\cdots\sum_{i_{n-2}=I_{n-2}}^{\infty}\sum_{i_{n-1}=0}^{I_{n-1}-1} G_n + \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\cdots\sum_{i_{n-2}=0}^{\infty}\sum_{i_{n-1}=I_{n-1}}^{\infty} G_n \quad (7)$$

whereas

$$G_n = \frac{(1-\rho^2)^m}{\Gamma(m)} \frac{\rho^{2i_1+2i_2+\cdots+2i_{n-1}}}{g_2 \prod_{j=1}^{n-1}[i_j!\Gamma(i_j+m)]} \times \gamma\left(i_1+m, \frac{R_1^2}{2(1-\rho^2)}\right) \times \gamma\left(i_1+i_2+m, \frac{R_2^2}{2}\left(\frac{\rho^2+1}{1-\rho^2}\right)\right)\ldots \times \gamma\left(i_{n-2}+i_{n-1}+m, \frac{R_{n-1}^2}{2}\left(\frac{\rho^2+1}{1-\rho^2}\right)\right) \times \gamma\left(i_{n-1}+m, \frac{R_n^2}{2(1-\rho^2)}\right)$$

and $I_1, I_2, \cdots, I_n$ is the number of terms required to be summed in (6), in order to meet a desired accuracy. An upper bound for the error $E_{I,n}$ can be found by replacing the incomplete gamma functions in G_n with their confluent hypergeometric function representation [15, (6.5.12)] and following a procedure similar to the one described in [12]. However, due to the space limitation of this letter, the formula for the n-variate case could not be cited. As an example, the trivariate case is approached and following the same mathematical analysis, the error bound for the n-variate case can be derived. The upper bound of the error for the trivariate case $(n = 3)$ can be evaluated as (see Appendix B)

$$E_{I,3} \leq \frac{(A_1 + A_2)}{(1-\rho^2)^{2m}\Gamma(m)} \quad (8)$$

with A_1, A_2 given on the next page. ${}_pF_q(a_1, a_2, \cdots, a_p; b_1, b_2, \cdots, b_q; z) = \sum_{k=0}^{\infty}(a_1)_k(a_2)_k\cdots(a_p)_k z^k/(b_1)_k(b_2)_k\cdots(b_q)_k k!$ is the generalized hypergeometric function [17, (9.14/1)] with $(x)_n$ the Pochammer symbols.

III. Numerical Results and Discussion

Equation (3) can be used in the performance analysis of linear antenna arrays with n correlated branches. Assuming that

[2]When $\rho \to 1$, it is easily verified that $\mathbf{\Sigma}$ is not a positive definite matrix (all eigenvalues are not greater than zero). In such a case, the multivariate normal pdf used in [13, (2.3)] cannot be defined [16] and consequently (3) does not hold.

TABLE I
COMPARISON OF INFINITE SERIES APPROACH FOR TRIVARIATE AND QUATRIVARIATE NAKAGAMI-m CDF WITH ADAPTIVE NUMERICAL INTEGRATION TECHNIQUE

ρ	m	Infinite Series	|Infinite Series - Adaptive|
R_1= 2, R_2= 3, R_3=5			
0.3	1	0.8562743 (I_1=I_2=7)	1.9498303x 10^{-10}
	3	0.2752449 (I_1=I_2=7)	4.7744947x10^{-10}
0.6	1	0.8599034 (I_1=I_2=13)	1.3217369 x10^{-9}
	3	0.2978637 (I_1=I_2=13)	3.8049051 x10^{-9}
0.9	1	0.8644688 (I_1=I_2=37)	1.8004442 x10^{-8}
	3	0.3223925 (I_1=I_2=37)	1.7167649x10^{-7}
R_1= 1, R_2= 3, R_3=4, R_4=5			
0.3	1	0.3902256 (I_1=I_2=I_3=7)	7.8639428x10^{-10}
	3	0.012299 (I_1=I_2=I_3=7)	3.9655428 x10^{-10}
0.6	1	0.3926822 (I_1=I_2=I_3=13)	2.3966878x10^{-8}
	3	0.0137375 (I_1=I_2=I_3=14)	1.7565906x10^{-8}
0.8	1	0.3934385 (I_1=I_2=I_3=26)	7.3792288x10^{-9}
	3	0.0143562 (I_1=I_2=I_3=26)	8.6165918 x10^{-9}

the correlation between the branches follows the exponential model, then the n-variate Nakagami-m pdf and cdf can be derived directly from (3) and (6), correspondingly.

In order to check the accuracy of the proposed mathematical analysis, the results obtained by setting $n = 3$ (trivariate) and $n = 4$ (quatrivariate) in (6) are compared in Table I with those from the adaptive integration technique [18], with regard to the accuracy and speed of calculations. This numerical integration technique is used to evaluate the triple and the quatrifold integral of (5). The numbers in brackets in the second column of Table I are the required terms to be summed in order to obtain accuracy at the seventh significant digit. To simplify the computations, it is assumed—without loss of generality—that $I_1 = I_2 = I_3$. As Table I indicates, an increase to the correlation also leads to an increase of the required terms that need to be summed in order to obtain a given accuracy. Furthermore, the number of the required terms depends strongly on the signals envelopes. An increase of the signals envelopes R_i increases the area under which the function is integrated, and correspondingly, the number of terms required in the summation of the series. Taking into consideration the accuracy of the computation, the adaptive numerical technique gives results that differ slightly from the infinite series approach $(< 10^{-8})$. Furthermore, it was observed that the infinite series representation is evaluated much faster (<1 s) compared to the numerical integration technique. Especially for the trivariate case, the CPU time for the adaptive method was approximately 2–4 s for all cases, and increased proportionally with the correlation ρ. For the quatrivariate case, the observed calculation time was about 1 s for the infinite series and about 15 s for the adaptive method. The above calculations were performed on an Athlon 1,2 Ghz PC

$$
\begin{aligned}
A_1 = &\frac{\left(R_1^2 R_2^2\right)^{I_1+m} R_3^{2m}}{2^{2I_1+3m} I_1! \Gamma(m+I_1)(m+I_1)^2} \left(\frac{\rho}{1-\rho^2}\right)^{2I_1} \\
&\times {}_1F_1\left(m+I_1, m+I_1+1, -\frac{R_1^2}{2(1-\rho^2)}\right) \\
&\times {}_1F_1\left(m+I_1, m+I_1+1, -\frac{(1+\rho^2)R_2^2}{2(1-\rho^2)}\right) \\
&\times {}_1F_2\left(1; m+I_1+1, I_1+1; \left(\frac{\rho R_1 R_2}{2(1-\rho^2)}\right)^2\right) \\
&\times \sum_{i_2=0}^{I_2-1} \frac{R_2^{2i_2} R_3^{2i_2}}{2^{2i_2} i_2! \Gamma(m+i_2)(m+i_2)} \left(\frac{\rho}{(1-\rho^2)}\right)^{2i_2} \\
&\times {}_1F_1\left(m+i_2, m+i_2+1, -\frac{R_3^2}{2(1-\rho^2)}\right) \\
A_2 = &\frac{R_1^m R_2^{m+2I_2} R_3^{2(m+I_2)}}{4^{m+I_2}(m+I_2)^2 I_2! \Gamma(m+I_2)} \left(\frac{\rho}{(1-\rho^2)}\right)^{2I_2-m} \\
&\times I_m\left(\frac{\rho R_1 R_2}{(1-\rho^2)}\right) {}_1F_1\left(m, m+1, -\frac{R_1^2}{2(1-\rho^2)}\right) \\
&\times {}_1F_1\left(m+I_2, m+I_2+1, -\frac{(1+\rho^2)R_2^2}{2(1-\rho^2)}\right) \\
&\times {}_1F_1\left(m+I_2, m+I_2+1, -\frac{R_3^2}{2(1-\rho^2)}\right) \\
&\times {}_1F_2\left(1; m+I_2+1, I_2+1; \left(\frac{\rho R_2 R_3}{2(1-\rho^2)}\right)^2\right).
\end{aligned}
$$

IV. CONCLUSIONS

In this letter, capitalizing on the proof of a theorem presented many years ago by Blumenson and Miller and extending the Tan and Beaulieu approach for the bivariate Nakagami-m cdf, useful formulae for the joint n-variate Nakagami-m pdf and cdf with exponential correlation are derived. The proposed formulation can be efficiently used in the performance analysis of space-diversity techniques. Moreover, it can be used, with care, to find transition probabilities in the Markov chain modeling of the Nakagami-m process.[3]

[3]Very recently, another useful work on the multivariate Rayleigh distribution was published [19].

APPENDIX A

DERIVATION OF THE MULTIVARIATE NAKAGAMI-m CDF WITH EXPONENTIAL CORRELATION

Substituting the Bessel function in (3) with its infinite series representation [17, (8.445)] results in

$$f(r_1,r_2,\cdots,r_n)=\frac{(r_1r_2\cdots r_n)^{2m-1}}{2^{n(m-1)}\Gamma(m)}\times\frac{e^{-\frac{\left(r_1^2+r_n^2\right)}{2(1-\rho^2)}-\sum_{k=2,\,n\geq 3}^{n-1}\frac{\rho^2+1}{2(1-\rho^2)}r_k^2}}{(1-\rho^2)^{(n-1)m}}\times\sum_{i_1,i_2,\ldots,i_{n-1}=0}^{\infty}\left(\frac{\rho}{1-\rho^2}\right)^{2i_1+2i_2+\ldots+2i_{n-1}}\times\frac{r_1^{2i_1}r_2^{2i_1+2i_2}\cdots r_{n-1}^{2i_{n-2}+2i_{n-1}}r_n^{2i_{n-1}}}{\prod_{j=1}^{n-1}\left[4^{i_j}i_j!\Gamma(m+i_j)\right]}. \quad (9)$$

Substituting (9) into (5), the cdf can be written as shown in (10) at the bottom of the page. Making the transformations

$$y_1=\frac{r_1^2}{2(1-\rho^2)},\ y_2=\frac{r_2^2}{2}\left(\frac{\rho^2+1}{1-\rho^2}\right),\ldots,y_n=\frac{r_n^2}{2(1-\rho^2)}$$

and using the definition of the incomplete gamma function [15, (6.52)], the n-variate Nakagami-m cdf is finally derived as shown in (6).

APPENDIX B

DERIVATION OF THE ERROR BOUND

The error in truncating the nested infinite series for the trivariate Nakagami-m cdf is produced setting $n=3$ in (7) as

$$E_{I,3}=\left(\sum_{i_1=I_1}^{\infty}\sum_{i_2=0}^{I_2-1}G_3+\sum_{i_1=0}^{\infty}\sum_{i_2=I_2}^{\infty}G_3\right) \quad (11)$$

$$F(R_1,R_2,\ldots,R_n)=\frac{1}{2^{n(m-1)}\Gamma(m)(1-\rho^2)^{(n-1)m}}\sum_{i_1,i_2,\ldots,i_{n-1}=0}^{\infty}\left(\frac{\rho}{1-\rho^2}\right)^{2i_1+2i_2+\ldots+2i_{n-1}}\frac{1}{\prod_{j=1}^{n-1}\left[4^{i_j}i_j!\Gamma(m+i_j)\right]}$$
$$\times\int_0^{R_1}r_1^{2i_1+2m-1}e^{-\frac{r_1^2}{2(1-\rho^2)}}dr_1\int_0^{R_2}r_2^{2i_1+2i_2+2m-1}e^{-\frac{r_2^2(\rho^2+1)}{2(1-\rho^2)}}dr_2\cdots$$
$$\int_0^{R_{n-1}}r_{n-1}^{2i_{n-2}+2i_{n-1}+2m-1}e^{-\frac{r_{n-1}^2(\rho^2+1)}{2(1-\rho^2)}}dr_{n-1}\int_0^{R_n}r_n^{2i_{n-1}+2m-1}e^{-\frac{r_n^2}{2(1-\rho^2)}}dr_n \quad (10)$$

$$G_3'.=\frac{\rho^{2i_1+2i_2}R_1^{2(m+i_1)}R_2^{2(m+i_1+i_2)}R_3^{2(m+i_2)}}{2^{2i_1+2i_2+3m}(1-\rho^2)^{2i_1+2i_2}i_1!i_2!\Gamma(m+i_1)\Gamma(m+i_2)(m+i_1)(m+i_1+i_2)(m+i_2)}$$
$$\times{}_1F_1\left(m+i_1,m+i_1+1,-\frac{R_1^2}{2(1-\rho^2)}\right)$$
$$\times{}_1F_1\left(m+i_1+i_2,m+i_1+i_2+1,-\frac{1}{2}\left(\frac{\rho^2+1}{1-\rho^2}\right)R_2^2\right){}_1F_1\left(m+i_2,m+i_2+1,-\frac{R_3^2}{2(1-\rho^2)}\right)$$

$$B_1=\frac{R_1^{2m}R_2^{2m}R_3^{2m}}{2^{3m}(m+I_1)}{}_1F_1\left(m+I_1,m+I_1+1,-\frac{R_1^2}{2(1-\rho^2)}\right){}_1F_1\left(m+I_1,m+I_1+1,-\frac{R_2^2}{2}\left(\frac{\rho^2+1}{1-\rho^2}\right)\right)$$
$$\times\sum_{i_1=I_1}^{\infty}\left[\left(\frac{\rho}{1-\rho^2}\right)^{2i_1}\frac{\left(R_1^2R_2^2\right)^{i_1}}{4^{i_1}i_1!\Gamma(m+i_1)(m+i_1)}\right]$$
$$\times\sum_{i_2=0}^{I_2-1}\left[\left(\frac{\rho}{1-\rho^2}\right)^{2i_2}\frac{R_2^{2i_2}R_3^{2i_2}}{4^{i_2}i_2!\Gamma(m+i_2)(m+i_2)}{}_1F_1\left(m+i_2,m+i_2+1,-\frac{R_3^2}{2(1-\rho^2)}\right)\right]$$

$$B_2=\frac{R_1^{2m}R_2^{2m}R_3^{2m}}{2^{3m}(m+I_2)}$$
$$\times{}_1F_1\left(m,m+1,-\frac{R_1^2}{2(1-\rho^2)}\right){}_1F_1\left(m+I_2,m+I_2+1,-\frac{R_2^2}{2}\left(\frac{\rho^2+1}{1-\rho^2}\right)\right){}_1F_1\left(m+I_2,m+I_2+1,-\frac{R_3^2}{2(1-\rho^2)}\right)$$
$$\times\sum_{i_1=0}^{\infty}\left[\left(\frac{\rho}{1-\rho^2}\right)^{2i_1}\frac{R_1^{2i_1}R_2^{2i_1}}{4^{i_1}i_1!\Gamma(m+i_1)(m+i_1)}\right]\sum_{i_2=I_2}^{\infty}\left[\left(\frac{\rho}{1-\rho^2}\right)^{2i_2}\frac{R_2^{2i_2}R_3^{2i_2}}{4^{i_2}i_2!\Gamma(m+i_2)(m+i_2)}\right]$$

where

$$G_3 = \frac{\rho^{2i_1+2i_2}(1-\rho^2)^m(\rho^2+1)^{-(i_1+i_2+m)}}{i_1!i_2!\Gamma(m)\Gamma(i_1+m)\Gamma(i_2+m)} \times \gamma\left(i_1+m, \frac{R_1^2}{2(1-\rho^2)}\right) \times \gamma\left(i_1+i_2+m, \frac{1}{2}\left(\frac{\rho^2+1}{1-\rho^2}\right)R_2^2\right) \times \gamma\left(i_2+m, \frac{R_3^2}{2(1-\rho^2)}\right).$$

Using [15, (6.5.12)], (11) can be written as

$$E_{I,3} = \frac{1}{\Gamma(m)(1-\rho^2)^{2m}}\left(\sum_{i_1=I_1}^{\infty}\sum_{i_2=0}^{I_2-1} G_3' + \sum_{i_1=0}^{\infty}\sum_{i_2=I_2}^{\infty} G_3'\right) \tag{12}$$

with G_3' given at the bottom of the previous page and ${}_1F_1(a,b;z)$ being the well-known confluent hypegeometric function. Since ${}_1F_1(a,1+a,-x)$ can be shown to be monotonically decreasing for all the positive values of a and x, $E_{1,3}$ can be upper bounded as

$$E_{I,3} \le \frac{(B_1+B_2)}{\Gamma(m)(1-\rho^2)^{2m}} \tag{13}$$

where B_1, B_2 are given at the bottom of the previous page. After algebraic manipulations, (8) is derived.

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