Two-Way AF Relaying in the Presence of Co-Channel Interference

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***Abstract*—In this paper, we investigate the performance of two-way interference-limited amplify-and-forward relaying systems over independent, non-identically distributed Nakagami-m fading channels. Our analysis generalizes several previous results, since it accounts for interference affecting all network nodes. In particular, tight lower bounds on the end-to-end outage and symbol error probability are derived in closed-form, while a useful expression is presented for the asymptotically low outage regime. Some special cases of practical interest (e.g., no interference power and interference-limited case) are also studied. Using the derived lower bounds as a starting point and for the case of Rayleigh fading, we formulate and solve analytically three practical optimization problems, namely, power allocation under fixed location for the relay, optimal relay position with fixed power allocation, and joint optimization of power allocation and relay position under a transmit power constraint. The numerical results provide important physical insights into the implications of model parameters on the system performance; for instance, it is demonstrated that relay position optimization offers significant performance enhancement over the non-optimized case for an asymmetric interference power profile, whilst the optimization gains are marginal for a symmetric one.**

***Index Terms*—Amplify-and-forward, interference limited systems, outage probability, two-way relaying.**

I. Introduction

THE increasing data rate demands of concurrent and future wireless applications have fostered the development of cooperative diversity schemes. Many cooperative strategies have been proposed in the literature based on different relaying techniques, such as amplify-and-forward (AF) and decode-and-forward (DF). More specifically, in AF relaying schemes, the relay simply amplifies the received signal from the source before retransmitting it to the destination (without performing any demodulating and decoding of the received signal) [1, 2]. On this basis, AF relaying systems have low implementation complexity and are anticipated to be deployed in future vehicular and sensor networks among others. For this reason, we henceforth assess the performance of AF relaying schemes.

As wireless networks evolve towards high load deployments with aggressive cellular frequency reuse, a dominant factor is inter-cell co-channel interference (CCI). For this reason, CCI has been recently investigated in the context of wireless relaying (see e.g., [3–7] and references therein). In [3–7], however, the main aim was to study the effect of CCI in unidirectional relay links. For example, in [3], the outage probability (OP) was derived in closed-form in the presence of CCI at the destination. Considering CCI at the relay, the OP was obtained in [4] for Rayleigh fading channels. An extension of [4] to Nakagami-m fading channels was presented in [5]. More recently, the impact of feedback delay with beamforming and CCI at the relay was investigated in [6]. In [7], the OP and bit error rate (BER), with a single interferer impairing both the relay and the destination, were analyzed. In [8], the OP for dual-hop interference-limited relaying systems was deduced, where both the relay and destination nodes are subject to interference.

The spectral efficiency (SE) in unidirectional relaying systems is inherently low since the communication occupies two time slots. Therefore, the coded bidirectional or two-way relaying techniques have recently received significant attention, since they can improve the SE [9]. In two-way relaying systems, in the first time slot, two nodes transmit simultaneously to the relay, and the relay will transmit data to the designated destinations in the second time slot. The authors in [10] obtained the OP and symbol error probability (SEP) of interference-limited systems over Rayleigh fading channels, where they worked with the upper bound of the harmonic mean. However, they introduced an assumption on independency of two dependent random variables (RVs) to derive their closed-form results for the OP and SEP. In [11], the authors derived the OP and SEP of two multiple-input multiple-output two-way relaying schemes over Nakagami-m fading channels, in an interference-free network. The authors in [12] investigated the SEP of two-way relaying systems using network coding schemes, though they did not consider any interference in the network. In [13], the authors derived an approximation for the end-to-end OP of a fixed gain AF two-way relay network, suffering from CCI and over Rayleigh fading channels. In [14], the authors examined the outage performance of dual-hop AF relaying systems with CCI over independent, non-identically distributed Nakagami-

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m fading channels. They extended their work to two-way relaying systems in [15], where they considered the impact of interference only at the relay; in addition, they approximated the probability distribution function (PDF) of the sum of interferers' powers by a gamma RV. In [16], the authors examined the OP of two-way AF relaying systems with CCI over Nakagami-m fading channels, where the relay was not subject to interference. While all previous works have improved our knowledge on the performance characterization of two-way interference-limited AF relaying, the most important differences between our work and [15, 16] are: 1) In [16], interference affects only the source nodes, whilst the relay is subject to noise only; moreover, all analytical results are limited to the OP and closed-from results were derived only for Rayleigh fading, 2) In [15], interference affects only the relay, while the relay gain does not contain the interferers' effect.

Motivated by the above mentioned limitations of [15] and [16], we herein pursue a detailed and generalized performance analysis of dual-hop two-way AF relaying systems, *where CCI is considered at all nodes in the network (i.e. both the source nodes and relay)*. In this light, we derive tight lower bounds on the OP and SEP of two-way interference limited AF relaying networks over Nakagami-m fading channels, at arbitrary signal to interference plus noise ratios (SINRs), along with asymptotic expressions in the low outage regime. The contributions of this paper can be summarized as follows:

- We consider a two-way dual-hop configuration, where the single-antenna source nodes and relay are affected by multiple interferers. This is a practical but complicated setup which has scarcely appeared in the literature. We focus on the Nakagami-m fading model, that has been extensively used for the performance analysis of wireless communication systems [17]. In particular, tight closed-form lower bounds on the OP and SEP are derived that extend and complement several previous results in the literature (e.g., those in [10, 15, 16]).
- Based on the derived lower bounds and for the case of Rayleigh fading, we formulate three interesting optimization problems which seek to minimize the OP. In particular, we consider power allocation under fixed location for the relay, optimal relay position with fixed power allocation, and joint optimization of power allocation and relay position under a transmit power constraint.
- In order to get some additional insights into the impact of system parameters, such as fading parameters and number of interferers, we consider the asymptotically low outage regime and obtain the diversity order and coding gain. Finally, we particularize our results to the cases of no interference power and interference-limited case.

The rest of the paper is organized as: Section II introduces the system model. In Section III, we derive closed-form lower bounds for the OP and SEP. The asymptotic analysis and optimization results are given in Section IV and V, respectively. Section VI particularizes the results of Section III to some special cases of interest. Finally, Section VII presents our numerical results, while Section VIII concludes the paper.

Notation: We use $f_h(.)$ and $F_h(.)$ to denote the PDF and cumulative distribution function (CDF) of a RV h, respectively. We consider that $\mathfrak{g}(a, b)$ demonstrates the Gamma distribution, with a and b being its shape and scale parameters. Recall that the PDF and CDF of a Gamma RV are respectively

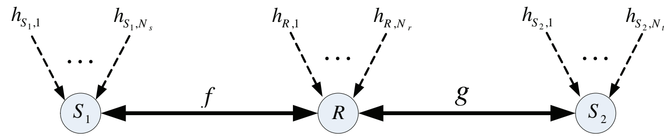

Fig. 1. Schematic illustration of the cooperative system under consideration.

$$f_\gamma(\gamma) = \frac{\gamma^{a-1}}{b^a \Gamma(a)} e^{-\frac{\gamma}{b}} \quad \text{and} \quad F_\gamma(\gamma) = 1 - \frac{\Gamma\left(a, \frac{\gamma}{b}\right)}{\Gamma(a)} \tag{1}$$

where $\Gamma(n) = \int_0^\infty e^{-t} t^{n-1}\, dt$ is the gamma function [18, Eq. (8.310.1)], and $\Gamma(b, x) = \int_x^\infty e^{-t} t^{b-1}\, dt$ is the upper incomplete gamma function [18, Eq. (8.350.2)]. The operator $\mathbb{E}[.]$ stands for expectation.

II. System Model and Fading Statistics

We consider a cooperative relaying system with two single-antenna source nodes (S_1 and S_2), which exchange information via the relay R (see Fig. 1). Moreover, R, S_1 and S_2 are impaired by N_r, N_s and N_t sources of CCI from other users in the network. Additionally, f is the channel coefficient between S_1 and R and viceversa (i.e., the $S_1 \to R$ and $R \to S_1$ links) and g is the channel coefficient between S_2 and R which is reciprocal (i.e., the $S_2 \to R$ and $R \to S_2$ links). Also, $h_{R,i}$, $h_{S_1,j}$ and $h_{S_2,k}$ are the channel coefficients between R, S_1 and S_2 and the i-th ($i = 1, ..., N_r$), j-th ($j = 1, ..., N_s$) and k-th ($k = 1, ..., N_t$) interferer at R, S_1 and S_2, respectively. Additionally, P_R, P_{S_1} and P_{S_2} are the transmitted powers of R, S_1 and S_2, respectively. Furthermore, P_{Ri}, $P_{S_1 j}$ and $P_{S_2 k}$ is the power of the i-th, j-th and k-th CCI signal impairing R, S_1 and S_2, respectively and σ^2 denotes the noise variance at all nodes. Hence, the instantaneous SNRs for the $S_1 \to R$, $S_2 \to R$, $R \to S_1$, and $R \to S_2$ links are given by $\gamma_1 = \frac{P_{S_1}|f|^2}{\sigma^2}$, $\gamma_2 = \frac{P_{S_2}|g|^2}{\sigma^2}$, $\gamma_3 = \frac{P_R|f|^2}{\sigma^2}$ and $\gamma_4 = \frac{P_R|g|^2}{\sigma^2}$, respectively. Also, the instantaneous interference-to-noise ratio for the i-th CCI at R, j-th CCI at S_1 and k-th CCI at S_2 is given by $\gamma_{R,i} = \frac{P_{Ri}|h_{R,i}|^2}{\sigma^2}$, $\gamma_{S_1,j} = \frac{P_{S_1 j}|h_{S_1,j}|^2}{\sigma^2}$ and $\gamma_{S_2,k} = \frac{P_{S_2 k}|h_{S_2,k}|^2}{\sigma^2}$, respectively.

As was previously mentioned, we assume that the amplitude of all links follows the Nakagami-m distribution, where $m \geq 0.5$ represents the fading severity parameter [19]. As such, the distribution of the corresponding SNRs are Gamma RVs, where the shape parameter is m and the scale parameter is Ω/m, where Ω is the mean value of Gamma RVs. As such, the distributions of $|f|^2$, $|g|^2$, $\gamma_{S_1,j}$, $\gamma_{S_2,k}$ and $\gamma_{R,i}$ can be expressed, via the corresponding parameters, as follows

$$\begin{aligned} &|f|^2 \overset{d}{\sim} \mathfrak{g}(m_1, 1/a), \qquad |g|^2 \overset{d}{\sim} \mathfrak{g}(m_2, 1/b), \\ &\gamma_{S_1,j} \overset{d}{\sim} \mathfrak{g}(m_s, 1/\alpha), \quad \gamma_{S_2,k} \overset{d}{\sim} \mathfrak{g}(m_t, 1/\eta), \\ &\gamma_{R,i} \overset{d}{\sim} \mathfrak{g}(m_r, 1/\beta) \end{aligned} \tag{2}$$

where the symbol $\overset{d}{\sim}$ denotes "distributed as". Moreover, $a \triangleq \frac{m_1}{\Omega_1}$, $b \triangleq \frac{m_2}{\Omega_2}$, $\alpha \triangleq \frac{m_s \sigma^2}{\Omega_s P_{S_1 j}}$, $\beta \triangleq \frac{m_r \sigma^2}{\Omega_r P_{Ri}}$ and $\eta \triangleq \frac{m_t \sigma^2}{\Omega_t P_{S_2 k}}$. The signal received at the relay is as follows

$$y_R = \sqrt{P_{S_1}} f x_{S_1} + \sqrt{P_{S_2}} g x_{S_2} + \sum_{i=1}^{N_r} \sqrt{P_{Ri}} h_{R,i} x_{R,i} + n_R \tag{3}$$

where x_{S_1}, x_{S_2} and $x_{R,i}$ are the signals generated from S_1, S_2 and the i-th interferer affecting the relay, respectively, while n_R is the additive white Gaussian noise (AWGN) at the relay. After amplification at the relay by a variable gain factor G, the signal received at S_1 can be written as

$$\widetilde{y}_{S_1} = \sqrt{P_r} G f y_R + \sum_{j=1}^{N_s} \sqrt{P_{S_1 j}} h_{S_1,j} x_{S_1,j} + n_{S_1} \tag{4}$$

where $x_{S_1,j}$ is the signal generated from the j-th interferer affecting S_1 and n_{S_1} is the AWGN noise at S_1, while

$$G^{-1} \triangleq \sqrt{P_{S_1}|f|^2 + P_{S_2}|g|^2 + \sum_{i=1}^{N_r} P_{Ri}|h_{R,i}|^2 + \sigma^2}. \tag{5}$$

Since S_1 knows its transmitted signal, it can eliminate the self-interference term.[1] Then, the signal received at S_1 becomes

$$y_{S_1} = \sqrt{P_R P_{S_2}} G f g x_{S_2} + G f \sqrt{P_R} \sum_{i=1}^{N_r} \sqrt{P_{Ri}} h_{R,i} x_{R,i} + \sum_{j=1}^{N_s} \sqrt{P_{S_1 j}} h_{S_1,j} x_{S_1,j} + \sqrt{P_R} f G n_R + n_{S_1}. \tag{6}$$

The received SINR at S_1 can then be expressed as

$$\gamma_{S_1} = \frac{P_R P_{S_2} G^2 |f|^2 |g|^2}{\left[G^2 P_R |f|^2 \sum_{i=1}^{N_r} P_{Ri} |h_{R,i}|^2 + \sum_{j=1}^{N_s} P_{S_1 j} |h_{S_1,j}|^2 + \sigma^2 \left(G^2 P_R |f|^2 + 1 \right) \right]}. \tag{7}$$

After some algebraic manipulations, the SINR at S_1 becomes

$$\gamma_{S_1} = \frac{\dfrac{P_R P_{S_2} |f|^2 |g|^2}{\left[\sum_{j=1}^{N_s} P_{S_1 j} |h_{S_1,j}|^2 + \sigma^2\right]\left[\sum_{i=1}^{N_r} P_{Ri} |h_{R,i}|^2 + \sigma^2\right]}}{\dfrac{P_{S_1}|f|^2 + P_{S_2}|g|^2}{\left[\sum_{i=1}^{N_r} P_{Ri}|h_{R,i}|^2 + \sigma^2\right]} + \dfrac{P_R |f|^2}{\left[\sum_{j=1}^{N_s} P_{S_1 j}|h_{S_1,j}|^2 + \sigma^2\right]} + 1}. \tag{8}$$

By assuming $P_{S_1} = P_{S_2} = P_S$ [20, 21], without significant loss of generality, the SINR at S_1 further simplifies to

$$\gamma_{S_1} = \frac{\dfrac{\varrho \gamma_1 \gamma_2}{\left[\sum_{j=1}^{N_s} \gamma_{S_1,j} + 1\right]\left[\sum_{i=1}^{N_r} \gamma_{R,i} + 1\right]}}{\dfrac{\gamma_1 + \gamma_2}{\left[\sum_{i=1}^{N_r} \gamma_{R,i} + 1\right]} + \dfrac{\varrho \gamma_1}{\left[\sum_{j=1}^{N_s} \gamma_{S_1,j} + 1\right]} + 1} \tag{9}$$

where $\varrho \triangleq \frac{P_R}{P_S}$. Likewise, the received SINR at S_2 is

$$\gamma_{S_2} = \frac{\dfrac{\varrho \gamma_1 \gamma_2}{\left[\sum_{k=1}^{N_t} \gamma_{S_2,k} + 1\right]\left[\sum_{i=1}^{N_r} \gamma_{R,i} + 1\right]}}{\dfrac{\gamma_1 + \gamma_2}{\left[\sum_{i=1}^{N_r} \gamma_{R,i} + 1\right]} + \dfrac{\varrho \gamma_2}{\left[\sum_{k=1}^{N_t} \gamma_{S_2,k} + 1\right]} + 1}. \tag{10}$$

Moreover, the instantaneous SNR for $S_1 \to R$ and $S_2 \to R$ are given by $\gamma_1 = \bar{\gamma}|f|^2$ and $\gamma_2 = \bar{\gamma}|g|^2$, respectively, where $\bar{\gamma} = \frac{P_S}{\sigma^2}$ is the average SNR per symbol. Note that in the following section, we analytically investigate the OP and SEP, starting from the expressions in (9) and (10).

[1]We have implicitly assumed that the channel coefficient between S_1 and R is known at S_1. Likewise, the channel coefficient between S_2 and R is known at S_2. For a detailed discussion about this assumption, see [9].

III. Performance Analysis

By setting $\gamma_S \triangleq \sum_{j=1}^{N_s} \gamma_{S_1,j} + 1$ and $\gamma_R \triangleq \sum_{i=1}^{N_r} \gamma_{R,i} + 1$, the received SINR at S_1 can be tightly upper bounded in the interference-limited regime, according to

$$\gamma_{S_1} \leq \frac{\frac{\varrho \gamma_1 \gamma_2}{\gamma_S \gamma_R}}{\frac{\gamma_1 + \gamma_2}{\gamma_R} + \frac{\varrho \gamma_1}{\gamma_S}} = \frac{\varrho \gamma_1 \gamma_2}{\gamma_1 (\gamma_S + \varrho \gamma_R) + \gamma_2 \gamma_S} = \frac{\varrho \frac{\gamma_1 \gamma_2}{(\gamma_S + \varrho \gamma_R)}}{\gamma_1 + \frac{\gamma_2}{(\gamma_S + \varrho \gamma_R)} \gamma_S} = \frac{\frac{\varrho \gamma_1 \gamma_2}{(\gamma_S + \varrho \gamma_R)\gamma_S}}{\frac{\gamma_1}{\gamma_S} + \frac{\gamma_2}{(\gamma_S + \varrho \gamma_R)}} = \varrho \frac{XY}{X+Y} \tag{11}$$

where $X \triangleq \frac{\gamma_1}{\gamma_S}$, $Y \triangleq \frac{\gamma_2}{\gamma_S + \varrho \gamma_R}$. Note that a similar expression can be derived for the received SINR at S_2 where $\gamma_T \triangleq \sum_{k=1}^{N_t} \gamma_{S_2,k} + 1$. It is well known that the $\min(X, Y)$ is a tight upper bound of $\frac{XY}{X+Y}$; in fact, as X and Y go to infinity the bound becomes exact. Hence, we use this bound for all derivations henceforth.[2] As such, the upper bounded SINR at S_1 and S_2 can be expressed as

$$\gamma_{S_1}^{\mathrm{up}} = \varrho \min \left(\frac{\gamma_1}{\gamma_S}, \frac{\gamma_2}{\gamma_S + \varrho \gamma_R} \right) \tag{12}$$

$$\gamma_{S_2}^{\mathrm{up}} = \varrho \min \left(\frac{\gamma_2}{\gamma_T}, \frac{\gamma_1}{\gamma_T + \varrho \gamma_R} \right). \tag{13}$$

Finally, the end-to-end SINR of this system can be written as

$$\gamma_{e2e} = \min(\gamma_{S_1}, \gamma_{S_2}) \leq \min\left(\gamma_{S_1}^{\mathrm{up}}, \gamma_{S_2}^{\mathrm{up}}\right) \triangleq \gamma_{e2e}^{\mathrm{up}}. \tag{14}$$

Note that some works, such as [10], derived analytical expressions based on γ_{S_1}, which is not the true end-to-end SINR of two-way relaying systems. To compute the OP of the end-to-end SINR, we first need to derive the CDFs of the intermediate variables X and Y. In general, it is known that the sum of L i.i.d. Gamma RVs with shape parameter k and scale parameter θ, is also a Gamma RV with parameters kL and θ. By defining $\gamma_s \triangleq \sum_{j=1}^{N_s} \gamma_{S_1,j}$, $\gamma_t \triangleq \sum_{k=1}^{N_t} \gamma_{S_2,k}$ and $\gamma_r \triangleq \sum_{i=1}^{N_r} \gamma_{R,i}$, the distributions of γ_s, γ_t and γ_r are respectively as $\mathfrak{g}(N_s m_s, 1/\alpha)$, $\mathfrak{g}(N_t m_t, 1/\eta)$ and $\mathfrak{g}(N_r m_r, 1/\beta)$. The CDFs of X and Y are given by the following proposition:

Proposition 1: The CDFs of X and Y are respectively

$$F_X(z) = 1 - \frac{\alpha^{N_s m_s}}{\Gamma(N_s m_s)} e^{-\frac{az}{\bar{\gamma}}} \sum_{i=0}^{m_1 - 1} \sum_{j=0}^{i} \binom{i}{j} \frac{(az)^i}{\bar{\gamma}^i i!} \times \frac{\Gamma(j + N_s m_s)}{\left(\frac{az}{\bar{\gamma}} + \alpha\right)^{j + N_s m_s}} \tag{15}$$

[2]Note that this is a standard methodology in the performance analysis of unidirectional and bidirectional relaying systems affected by interference, which facilitates the, otherwise tedious, mathematical manipulations [10, 15]. More importantly, it guarantees that our asymptotic results are exact.

$$F_Y(z)=1-\frac{\alpha^{N_s m_s}}{\Gamma(N_s m_s)}\frac{\beta^{N_r m_r}}{\Gamma(N_r m_r)}\sum_{i=0}^{m_2-1}\sum_{j=0}^{i}\sum_{k=0}^{i-j}\binom{i}{j}\binom{i-j}{k}\frac{\varrho^k (az)^i}{\bar{\gamma}^i i!}\frac{(\varrho+1)^{i-j-k}}{e^{\frac{2bz}{\bar{\gamma}}}}\frac{\Gamma(j+N_s m_s)}{\left(\frac{bz}{\bar{\gamma}}+\alpha\right)^{j+N_s m_s}}\frac{\Gamma(k+N_r m_r)}{\left(\frac{\varrho bz}{\bar{\gamma}}+\beta\right)^{k+N_r m_r}}. \quad (16)$$

Proof: See Appendix I. ■

From (15) and (16), it is clear that m_1 and m_2 should be integers. After computing the CDFs of X and Y, we now proceed to derive the CDFs of $\gamma_{S_1}^{\text{up}}$ and $\gamma_{S_2}^{\text{up}}$:

Proposition 2: The CDFs of $\gamma_{S_1}^{\text{up}}$ and $\gamma_{S_2}^{\text{up}}$ are given by

$$F_{\gamma_{S_1}^{\text{up}}}(z)=1-\frac{\alpha^{m_s N_s}}{\Gamma(m_s N_s)}\frac{\beta^{m_r N_r}}{\Gamma(m_r N_r)}e^{-((\varrho+1)b+a)\frac{z}{\varrho\bar{\gamma}}}\sum_{i=0}^{m_1-1}\sum_{j=0}^{m_2-1}\sum_{k=0}^{j}\sum_{l=0}^{i}\sum_{t=0}^{j-k}\frac{(\varrho+1)^{j-k-t}}{i!j!}\binom{j}{k}\binom{j-k}{t}\binom{i}{l}\frac{(az)^i (bz)^j}{\varrho^{i+j-t}\bar{\gamma}^{i+j}}\frac{\Gamma(m_s N_s+l+k)}{\left(\frac{az}{\varrho\bar{\gamma}}+\frac{bz}{\varrho\bar{\gamma}}+\alpha\right)^{m_s N_s+l+k}}\frac{\Gamma(m_r N_r+t)}{\left(\beta+\frac{bz}{\bar{\gamma}}\right)^{m_r N_r+t}}. \quad (17)$$

$$F_{\gamma_{S_2}^{\text{up}}}(z)=1-\frac{\eta^{m_t N_t}}{\Gamma(m_t N_t)}\frac{\beta^{m_r N_r}}{\Gamma(m_r N_r)}e^{-((\varrho+1)a+b)\frac{z}{\varrho\bar{\gamma}}}\sum_{i=0}^{m_2-1}\sum_{j=0}^{m_1-1}\sum_{k=0}^{j}\sum_{l=0}^{i}\sum_{t=0}^{j-k}\frac{(\varrho+1)^{j-k-t}}{i!j!}\binom{j}{k}\binom{j-k}{t}\binom{i}{l}\frac{(az)^j (bz)^i}{\varrho^{i+j-t}\bar{\gamma}^{i+j}}\frac{\Gamma(m_t N_t+l+k)}{\left(\frac{az}{\varrho\bar{\gamma}}+\frac{bz}{\varrho\bar{\gamma}}+\eta\right)^{m_t N_t+l+k}}\frac{\Gamma(m_r N_r+t)}{\left(\beta+\frac{az}{\bar{\gamma}}\right)^{m_r N_r+t}}. \quad (18)$$

Proof: See Appendix II. ■

As before, from (17) and (18), it is clear that m_1 and m_2 should be integers. With these results in our hands, we can now evaluate the CDF of the upper bounded end-to-end SINR.

Proposition 3: The CDF of the upper bounded end-to-end SINR, $\gamma_{\text{e2e}}^{\text{up}}$, is given by

$$F_{\gamma_{e2e}^{\text{up}}}(z)=1-\left(\mathcal{P}_{11}(z)+\mathcal{P}_{12}(z)\right)\left(\mathcal{P}_{21}(z)+\mathcal{P}_{22}(z)\right) \quad (19)$$

where $\mathcal{P}_{11}(z)$, $\mathcal{P}_{12}(z)$, $\mathcal{P}_{21}(z)$ and $\mathcal{P}_{22}(z)$ are defined in (20)-(23) at the top of next page.

Proof: See Appendix III. ■

Note that m_1, m_2, $m_s N_s$ and $m_t N_t$ should have integer values. Hereafter, we investigate the most important performance metrics i.e. the lower bounded OP and SEP for two-way interference-limited systems based on (19).

A. Outage Probability

The OP is the probability that either the S_1-to-relay link SINR or the S_2-to-relay link SINR falls bellow a certain threshold, $\gamma_{\text{th}}=\frac{\gamma_0}{\bar{\gamma}}$. By using (19), we can now obtain the following lower bound on the exact OP of the system

$$P_{\text{out}}(\gamma_{\text{th}}) \geq P_{\text{out}}^{\text{lb}}(\gamma_{\text{th}})=1-\mathcal{P}_1(\gamma_0)\mathcal{P}_2(\gamma_0) \quad (24)$$

where $\mathcal{P}_1(\gamma_0)\triangleq\mathcal{P}_{11}(\gamma_0)+\mathcal{P}_{12}(\gamma_0)$, $\mathcal{P}_2(\gamma_0)\triangleq\mathcal{P}_{21}(\gamma_0)+\mathcal{P}_{22}(\gamma_0)$. Note that (24) can be efficiently evaluated, as it includes finite summations of elementary functions.

B. Symbol Error Probability

We now turn our attention to the SEP, which for most digital communication modulations can be expressed as [17]

$$\overline{P_e^{\text{lb}}}=\frac{c}{2}\sqrt{\frac{d}{\pi}}\int_0^\infty \frac{e^{-d\gamma}}{\sqrt{\gamma}}F_{\gamma_{e2e}^{\text{up}}}(\gamma)(\gamma)d\gamma \quad (25)$$

where the constants c and d depend on the type of the modulation. For example, we have

$$T=\frac{\left(\frac{a}{\bar{\gamma}}\right)^{m_s N_s+m_r N_r}}{\left(\alpha+\frac{az}{\bar{\gamma}}\right)^{m_s N_s}\left(\alpha+\frac{az}{\bar{\gamma}}+\beta\right)^{m_r N_r}}=\sum_{i_1=1}^{m_s N_s}\frac{\tau_{i_1}}{\left(z+\frac{\alpha\bar{\gamma}}{a}\right)^{i_1}}+\sum_{i_2=1}^{m_r N_r}\frac{\lambda_{i_2}}{\left(z+\frac{(\alpha+\beta)\bar{\gamma}}{a}\right)^{i_2}} \quad (26)$$

where

$$\tau_{i_1}\triangleq\lim_{z\to-\frac{\alpha\bar{\gamma}}{a}}\frac{\partial^{m_s N_s-i_1}}{(m_s N_s-i_1)!\partial z^{m_s N_s-i_1}}\left(z+\frac{\alpha\bar{\gamma}}{a}\right)^{m_s N_s}T$$

$$\lambda_{i_2}\triangleq\lim_{z\to-\frac{(\alpha+\beta)\bar{\gamma}}{a}}\frac{\partial^{m_r N_r-i_2}}{(m_r N_r-i_2)!\partial z^{m_r N_r-i_2}}\left(z+\frac{(\alpha+\beta)\bar{\gamma}}{a}\right)^{m_r N_r}T.$$

We also invoke the following identity [22, Eq. (2.1.3.1)]

$$\int_0^\infty \frac{x^\mu e^{-sx}}{(x+z)^v}dx=\Gamma(\mu+1)\,z^{\mu-v+1}\Psi(\mu+1,\mu-v+2;sz) \quad (27)$$

where $\Psi(\cdot;\cdot;\cdot)$ is the Tricomi confluent hypergeometric function [18, Eq. (9.210.2)]. By substituting (19) into (25), using partial fractions expansions as in (26) and utilizing (27), a lower bound on the SEP of two-way interference-limited relaying systems can be derived in closed-from; however, the derived expression is omitted due to space limitations. The SEP for some special cases is investigated in Section IV.

IV. Asymptotic Outage Analysis

Since the exact results of the previous section provide limited physical insights, we now focus on the low outage regime. The threshold value of the OP is defined as $\gamma_{\text{th}}=\frac{\gamma_0}{\bar{\gamma}}$, where γ_0 is a finite threshold value. As SNR $\bar{\gamma}$ increases, γ_{th} tends to zero and we can approximate the PDF distribution of the end-to-end SINR around the origin via a Taylor's series. The following approximations will be useful in our analysis

$$(1+\gamma_{\text{th}})^{-n}=\sum_{i=0}^{K}\binom{-n}{i}\gamma_{\text{th}}^i+o\left(\gamma_{\text{th}}^K\right) \quad (28)$$

$$e^{\gamma_{\text{th}}}=\sum_{i=0}^{K}\frac{\gamma_{\text{th}}^i}{i!}+o\left(\gamma_{\text{th}}^K\right), \quad \text{as } \gamma_{\text{th}}\to 0 \quad (29)$$

where n and K are positive integers. Recall that, in the low outage regime the proposed upper bound on the end-to-end SINR becomes exact and we can precisely predict the diversity order and coding gain, with the aid of (29):

Proposition 4: The asymptotic OP of the end-to-end SINR is given by

$$P_{\text{out}}^{\infty}(\gamma_{\text{th}})=G_c\gamma_{\text{th}}^{G_d}+o\left(\gamma_{\text{th}}^{G_d}\right) \quad (30)$$

$$\mathcal{P}_{11}(z) \triangleq \frac{\beta^{m_rN_r}}{\Gamma(m_rN_r)}\frac{\eta^{m_tN_t}}{\Gamma(m_tN_t)}\frac{\alpha^{m_sN_s}}{\Gamma(m_sN_s)}e^{-\left(\frac{az}{\bar{\gamma}}\left(\frac{1}{\varrho}+1\right)+\varrho\alpha\right)}\sum_{l=0}^{m_1-1}\sum_{r=0}^{l}\sum_{i=0}^{m_sN_s+r-1}\sum_{j=0}^{i}\sum_{k=0}^{i-j}\binom{l}{r}\binom{i}{j}\binom{i-j}{k}\frac{a^lz^l}{\varrho^{l+k-i}\bar{\gamma}^l l!i!}$$
$$\times\frac{\Gamma(m_sN_s+r)}{\left(\alpha+\frac{az}{\varrho\bar{\gamma}}\right)^{m_sN_s+r-i}}\frac{\Gamma(m_rN_r+j)}{\left(\varrho\alpha+\frac{az}{\bar{\gamma}}+\beta\right)^{m_rN_r+j}}\frac{\Gamma(m_tN_t+k)}{\left(\alpha+\frac{az}{\varrho\bar{\gamma}}+\eta\right)^{m_tN_t+k}} \tag{20}$$

$$\mathcal{P}_{12}(z) \triangleq \frac{\beta^{m_rN_r}}{\Gamma(m_rN_r)}\frac{\eta^{m_tN_t}}{\Gamma(m_tN_t)}e^{-\frac{az}{\bar{\gamma}}\left(\frac{1}{\varrho}+1\right)}\sum_{l=0}^{m_1-1}\sum_{j=0}^{l}\sum_{k=0}^{l-j}\binom{l}{j}\binom{l-j}{k}\frac{a^lz^l(\varrho+1)^{l-j-k}}{\varrho^{l-j}\bar{\gamma}^l l!}\frac{\Gamma(m_rN_r+j)}{\left(\frac{az}{\bar{\gamma}}+\beta\right)^{m_rN_r+j}}\frac{\Gamma(m_tN_t+k)}{\left(\frac{az}{\varrho\bar{\gamma}}+\eta\right)^{m_tN_t+k}}$$
$$-\frac{\eta^{m_tN_t}}{\Gamma(m_tN_t)}\frac{\beta^{m_rN_r}}{\Gamma(m_rN_r)}e^{-\left(\frac{az}{\bar{\gamma}}\left(\frac{1}{\varrho}+1\right)+\varrho\alpha\right)}\sum_{l=0}^{m_1-1}\sum_{r=0}^{l}\sum_{i=0}^{m_sN_s-1}\sum_{j=0}^{i}\sum_{w=0}^{l-r}\sum_{k=0}^{i-j}\binom{i}{j}\binom{l}{r}\binom{l-r}{w}\binom{i-j}{k}$$
$$\times\frac{a^lz^l\alpha^i(\varrho+1)^{l-r-w}}{\varrho^{l+k-i-r}\bar{\gamma}^l l!i!}\frac{\Gamma(m_rN_r+j+r)}{\left(\frac{az}{\bar{\gamma}}+\varrho\alpha+\beta\right)^{m_rN_r+j+r}}\frac{\Gamma(m_tN_t+k+w)}{\left(\frac{az}{\varrho\bar{\gamma}}+\alpha+\eta\right)^{m_tN_t+k+w}} \tag{21}$$

$$\mathcal{P}_{21}(z) \triangleq \frac{\beta^{m_rN_r}}{\Gamma(m_rN_r)}\frac{\alpha^{m_sN_s}}{\Gamma(m_sN_s)}e^{-\frac{2bz}{\bar{\gamma}}\left(\frac{1}{\varrho}+1\right)}\sum_{l=0}^{m_2-1}\sum_{j=0}^{l}\sum_{k=0}^{l-j}\binom{l}{j}\binom{l-j}{k}\frac{b^lz^l(\varrho+1)^{l-j-k}}{\varrho^{l-j}\bar{\gamma}^l l!}\frac{\Gamma(m_rN_r+j)}{\left(\frac{bz}{\bar{\gamma}}+\beta\right)^{m_rN_r+j}}\frac{\Gamma(m_sN_s+k)}{\left(\frac{bz}{\varrho\bar{\gamma}}+\alpha\right)^{m_sN_s+k}}$$
$$-\frac{\beta^{m_rN_r}}{\Gamma(m_rN_r)}\frac{\alpha^{m_sN_s}}{\Gamma(m_sN_s)}e^{-\left(\frac{bz}{\bar{\gamma}}\left(\frac{1}{\varrho}+1\right)+\varrho\eta\right)}\sum_{l=0}^{m_2-1}\sum_{r=0}^{l}\sum_{i=0}^{m_tN_t-1}\sum_{j=0}^{i}\sum_{w=0}^{l-r}\sum_{k=0}^{i-j}\binom{i}{j}\binom{l}{r}\binom{l-r}{w}\binom{i-j}{k}$$
$$\times\frac{b^lz^l\eta^i(\varrho+1)^{l-r-w}}{\varrho^{l+k-i-r}\bar{\gamma}^l l!i!}\frac{\Gamma(m_rN_r+j+r)}{\left(\frac{bz}{\bar{\gamma}}+\varrho\eta+\beta\right)^{m_rN_r+j+r}}\frac{\Gamma(m_sN_s+k+w)}{\left(\frac{bz}{\varrho\bar{\gamma}}+\alpha+\eta\right)^{m_sN_s+k+w}} \tag{22}$$

$$\mathcal{P}_{22}(z) \triangleq \frac{\beta^{m_rN_r}}{\Gamma(m_rN_r)}\frac{\eta^{m_tN_t}}{\Gamma(m_tN_t)}\frac{\alpha^{m_sN_s}}{\Gamma(m_sN_s)}e^{-\left(\frac{bz}{\bar{\gamma}}\left(\frac{1}{\varrho}+1\right)+\varrho\eta\right)}\sum_{l=0}^{m_2-1}\sum_{r=0}^{l}\sum_{i=0}^{m_tN_t+r-1}\sum_{j=0}^{i}\sum_{k=0}^{i-j}\binom{l}{r}\binom{i}{j}\binom{i-j}{k}\frac{b^lz^l}{\varrho^{l+k-i}\bar{\gamma}^l l!i!}$$
$$\times\frac{\Gamma(m_tN_t+r)}{\left(\eta+\frac{bz}{\varrho\bar{\gamma}}\right)^{m_tN_t+r-i}}\frac{\Gamma(m_rN_r+j)}{\left(\varrho\eta+\frac{bz}{\bar{\gamma}}+\beta\right)^{m_rN_r+j}}\frac{\Gamma(m_sN_s+k)}{\left(\alpha+\frac{bz}{\varrho\bar{\gamma}}+\eta\right)^{m_sN_s+k}}. \tag{23}$$

where the diversity order and coding gain are respectively

$$G_d = \min(m_1, m_2)$$
$$G_c = \begin{cases} G_{c_1} & m_2 > m_1 \\ G_{c_2} & m_1 > m_2 \\ G_{c_1}+G_{c_2} & m_1 = m_2 \end{cases} \tag{31}$$

while G_{c_1} and G_{c_2} are given at the top of next page.

Proof: At high SNRs, where $\overline{\gamma} \to \infty$, we have $1 - \frac{\Gamma\left(m,\frac{az}{\overline{\gamma}}\right)}{\Gamma(m)} \to \frac{\left(\frac{az}{\overline{\gamma}}\right)^m}{\Gamma(m+1)}$ [22, Eq. (06.06.06.0001.02)]. Following the proof of Proposition 3, we can get (30). ■

The above result implies that, when all interferers' powers (i.e. P_{Ri}, P_{S_1j} and P_{S_2k}) are kept constant, interference does not affect the diversity order. However, when the interference power is growing large while the ratio of transmit powers of both sources versus the interferers' powers is kept constant (a scenario corresponding to the special case b in Section VI), the performance of the system cannot be improved due to the interference becoming dominant; as such, the diversity order in this case is equal to 0. These results are consistent with [10] and [15]. For the sake of simplicity, we now elaborate on the case of Rayleigh fading, by setting all m parameters to one, while the interference power is assumed constant; then, the asymptotic OP becomes equal to

$$P_{\text{out}}^{\infty}(\gamma_{\text{th}}) = \left((a+b)\left(\frac{\varrho+1}{\varrho}+\frac{N_r}{\beta}\right)+\frac{aN_t}{\varrho\eta}+\frac{bN_s}{\varrho\alpha}\right.$$
$$+\frac{\beta^{N_r}}{\Gamma(N_r)}\frac{\eta^{N_t}}{\Gamma(N_t)}e^{-\varrho\alpha}\sum_{i=0}^{N_s-1}\sum_{j=0}^{i}\sum_{k=0}^{i-j}\frac{a\alpha^{i-1}(N_s-i)}{\varrho^{k-i+1}i!}\binom{i}{j}$$
$$\times\binom{i-j}{k}\frac{\Gamma(N_r+j)}{(\varrho\alpha+\beta)^{N_r+j}}\frac{\Gamma(N_t+k)}{(\eta+\alpha)^{N_t+k}}+\frac{\beta^{N_r}\alpha^{N_s}}{\Gamma(N_r)\Gamma(N_s)}$$
$$\times e^{-\varrho\eta}\sum_{i=0}^{N_t-1}\sum_{j=0}^{i}\sum_{k=0}^{i-j}\frac{b\eta^{i-1}(N_t-i)}{\varrho^{k-i+1}i!}\binom{i}{j}\binom{i-j}{k}$$
$$\left.\times\frac{\Gamma(N_r+j)}{(\varrho\eta+\beta)^{N_r+j}}\frac{\Gamma(N_s+k)}{(\alpha+\eta)^{N_s+k}}\right)\gamma_{\text{th}}+o(\gamma_{\text{th}}). \tag{34}$$

Note that (34) is different from [10, Eq. (12)] since, as previously mentioned, the authors therein worked on the OP of γ_{S_1} and assumed that X and Y are independent. In this paper, however, this assumption has been relaxed. In the asymptotic regime, by substituting (34) into (25), the SEP for Rayleigh

$$G_{c_1} = \frac{\beta^{m_rN_r}}{\Gamma(m_rN_r)}\frac{\eta^{m_tN_t}}{\Gamma(m_tN_t)}\frac{a^{m_1}}{\Gamma(m_1+1)}\left[\sum_{i=0}^{m_sN_s-1}\sum_{j=0}^{i}\sum_{l=0}^{m_1}\sum_{k=0}^{i-j}\sum_{w=0}^{m_1-l}\binom{i}{j}\binom{m_1}{l}\binom{i-j}{k}\right.$$
$$\times\binom{m_1-l}{w}\frac{\alpha^i(\varrho+1)^{m_1-l-w}}{\varrho^{m_1+k-i-l}i!}e^{-\alpha\varrho}\frac{\Gamma(m_rN_r+l+j)}{(\alpha\varrho+\beta)^{m_rN_r+l+j}}\frac{\Gamma(m_tN_t+k+w)}{(\alpha+\eta)^{m_tN_t+k+w}}-\sum_{l=0}^{m_1}\sum_{w=0}^{m_1-l}\binom{m_1-l}{w}$$
$$\times\frac{(\varrho+1)^{m_1-l-w}}{\varrho^{m_1-l}}\frac{\Gamma(m_rN_r+l)}{\beta^{m_rN_r+l}}\frac{\Gamma(m_tN_t+w)}{\eta^{m_tN_t+w}}-\frac{\alpha^{m_sN_s}}{\Gamma(m_sN_s)}\sum_{r=0}^{m_1}\sum_{i=0}^{m_sN_s+r-1}\sum_{j=0}^{i}\sum_{k=0}^{i-j}\binom{i}{j}$$
$$\left.\times\binom{m_1}{r}\binom{i-j}{k}\frac{\alpha^i}{\varrho^{m_1+k-i}i!}e^{-\alpha\varrho}\frac{\Gamma(m_sN_s+r)}{\alpha^{m_sN_s+r}}\frac{\Gamma(m_rN_r+j)}{(\alpha\varrho+\beta)^{m_rN_r+j}}\frac{\Gamma(m_tN_t+k)}{(\alpha+\eta)^{m_tN_t+k}}\right] \tag{32}$$

$$G_{c_2} = \frac{\beta^{m_rN_r}}{\Gamma(m_rN_r)}\frac{\alpha^{m_sN_s}}{\Gamma(m_sN_s)}\frac{b^{m_2}}{\Gamma(m_2+1)}\left[\sum_{i=0}^{m_tN_t-1}\sum_{j=0}^{i}\sum_{l=0}^{m_2}\sum_{k=0}^{i-j}\sum_{w=0}^{m_2-l}\binom{i}{j}\binom{m_2}{l}\binom{i-j}{k}\right.$$
$$\times\binom{m_2-l}{w}\frac{\eta^i(\varrho+1)^{m_2-l-w}}{\varrho^{m_2+k-i-l}i!}e^{-\eta\varrho}\frac{\Gamma(m_rN_r+l+j)}{(\eta\varrho+\beta)^{m_rN_r+l+j}}\frac{\Gamma(m_sN_s+k+w)}{(\alpha+\eta)^{m_sN_s+k+w}}-\sum_{l=0}^{m_2}\sum_{w=0}^{m_2-l}\binom{m_2-l}{w}$$
$$\times\frac{(\varrho+1)^{m_2-l-w}}{\varrho^{m_2-l}}\frac{\Gamma(m_rN_r+l)}{\beta^{m_rN_r+l}}\frac{\Gamma(m_sN_s+w)}{\alpha^{m_sN_s+w}}-\frac{\eta^{m_tN_t}}{\Gamma(m_tN_t)}\sum_{r=0}^{m_2}\sum_{i=0}^{m_tN_t+r-1}\sum_{j=0}^{i}\sum_{k=0}^{i-j}\binom{i}{j}$$
$$\left.\times\binom{m_2}{r}\binom{i-j}{k}\frac{\eta^i}{\varrho^{m_2+k-i}i!}e^{-\eta\varrho}\frac{\Gamma(m_tN_t+r)}{\eta^{m_tN_t+r}}\frac{\Gamma(m_rN_r+j)}{(\eta\varrho+\beta)^{m_rN_r+j}}\frac{\Gamma(m_sN_s+k)}{(\alpha+\eta)^{m_sN_s+k}}\right]. \tag{33}$$

fading channels can be obtained as

$$\overline{P_e^{\infty}} = \frac{c}{4d\bar{\gamma}}\left((a+b)\left(\frac{\varrho+1}{\varrho}+\frac{N_r}{\beta}\right)+\frac{aN_t}{\varrho\eta}+\frac{bN_s}{\varrho\alpha}\right.$$
$$+\frac{\beta^{N_r}}{\Gamma(N_r)}\frac{\eta^{N_t}}{\Gamma(N_t)}e^{-\varrho\alpha}\sum_{i=0}^{N_s-1}\sum_{j=0}^{i}\sum_{k=0}^{i-j}\frac{a\alpha^{i-1}(N_s-i)}{\varrho^{k-i+1}i!}\binom{i}{j}$$
$$\times\binom{i-j}{k}\frac{\Gamma(N_r+j)}{(\varrho\alpha+\beta)^{N_r+j}}\frac{\Gamma(N_t+k)}{(\eta+\alpha)^{N_t+k}}+\frac{\beta^{N_r}\alpha^{N_s}}{\Gamma(N_r)\Gamma(N_s)}$$
$$\times e^{-\varrho\eta}\sum_{i=0}^{N_t-1}\sum_{j=0}^{i}\sum_{k=0}^{i-j}\frac{b\eta^{i-1}(N_t-i)}{\varrho^{k-i+1}i!}\binom{i}{j}\binom{i-j}{k}$$
$$\left.\times\frac{\Gamma(N_r+j)}{(\varrho\eta+\beta)^{N_r+j}}\frac{\Gamma(N_s+k)}{(\alpha+\eta)^{N_s+k}}\right)+o\left(\bar{\gamma}^{-1}\right). \tag{35}$$

As anticipated, the diversity order is equal to 1 for the particular Rayleigh case under consideration. When interference exists only at the relay (i.e. $N_s = N_t = 0$), (34) and (35) reduce respectively to

$$P_{\text{out}}^{\infty}(\gamma_{\text{th}}) = (a+b)\left(\frac{\varrho+1}{\varrho}+\frac{N_r}{\beta}\right)\gamma_{\text{th}}+o(\gamma_{\text{th}})$$
$$\overline{P_e^{\infty}} = \frac{c(a+b)\left(\frac{\varrho+1}{\varrho}+\frac{N_r}{\beta}\right)}{4d\bar{\gamma}}+o\left(\bar{\gamma}^{-1}\right). \tag{36}$$

The above expressions reveal straightforwardly the impact of the model parameters on the system performance. More specifically, we can see that by increasing N_r and the nodes' power or by reducing the interference power, the OP and SEP will reduce. For the interference-free system (i.e. $N_s = N_r = N_t = 0$), (34) and (35) reduce respectively to

$$P_{\text{out}}^{\infty} = \frac{\varrho+1}{\varrho}(a+b)\gamma_{\text{th}}+o(\gamma_{\text{th}}) \tag{37}$$

$$\overline{P_e^{\infty}} = \frac{c\frac{\varrho+1}{\varrho}(a+b)}{4d\bar{\gamma}}+o\left(\bar{\gamma}^{-1}\right). \tag{38}$$

Note that, in this case $\gamma_{e2e}^{\text{up}} = \varrho\min\left(\frac{\gamma_1}{\varrho+1},\frac{\gamma_2}{\varrho+1}\right)$, while the diversity order is 1, which is in agreement with $\min(m_1, m_2)$, predicted by Proposition 4.

V. Relay position Optimization

The optimal placement of relays and/or the optimal power allocation for improving the system performance (e.g., minimizing the OP) has been a very hot area of research. For example, recently the authors in [23] investigated an optimization problem based on the error probability for one-way relaying networks over Rayleigh fading channels. In [24], they extended their work to Nakagami-m fading channels, where they worked with the high SNR approximation of the OP. In the context of two-way relaying, the authors in [25] minimized the individual OPs of the source nodes at high SNRs. The authors in [26] derived the optimal achievable end-to-end rate of two-way interference-free systems. In [27], the authors presented optimal power allocation results in order to maximize the sum of the achievable rates of both hops, when interference is present in the network. The authors in [28] worked out some optimal power allocation results for the OP of DF two-way relay networks. Note that, to the best of our knowledge, the work presented here on two-way interference-limited relaying networks, where CCI affect all nodes, is new. In the following, we consider some optimization problems which seek to optimally allocate power to the network nodes and also find the optimal relay position, in order to minimize the OP for constant interference power. Note that an arbitrary SINR analysis is intractable and for this reason we elaborate on the asymptotically low outage regime and assume the case of Rayleigh fading for the $S_1 \to R$ and $R \to S_2$ links (i.e. $m_1 = m_2 = 1$).[3]

A. Optimization of OP at S_1 and S_2

In our first optimization formulation we seek to minimize the OP at S_1.[4] Referring back to (17), the lower bound on the OP at S_1, $P^{\rm lb}_{{\rm out}_{S_1}}$, should be minimized according to

$$\min_{d_{S_1,R},\, d_{S_2,R}} P^{\rm lb}_{{\rm out}_{S_1}}(\gamma_{\rm th})$$
$$\text{subject to} \quad 0 < d_{S_1,R},\ 0 < d_{S_2,R},\ 2P_S + P_R = P_{\tt tot} \quad (39)$$

where $d_{S_1,R}$ and $d_{S_2,R}$ are the distances between S_1 and the relay and S_2 and the relay, respectively, while $P_{\tt tot}$ is the total power of the system. We now make the standard assumption that the average SNR, Ω, decays exponentially with the distance such that $a = \frac{m_1}{P_S d_{S_1,R}^{-v}}$, $b = \frac{m_2}{P_S d_{S_2,R}^{-v}}$. Note that v is the path-loss exponent, whose value is normally in the range of 2 to 6. By setting $m_1 = m_2 = 1$ in (17), we have $i = j = k = l = t = 0$. By doing so, the lower bound on the OP at node S_1 can be simplified as

$$P^{\rm lb}_{{\rm out}_{S_1}}(\gamma_{\rm th}) = 1 - \frac{\alpha^{m_s N_s} \beta^{m_r N_r} e^{-\left(b\left(\frac{1}{\varrho}+1\right)+\frac{a}{\varrho}\right)\gamma_{\rm th}}}{\left(\frac{a+b}{\varrho}\gamma_{\rm th} + \alpha\right)^{m_s N_s} (b\gamma_{\rm th} + \beta)^{m_r N_r}}. \quad (40)$$

In the low outage regime, $\frac{a+b}{\alpha}$ and $\frac{b}{\beta}$ tend to zero. Thus, using (29), and by keeping only the dominant term, the lower bound in (40) becomes exact and equal to

$$P^{\infty}_{{\rm out}_{S_1}}(\gamma_{\rm th}) \triangleq \mathcal{L}(\gamma_{\rm th}) = \frac{(\varrho+1)b + a}{\varrho}\gamma_{\rm th} + \frac{a+b}{\varrho\alpha} m_s N_s \gamma_{\rm th} + \frac{b}{\beta} m_r N_r \gamma_{\rm th} + o(\gamma_{\rm th}). \quad (41)$$

Hereafter, our objective function is $\mathcal{L}(\gamma_{\rm th})$ which, fortunately, lends itself to algebraic manipulations;[5]

1) Power Allocation Optimization under Fixed Relay Location: By substituting the path-loss based definition of a and b and assuming a fixed location for the relay, (41) can be rewritten as

$$\mathcal{L}(\gamma_{\rm th}) = A_1 \frac{d^v_{S_1,R}}{P_R} + A_1 \frac{d^v_{S_2,R}}{P_R} + B_1 \frac{d^v_{S_2,R}}{P_S} \quad (42)$$

where $A_1 \triangleq \left(1 + \frac{m_s N_s}{\alpha}\right)\gamma_{\rm th}$ and $B_1 \triangleq \left(1 + \frac{m_r N_r}{\beta}\right)\gamma_{\rm th}$. The second derivative of (42) with respect to P_S is equal to

$$\frac{\partial^2 \mathcal{L}(\gamma_{\rm th})}{\partial P_S^2} = 8A_1 \frac{d^v_{S_1,R}}{P_R^3} + 8A_1 \frac{d^v_{S_2,R}}{P_R^3} + 2B_1 \frac{d^v_{S_2,R}}{P_S^3} > 0 \quad (43)$$

which is, of course, strictly positive. As such, the optimization problem is convex. By introducing the Lagrangian multipliers and applying the Karush-Kuhn-Tucker (KKT) conditions, we can obtain the optimal powers as

$$\frac{\partial \mathcal{L}(\gamma_{\rm th})}{\partial P_S} = 2A_1 \frac{d^v_{S_1,R}}{P_R^2} + 2A_1 \frac{d^v_{S_2,R}}{P_R^2} - B_1 \frac{d^v_{S_2,R}}{P_S^2} = 0$$
$$\Rightarrow P_S^* = \frac{P_{\tt tot}}{2 + \sqrt{\frac{2A_1\left(d^v_{S_1,R} + d^v_{S_2,R}\right)}{B_1 d^v_{S_2,R}}}}. \quad (44)$$

It can be seen that, if the relay is closer to S_2, S_1 needs more power to achieve the same performance. Interestingly, the optimal power allocation is independent of $\gamma_{\rm th}$. For i.i.d. fading and symmetric interference channels (i.e., $m_s = m_r = m_t$, $\Omega_s = \Omega_r = \Omega_t$ and $N_s = N_r = N_t$), where the optimum relay position is in the middle of the two source nodes' distance [23], the optimal power allocation solution becomes $P_R^* = 2P_S^* = \frac{P_{\tt tot}}{2}$. By substituting the optimal values from (44) into (42) and setting $d_{S_1,R} = d_{S_2,R} = 0.5$, the minimum value of OP at $S1$, can be written as

$$\mathcal{L}^*_{\tt sc1}(\gamma_{\rm th}) = A_1 \frac{d^v_{S_1,R}}{P_R^*} + A_1 \frac{d^v_{S_2,R}}{P_R^*} + B_1 \frac{d^v_{S_2,R}}{P_S^*} = \frac{\sqrt{A_1}\left(\sqrt{A_1} + \sqrt{B_1}\right)}{2^{v-1} P_{\tt tot}} + \frac{\sqrt{B_1}\left(\sqrt{A_1} + \sqrt{B_1}\right)}{2^{v-1} P_{\tt tot}}. \quad (45)$$

The minimum value of OP at S_1, for this first scenario, is

$$\mathcal{L}^*_{\tt sc1}(\gamma_{\rm th}) = \frac{A_1 + 2\sqrt{A_1 B_1} + B_1}{2^{v-1} \mathrm{P}_{\tt tot}}. \quad (46)$$

It is obvious that by decreasing m_s, m_r, N_s, N_r and $\gamma_{\rm th}$ and also by increasing α, β, v and $P_{\tt tot}$, $\mathcal{L}^*_{\tt sc1}(\gamma_{\rm th})$ will decrease.

2) Relay Position Optimization under Fixed Power Allocation: To minimize the effects of path-loss, it is obvious that the relay should be aligned between S_1 and S_2. This is reasonable since the direct line link between any two nodes minimizes the loss of transmitted power [24]. By setting $d_{S_1,R} = d$ and $d_{S_2,R} = 1 - d$, the OP in (41) can be simplified as

$$\mathcal{L}(\gamma_{\rm th}) = A_1 \frac{d^v + (1-d)^v}{P_R} + B_1 \frac{(1-d)^v}{P_S} + o(\gamma_{\rm th}). \quad (47)$$

The second derivative of (47) with respect to d is equal to

$$\frac{\partial^2 \mathcal{L}(\gamma_{\rm th})}{\partial d^2} = A_1 \frac{v(v-1)\, d^{v-2}}{P_R} + A_1 \frac{v(v-1)(1-d)^{v-2}}{P_R} + B_1 \frac{v(v-1)(1-d)^{v-2}}{P_S} > 0 \quad (48)$$

which is, of course, strictly positive. By introducing the Lagrangian multipliers and applying the KKT conditions, we obtain the optimal relay position as

$$\frac{\partial \mathcal{L}(\gamma_{\rm th})}{\partial d} = A_1 \frac{v d^{v-1}}{P_R} - A_1 \frac{v(1-d)^{v-1}}{P_R} - B_1 \frac{v(1-d)^{v-1}}{P_S} = 0$$
$$\Rightarrow d^* = \frac{1}{1 + \left(\frac{\frac{A_1}{P_R}}{\frac{A_1}{P_R} + \frac{B_1}{P_S}}\right)^{\frac{1}{v-1}}}. \quad (49)$$

By observing (49), it is obvious that if the allocated power to S_1 and S_2 decreases and also if m_r or N_r increase, the relay should be closer to S_2; this is reasonable since the quality of the $R \to S_2$ link needs to be improved to avoid being in outage. An interesting result appears for i.i.d. fading and symmetric interference channels (i.e., $m_s = m_r = m_t$, $\Omega_s =$

[3] We emphasize the fact that in order to analytically find the optimal values for the case of arbitrary m_1 and m_2, the optimization problem can be solved only numerically.

[4] We should note that several previous works have considered deriving the OP at S_1 as well [10, 12, 29, 30, 31].

[5] Note that a similar optimization problem can be defined based on SEP by using (25), where the optimization results are the same with the only difference pertaining to the replacement of A_1, B_1 with $\bar{A}_1 = \frac{c}{4} d A_1$, $\bar{B}_1 = \frac{c}{4} d B_1$.

$\Omega_r = \Omega_t$ and $N_s = N_r = N_t$), for which $A_1 = B_1$; this states that the optimal relay location is near to in the S_2, which is in agreement with [23]. This is rather intuitive: referring back to (40), we can see that the OP at S_1 includes exponential terms depending on the $S_1 \rightarrow R$ and $S_2 \rightarrow R$ links. Given that the former term scales twice as fast as the latter, the relay node should be close to S_2. By substituting the optimal value from (49) into (47), and setting $P_R = 2P_S = \frac{P_{\tt tot}}{2}$, the minimum value of OP at S_1 can be rewritten as

$$\mathcal{L}^*_{\tt sc2}(\gamma_{\rm th}) = A_1 \frac{d^{*v} + (1-d^*)^v}{P_R} + B_1 \frac{(1-d^*)^v}{P_S}$$
$$= \frac{2A_1}{P_{\tt tot}\left[1 + \left(\frac{A_1}{A_1+2B_1}\right)^{\frac{1}{v-1}}\right]^v} + \frac{2\left(\frac{A_1}{A_1+2B_1}\right)^{\frac{v}{v-1}}(A_1+2B_1)}{P_{\tt tot}\left[1 + \left(\frac{A_1}{A_1+2B_1}\right)^{\frac{1}{v-1}}\right]^v}. \tag{50}$$

Hence, the minimum value of OP at S_1 can be obtained as

$$\mathcal{L}^*_{\tt sc2}(\gamma_{\rm th}) = \frac{2A_1^{\frac{v}{v-1}}(A_1+2B_1) + 2A_1(A_1+2B_1)^{\frac{v}{v-1}}}{P_{\tt tot}\left[A_1^{\frac{1}{v-1}} + (A_1+2B_1)^{\frac{1}{v-1}}\right]^v}. \tag{51}$$

We can see that by decreasing m_s, m_r, N_s, N_r and $\gamma_{\rm th}$ and also by increasing α, β, v and $P_{\tt tot}$, the minimal value of OP in (51) will be reduced.

3) Joint Power Allocation and Relay Position Optimization: The joint optimization problem can be expressed as

$$\min_{P_S,\, P_R,\, d} \mathcal{L}(\gamma_{\rm th}) = A_1 \frac{d^v}{P_R} + A_1 \frac{(1-d)^v}{P_R} + B_1 \frac{(1-d)^v}{P_S}$$
$$\text{subject to} \quad 2P_S + P_R = P_{\tt tot}, \ \text{and}\ 0 < P_S, P_R,\ 0 < d < 1. \tag{52}$$

Proposition 5: The optimization problem in (52), when $v \geq 2$, $d \in [0,1]$ and $P_S \in [0, P_{\tt tot}]$, is convex.

Proof: By differentiating the objective function in (52) twice with respect to P_S and d, we can show that the Hessian matrix is positive semi-definite when $v \geq 2$, $d \in [0,1]$ and $P_S \in [0, P_{\tt tot}]$. ■

By differentiating (41) with respect to P_S and d and setting both derivatives equal to zero, we end up with

$$\frac{d^{v-1}}{(1-d)^{v-1}} = 1 + \frac{B_1 P_R}{A_1 P_S}, \quad \frac{B_1 P_R^2}{2A_1 P_S^2} - 1 = \frac{d^v}{(1-d)^v}. \tag{53}$$

We can now obtain the optimal power allocation and relay position by numerically solving the following equations

$$\left[\frac{d^{v-1}}{(1-d)^{v-1}} - 1\right]^2 = \frac{2B_1}{A_1}\left[\frac{d^v}{(1-d)^v} + 1\right],$$
$$P_S^* = \frac{P_{\tt tot}}{\frac{A_1}{B_1}\left[\frac{d^{v-1}}{(1-d)^{v-1}} - 1\right] + 2}. \tag{54}$$

Our numerical results show that joint optimization and relay position optimization yield almost the same performance.

B. Optimization of OP at S_1 with constrained OP at S_2

The previous optimization problem focuses on the OP at S_1 by ignoring the OP at S_2. Alternatively, we can define a new problem which seeks to minimize the OP at S_1 by considering the OP at S_2. The main advantage of this approach is that we can adaptively select this threshold to meet some stipulated OP tolerance. By assuming $\varrho = 1$, we now have

$$\min_d P^{\infty}_{{\rm out}_{S_1}}(\gamma_{\rm th}) = \frac{A_1}{P_S} d^v + \frac{C_1}{P_S}(1-d)^v$$
$$\text{subject to} \quad P^{\infty}_{{\rm out}_{S_2}}(\gamma_{\rm th}) = \frac{C_2}{P_S} d^v + \frac{A_2}{P_S}(1-d)^v \leq {\rm OP}_{S_2},$$
$$\text{and},\ 0 < d < 1 \tag{55}$$

where $A_2 \triangleq \left(1 + \frac{m_t N_t}{\eta}\right)\gamma_{\rm th}$, $C_1 \triangleq A_1 + B_1$ and $C_2 \triangleq A_2 + B_1$, while ${\rm OP}_{S_2}$ is the maximum tolerable OP at S_2. Note that $P^{\infty}_{{\rm out}_{S_2}}(\gamma_{\rm th})$ is derived by using (18) and in a similar way as (41). Using the Lagrange multiplier definition, we can prove that the optimization problem in (55) is convex.

$$\mathcal{L}(\gamma_{\rm th}) = \frac{A_1}{P_S} d^v + \frac{C_1}{P_S}(1-d)^v + \lambda\left(\frac{C_2}{P_S} d^v + \frac{A_2}{P_S}(1-d)^v - {\rm OP}_{S_2}\right) \tag{56}$$

$$\frac{\partial^2 \mathcal{L}(\gamma_{\rm th})}{\partial d^2} = v(v-1)\left[\left(\frac{A_1}{P_S} + \frac{\lambda C_2}{P_S}\right) d^{v-2} + \left(\frac{C_1}{P_S} + \frac{\lambda A_2}{P_S}\right)(1-d)^{v-2}\right] > 0. \tag{57}$$

Hence, by using the Lagrange multiplier, the KKT conditions are as follows:

$$(A_1 + \lambda C_2)\, d^{v-1} = (C_1 + \lambda A_2)(1-d)^{v-1} \tag{58}$$
$$\lambda \geq 0, \quad 0 < d < 1 \tag{59}$$
$$\lambda\left(\frac{C_2}{P_S} d^v + \frac{A_2}{P_S}(1-d)^v - {\rm OP}_{S_2}\right) = 0 \tag{60}$$

where λ is the Lagrangian coefficient. Here, we have two cases:

1. $\lambda = 0 \Rightarrow d^* = \frac{1}{1 + \left(\frac{A_1}{C_1}\right)^{\frac{1}{v-1}}}$.
2. $\lambda \neq 0 \Rightarrow d^* = \arg_d \left(\frac{C_2}{P_S} d^v + \frac{A_2}{P_S}(1-d)^v - {\rm OP}_{S_2} = 0\right)$.

Assuming $v = 4$, the optimal value is the solution to the following 4-th order polynomial

$$d^4 - \frac{4A_2}{A_2 + C_2} d^3 + \frac{6A_2}{A_2 + C_2} d^2 - \frac{4A_2}{A_2 + C_2} d + \frac{A_2 - P_S OP_{S_2}}{A_2 + C_2} = 0. \tag{61}$$

Using [32, Sec. (5)], we can solve this polynomial analytically, though the results are omitted due to space limitations.

We can now take a closer look into the actual impact of relay position optimization. We begin with the optimization scheme in (47). For i.i.d. fading and symmetric interference channels (i.e. $m_s = m_r = m_t$, $N_s = N_r = N_t$ and $\Omega_s = \Omega_r = \Omega_t$) for which $A_1 = B_1$, we have

$$\mathcal{L}^{\rm non-opt}(\gamma_{\rm th}) = \mathcal{L}^*_{\tt sc1}(\gamma_{\rm th}) = \frac{A_1}{P_{\tt tot} 2^{v-3}} \overset{v=4}{=} \frac{0.5A_1}{P_{\tt tot}}$$
$$\mathcal{L}^*_{\tt sc2}(\gamma_{\rm th}) = \frac{6A_1}{P_{\tt tot}\left(1 + 3^{\frac{1}{v-1}}\right)^{v-1}} \overset{v=4}{\approx} \frac{0.41A_1}{P_{\tt tot}} \tag{62}$$

where $\mathcal{L}^{\rm non-opt}(\gamma_{\rm th})$ indicates the OP at S_1 where no optimization is being performed; in this non-optimized case, we set $P_R = 2P_S = \frac{P_{\tt tot}}{2}$ and $d_{S_1,R} = d_{S_2,R} = 0.5$ in (47). Note that (62) implies that in the case of identical fading conditions, the system performance cannot improve via power optimization.

We can now compare the cases of symmetric and asymmetric interference power with the same total interference power constraint. In the case of symmetric interference power profile, where $N_s = N_r, \Omega_s = \Omega_r$, we have $A_1 = B_1$ and as such there is no need for optimization. On the other hand, in the asymmetric interference power profile, if we consider that the interference at the relay is stronger than at S_1 (i.e., we increase P_{Ri} while P_{S_1j} is kept fixed, and hence B_1 is increasing while A_1 is kept fixed), (47) and (51) simplify respectively to

$$\mathcal{L}^{\texttt{non-opt}}(\gamma_{\rm th}) \approx \frac{B_1}{P_{\texttt{tot}}2^{v-2}}, \quad \mathcal{L}^*_{\texttt{sc1}}(\gamma_{\rm th}) \approx \frac{B_1}{2^{v-1}P_{\texttt{tot}}}$$
$$\mathcal{L}^*_{\texttt{sc2}}(\gamma_{\rm th}) \approx \frac{2A_1}{P_{\texttt{tot}}}. \tag{63}$$

Note that for the non-optimized case we have assumed $d = 0.5$ and $2P_{S_2} = P_R = 2P_{S_1} = \frac{P_{\texttt{tot}}}{2}$. As can be observed from (63), significant performance enhancement can be attained via the use of relay position optimization. This verifies the importance of optimization for the asymmetric interference profile case. In fact, by increasing B_1, while A_1 is kept fixed (i.e. by increasing the SNR), the OPs in (63) are proportional to B_1, B_1 and B_1^0, respectively.

VI. Practical Cases of Interest

In this section, we particularize the previously reported results to some practical cases of interest.

a) Interference-free ($N_s = N_r = N_t = 0$)

When we set ($P_{S_1j} = P_{S_2k} = P_{Ri} = P_I = 0$), (24) simplifies to

$$P_{\rm out}^{\rm lb}(\gamma_{\rm th}) = 1 - e^{-(\varrho+1)(a+b)\gamma_{\rm th}} \sum_{i=0}^{m_1-1}\sum_{j=0}^{m_2-1} \frac{(\varrho+1)^{i+j}a^i b^j \gamma_{\rm th}^{i+j}}{i!j!} \tag{64}$$

where, in this case, $\gamma_{S_1}^{\rm up} = \varrho \min\left(\gamma_1, \frac{\gamma_2}{\varrho+1}\right)$, which is a tight upper bound for $\frac{\varrho\gamma_1\gamma_2}{(\varrho+1)\gamma_1+\gamma_2}$ while $\gamma_{S_2}^{\rm up} = \varrho\min\left(\gamma_2, \frac{\gamma_1}{\varrho+1}\right)$. Note that the derived CDF in (17), when $N_s = N_r = 0$, is a tight lower bound for [29, Eq. (4)].

b) Interference-limited case: For simplicity, we assume that $a = b, \alpha = \beta = \eta, m_1 = m_2, m_sN_s = m_tN_t$. By setting $P_{S_1j} = P_{S_2k} = P_{Ri} = P_I$, $P_S = P_R$ and $\frac{P_S}{P_I} = \rho \geq 1$ where $P_S, P_I \to \infty$, (24) after some manipulations simplifies to

$$P_{\rm out}^{\rm lb}(\gamma_{\rm th}) = 1 - \left(\mathcal{P}_3(\gamma_{\rm th}) + \mathcal{P}_4(\gamma_{\rm th})\right)^2 \tag{65}$$

where

$$\mathcal{P}_3(\gamma_{\rm th}) \triangleq \frac{1}{\Gamma(m_rN_r)}\frac{1}{[\Gamma(m_sN_s)]^2}\sum_{l=0}^{m_1-1}\sum_{i=0}^{m_sN_s+l-1}\sum_{j=0}^{i}\binom{i}{j}\frac{\left(\frac{m_1\Omega_s}{\rho m_s\Omega_f}\right)^l\gamma_{\rm th}^l}{l!i!}\frac{\Gamma(m_sN_s+l)}{\left(1+\frac{m_1\Omega_s}{\rho m_s\Omega_f}\gamma_{\rm th}\right)^{m_sN_s+l-j-k}}$$
$$\times\frac{\Gamma(m_rN_r+j)}{\left(2+\frac{m_1\Omega_s}{\rho m_s\Omega_f}\gamma_{\rm th}\right)^{m_rN_r+j}}\frac{\Gamma(m_sN_s+i-j)}{\left(2+\frac{m_1\Omega_s}{\rho m_s\Omega_f}\gamma_{\rm th}\right)^{m_sN_s+i-j}}$$

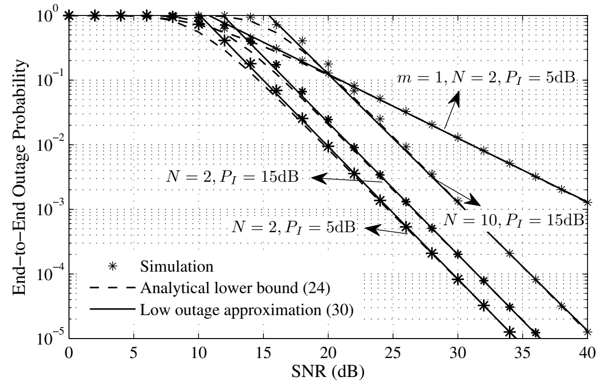

Fig. 2. Outage probability at S_1 (P_I is a constant) for $m_1 = m_t = 2, m_2 = m_s = 3, m_r = 2.5, \Omega_1 = \Omega_2 = 1, \Omega_s = \Omega_r = \Omega_t = 0.01, \gamma_0 = 3$.

$$\mathcal{P}_4(\gamma_{\rm th}) \triangleq \frac{1}{\Gamma(m_rN_r)}\frac{1}{\Gamma(m_sN_s)}\left(\sum_{l=0}^{m_1-1}\sum_{j=0}^{l}\binom{l}{j}\frac{\gamma_{\rm th}^l}{l!}\right.$$
$$\times\frac{\Gamma(m_rN_r+j)}{\left(\frac{m_1\Omega_s}{\rho m_s\Omega_f}\gamma_{\rm th}+1\right)^{m_rN_r+j}}\frac{\Gamma(m_sN_s+l-j)}{\left(\frac{m_1\Omega_s}{\rho m_s\Omega_f}\gamma_{\rm th}+1\right)^{m_sN_s+l-j}}$$
$$\times\left(\frac{m_1\Omega_s}{\rho m_s\Omega_f}\right)^l - \sum_{l=0}^{m_1-1}\sum_{r=0}^{l}\sum_{i=0}^{m_sN_s-1}\sum_{j=0}^{i}\binom{i}{j}\binom{l}{r}$$
$$\times\frac{\left(\frac{m_1\Omega_s}{\rho m_s\Omega_f}\right)^l\gamma_{\rm th}^l}{l!i!}\frac{\Gamma(m_rN_r+j+r)}{\left(\frac{m_1\Omega_s}{\rho m_s\Omega_f}\gamma_{\rm th}+2\right)^{m_rN_r+j+r}}$$
$$\left.\times\frac{\Gamma(m_sN_s+l-r+i-j)}{\left(\frac{m_1\Omega_s}{\rho m_s\Omega_f}\gamma_{\rm th}+2\right)^{m_sN_s+l-r+i-j}}\right)$$

where $\Omega_s \triangleq \mathbb{E}[|h_{S_1,j}|^2]$ and $\Omega_f \triangleq \mathbb{E}[|f|^2]$. The diversity order is equal to 0 which means that the OP will saturate when the ratio of signal to interference power is constant.

VII. Simulation Results

In this section, the presented theoretical results are validated by a set of Monte-Carlo simulations, where we assume that $N_t = N_r = N_s = N$. Note that all curves are plotted as a function of the average SNR $\bar{\gamma}$.

Figure 2 demonstrates the analytical lower bound for the OP in (24) along with the low outage approximation in (30), where P_I is a constant (i.e. a scenario corresponding to high SNR as well). For the sake of completeness, we also consider the case of Rayleigh fading, where all m parameters are equal to 1 while $\varrho = 1$. As expected, the diversity order for Nakagami-m and Rayleigh fading channels is respectively equal to 2 and 1 (i.e. minimum of m_1 and m_2). As can be seen, the OP increases as the power of interference, P_I, increases. The proposed analytical lower bound yields excellent tightness across the entire SNR range and becomes exact at high SNRs. Likewise, the asymptotic outage approximation can very efficiently predict the exact OP.

Figure 3 illustrates the analytical lower bound for the OP, where P_S/P_I is kept constant, while $\varrho = 1$. We observe that by increasing the number of interferers, the OP increases too. As the SNR increases, the OP reaches an error floor since

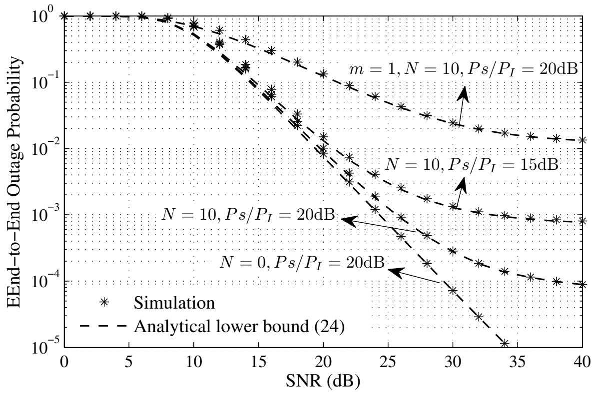

Fig. 3. Outage probability at S_1 (P_S/P_I is a constant) for $m_1 = m_t = 2, m_2 = m_s = 3, m_r = 2.5, \Omega_1 = \Omega_2 = 1, \Omega_s = \Omega_r = \Omega_t = 0.01, \gamma_0 = 3$.

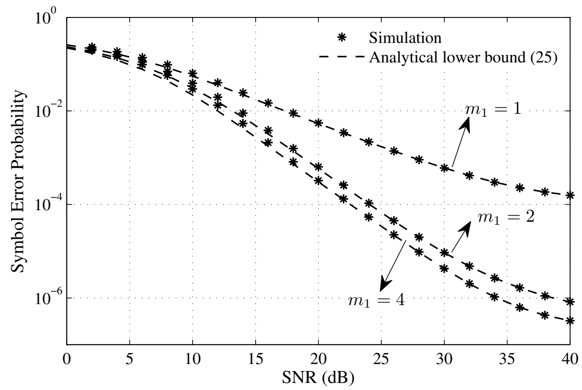

Fig. 4. Symbol error probability at S_1 for $m_2 = 2, m_s = m_r = m_t = 0.5, \Omega_1 = \Omega_2 = 1, \Omega_s = \Omega_r = \Omega_t = 0.01, N = 2, P_S/P_I = 20$dB.

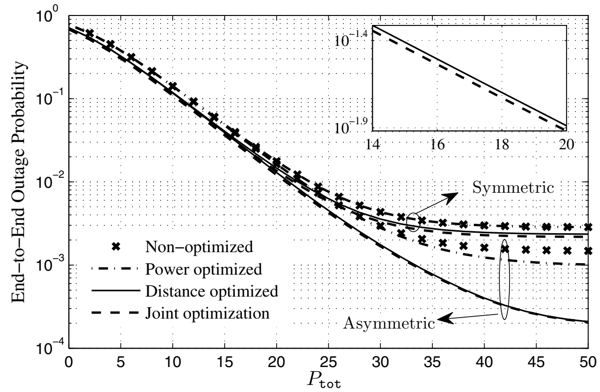

Fig. 5. Outage probability optimization at S_1 for $m_1 = m_2 = m_s = m_r = 1, N_s = N_r = 6, \gamma_0 = 3, \upsilon = 4$.

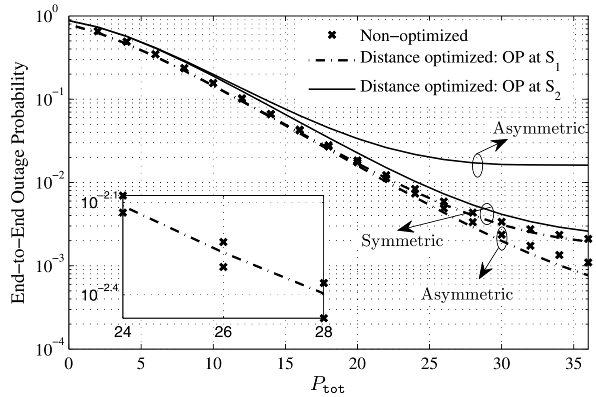

Fig. 6. Outage probability optimization at S_1 for $m_1 = m_2 = m_s = m_r = m_t = 1, N_s = N_r = 1, \gamma_0 = 3, \upsilon = 4$.

the effect of interference becomes dominant, while for the interference-free case the error floor does not occur. Note that the diversity order in the former case is equal to 0.

Figure 4 shows the analytical lower bound for the SEP for BPSK modulation ($c = d = 1$) and different values of the m_1 parameter where we have assumed that $\varrho = 1$. As observed, by increasing m_1 the SEP reduces systematically. Also, it can be seen that the relative distance between the curves reduces for higher values of the m_1 factor. This implies that the impact of m_1 becomes increasingly less pronounced. This is consistent with the results of [33]. Similar observations can be made when m_2 increases and m_1 is kept fixed. For example, $m_2 = 2m_1 = 2$ has the same SEP value as $m_1 = 2m_2 = 2$.

Figure 5 elaborates on the optimization scenarios outlined in Section V, where $\frac{P_S}{P_I}$ is a constant. Note that, in this case, all curves are plotted against $P_{\mathtt{tot}}$. Since the objective function is the OP at S_1, we have that $2P_{S_1} = 2P_{S_2} = 2P_S = P_R = P_{\mathtt{tot}}/2$ which means $\varrho = 2$. In the first case $P_S - P_{S_1j} = 30$ dB, $P_S - P_{Ri} = 0$ dB, $P_S - P_{S_2k} = 0$ dB, which represents an asymmetric interference power profile, while in the other case $P_S - P_{S_1j} = 15$ dB, $P_S - P_{Ri} = 15$ dB, $P_S - P_{S_2k} = 15$ dB, which represents a symmetric interference power profile. In the asymmetric case, the OP of relay position optimization is decreased by approximately 10 orders of magnitude compared to the non-optimized case.

Our results show that the asymmetric power case outperforms the symmetric case according to (62), Also, in the symmetric case, the power allocation optimization and non-optimized case ($d = 0.5, P_R = 2P_S = P_{\mathtt{tot}}/2$) have the same values. This means that performing optimization, leaves the OP unaffected. As a result, in the low SNR regime, if we want to optimize the OP at S_1, the best choice of powers and relay position are $d = 0.5, P_R = 2P_S = P_{\mathtt{tot}}/2$.

Figure 6 presents the outage optimization results at S_1, where the OP at S_2 is kept under a fixed threshold $\mathrm{OP}_{S_2} = \frac{10^3}{\bar{\gamma}}$. In this case, $P_{S_1} = P_{S_2} = P_S = P_R = \frac{P_{\mathtt{tot}}}{3}$ ($\varrho = 1$). The graph also depicts the following curves: the OP at S_2, which is obtained by substituting the optimal value of d into the constraint in (55), and the non-optimized OP at S_1 by setting $d = 0.5$ in the objective function in (55). In the first case $P_S - P_{S_1j} = 20$ dB, $P_S - P_{Ri} = 10$ dB, $P_S - P_{S_2k} = 0$ dB, which represents an asymmetric interference power profile, while in the other case $P_S - P_{S_1j} = P_S - P_{Ri} = P_S - P_{S_2k} = 10$ dB, which represents a symmetric interference power profile. In the former case, the OP at S_1 is smaller than in the symmetric case. For the symmetric profile, the optimized OP at S_1 has a marginal performance gain against the non-optimized one.

VIII. Conclusion

We assessed the performance of a dual-hop two-way AF relaying system over Nakagami-m fading channels, where all nodes are impaired by CCI. More specifically, we have derived new tight lower bounds for the OP and SEP of the system at arbitrary SINRs. Moreover, simplified asymptotic results in the low outage regime were deduced. For the case of Rayleigh fading, we examined three practical optimization problems, where we observed that relay position optimization yields substantial performance improvement. Finally, the distribution of the interferers' power can significantly affect the OP, since the asymmetric interference profile case yields lower OP than the symmetric profile case in the low outage regime. Note that all the presented expressions herein can be easily evaluated and efficiently programmed. We finally point out that the presented results complement and extend several previous results reported in the literature over the past years.

APPENDIX I: PROOF OF PROPOSITION 1

To begin with, the CDFs of X and Y are equal to

$$F_X(z) = \Pr(X \le z) = \Pr(\gamma_1 \le z\gamma_S | \gamma_S) = F_{\gamma_1}(z\gamma_S | \gamma_S)$$

$$\begin{aligned} F_Y(z) &= \Pr(Y \le z) \\ &= \Pr(\gamma_2 \le z(\gamma_S + \varrho\gamma_R) \mid \gamma_S, \gamma_R) \\ &= F_{\gamma_2}(z(\gamma_S + \varrho\gamma_R) \mid \gamma_S, \gamma_R). \end{aligned}$$

The CDFs of X and Y can be written in integral form as

$$F_X(z) = \int_0^\infty F_{\gamma_1}(z\gamma_S | \gamma_S) f_{\gamma_S}(\gamma_S) d\gamma_S \tag{66}$$

$$F_Y(z) = \int_0^\infty \int_0^\infty F_{\gamma_2}(z(\gamma_S + \varrho\gamma_R) | \gamma_S\gamma_R) \times f_{\gamma_S}(\gamma_S) f_{\gamma_R}(\gamma_R) d\gamma_S d\gamma_R. \tag{67}$$

Defining $\gamma_S = \gamma_s + 1$ and $\gamma_R = \gamma_r + 1$, substituting the PDFs of γ_s and γ_r and the CDF of γ_1 from (2) into (66), we get

$$F_X(z) = \frac{\alpha^{N_s m_s}}{\Gamma(N_s m_s)} \int_0^\infty \left[1 - \frac{\Gamma\left(m_1, \frac{az}{\bar{\gamma}}(x+1)\right)}{\Gamma(m_1)}\right] \frac{x^{N_s m_s - 1}}{e^{\alpha x}} dx. \tag{68}$$

Using [18, Eq. (8.352.2)] for integer m_1, (68) becomes

$$F_X(z) = 1 - \frac{\alpha^{N_s m_s}}{\Gamma(N_s m_s)} \sum_{i=0}^{m_1-1} \frac{(az)^i}{\bar{\gamma}^i i!} \times \int_0^\infty e^{-\frac{az}{\bar{\gamma}}(x+1)} (x+1)^i x^{N_s m_s - 1} e^{-\alpha x} dx. \tag{69}$$

Using the definition of binomial coefficients, (69) becomes

$$F_X(z) = 1 - \frac{\alpha^{N_s m_s}}{\Gamma(N_s m_s)} \sum_{i=0}^{m_1-1} \sum_{j=0}^{i} \frac{(az)^i}{\bar{\gamma}^i i!} \binom{i}{j} e^{-\frac{az}{\bar{\gamma}}} \times \int_0^\infty e^{-\left(\frac{az}{\bar{\gamma}} + \alpha\right)x} x^{j + N_s m_s - 1} dx. \tag{70}$$

Using [18, Eq. (17.13.3)], (70) can be written as in (15). Defining $\gamma_T = \gamma_t + 1$, (67) can be expanded, for integer m_2, as

$$F_Y(z) = 1 - \frac{\alpha^{N_s m_s}}{\Gamma(N_s m_s)} \frac{\beta^{N_r m_r}}{\Gamma(N_r m_r)} \sum_{i=0}^{m_2-1} \sum_{j=0}^{i} \frac{(bz)^i}{\bar{\gamma}^i i!} \binom{i}{j} \times \frac{\Gamma(j + N_s m_s)}{\left(\frac{bz}{\bar{\gamma}} + \alpha\right)^{j + N_s m_s}} \int_0^\infty \frac{(\varrho y + \varrho + 1)^{i-j}}{e^{\frac{bz}{\bar{\gamma}}(\varrho y + \varrho + 1)}} \frac{y^{N_r m_r - 1}}{e^{\beta y}} dy. \tag{71}$$

Once more, by using the definition of binomial coefficients and [18, Eq. (17.13.3)], (71) simplifies to (16).

APPENDIX II: PROOF OF PROPOSITION 2

The CDF of S_1, by considering $r = \frac{\gamma_1}{\gamma_s + 1}$ and $w = \frac{\gamma_2}{\gamma_s + \varrho\gamma_r + \varrho + 1}$, can be mathematically expressed as

$$\begin{aligned} F_{\gamma_{S_1}^{\rm up}}(z) &= \Pr(\varrho \min(r, w) < z) \\ &= 1 - \Pr(\varrho \min(r, w) > z) = 1 - \Pr(\varrho r > z, \varrho w > z). \end{aligned}$$

The above probability can be alternatively evaluated as

$$\begin{aligned} F_{\gamma_{S_1}^{\rm up}}(z) &= 1 - \mathbb{E}_{\gamma_s, \gamma_r}\left[\Pr\left(\varrho\gamma_1 > z(\gamma_s + 1) \middle| \gamma_s, \gamma_r\right) \times \Pr\left(\varrho\gamma_2 > z(\gamma_s + \varrho\gamma_r + \varrho + 1) \middle| \gamma_s, \gamma_r\right)\right] \\ &= 1 - \left(1 - \mathbb{E}_{\gamma_s, \gamma_r}\left[F_{\gamma_1}\left(\varrho^{-1} z(\gamma_s + 1)\right)\right]\right) \\ &\quad \times \left(1 - \mathbb{E}_{\gamma_s, \gamma_r}\left[F_{\gamma_2}\left(\varrho^{-1} z(\gamma_s + \varrho\gamma_r + \varrho + 1)\right)\right]\right). \end{aligned} \tag{72}$$

By assuming integer values for m_1 and m_2, we get

$$F_{\gamma_{S_1}^{\rm up}}(z) = 1 - \mathbb{E}_{\gamma_s, \gamma_r}\left[e^{-\frac{az}{\varrho\bar{\gamma}}(\gamma_s + 1) - \frac{bz}{\varrho\bar{\gamma}}(\gamma_s + \varrho\gamma_r + \varrho + 1)} \sum_{i=0}^{m_1-1} \sum_{j=0}^{m_2-1} \frac{(az)^i (bz)^j}{(\varrho\bar{\gamma})^{i+j} i! j!} (\gamma_s + 1)^i (\gamma_s + \varrho\gamma_r + \varrho + 1)^j\right]. \tag{73}$$

Using the definition of binomial coefficients, (73) becomes

$$F_{\gamma_{S_1}^{\rm up}}(z) = 1 - \mathbb{E}_{\gamma_s, \gamma_r}\left[e^{-\frac{az}{\varrho\bar{\gamma}}(\gamma_s + 1) - \frac{bz}{\varrho\bar{\gamma}}(\gamma_s + \varrho\gamma_r + \varrho + 1)} \sum_{i=0}^{m_1-1} \sum_{j=0}^{m_2-1} \sum_{l=0}^{i} \sum_{k=0}^{j} \sum_{t=0}^{j-k} \binom{i}{l} \binom{j}{k} \binom{j-k}{t} \times \frac{(az)^i (bz)^j}{(\varrho\bar{\gamma})^{i+j} i! j!} \gamma_s^{l+k} \varrho^t \gamma_r^t (\varrho + 1)^{j-k-t}\right]. \tag{74}$$

By integrating (74) over γ_s and γ_r, (17) is derived. The proof for $F_{\gamma_{S_2}^{\rm up}}(z)$ follows a similar line of reasoning.

APPENDIX III: PROOF OF PROPOSITION 3

Since X and Y are dependent to each other, we utilize the following methodology,

$$\begin{aligned} F_{\gamma_{e2e}^{\mathrm{up}}}(z) &= 1-\Pr\left(\min\left(\gamma_{S_1}^{\mathrm{up}},\gamma_{S_2}^{\mathrm{up}}\right)\geq z\right)\\ &= 1-\Pr\left(\gamma_{S_1}^{\mathrm{up}}\geq z,\gamma_{S_2}^{\mathrm{up}}\geq z\right)\\ &= 1-\Pr\left(\min\left(\frac{\varrho\gamma_1}{\gamma_s+1},\frac{\varrho\gamma_2}{\gamma_s+\varrho\gamma_r+\varrho+1}\right)\geq z,\right.\\ &\qquad\left.\min\left(\frac{\varrho\gamma_2}{\gamma_t+1},\frac{\varrho\gamma_1}{\gamma_t+\varrho\gamma_r+\varrho+1}\right)\geq z\right)\\ &= 1-\Pr\left(\frac{\varrho\gamma_1}{\gamma_s+1}\geq z,\frac{\varrho\gamma_2}{\gamma_s+\varrho\gamma_r+\varrho+1}\geq z,\right.\\ &\qquad\left.\frac{\varrho\gamma_2}{\gamma_t+1}\geq z,\frac{\varrho\gamma_1}{\gamma_t+\varrho\gamma_r+\varrho+1}\geq z\right)\\ &= 1-\mathbb{E}_{\gamma_s,\gamma_t,\gamma_r}\left[\Pr\left(\varrho\gamma_1\geq z(\gamma_s+1),\right.\right.\\ &\qquad\left.\left.\varrho\gamma_1\geq z(\gamma_t+\varrho\gamma_r+\varrho+1)\middle|\gamma_s,\gamma_t,\gamma_r\right)\right]\\ &\quad\times\mathbb{E}_{\gamma_s,\gamma_t,\gamma_r}\left[\Pr\left(\varrho\gamma_2\geq z(\gamma_s+\varrho\gamma_r+\varrho+1),\right.\right.\\ &\qquad\left.\left.\varrho\gamma_2\geq z(\gamma_t+1)\middle|\gamma_s,\gamma_t,\gamma_r\right)\right]\\ &= 1-\mathcal{P}_1(z)\mathcal{P}_2(z) \end{aligned} \tag{75}$$

where $\mathcal{P}_1(z)$ and $\mathcal{P}_2(z)$ can be expressed as

$$\begin{aligned} \mathcal{P}_1(z) &= \mathbb{E}_{\gamma_s,\gamma_t,\gamma_r}\left[\Pr\left(\varrho\gamma_1\geq z(\gamma_s+1),\right.\right.\\ &\qquad\left.\left.\gamma_s+1\geq\gamma_t+\varrho\gamma_r+\varrho+1\middle|\gamma_s,\gamma_t,\gamma_r\right)\right]\\ &\quad+\mathbb{E}_{\gamma_s,\gamma_t,\gamma_r}\left[\Pr\left(\varrho\gamma_1\geq z(\gamma_t+\varrho\gamma_r+\varrho+1),\right.\right.\\ &\qquad\left.\left.\gamma_t+\gamma_r+\varrho+1\geq\gamma_s+1\middle|\gamma_s,\gamma_t,\gamma_r\right)\right] \end{aligned}$$

$$\begin{aligned} \mathcal{P}_2(z) &= \mathbb{E}_{\gamma_s,\gamma_t,\gamma_r}\left[\Pr\left(\varrho\gamma_2\geq z(\gamma_s+\varrho\gamma_r+\varrho+1),\right.\right.\\ &\qquad\left.\left.\gamma_s+\varrho\gamma_r+\varrho+1\geq\gamma_t+1\middle|\gamma_s,\gamma_t,\gamma_r\right)\right]\\ &\quad+\mathbb{E}_{\gamma_s,\gamma_t,\gamma_r}\left[\Pr\left(\varrho\gamma_2\geq z(\gamma_t+1),\right.\right.\\ &\qquad\left.\left.\gamma_t+1\geq\gamma_s+\varrho\gamma_r+\varrho+1\middle|\gamma_s,\gamma_t,\gamma_r\right)\right]. \end{aligned}$$

Now, $\mathcal{P}_1(z)$ can be written in integral form according to

$$\begin{aligned} \mathcal{P}_1(z) &= \mathbb{E}_{\gamma_t,\gamma_r}\left[\int_{\gamma_t+\varrho\gamma_r+\varrho}^{\infty}\int_{\varrho^{-1}z(y+1)}^{\infty}f_{\gamma_1}(x)f_{\gamma_s}(y)\,dxdy\right]\\ &\quad+\mathbb{E}_{\gamma_t,\gamma_r}\left[\int_{0}^{\gamma_t+\varrho\gamma_r+\varrho}\int_{\varrho^{-1}z(\gamma_t+\varrho\gamma_r+\varrho+1)}^{\infty}f_{\gamma_1}(x)f_{\gamma_s}(y)\,dxdy\right]. \end{aligned} \tag{76}$$

Utilizing [18, Eq. (2.33.10)] and [18, Eq. (8.350.4)], the first term in (76) can be obtained as

$$\begin{aligned} \mathcal{P}_{11}(z) &= \mathbb{E}_{\gamma_t,\gamma_r}\left[\frac{\alpha^{m_sN_s}}{\Gamma(m_sN_s)}\frac{a^{m_1}}{\bar{\gamma}^{m_1}\Gamma(m_1)}\right.\\ &\qquad\left.\int_{\gamma_t+\varrho\gamma_r+\varrho}^{\infty}\int_{\varrho^{-1}z(y+1)}^{\infty}x^{m_1-1}e^{-\frac{ax}{\bar{\gamma}}}dxy^{m_sN_s-1}e^{-\alpha y}dy\right]\\ &= \mathbb{E}_{\gamma_t,\gamma_r}\left[\frac{\alpha^{m_sN_s}}{\Gamma(m_sN_s)}\right.\\ &\qquad\left.\int_{\gamma_t+\varrho\gamma_r+\varrho}^{\infty}\frac{\Gamma\left(m_1,\frac{az}{\varrho\bar{\gamma}}(y+1)\right)}{\Gamma(m_1)}y^{m_sN_s-1}e^{-\alpha y}dy\right]. \end{aligned}$$

For integer values of m_1, the above expression becomes

$$\begin{aligned} \mathcal{P}_{11}(z) &= \mathbb{E}_{\gamma_t,\gamma_r}\left[\frac{\alpha^{m_sN_s}}{\Gamma(m_sN_s)}\sum_{l=0}^{m_1-1}\frac{a^lz^l}{\varrho^l\bar{\gamma}^l l!}\right.\\ &\qquad\left.\int_{\gamma_t+\varrho\gamma_r+\varrho}^{\infty}(y+1)^le^{-\frac{az}{\varrho\bar{\gamma}}(y+1)}y^{m_sN_s-1}e^{-\alpha y}dy\right]\\ &= \mathbb{E}_{\gamma_t,\gamma_r}\left[\frac{\alpha^{m_sN_s}e^{-\frac{az}{\varrho\bar{\gamma}}}}{\Gamma(m_sN_s)}\sum_{l=0}^{m_1-1}\sum_{r=0}^{l}\binom{l}{r}\frac{a^lz^l}{\varrho^l\bar{\gamma}^l l!}\right.\\ &\qquad\left.\int_{\gamma_t+\gamma_r+1}^{\infty}e^{-\left(\frac{az}{\varrho\bar{\gamma}}+\alpha\right)y}y^{m_sN_s+r-1}dy\right] \end{aligned} \tag{77}$$

where the second equality is obtained by using the definition of binomial coefficients. By solving the integral in (77) utilizing [18, Eq. (2.33.10)], and assuming integer values for m_sN_s and applying [18, Eq. (8.352.2)]), we have

$$\begin{aligned} \mathcal{P}_{11}(z) &= \mathbb{E}_{\gamma_t,\gamma_r}\left[\frac{\alpha^{m_sN_s}e^{-\frac{az}{\varrho\bar{\gamma}}}}{\Gamma(m_sN_s)}\sum_{l=0}^{m_1-1}\sum_{r=0}^{l}\binom{l}{r}\frac{a^lz^l}{\varrho^l\bar{\gamma}^l l!}\right.\\ &\qquad\left.\frac{\Gamma\left(m_sN_s+r,\left(\alpha+\frac{az}{\varrho\bar{\gamma}}\right)(\gamma_t+\varrho\gamma_r+\varrho)\right)}{\left(\alpha+\frac{az}{\varrho\bar{\gamma}}\right)^{m_sN_s+r}}\right]\\ &= \mathbb{E}_{\gamma_t,\gamma_r}\left[\sum_{l=0}^{m_1-1}\sum_{r=0}^{l}\sum_{i=0}^{m_sN_s+r-1}\frac{1}{i!}\binom{l}{r}\frac{a^lz^l}{\varrho^l\bar{\gamma}^l l!}\frac{\alpha^{m_sN_s}e^{-\frac{az}{\varrho\bar{\gamma}}}}{\Gamma(m_sN_s)}\right.\\ &\qquad\left.\frac{(\gamma_t+\varrho\gamma_r+\varrho)^i\Gamma(m_sN_s+r)}{\left(\alpha+\frac{az}{\varrho\bar{\gamma}}\right)^{m_sN_s+r-i}}e^{-\left(\alpha+\frac{az}{\varrho\bar{\gamma}}\right)(\gamma_t+\varrho\gamma_r+\varrho)}\right]. \end{aligned} \tag{78}$$

By using [34, Eq. (2.1.3.2)], (78) can be written as

$$\mathcal{P}_{11}(z) = \mathbb{E}_{\gamma_t}\left[\frac{\beta^{m_r N_r}}{\Gamma(m_r N_r)} \sum_{l=0}^{m_1-1} \sum_{r=0}^{l} \sum_{i=0}^{m_s N_s + r - 1} \sum_{j=0}^{i} \binom{i}{j} \binom{l}{r} \frac{a^l z^l}{\varrho^{l-j} \bar{\gamma}^l l! i!} \frac{\alpha^{m_s N_s} e^{-\frac{az}{\varrho\bar{\gamma}} - \left(\alpha + \frac{az}{\varrho\bar{\gamma}}\right)(\gamma_t + \varrho)}}{\Gamma(m_s N_s)} \Gamma(m_s N_s + r) \times \frac{(\gamma_t + \varrho)^{i-j}}{\left(\alpha + \frac{az}{\varrho\bar{\gamma}}\right)^{m_s N_s + r - i}} \int_0^\infty e^{-\left(\alpha\varrho + \frac{az}{\bar{\gamma}} + \beta\right)x} x^{m_r N_r + j - 1} dx\right].$$

The above expression admits the following manipulations

$$\mathcal{P}_{11}(z) = \mathbb{E}_{\gamma_t}\left[\frac{\beta^{m_r N_r}}{\Gamma(m_r N_r)} \sum_{l=0}^{m_1-1} \sum_{r=0}^{l} \sum_{i=0}^{m_s N_s + r - 1} \sum_{j=0}^{i} \binom{i}{j} \binom{l}{r} \times \frac{a^l z^l}{\varrho^{l-j} \bar{\gamma}^l l! i!} \frac{\alpha^{m_s N_s} e^{-\frac{az}{\varrho\bar{\gamma}} - \left(\alpha + \frac{az}{\varrho\bar{\gamma}}\right)(\gamma_t + \varrho)}}{\Gamma(m_s N_s)} \frac{(\gamma_t + \varrho)^{i-j}}{\left(\alpha + \frac{az}{\varrho\bar{\gamma}}\right)^{m_s N_s + r - i}} \times \frac{\Gamma(m_r N_r + j)}{\left(\varrho\alpha + \frac{az}{\bar{\gamma}} + \beta\right)^{m_r N_r + j}}\right] = \frac{\beta^{m_r N_r}}{\Gamma(m_r N_r)} \frac{\eta^{m_t N_t}}{\Gamma(m_t N_t)} \frac{\alpha^{m_s N_s}}{\Gamma(m_s N_s)} \times e^{-\frac{az}{\varrho\bar{\gamma}} - \left(\varrho\alpha + \frac{az}{\bar{\gamma}}\right)} \sum_{l=0}^{m_1-1} \sum_{r=0}^{l} \sum_{i=0}^{m_s N_s + r - 1} \sum_{j=0}^{i} \sum_{k=0}^{i-j} \binom{l}{r} \binom{i}{j} \times \binom{i-j}{k} \frac{\Gamma(m_s N_s + r)}{\left(\alpha + \frac{az}{\varrho\bar{\gamma}}\right)^{m_s N_s + r - i}} \frac{\Gamma(m_r N_r + j)}{\left(\varrho\alpha + \frac{az}{\bar{\gamma}} + \beta\right)^{m_r N_r + j}} \times \frac{a^l z^l}{\varrho^{l+k-i} \bar{\gamma}^l l! i!} \int_0^\infty e^{-\left(\alpha + \frac{az}{\varrho\bar{\gamma}} + \eta\right)y} y^{m_t N_t + k - 1} dy. \quad (79)$$

By integrating over γ_t in (79) using [18, Eq. (3.351.3)], we arrive to $\mathcal{P}_{11}(z)$ in (19). Likewise, $\mathcal{P}_{12}(z)$, $\mathcal{P}_{21}(z)$ and $\mathcal{P}_{22}(z)$ can be derived for integer m_2 and $m_t N_t$.

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