Performance analysis of switched diversity receivers in Weibull fading

N.C. Sagias, D.A. Zogas, G.K. Karagiannidis and G.S. Tombras

A novel approach to the performance analysis of switched-and-stay combining diversity receivers over independent Weibull fading channels is presented. Closed-form expressions are extracted for important performance parameters, such as the average output signal-to-noise ratio, the amount of fading, the outage probability, and the switching rate.

Introduction: Among all well-known diversity techniques, switched-and-stay combining (SSC) is simpler and cheaper though less efficient. In a dual-branch SSC system, if the instantaneous signal-to-noise ratio (SNR) of the first branch falls below a predefined switching threshold, the second branch is immediately selected, regardless of whether or not the SNR of that branch is above or below the predetermined threshold [1]. The performance of SSC receivers has been studied extensively in the literature for several well-known fading statistical models, such as Rayleigh, Rice, Nakagami-m and Nakagami-q [1–3]. However, another fading channel model, namely the Weibull model, has not received as much attention as the others, despite the fact that it exhibits an excellent fit to experimental fading channel measurements, for indoor [4] and outdoor environments [5]. In this Letter, useful formulae are presented for statistical parameters of the SSC output SNR, over Weibull fading, such as the cumulative distribution function (cdf) and the probability density function (pdf) and the moments. These formulae are used to express in closed-form the average output SNR, the amount of fading (AoF), the outage probability, the optimum switching threshold for maximum SNR and the switching rate (SR) of the combiner. Several numerical results are also presented to illustrate the proposed mathematical analysis.

Statistics of output SNR: We consider a dual-branch SSC diversity receiver operating in a flat Weibull fading environment. The cdf of the independent distributed Weibull fading envelopes, r_1 and r_2, is given by [6]

$$F_{r_i}(r_i) = 1 - e^{-(r_i/\omega_i)^{\beta}}, \quad i = 1 \text{ and } 2 \tag{1}$$

where ω_i is a scale factor defined as $\omega_i = \sqrt{(\overline{r_i^2}/\Gamma(b_1))}$ with $\overline{r_i^2}$ being the average signal power, $\Gamma(\cdot)$ is the Gamma function, $b_n = 1 + 2n/\beta$ with n being positive integer and β is the shape factor of the Weibull distribution ($\beta > 0$). As β increases the severity of the fading decreases, while for $\beta = 2$, (1) reduces to the well-known Rayleigh cdf. The instantaneous SNR per symbol and per diversity branch is defined as $\zeta_i = r_i^2 E_s/N_0$ and the corresponding average SNR is $\bar{\zeta}_i = \omega_i^2\Gamma(b_1)E_s/N_0$, where E_s is the symbol-energy and N_0 the double-sided noise power spectral density. Setting $a = 1/\Gamma(b_1)$, the cdf of the SNR per symbol is obtained directly from (1) as

$$F_{\zeta_i}(\zeta_i) = 1 - e^{-(\zeta_i/a\bar{\zeta}_i)^{\beta/2}} \tag{2}$$

Assuming without loss of generality that the switching threshold ζ_τ is the same for both input branches and defining $P_i = F_{\zeta_i}(\zeta_i = \zeta_\tau)$, the cdf of the SSC receiver output SNR, ζ_{SCC}, is derived using [3] as

$$F_{\zeta_{SCC}}(\zeta_{SCC}) = \begin{cases} \dfrac{P_1P_2}{P_1+P_2}[2 - e^{-(\zeta_{SCC}/a\bar{\zeta}_1)^{\beta/2}} - e^{-(\zeta_{SCC}/a\bar{\zeta}_2)^{\beta/2}}], & \zeta_{SCC} \le \zeta_\tau \\ 1 - \dfrac{(1+P_1)P_2}{P_1+P_2} e^{-(\zeta_{SCC}/a\bar{\zeta}_1)^{\beta/2}} - \dfrac{(1+P_2)P_1}{P_1+P_2} e^{-(\zeta_{SCC}/a\bar{\zeta}_2)^{\beta/2}}, & \zeta_{SCC} > \zeta_\tau \end{cases} \tag{3}$$

Differentiating (3) and after manipulations the pdf of the SSC output SNR can be expressed as

$$f_{\zeta_{SCC}}(\zeta_{SCC}) = \begin{cases} \dfrac{\beta}{2}\dfrac{P_1P_2}{P_1+P_2}(\xi_1 + \xi_2), & \zeta_{SCC} \le \zeta_\tau \\ \dfrac{\beta}{2}\left[\dfrac{P_2(1+P_1)}{P_1+P_2}\xi_1 + \dfrac{P_1(1+P_2)}{P_1+P_2}\xi_2\right], & \zeta_{SCC} > \zeta_\tau \end{cases} \tag{4}$$

where $\xi_i = [\zeta_{SCC}/(a\bar{\zeta}_i)]^{\beta/2-1} e^{-[\zeta_{SCC}/(a\bar{\zeta}_i)]^{\beta/2}}/(a\bar{\zeta}_i)$, $i = 1$ and 2.

The nth moment of the combiner output SNR is by definition [6]

$$E\langle\zeta_{SCC}^n\rangle = \int_0^\infty \zeta_{SCC}^n f_{\zeta_{SCC}}(\zeta_{SCC})\, d\zeta_{SCC} \tag{5}$$

where by $E\langle\cdot\rangle$ means expectation. Using (4) and [7], (5) is written in closed-form as

$$E\langle\zeta_{SCC}^n\rangle = \lambda_2 \frac{\Gamma(b_n, c_1) + P_1\Gamma(b_n)}{\Gamma^n(b_1)}\bar{\zeta}_1^n + \lambda_1 \frac{\Gamma(b_n, c_2) + P_2\Gamma(b_n)}{\Gamma^n(b_1)}\bar{\zeta}_2^n \tag{6}$$

where $c_i = [\zeta_\tau/(a\zeta_i)]^{\beta/2}$, $\lambda_i = P_i/(P_1 + P_2)$ and $\Gamma(\cdot, \cdot)$ being the incomplete Gamma function [7].

Performance analysis: One of the most important performance criterion serving as an excellent indicator of the system's fidelity and measured at the output of the SSC diversity receiver, is the average SNR. In our case, it can be obtained by setting $n = 1$ in (6) as

$$\bar{\zeta}_{SCC} = \lambda_2 \frac{\Gamma(b_1, c_1) + P_1\Gamma(b_1)}{\Gamma(b_1)}\bar{\zeta}_1 + \lambda_1 \frac{\Gamma(b_1, c_2) + P_2\Gamma(b_1)}{\Gamma(b_1)}\bar{\zeta}_2 \tag{7}$$

The optimum common switching threshold, for maximum average output SNR, is derived solving the equation which is obtained by differentiating (7) with respect to ζ_τ and setting the result equal to zero. It can be easily shown that the optimum switching threshold is $\zeta_\tau^* = \bar{\zeta}$, for independent and identical distributed (i.i.d.) average input SNRs ($\bar{\zeta}_1 = \bar{\zeta}_2 = \bar{\zeta}$). If the average input SNRs are not identically distributed, the optimum switching threshold can be derived using numerical methods, available in most of the well-known mathematical software packets.

Outage probability, P_{out}, which is defined as the probability that the combiner's output SNR falls below a given outage threshold, ζ_{th}, can be obtained by replacing ζ_{SCC} with ζ_{th} in (3) as

$$P_{\text{out}}(\zeta_{th}) = F_{\zeta_{th}}(\zeta_{th}) \tag{8}$$

Amount of fading (AoF) is defined as the ratio of the variance of ζ_{SCC} to the square of $\bar{\zeta}_{SCC}$ and is considered as a unified measure of the severity of fading [1]. The AoF of SSC output SNR can be expressed as $\text{AoF} = E\langle\zeta_{SCC}^2\rangle/\bar{\zeta}_{SCC}^2 - 1$ and using (6) and (7), can be further written in closed-form as

$$\text{AoF} = \frac{\lambda_2[\Gamma(b_2, c_1) + P_1\Gamma(b_2)]\bar{\zeta}_1^2 + \lambda_1[\Gamma(b_2, c_2) + P_2\Gamma(b_2)]\bar{\zeta}_2^2}{\{\lambda_2[\Gamma(b_1, c_1) + P_1\Gamma(b_1)]\bar{\zeta}_1 + \lambda_1[\Gamma(b_1, c_2) + P_2\Gamma(b_1)]\bar{\zeta}_2\}^2} - 1 \tag{9}$$

Switching rate (SR) is of great importance and fundamental for the design process, especially when the SSC is employed in mobile terminals with limited battery life time and is defined as the rate of switches between the two branches. So, the mean time between switching, as a function of ζ_τ, is $T(\zeta_\tau) = \pi_1/P_1 + \pi_2/P_2$, where $\pi_i = 0.5$, $i = 1$ and 2 is the probability the switcher be in the ith branch. Hence, the SR, which is the reciprocal of $T(\zeta_\tau)$, can be written as

$$\text{SR}(\zeta_\tau) = 2\frac{P_1P_2}{P_1 + P_2} \tag{10}$$

Numerical results: We have numerically evaluated (7), (8) and (10) for i.i.d. input branches and the results are shown in Figs. 1 and 2. In Fig. 1, P_{out} is plotted against the normalised outage threshold, ζ_{th}/ζ_τ^*, for β=2, 3.3, 4.3 and 5. For fixed values of ζ_{th}/ζ_τ^* an increase of β leads to a decrease of P_{out} and better outage performance is obtained. In Fig. 2, the SR and the normalised output SNR, $\bar{\zeta}_{SCC}/\bar{\zeta}$, is plotted against the normalised switching threshold, $\zeta_\tau/\bar{\zeta}$, for $\beta = 2$, 3 and 4.3. For fixed values of $\zeta_\tau/\bar{\zeta}$, the SR and $\bar{\zeta}_{SCC}/\bar{\zeta}$ increase, as the severity of fading increases (i.e. β decreases). As is clear, the optimum common switching threshold, for maximum output SNR, is obtained when $\zeta_\tau^* = \bar{\zeta}$, independently of the Weibull β parameter.

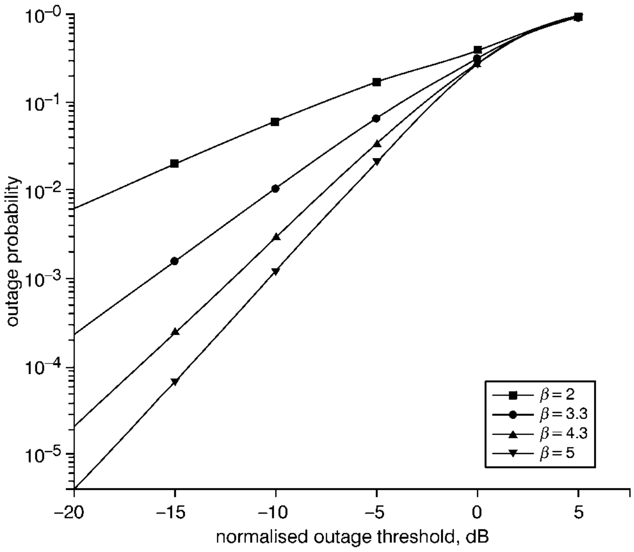

Fig. 1 *Outage probability against normalised outage threshold,* $\zeta_{th}/\zeta_{\tau}^{*}$

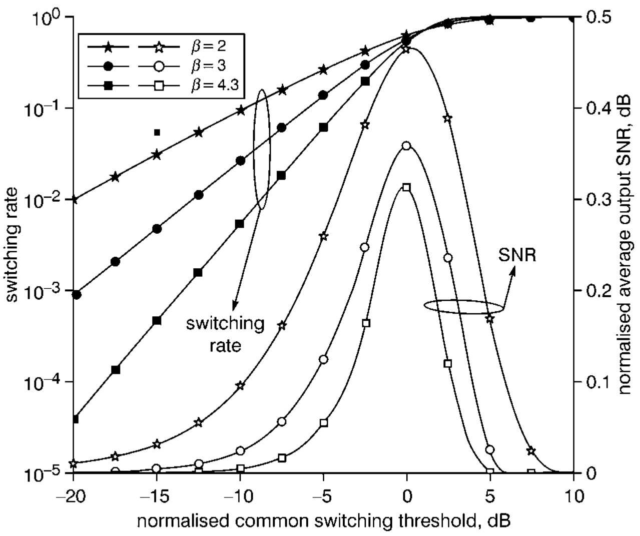

Fig. 2 *Switching rate and normalised average output SNR against normalised switching threshold,* $\zeta_{\tau}/\bar{\zeta}$

Conclusions: An approach to the performance analysis of SSC diversity receivers over independent Weibull fading channels is presented. Important performance parameters, such as the average output SNR, the AoF, the outage probability, the optimum switching threshold for maximum SNR and the SR, are expressed in useful closed-form expressions. Several numerical results are also presented to illustrate the proposed mathematical analysis and to point out the effect of fading severity to the performance of the combiner.

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