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Cooperative spectrum sharing systems with relay selection in the presence of multiple primary receivers

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Abstract: In this study, the outage performance of a multi-relay cooperative network operating in a spectrum sharing environment is analysed. Focusing on the secondary communication process, a relay selection procedure is performed prior to the communication process, where the selected relay is the one maximising the instantaneous end-to-end signal-to-noise ratio (SNR) and, at the same time, satisfying the power restrictions imposed by the primary receivers. The secondary destination employs a selection combining strategy, selecting the best path (direct link or relaying link) with the highest instantaneous SNR. Two kinds of relaying protocols are considered: decode-and-forward and amplify-and-forward. For the former, an exact closed-form expression for the outage probability (OP) is derived, whereas for the latter an accurate approximation for the OP is obtained. Then, an asymptotic analysis is carried out and reveals that the system diversity order is $N+1$ for both relaying protocols, with N denoting the number of relays. Monte Carlo simulations assess the accuracy of the authors' results.

1 Introduction

Cooperative diversity (CD) [1, 2] is nowadays a well-established technique to enhance the performance of wireless systems by emulating a virtual multiple-input multiple-output system utilising solely single-antenna devices. On the other hand, cognitive radio (CR) [3] is a popular approach to improve the radio spectrum access and efficiency [4] by allowing secondary users (SUs) to access spectrum portions initially allocated to a primary user (PU); as long as defined power and/or interference constraints are satisfied. Owing to the performance gains obtained with the use of CD and CR concepts, several works have investigated the combination of both techniques.

In [5], assuming Nakagami-m fading and the absence of a direct link between the SU source and SU destination, the authors investigated the performance of a dual-hop amplify-and-forward (AF) cognitive relay network (CRN) in a spectrum sharing environment with only one PU. A similar cooperative relay network [6] analysed the outage performance of decode-and-forward (DF) relaying protocol, in which the 'saturation' phenomenon of the outage probability (OP) was observed. In [7], considering a multi-relay CRN with no direct link, the authors performed a suitable relay selection for the underlay spectrum access approach. Assuming the presence of the direct link and employing selection combining (SC) at the SU destination, Duong *et al.* [8] derived an expression for the OP of CRNs with one AF relay. A dual-hop CRN with multiple AF relays subject to Nakagami-m fading was examined in [9]. This work was extended later in [10], where the authors considered DF with the presence of the direct link. Finally, the outage performance of DF CRNs was studied in [11] with the presence of multiple relays, but the authors did not consider channel state information (CSI) between the source and relays during the relay selection procedure.

In this paper, we study the outage behaviour of multi-relay cooperative networks in a more general spectrum sharing setup with the presence of multiple primary receivers. The SU network consists of one secondary source, N secondary DF or AF relays and one secondary destination. We assume that the direct link between the source and destination is available. Each primary receiver imposes a power restriction on the SU nodes; their transmit powers are hence limited by the PUs' interference thresholds and their own peak transmit powers. Focusing on the SU communication process, a relay selection is first performed before the beginning of the communication, where the selected relay is the one which maximises the instantaneous end-to-end signal-to-noise ratio (SNR) while satisfying the power restrictions imposed by the PU receivers. An SC is performed at the SU destination to select the best path between the direct and the relaying links.

We derive an exact closed-form expression for the OP of DF relaying protocol, and an accurate approximation of the OP when AF relaying is adopted. Then, we carry out an asymptotical analysis of the obtained expressions which reveals that the system diversity order is $N+1$ for both relaying protocols. Finally, our results are validated by means of Monte Carlo simulations in different illustrative scenarios.

Throughout this paper, $f_Z(\cdot)$ and $F_Z(\cdot)$ denote, respectively, the probability density function and the cumulative distribution function (CDF) of an arbitrary random variable (RV) Z, and $\mathbb{E}[\cdot]$ stands for mathematical expectation.

2 System model

We consider a cooperative network operating in a spectrum sharing environment composed by one SU source S, N SU relays R_n ($n = 1, \ldots, N$), one SU destination D and M PU receivers P_m ($m = 1, \ldots, M$), as shown in Fig. 1. We assume that the PU transmitters are located far enough from the SU nodes that they cause no interference with the SU communication. All nodes are single-antenna devices and they operate in a half-duplex mode. The direct link from SU source to SU destination is available and all channel coefficients, h_{ij}, between two arbitrary nodes i and j, experience Rayleigh block fading, that is, the channel remains constant during a symbol block and changes independently between blocks. The noise terms are additive white Gaussian noise signals with mean power N_0.

Owing to the spectrum sharing environment, the secondary network can access the spectrum initially allocated to the PU for transmitting its information. However, to avoid interference with the primary communication, SU nodes adjust their respective transmit powers according to the interference constraint Q_m imposed by each PU as well as by their own peak transmit power. More specifically, the transmit powers of the SU source and SU relays can be written, respectively, as [12]

$$W_S \leq \min\left(\min_m \frac{Q_m}{|h_{SP_m}|^2}, \quad \overline{W}_S\right)$$
$$W_{R_n} \leq \min\left(\min_m \frac{Q_m}{|h_{R_nP_m}|^2}, \quad \overline{W}_{R_n}\right), \quad n = 1, \ldots, N$$

where $\overline{W}_S$ and $\overline{W}_{R_n}$ are the maximum transmit powers of the SU source and SU relays, respectively [Herein, we assume that the SU source and each SU relay have perfect CSI of the channel to the PU. In practice, this can be implemented in several ways. One possibility is through a direct feedback from the PU or through an indirect feedback by a band manager, which mediates the exchange of information between the primary and secondary networks. Another strategy would be to have the relay periodically sense the transmitted signal from the PU, provided that time-division duplex is employed during the primary transmission.].

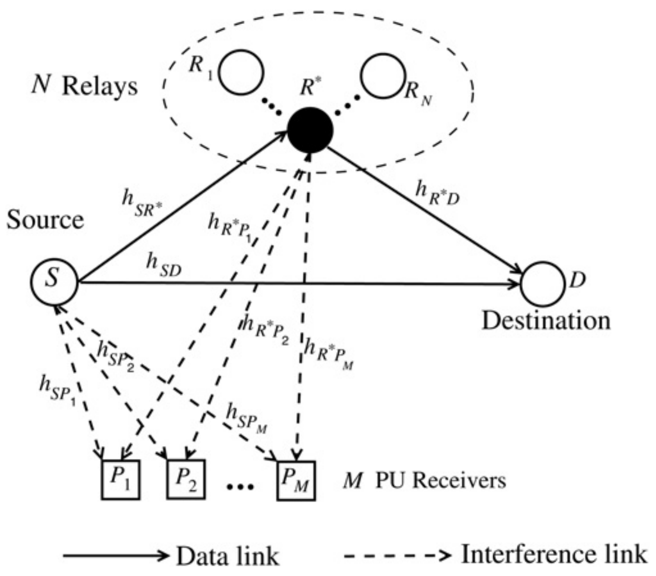

Fig. 1 *System model*

The SU communication comprises of two phases. Before the communication starts, a relay selection process is performed in such a way that the chosen relay R^* maximises the dual-hop instantaneous SNR and, at the same time, satisfies the transmit power requirements imposed by each PU receiver. Here, we focus our analysis on two relaying protocols: DF and variable-gain AF.

For the DF protocol, in phase I, the source broadcasts its information to the 'best relay' R^* and the destination D with transmit power W_S where, in this case, the best relay is chosen according to

$$R^* = \arg\max_n \left[\min\left[\gamma_{SR_n}, \ \gamma_{R_nD}\right]\right] \tag{1}$$

where γ_{SR_n} and γ_{R_nD} denote the instantaneous source–relay and relay–destination SNR, respectively, which are given by

$$\gamma_{SR_n} = W_S|h_{SR_n}|^2/N_0 \text{ and } \gamma_{R_nD} = W_{R_n}|h_{R_nD}|^2/N_0 \tag{2}$$

In phase II, assuming that the chosen relay can always fully decode the signal received from the source, R^* decodes the information received from the source, re-encodes it and forwards it to the destination with an average transmission power W_{R_n}. With SC at the SU destination, the end-to-end SNR for the DF case can be written as

$$\gamma_{\text{e2e}}^{\text{DF}} = \max\left[\gamma_{SD}, \ \max_n\left[\min\left[\gamma_{SR_n}, \ \gamma_{R_nD}\right]\right]\right] \tag{3}$$

where $\gamma_{SD} = W_S|h_{SD}|^2/N_0$ is the instantaneous SNR over the direct link.

For the AF relaying protocol, the phase I is similar to the DF case, that is, the source transmits its signal to both D and R^*, where the selection procedure is here given by

$$R^* = \arg\max_n \left[\frac{\gamma_{SR_n}\gamma_{R_nD}}{1 + \gamma_{SR_n} + \gamma_{R_nD}}\right] \tag{4}$$

Note that, by considering a variable-gain AF, the relay amplifies the received signal (and also the noise) by a factor determined by the instantaneous CSI of the source–relay link. In phase II, the relay forwards the signal to the SU destination. In this case, the end-to-end SNR after SC can be written as

$$\gamma_{\text{e2e}}^{\text{AF}} = \max\left[\gamma_{SD}, \ \max_n\left[\frac{\gamma_{SR_n}\gamma_{R_nD}}{1 + \gamma_{SR_n} + \gamma_{R_nD}}\right]\right] \tag{5}$$

3 OP analysis

3.1 DF relaying

The OP can be defined as the probability that the instantaneous end-to-end SNR γ_{e2e} falls below a given threshold, γ_{th}, that is, $P_{\text{out}} = \Pr(\gamma_{\text{e2e}} < \gamma_{\text{th}})$. Next, for the sake of simplicity, we assume that $Q_m = Q$, $m = 1, \ldots, M$. Noting that the terms inside the maximisation operator in (3) are not statistically independent because of the common RV $|h_{SP_m}|^2$. To deal with this dependence, we first write the conditional OP [8] as

$$Pr\left(\gamma_{\text{e2e}}^{\text{DF}} < \gamma_{\text{th}}|X\right) = \underbrace{Pr\left(\gamma_{SD} < \gamma_{\text{th}}|X\right)}_{\xi} \cdot \underbrace{Pr\left(\max_n\left[\min\left[\gamma_{SR_n}, \ \gamma_{R_nD}\right]\right] < \gamma_{\text{th}}|X\right)}_{\theta} \tag{6}$$

where $X = \max_m |h_{SP_m}|^2$. Thus, knowing that $Q_m = Q$, it follows that $\min_m\left(Q_m/|h_{TP_m}|^2\right)$ is equivalent to $Q/\left(\max_m |h_{TP_m}|^2\right)$, for $T \in \{S, R_1, \ldots, R_N\}$.

Considering the exponentially distributed instantaneous SNRs over the Rayleigh fading links, ξ in (6) is given by

$$\xi = 1 - \mathrm{e}^{-\gamma_{\text{th}}\,\overline{\gamma}_{SD}} \tag{7}$$

where $\overline{\gamma}_{SD} \triangleq 1/\mathbb{E}\left[\gamma_{SD}\right]$, and using the CDF of the minimum of two RVs [13], θ in (6) can be written as (see (8))

$$\theta = \prod_{n=1}^{N}\left[\left(1 - \mathrm{e}^{-\gamma_{\text{th}}\,\overline{\gamma}_{SR_n}}\right) + \left(1 - \mathrm{e}^{-\gamma_{\text{th}}\,\overline{\gamma}_{R_nD}}\right) - \left(1 - \mathrm{e}^{-\gamma_{\text{th}}\,\overline{\gamma}_{SR_n}}\right) \cdot \left(1 - \mathrm{e}^{-\gamma_{\text{th}}\,\overline{\gamma}_{R_nD}}\right)\right] \tag{8}$$

where $\overline{\gamma}_{SR_n} \triangleq 1/\mathbb{E}\left[\gamma_{SR_n}\right]$ and $\overline{\gamma}_{R_nD} \triangleq 1/\mathbb{E}\left[\gamma_{R_nD}\right]$.

Finally, the end-to-end OP can be formulated as

$$P_{\text{out}}^{\text{DF}} = \int_0^{\infty}\int_0^{\infty} F_{\gamma_{SD}}\left(\gamma_{\text{th}}|X\right) \cdot F_{\gamma_{SR^*D}}\left(\gamma_{\text{th}}|X, \ Y\right) \cdot f_X(x) \cdot f_Y(y)\mathrm{d}x\mathrm{d}y \tag{9}$$

where

$$\gamma_{SR^*D} = \max_n\left[\min\left[\gamma_{SR_n}, \ \gamma_{R_nD}\right]\right], \quad Y = \max_m |h_{R_nP_m}|^2$$

and

$$f_X(x) = M\,\overline{\gamma}_{SP}\ \mathrm{e}^{-x\,\overline{\gamma}_{SP}}\left(1 - \mathrm{e}^{-x\,\overline{\gamma}_{SP}}\right)^{M-1}$$
$$f_Y(y) = M\,\overline{\gamma}_{R_nP}\ \mathrm{e}^{-y\,\overline{\gamma}_{R_nP}}\left(1 - \mathrm{e}^{-y\,\overline{\gamma}_{R_nP}}\right)^{M-1} \tag{10}$$

with $\overline{\gamma}_{TP} \triangleq 1/\mathbb{E}\left[|h_{TP_m}|^2\right]$, for $T \in \{S, R_1, \ldots, R_N\}$.

It is important to note that

$$\min\left(\frac{Q}{\Lambda}, \ W\right) = \begin{cases} W, & \text{if } \Lambda \le Q/W \\ Q/\Lambda, & \text{if } \Lambda > Q/W \end{cases} \tag{11}$$

To simplify the derivation of (9), we subdivide the OP expression into a sum of four terms, that is, $P_{\text{out}}^{\text{DF}} = \varphi_1 + \varphi_2 + \varphi_3 + \varphi_4$, where

$$\varphi_1 = \int_0^{Q/W}\int_0^{Q/W} F_{\gamma_{SD}}\left(\gamma_{\text{th}}|X\right) \cdot F_{\gamma_{SR^*D}}\left(\gamma_{\text{th}}|X, \ Y\right) \cdot f_X(x) \cdot f_Y(y)\mathrm{d}x\mathrm{d}y \tag{12}$$

$$\varphi_2 = \int_0^{Q/W}\int_{Q/W}^{\infty} F_{\gamma_{SD}}\left(\gamma_{\text{th}}|X\right) \cdot F_{\gamma_{SR^*D}}\left(\gamma_{\text{th}}|X, \ Y\right) \cdot f_X(x) \cdot f_Y(y)\mathrm{d}x\mathrm{d}y \tag{13}$$

$$\varphi_3 = \int_{Q/W}^{\infty}\int_0^{Q/W} F_{\gamma_{SD}}\left(\gamma_{\text{th}}|X\right) \cdot F_{\gamma_{SR^*D}}\left(\gamma_{\text{th}}|X, \ Y\right) \cdot f_X(x) \cdot f_Y(y)\mathrm{d}x\mathrm{d}y \tag{14}$$

$$\varphi_4 = \int_{Q/W}^{\infty}\int_{Q/W}^{\infty} F_{\gamma_{SD}}\left(\gamma_{\text{th}}|X\right) \cdot F_{\gamma_{SR^*D}}\left(\gamma_{\text{th}}|X, \ Y\right) \cdot f_X(x) \cdot f_Y(y)\mathrm{d}x\mathrm{d}y \tag{15}$$

where we assume, without loss of generality, that $W_S = W_{R_n} = W$.

The first term φ_1 can be calculated as

$$\varphi_1 = \int_0^{Q/W}\int_0^{Q/W}\prod_{n=1}^{N}\left[\left(1 - \mathrm{e}^{-\gamma_{\text{th}}\overline{\gamma}_{SR_n}^{W}}\right) + \left(1 - \mathrm{e}^{-\gamma_{\text{th}}\overline{\gamma}_{R_nD}^{W}}\right) - \left(1 - \mathrm{e}^{-\gamma_{\text{th}}\overline{\gamma}_{SR_n}^{W}}\right) \cdot \left(1 - \mathrm{e}^{-\gamma_{\text{th}}\overline{\gamma}_{R_nD}^{W}}\right)\right] \times \left(1 - \mathrm{e}^{-\gamma_{\text{th}}\,\overline{\gamma}_{SD}^{W}}\right) \cdot f_X(x) \cdot f_Y(y)\mathrm{d}x\mathrm{d}y \tag{16}$$

$$= \left(1 - \mathrm{e}^{-\gamma_{\text{th}}\,\overline{\gamma}_{SD}^{W}}\right)\prod_{n=1}^{N}\left(1 - \mathrm{e}^{-\gamma_{\text{th}}\left(\overline{\gamma}_{SR_n}^{W} + \overline{\gamma}_{R_nD}^{W}\right)}\right) \times M^2\sum_{m=0}^{M-1}\sum_{t=0}^{M-1}(-1)^m(-1)^t\binom{M-1}{m}\binom{M-1}{t} \times \frac{1 - \mathrm{e}^{\left(-(Q/W)\,\overline{\gamma}_{SP}\,(m+1)\right)}}{\overline{\gamma}_{SP}\,(m+1)} \cdot \frac{1 - \mathrm{e}^{\left(-(Q/W)\,\overline{\gamma}_{R_nP}\,(t+1)\right)}}{\overline{\gamma}_{R_nP}\,(t+1)} \tag{17}$$

where

$$\overline{\gamma}_{IJ}^{W} \triangleq \frac{1}{\mathbb{E}\left[W|h_{IJ}|^2/N_0\right]}$$

with $I \in \{S, R_n\}$ and $J \in \{R_n, D\}$.

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Following the same procedure, and making the appropriate substitutions in (13), we have

$$\varphi_2 = \int_0^{Q/W}\int_{Q/W}^{\infty}\left(1-\mathrm{e}^{-\gamma_{\mathrm{th}}\overline{\gamma}_{SD}^{W}}\right)\prod_{n=1}^{N}\left(1-\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\overline{\gamma}_{SR_n}^{W}+y\overline{\gamma}_{R_nD}^{Q}\right)}\right) \times M\left(1-\mathrm{e}^{-x\overline{\gamma}_{SP}}\right)^{M-1}\overline{\gamma}_{\mathrm{SP}}\ \mathrm{e}^{-x\overline{\gamma}_{SP}} \times M\left(1-\mathrm{e}^{-y\overline{\gamma}_{R_nP}}\right)^{M-1}\overline{\gamma}_{R_nP}\ \mathrm{e}^{-y\overline{\gamma}_{R_nP}}\mathrm{d}x\mathrm{d}y \tag{18}$$

where

$$y\overline{\gamma}_{IJ}^{Q} \triangleq 1/\mathbb{E}\left[(Q/y)\left(|h_{IJ}|^2/N_0\right)\right]$$

with $I \in \{S, R_n\}$ and $J \in \{R_n, D\}$. To get a closed-form of this integral, we make use of the following identity

$$\prod_{k=1}^{K}(1-x_k) = \sum_{k=0}^{K}\frac{(-1)^k}{k!}\sum_{\substack{n_1=\cdots=n_k\\ n_1<\cdots<n_k}}^{K}\cdots\sum^{K}\prod_{t=1}^{k}x_{n_t} \tag{19}$$

yielding (see (20))

As in [14], we assume that all SU relay nodes are clustered together and can be selected by a long-term routing process, implying that all links from S to R_n have the same average SNR, that is, $\overline{\gamma}_{SR_n} = \overline{\gamma}_{SR}$, for $n = 1,\ldots, N$. Similarly, we also assume that the links from R_n to D undergo independent and identically distributed Rayleigh fading with the same average SNR. Therefore (20) can be expressed as

$$\varphi_2 = M^2\sum_{m=0}^{M-1}\sum_{t=0}^{M-1}\sum_{n=0}^{N}(-1)^{m+t+n}\binom{M-1}{m}\binom{M-1}{t}\binom{N}{n}\overline{\gamma}_{R_nP} \left(1-\mathrm{e}^{-\gamma_{\mathrm{th}}\overline{\gamma}_{SD}^{W}}\right)\frac{\left(1-\mathrm{e}^{(-(Q/W)\overline{\gamma}_{SP}(m+1))}\right)}{\overline{\gamma}_{SP}(m+1)} \times\frac{\exp\left(-\gamma_{\mathrm{th}}n\left(\overline{\gamma}_{SR_n}^{W}+(Q/W)\overline{\gamma}_{R_nD}^{Q}\right)-(Q/W)\overline{\gamma}_{R_nP}(t+1)\right)}{\gamma_{\mathrm{th}}n\overline{\gamma}_{R_nD}^{Q}+\overline{\gamma}_{R_nP}(t+1)} \tag{21}$$

Again, following the same steps described above, φ_3 and φ_4 in (14) and (15), respectively, can be straightforwardly derived as

$$\varphi_3 = \sum_{l=0}^{1}\sum_{m=0}^{M-1}\sum_{t=0}^{M-1}\sum_{n=0}^{N}M^2(-1)^{(m+t+n+l)}\binom{M-1}{m}\binom{M-1}{t} \times\binom{N}{n}\overline{\gamma}_{R_nP}\frac{\left(1-\mathrm{e}^{\left(-(Q/W)\overline{a}_{R_nP}(t+1)\right)}\right)}{\overline{\gamma}_{R_nP}(t+1)} \times\exp\left(-l\frac{Q}{W}\gamma_{\mathrm{th}}\overline{\gamma}_{SD}^{Q}\right)\overline{\gamma}_{SP} \frac{\exp\left(-n\gamma_{\mathrm{th}}\left((Q/W)\overline{\gamma}_{SR_n}^{Q}+\overline{\gamma}_{R_nD}^{W}\right)-(Q/W)\overline{\gamma}_{SP}(m+1)\right)}{l\gamma_{\mathrm{th}}\overline{\gamma}_{SD}^{Q}+n\gamma_{\mathrm{th}}\overline{\gamma}_{SR_n}^{Q}+\overline{\gamma}_{SP}(m+1)} \tag{22}$$

(see (23))

Finally, by summing (17), (21)–(23), a closed-form expression for the OP $P_{\mathrm{out}}^{\mathrm{DF}}$ of the DF relaying protocol can be obtained.

3.2 AF relaying

As mentioned in the previous subsection, the terms in the maximisation in (5) are not statistically independent. Thus, following a similar approach as in Section 3.1, it follows that

$$\Pr\left(\gamma_{\mathrm{e2e}}^{\mathrm{AF}}<\gamma_{\mathrm{th}}|X\right) = \underbrace{\Pr\left(\gamma_{SD}<\gamma_{\mathrm{th}}|X\right)}_{\phi}\cdot \underbrace{\Pr\left(\max_n\left[\left(\left(\gamma_{SR_n}\gamma_{R_nD}\right)/\left(1+\gamma_{SR_n}+\gamma_{R_nD}\right)\right)\right]<\gamma_{\mathrm{th}}|X\right)}_{\Phi} \tag{24}$$

where ϕ is given by (7), and we use the results in [15] for the

$$\varphi_2 = \int_0^{Q/W}\int_{Q/W}^{\infty}\left(1-\mathrm{e}^{-\gamma_{\mathrm{th}}\overline{\gamma}_{SD}^{W}}\right)\sum_{k=0}^{K}\frac{(-1)^k}{k!}\sum_{\substack{n_1=\cdots=n_k\\ n_1<\cdots<n_k}}^{K}\cdots\sum^{K}\prod_{t=1}^{k}\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\overline{\gamma}_{SR_{n_t}}^{W}+y\overline{\gamma}_{R_{n_t}D}^{Q}\right)}M\left(1-\mathrm{e}^{-x\overline{\gamma}_{SP}}\right)^{M-1} \times M\overline{\gamma}_{\mathrm{SP}}\ \mathrm{e}^{-x\overline{\gamma}_{SP}}\left(1-\mathrm{e}^{-y\overline{\gamma}_{R_nP}}\right)^{M-1}\overline{\gamma}_{R_nP}\ \mathrm{e}^{-y\overline{\gamma}_{R_nP}}\mathrm{d}x\mathrm{d}y \tag{20}$$

$$\varphi_4 = \sum_{l=0}^{1}\sum_{m=0}^{M-1}\sum_{t=0}^{M-1}\sum_{n=0}^{N}M^2(-1)^{(m+t+n+l)}\binom{M-1}{m}\binom{M-1}{t}\binom{N}{n}\overline{\gamma}_{R_nP}\exp\left(-l(Q/W)\gamma_{\mathrm{th}}\overline{\gamma}_{SD}^{Q}\right) \times\exp\left(-\frac{Q}{W}\overline{\gamma}_{R_nP}(t+1)\right)\frac{1}{n\gamma_{\mathrm{th}}\overline{\gamma}_{R_nD}^{Q}+\overline{\gamma}_{R_nP}(t+1)} \times\frac{\exp\left(-n\gamma_{\mathrm{th}}(Q/W)\left(\overline{\gamma}_{SR_n}^{Q}+\overline{\gamma}_{R_nD}^{Q}\right)-(Q/W)\overline{\gamma}_{SP}(m+1)\right)}{l\gamma_{\mathrm{th}}\overline{\gamma}_{SD}^{Q}+n\gamma_{\mathrm{th}}\overline{\gamma}_{SR_n}^{Q}+\overline{\gamma}_{SP}(m+1)} \tag{23}$$

single antenna case to express Φ as

$$\Phi = 1 - 2\,\overline{\gamma}_{SR_n}\cdot \mathrm{e}^{-\gamma_{\text{th}}\left(\overline{\gamma}_{SR_n}+\overline{\gamma}_{R_nD}\right)}\cdot\sqrt{\frac{\gamma_{\text{th}}\left(\gamma_{\text{th}}+1\right)\overline{\gamma}_{R_nD}}{\overline{\gamma}_{SR_n}}}\cdot K_1\left(2\sqrt{\gamma_{\text{th}}\left(\gamma_{\text{th}}+1\right)\overline{\gamma}_{R_nD}\,\overline{\gamma}_{SR_n}}\right) \tag{25}$$

where $K_n(\cdot)$ is the nth order modified Bessel function of the second kind [16, eq. (8.432.6)]. Thus, the unconditioned OP is given by (9), with $F_{\gamma_{SR^*D}}\left(\gamma_{\text{th}}|X,\ Y\right)\triangleq\Phi$. Following the same procedure as in the DF case, the OP for AF relaying can be rewritten as $P_{\text{out}}^{\text{AF}}=\chi_1+\chi_2+\chi_3+\chi_4$, which are given by (12)–(15). Making the appropriate substitutions, χ_1 is given by

$$\chi_1 = M^2\sum_{m=0}^{M-1}\sum_{t=0}^{M-1}(-1)^{m+t}\binom{M-1}{m}\binom{M-1}{t}\left(1-\mathrm{e}^{-\gamma_{\text{th}}\overline{\gamma}_{SD}^{W}}\right)\times\prod_{n=1}^{N}\left(1-2\overline{\gamma}_{SR_n}^{W}\cdot\mathrm{e}^{-\gamma_{\text{th}}\left(\overline{\gamma}_{SR_n}^{W}+\overline{\gamma}_{R_nD}^{W}\right)}\cdot\sqrt{\frac{\gamma_{\text{th}}\left(\gamma_{\text{th}}+1\right)\overline{\gamma}_{R_nD}}{\overline{\gamma}_{SR_n}}}\cdot K_1\left(2\sqrt{\gamma_{\text{th}}\left(\gamma_{\text{th}}+1\right)\overline{\gamma}_{R_nD}\,\overline{\gamma}_{SR_n}}\right)\right)\times\frac{\left(1-\mathrm{e}^{\left(-(Q/W)\overline{\gamma}_{SP}(m+1)\right)}\right)}{\overline{\gamma}_{SP}(m+1)}\frac{\left(1-\mathrm{e}^{\left(-(Q/W)\overline{\gamma}_{R_nP}(t+1)\right)}\right)}{\overline{\gamma}_{R_nP}(t+1)} \tag{26}$$

Similarly, χ_2 can be expressed as

$$\chi_2=\int_0^{Q/W}\int_{Q/W}^{\infty}\left(1-\mathrm{e}^{-\gamma_{\text{th}}\overline{\gamma}_{SD}^{W}}\right)M\left(1-\mathrm{e}^{-x\overline{\gamma}_{SP}}\right)^{(M-1)}\times\prod_{n=1}^{N}\left(1-2\overline{\gamma}_{SR_n}^{W}\,\mathrm{e}^{-\gamma_{\text{th}}\left(\overline{\gamma}_{SR_n}^{W}+y\overline{\gamma}_{R_nD}^{Q}\right)}M\sqrt{\frac{y\gamma_{\text{th}}\left(\gamma_{\text{th}}+1\right)\overline{\gamma}_{R_nD}^{Q}}{\overline{\gamma}_{SR_n}}}\cdot K_1\left(2\sqrt{y\gamma_{\text{th}}\left(\gamma_{\text{th}}+1\right)\overline{\gamma}_{R_nD}^{Q}\overline{\gamma}_{SR_n}}\right)\right)\times\overline{\gamma}_{SP}\,\mathrm{e}^{-x\overline{\gamma}_{SP}}\left(1-\mathrm{e}^{-y\overline{\gamma}_{R_nP}}\right)^{M-1}\overline{\gamma}_{R_nP}\,\mathrm{e}^{-y\overline{\gamma}_{R_nP}}\mathrm{d}x\mathrm{d}y \tag{27}$$

Since it is difficult to obtain a closed-form solution for the integral above, we rely on the approximation [This approximation is valid for small values of ζ. In our case, the simulation results we present later in Section 5 show that this approximation is very accurate.] $K_1(\zeta)\simeq 1/\zeta$ [16, eq. (9.6.9)]. Thus, χ_2 can be approximated by (18), resulting in (21). Following the same approach for χ_3 and χ_4, these terms can be approximated by (22) and (23), respectively.

Finally, substituting (21)–(23), and (26) in $P_{\text{out}}^{\text{AF}}\simeq\chi_1+\chi_2+\chi_3+\chi_4$, an accurate approximation for the OP with AF relaying protocol is obtained.

4 Asymptotic analysis

To get further insight into the outage behaviour of the analysed system from the obtained expressions in Section 3, we extend the analysis to the asymptotical limits. To this end, let $\overline{\gamma}\triangleq 1/N_0$ be the system SNR [17, 18] and $Q/W=\mu$, where $\mu>0$ is a constant. Note that, when $\overline{\gamma}\to\infty$, it follows that $\overline{\gamma}_{SP}\gg\gamma_{\text{th}}/\overline{\gamma}$ and $\overline{\gamma}_{R_nP}\gg\gamma_{\text{th}}/\overline{\gamma}$. Thus, using the McLaurin series of exponential functions and the approximation $K_1(\zeta)\simeq 1/\zeta$ for χ_1 in the AF case, we have the following equations.

4.1 DF relaying

$$P_{\text{out}}^{\xi_1}\simeq\left(\gamma_{\text{th}}\,\overline{\gamma}_{SD}^{W}\right)M^2\sum_{m=0}^{M-1}\sum_{t=0}^{M-1}(-1)^{m+t}\binom{M-1}{m}\binom{M-1}{t}\times\frac{\left(1-\mathrm{e}^{\left(-\mu\overline{\gamma}_{SP}(m+1)\right)}\right)}{\overline{\gamma}_{SP}(m+1)}\frac{\left(1-\mathrm{e}^{\left(-\mu\overline{\gamma}_{R_nP}(t+1)\right)}\right)}{\overline{\gamma}_{R_nP}(t+1)}\times\prod_{n=1}^{N}\left[\gamma_{\text{th}}\left(\overline{\gamma}_{SR_n}^{W}+\overline{\gamma}_{R_nD}^{W}\right)\right]\propto\left(\frac{1}{\overline{\gamma}}\right)^{N+1} \tag{28}$$

$$P_{\text{out}}^{\xi_2}\simeq\left(\gamma_{\text{th}}\,\overline{\gamma}_{SD}^{W}\right)M^2\sum_{m=0}^{M-1}\sum_{t=0}^{M-1}(-1)^{m+t}\binom{M-1}{m}\binom{M-1}{t}\times\frac{\left(1-\mathrm{e}^{\left(-\mu\overline{\gamma}_{SP}(m+1)\right)}\right)}{\overline{\gamma}_{SP}(m+1)}\frac{\mathrm{e}^{\left(-\mu\overline{\gamma}_{R_nP}(t+1)\right)}}{\overline{\gamma}_{R_nP}(t+1)}\times\prod_{n=1}^{N}\left[\gamma_{\text{th}}\left(\overline{\gamma}_{SR_n}^{W}+\mu\,\overline{\gamma}_{R_nD}^{Q}\right)\right]\propto\left(\frac{1}{\overline{\gamma}}\right)^{N+1} \tag{29}$$

$$P_{\text{out}}^{\xi_3}\simeq\left(\mu\gamma_{\text{th}}\,\overline{\gamma}_{SD}^{Q}\right)M^2\sum_{m=0}^{M-1}\sum_{t=0}^{M-1}(-1)^{m+t}\binom{M-1}{m}\binom{M-1}{t}\times\frac{\left(1-\mathrm{e}^{\left(-\mu\overline{\gamma}_{R_nP}(t+1)\right)}\right)}{\overline{\gamma}_{R_nP}(t+1)}\frac{\mathrm{e}^{\left(-\mu\overline{\gamma}_{SP}(m+1)\right)}}{\overline{\gamma}_{SP}(m+1)}\times\prod_{n=1}^{N}\left[\gamma_{\text{th}}\left(\mu\,\overline{\gamma}_{SR_n}^{Q}+\overline{\gamma}_{R_nD_l}^{W}\right)\right]\propto\left(\frac{1}{\overline{\gamma}}\right)^{N+1} \tag{30}$$

$$P_{\text{out}}^{\xi_4}\simeq\left(\mu\gamma_{\text{th}}\,\overline{\gamma}_{SD}^{Q}\right)M^2\sum_{m=0}^{M-1}\sum_{t=0}^{M-1}(-1)^{m+t}\binom{M-1}{m}\binom{M-1}{t}\times\frac{\left(\mathrm{e}^{\left(-\mu\overline{\gamma}_{R_nP}(t+1)\right)}\right)}{\overline{\gamma}_{R_nP}(t+1)}\frac{\mathrm{e}^{\left(-\mu\overline{\gamma}_{SP}(m+1)\right)}}{\overline{\gamma}_{SP}(m+1)}\times\prod_{n=1}^{N}\left[\mu\gamma_{\text{th}}\left(\overline{\gamma}_{SR_n}^{Q}+\overline{\gamma}_{R_nD}^{Q}\right)\right]\propto\left(\frac{1}{\overline{\gamma}}\right)^{N+1} \tag{31}$$

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4.2 AF relaying

$$P_{\text{out}}^{\chi_1} \simeq \left(\gamma_{\text{th}}\,\overline{\gamma}_{SD}^{W}\right)M^2 \sum_{m=0}^{M-1}\sum_{t=0}^{M-1}(-1)^{m+t}\binom{M-1}{m}\binom{M-1}{t} \times \frac{\left(1-\mathrm{e}^{\left(-\mu\,\overline{\gamma}_{SP}\,(m+1)\right)}\right)}{\overline{\gamma}_{SP}\,(m+1)}\,\frac{\left(1-\mathrm{e}^{\left(-\mu\,\overline{\gamma}_{R_nP}\,(t+1)\right)}\right)}{\overline{\gamma}_{R_nP}\,(t+1)} \times \prod_{n=1}^{N}\left[\gamma_{\text{th}}\left(\overline{\gamma}_{SR_n}^{W}+\overline{\gamma}_{R_nD}^{W}\right)\right] \propto \left(\frac{1}{\overline{\gamma}}\right)^{N+1} \quad (32)$$

$$P_{\text{out}}^{\chi_2} = P_{\text{out}}^{\xi_2}, \quad P_{\text{out}}^{\chi_3} = P_{\text{out}}^{\xi_3}, \text{ and } P_{\text{out}}^{\chi_4} = P_{\text{out}}^{\xi_4} \quad (33)$$

From the analytical expressions above, it is easy to see that the system achieves full diversity order (i.e. $N+1$) for both DF and AF protocols. It is worth noting that the interference constraints have no influence on the diversity gain.

5 Numerical plots and discussions

In this section, we present a few illustrative numerical results in order to assess the accuracy of our analysis. All nodes are placed in a two-dimensional plane, where the SU source is placed at (0, 0), the SU destination is located at (1, 0), the SU relays nodes are clustered together and placed at (0.5, 0) and the PU receivers are also clustered together and located at (0, 1). The distance among the nodes determine the statistical average of the channel gains, the pathloss exponent is set to 4 and the outage threshold is set to $\gamma_{\text{th}} = 3$ dB.

Figs. 2 and 3 depict the OP against the system SNR for both DF and AF relaying protocols, respectively. We can observe that, in both cases, when the number of SU relays increases, the outage performance and the system diversity order also increase, as expected. In addition, by increasing the number of PU receivers, the outage performance gets worse but, for a fixed number of SU relays, the diversity order remains the same, which confirms that the number of PU receivers does not have any influence in the diversity

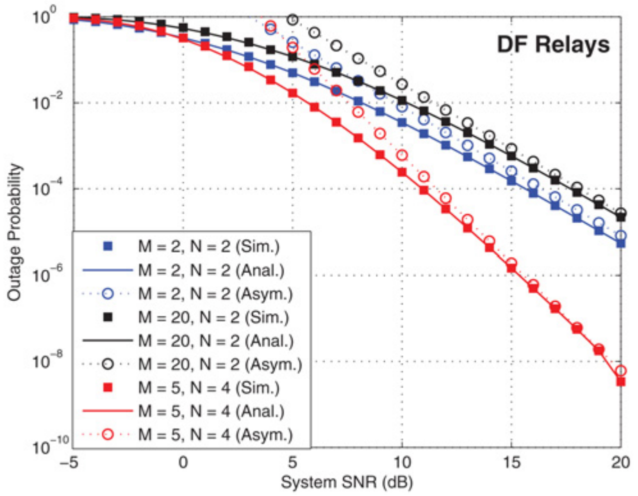

Fig. 2 *OP and asymptotical behaviour against system SNR using DF strategy for different numbers of SU relays and PU receivers (W = Q = 0.5)*

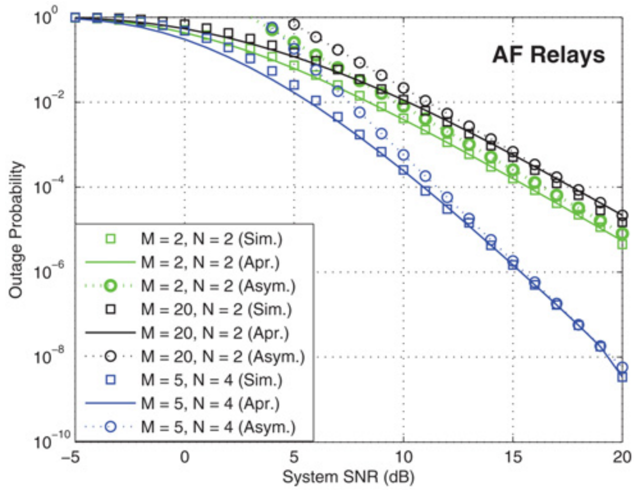

Fig. 3 *OP and asymptotical behaviour against system SNR using AF strategy for different numbers of SU relays PU receivers (W = Q = 0.5)*

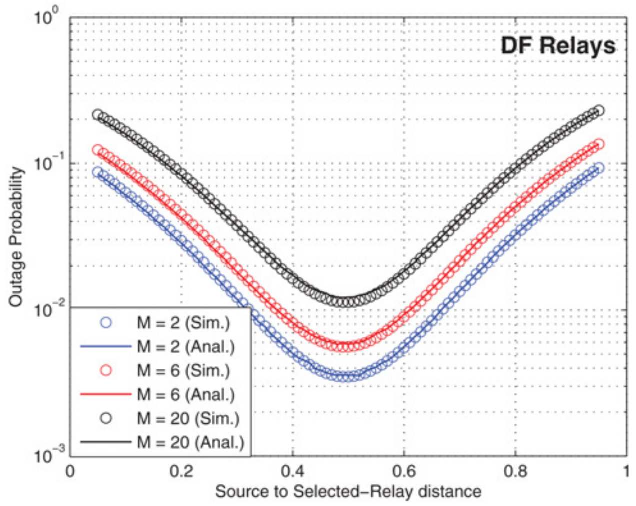

Fig. 4 *Impact of chosen relay position on the OP using DF strategy for different numbers of PU receivers (N = 2)*

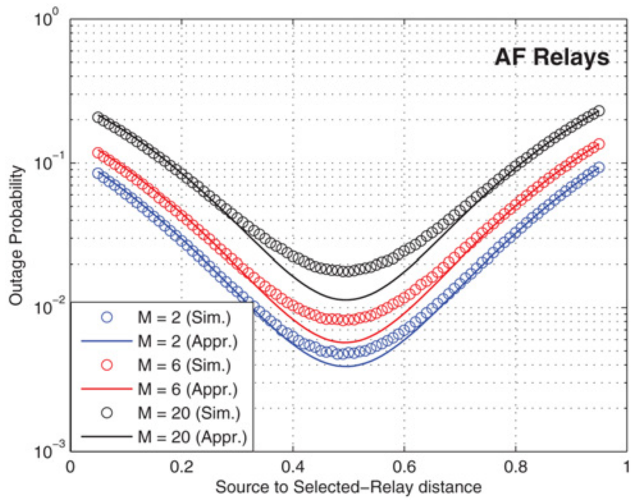

Fig. 5 *Impact of chosen relay position on the OP using AF strategy for different numbers of PU receivers (N = 2)*

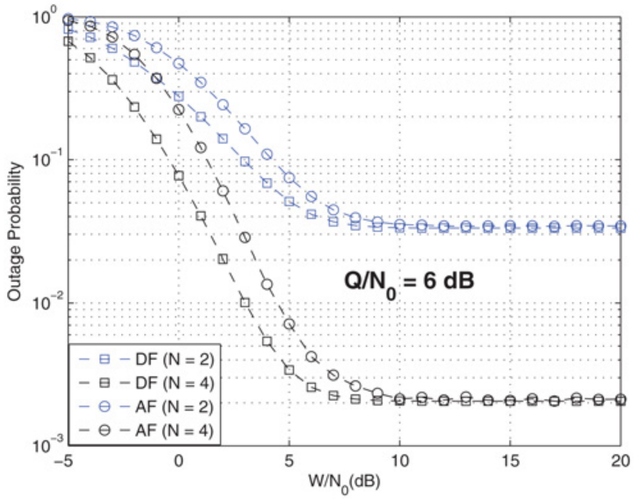

Fig. 6 *Comparative outage analysis between DF and AF relaying protocols for different numbers of relays with $M = 5$*

order of the system. Moreover, the asymptotical results are very close to simulation results in the high-SNR regions, revealing that the diversity order of the considered system is indeed $N+1$, confirming the correctness of the proposed analysis.

In the case of a DF relaying protocol, Fig. 4 shows the impact of the number of PU receivers on the best SU relay position. In this plot, we set $N=2$, and the system SNR to 10 dB. By increasing the number of primary receivers, the best relay position remains at the middle point between the source and the destination. One possible explanation is that the number of PU receivers just imposes a threshold for the transmit powers of the secondary nodes, whereas the PU receivers have no impact on the channels quality in the SU network. In other words, the DF relays will always be able to decode the received information such that the middle point will balance the quality of the channels. By assuming these same conditions, the effect of the best relay position for AF relays is observed in Fig. 5. As observed in the DF case, the best position remains close to the middle point between the SU source and SU destination in order to balance the channel quality between the first and second

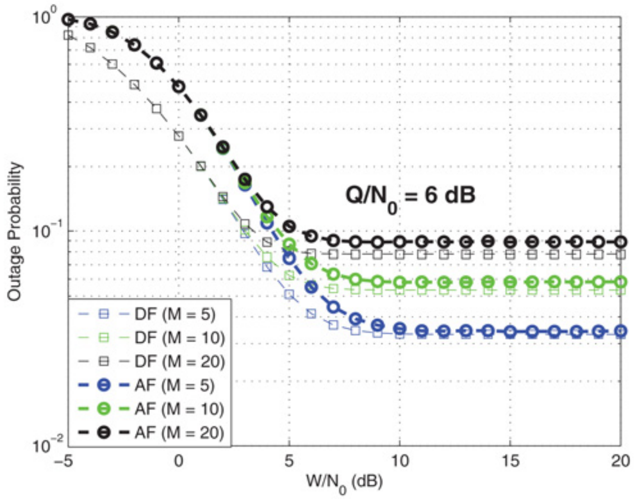

Fig. 7 *Comparative analysis between DF and AF relaying protocols for different numbers of PUs with $N = 2$*

hop. Moreover, we can see that the derived outage approximation for the AF case is very tight when compared with the simulated plots for low values of M.

Figs. 6 and 7 show the saturation phenomenon of the OP when we have different number of relays and primary receivers, respectively. When N increases, for the same number of PUs (i.e. $M=5$), the outage performance gets better and the performance gap between DF and AF increases in the 'unsaturated' regime. When we vary the number M of primary receivers, for a fixed number of relays (i.e. $N=2$), the gap between the considered relaying protocols increases in the 'saturated' regime. This could be explained by the fact that the available transmit power at the SU nodes will decrease when the number of PU receivers increases. The AF performance will hence suffer a degradation.

6 Conclusions

In this work, we have studied the performance of a multi-relay cooperative network in a spectrum sharing environment with best relay selection. The presence of multiple PU receivers imposes a stringent interference constraint on the transmission power of the SU nodes. These constraints do not impact the diversity order of the considered system, which is $N+1$, achieving, thus, full diversity order. Moreover, the new global temperature interference does not change the best relay position, but degrades the system performance and increases the gap between DF and AF in the outage floor region when the number of PUs is increasing. Finally, we again observe the saturation phenomenon of the OP because of the restrictions imposed by each PU receiver.

7 Acknowledgment

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8 References

1 Sendonaris, A., Erkip, E., Aazhang, B.: 'User cooperation diversity – part I: system description', *IEEE Trans. Commun.*, 2003, **51**, (11), pp. 1927–1938
2 Laneman, J.N., Tse, D.N.C., Wornell, G.W.: 'Cooperative diversity in wireless networks: efficient protocols and outage behavior', *IEEE Trans. Inf. Theory*, 2004, **50**, (12), pp. 3062–3080
3 Mitola, J.III: 'Cognitive radio: making software radios more personal', *IEEE Pers. Commun.*, 1999, **6**, (4), pp. 13–18
4 Ghasemi, A., Sousa, E.S.: 'Fundamental limits of spectrum-sharing in fading environments', *IEEE Trans. Wirel. Commun.*, 2007, **6**, (2), pp. 649–658
5 Asghari, V., Aissa, S.: 'Performance of cooperative spectrum-sharing systems with amplify-and-forward relaying', *IEEE Trans. Wirel. Commun.*, 2012, **11**, (4), pp. 1295–1300
6 Zhong, C., Ratnarajah, T., Wong, K.-K.: 'Outage analysis of decode-and-forward cognitive dual-hop systems with the interference constraint in Nakagami-*m* fading channels', *IEEE Trans. Veh. Technol.*, 2011, **60**, (6), pp. 2875–2879
7 Lee, J., Wang, H., Andrews, J.G., Hong, D.: 'Outage probability of cognitive relay networks with interference constraints', *IEEE Trans. Wirel. Commun.*, 2011, **10**, (2), pp. 390–395
8 Duong, T.Q., Bao, V.N.Q., Alexandropoulos, G.C., Zepernick, H.-J.: 'Cooperative spectrum sharing networks with AF relay and selection diversity', *IET Electron. Lett.*, 2011, **47**, (20), pp. 1149–1151
9 Doung, T.Q., da Costa, D.B., Elkashlan, M., Bao, V.N.Q.: 'Cognitive amplify-and-forward relay networks over Nakagami-*m* fading', *IEEE Trans. Veh. Technol.*, 2012, **61**, (5), pp. 2368–2374
10 Duong, T.Q., da Costa, D.B., Tsiftsis, T.A., Zhong, C., Nallanathan, A.: 'Outage and diversity of cognitive relaying systems under spectrum sharing environments in Nakagami-m fading', *IEEE Commun. Lett.*, 2012, **16**, (12), pp. 2075–2078

IET Commun., 2014, Vol. 8, Iss. 4, pp. 546–553
doi: 10.1049/iet-com.2013.0575

11 Luo, L., Zhang, P., Zhang, G., Qin, J.: 'Outage performance for cognitive relay networks with underlay spectrum sharing', *IEEE Commun. Lett.*, 2011, **15**, (7), pp. 710–712
12 Guimarães, F.R.V., da Costa, D.B., Benjillali, M., Tsiftsis, T.A., Karagiannidis, G.K.: 'Best relay selection in cooperative spectrum sharing systems with multiple primary users'. IEEE Int. Conf. Communications (ICC), Budapest, Hungary, June 2013
13 Papoulis, A.: 'Probability, random variables, and stochastic processes' (McGraw-Hill, 2002, 4th edn.)
14 da Costa, D.B., Aissa, S.: 'End-to-end performance of dual-hop semi-blind relaying systems with partial relay selection', *IEEE Trans. Wirel. Commun.*, 2009, **8**, (8), pp. 4306–4315
15 de Melo, M.A.B., da Costa, D.B.: 'An efficient relay-destination selection scheme for multiuser multirelay downlink cooperative networks', *IEEE Trans. Veh. Technol.*, 2012, **61**, (5), pp. 2354–2360
16 Abramowitz, M., Stegun, I.A.: 'Handbook of mathematical functions with formulas, graphs, and mathematical tables' (Wiley, New York, 1972)
17 Zhao, Y., Adve, R., Lim, T.J.: 'Improving amplify-and-forward relay networks: optimal power allocation versus selection', *IEEE Trans. Wirel. Commun.*, 2007, **6**, (8), pp. 3114–3123
18 Zhao, Y., Adve, R., Lim, T.J.: 'Symbol error rate of selection amplify-and-forward relay system', *IEEE Commun. Lett.*, 2006, **10**, (11), pp. 757–759